In section 3.1, we reviewed some basic knowledge of simple harmonic motion. We started with the force in the vertical direction, $F_\theta = -mg\, sin\,\theta$, the force in the vertical direction reaches the maximum when θ = 0. With Newton's second law, we can get $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$, an important assumption is that θ is small so that sin(θ) ≈ θ.

From here, we can get the general solution, $\theta = \theta_0\, sin(\Omega t + \phi)$, where $\Omega = \sqrt{g\,/\,l}$.

The total energy is $E = \frac{1}{2}ml^2\omega^2 + mgl(1 - cos\,\theta)$. This can be converted into $E = \frac{1}{2}ml^2\left(\omega^2 + \frac{g}{l}\theta^2\right)$.

In section 3.2, we start to make the pendulum more real and start to consider the dissipation, nonlinearity, and driving force. All of the forces above can be determined as the damping force, which can be written as -q(dθ/dt). The new equation of motion is $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$. The relation of θ(t) is, $\theta(t) = \theta_0 e^{-qt/2} \cdot sin\left(\sqrt{\Omega^2 - q^2\,/\,4}t + \emptyset\right)$. The greater the value of q, the smaller the vibration. This section also explored different damping regimes, underdamping, overdamping, and critical damping.

In section 3.3, we learned the behavior of the harmonic oscillation and discussed the resonance phenomenon, where the system's amplitude response becomes significant when the driving frequency matches the natural frequency. Used the numerical method to analyze the system's response under different conditions.