Probability-based Learning

Chapter 6 – Naive Bayes' Classifier

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[1] Original slides from John Kelleher, Brian Mac Namee, and Aoife D'Arcy

Naive Bayes' Classifier

$$\mathbb{M}(\mathbf{q}) = \underset{l \in levels(t)}{\operatorname{argmax}} \left(\prod_{i=1}^{m} P(\mathbf{q}[i] \mid t = l) \right) \times P(t = l)$$

Naive Bayes' is simple to train!

1. calculate the priors for each of the target levels
2. calculate the conditional probabilities for each feature given each target level.

Table: A dataset from a loan application fraud detection domain.

ID	Credit History	Guarantor/ CoApplicant	Accomodation	Fraud
1	current	none	own	true
2	paid	none	own	false
3	paid	none	own	false
4	paid	guarantor	rent	true
5	arrears	none	own	false
6	arrears	none	own	true
7	current	none	own	false
8	arrears	none	own	false
9	current	none	rent	false
10	none	none	own	true
11	current	coapplicant	own	false
12	current	none	own	true
13	current	none	rent	true
14	paid	none	own	false
15	arrears	none	own	false
16	current	none	own	false
17	arrears	coapplicant	rent	false
18	arrears	none	free	false
19	arrears	none	own	false
20	paid	none	own	false

$P(fr)$	$=$	0.3	$P(\neg fr)$	$=$	0.7
$P(CH = \textit{'none'} \mid fr)$	$=$	0.1666	$P(CH = \textit{'none'} \mid \neg fr)$	$=$	0
$P(CH = \textit{'paid'} \mid fr)$	$=$	0.1666	$P(CH = \textit{'paid'} \mid \neg fr)$	$=$	0.2857
$P(CH = \textit{'current'} \mid fr)$	$=$	0.5	$P(CH = \textit{'current'} \mid \neg fr)$	$=$	0.2857
$P(CH = \textit{'arrears'} \mid fr)$	$=$	0.1666	$P(CH = \textit{'arrears'} \mid \neg fr)$	$=$	0.4286
$P(GC = \textit{'none'} \mid fr)$	$=$	0.8334	$P(GC = \textit{'none'} \mid \neg fr)$	$=$	0.8571
$P(GC = \textit{'guarantor'} \mid fr)$	$=$	0.1666	$P(GC = \textit{'guarantor'} \mid \neg fr)$	$=$	0
$P(GC = \textit{'coapplicant'} \mid fr)$	$=$	0	$P(GC = \textit{'coapplicant'} \mid \neg fr)$	$=$	0.1429
$P(ACC = \textit{'own'} \mid fr)$	$=$	0.6666	$P(ACC = \textit{'own'} \mid \neg fr)$	$=$	0.7857
$P(ACC = \textit{'rent'} \mid fr)$	$=$	0.3333	$P(ACC = \textit{'rent'} \mid \neg fr)$	$=$	0.1429
$P(ACC = \textit{'free'} \mid fr)$	$=$	0	$P(ACC = \textit{'free'} \mid \neg fr)$	$=$	0.0714

Table: The probabilities needed by a Naive Bayes prediction model calculated from the dataset. Notation key: FR=FRAUDULENT, CH=CREDIT HISTORY, GC = GUARANTOR/COAPPLICANT, ACC = ACCOMODATION, T=*'true'*, F=*'false'*.

$$
\begin{array}{rclrcl}
P(fr) & = & 0.3 & P(\neg fr) & = & 0.7 \\
P(\mathrm{CH} = \textit{'none'} \mid fr) & = & 0.1666 & P(\mathrm{CH} = \textit{'none'} \mid \neg fr) & = & 0 \\
P(\mathrm{CH} = \textit{'paid'} \mid fr) & = & 0.1666 & P(\mathrm{CH} = \textit{'paid'} \mid \neg fr) & = & 0.2857 \\
P(\mathrm{CH} = \textit{'current'} \mid fr) & = & 0.5 & P(\mathrm{CH} = \textit{'current'} \mid \neg fr) & = & 0.2857 \\
P(\mathrm{CH} = \textit{'arrears'} \mid fr) & = & 0.1666 & P(\mathrm{CH} = \textit{'arrears'} \mid \neg fr) & = & 0.4286 \\
P(\mathrm{GC} = \textit{'none'} \mid fr) & = & 0.8334 & P(\mathrm{GC} = \textit{'none'} \mid \neg fr) & = & 0.8571 \\
P(\mathrm{GC} = \textit{'guarantor'} \mid fr) & = & 0.1666 & P(\mathrm{GC} = \textit{'guarantor'} \mid \neg fr) & = & 0 \\
P(\mathrm{GC} = \textit{'coapplicant'} \mid fr) & = & 0 & P(\mathrm{GC} = \textit{'coapplicant'} \mid \neg fr) & = & 0.1429 \\
P(\mathrm{ACC} = \textit{'own'} \mid fr) & = & 0.6666 & P(\mathrm{ACC} = \textit{'own'} \mid \neg fr) & = & 0.7857 \\
P(\mathrm{ACC} = \textit{'rent'} \mid fr) & = & 0.3333 & P(\mathrm{ACC} = \textit{'rent'} \mid \neg fr) & = & 0.1429 \\
P(\mathrm{ACC} = \textit{'free'} \mid fr) & = & 0 & P(\mathrm{ACC} = \textit{'free'} \mid \neg fr) & = & 0.0714
\end{array}
$$

CREDIT HISTORY	GUARANTOR/COAPPLICANT	ACCOMODATION	FRAUDULENT
paid	none	rent	?

$P(fr)$	$=$	0.3	$P(\neg fr)$	$=$	0.7
$P(\text{CH} = \textit{'paid'} \mid fr)$	$=$	0.1666	$P(\text{CH} = \textit{'paid'} \mid \neg fr)$	$=$	0.2857
$P(\text{GC} = \textit{'none'} \mid fr)$	$=$	0.8334	$P(\text{GC} = \textit{'none'} \mid \neg fr)$	$=$	0.8571
$P(\text{ACC} = \textit{'rent'} \mid fr)$	$=$	0.3333	$P(\text{ACC} = \textit{'rent'} \mid \neg fr)$	$=$	0.1429

$$\left(\prod_{k=1}^{m} P\left(\mathbf{q}\left[k\right] \mid fr\right)\right) \times P\left(fr\right) = 0.0139$$

$$\left(\prod_{k=1}^{m} P\left(\mathbf{q}\left[k\right] \mid \neg fr\right)\right) \times P(\neg fr) = 0.0245$$

CREDIT HISTORY	GUARANTOR/COAPPLICANT	ACCOMODATION	FRAUDULENT
paid	none	rent	?

$P(fr)$	$=$	0.3	$P(\neg fr)$	$=$	0.7
$P(\text{CH} = \textit{'paid'} \mid fr)$	$=$	0.1666	$P(\text{CH} = \textit{'paid'} \mid \neg fr)$	$=$	0.2857
$P(\text{GC} = \textit{'none'} \mid fr)$	$=$	0.8334	$P(\text{GC} = \textit{'none'} \mid \neg fr)$	$=$	0.8571
$P(\text{ACC} = \textit{'rent'} \mid fr)$	$=$	0.3333	$P(\text{ACC} = \textit{'rent'} \mid \neg fr)$	$=$	0.1429

$$\left(\prod_{k=1}^{m} P\left(\mathbf{q}\left[k\right] \mid fr\right)\right) \times P\left(fr\right) = 0.0139$$

$$\left(\prod_{k=1}^{m} P\left(\mathbf{q}\left[k\right] \mid \neg fr\right)\right) \times P(\neg fr) = 0.0245$$

CREDIT HISTORY	GUARANTOR/COAPPLICANT	ACCOMODATION	FRAUDULENT
paid	none	rent	*'false'*

The model is generalizing beyond the dataset!

ID	Credit History	Guarantor/ CoApplicant	Accommodation	Fraud
1	current	none	own	true
2	paid	none	own	false
3	paid	none	own	false
4	paid	guarantor	rent	true
5	arrears	none	own	false
6	arrears	none	own	true
7	current	none	own	false
8	arrears	none	own	false
9	current	none	rent	false
10	none	none	own	true
11	current	coapplicant	own	false
12	current	none	own	true
13	current	none	rent	true
14	paid	none	own	false
15	arrears	none	own	false
16	current	none	own	false
17	arrears	coapplicant	rent	false
18	arrears	none	free	false
19	arrears	none	own	false
20	paid	none	own	false

Smoothing

$$
\begin{array}{rclrcl}
P(fr) & = & 0.3 & P(\neg fr) & = & 0.7 \\
P(\text{CH} = \textit{'none'} \mid fr) & = & 0.1666 & P(\text{CH} = \textit{'none'} \mid \neg fr) & = & 0 \\
P(\text{CH} = \textit{'paid'} \mid fr) & = & 0.1666 & P(\text{CH} = \textit{'paid'} \mid \neg fr) & = & 0.2857 \\
P(\text{CH} = \textit{'current'} \mid fr) & = & 0.5 & P(\text{CH} = \textit{'current'} \mid \neg fr) & = & 0.2857 \\
P(\text{CH} = \textit{'arrears'} \mid fr) & = & 0.1666 & P(\text{CH} = \textit{'arrears'} \mid \neg fr) & = & 0.4286 \\
P(\text{GC} = \textit{'none'} \mid fr) & = & 0.8334 & P(\text{GC} = \textit{'none'} \mid \neg fr) & = & 0.8571 \\
P(\text{GC} = \textit{'guarantor'} \mid fr) & = & 0.1666 & P(\text{GC} = \textit{'guarantor'} \mid \neg fr) & = & 0 \\
P(\text{GC} = \textit{'coapplicant'} \mid fr) & = & 0 & P(\text{GC} = \textit{'coapplicant'} \mid \neg fr) & = & 0.1429 \\
P(\text{ACC} = \textit{'own'} \mid fr) & = & 0.6666 & P(\text{ACC} = \textit{'own'} \mid \neg fr) & = & 0.7857 \\
P(\text{ACC} = \textit{'rent'} \mid fr) & = & 0.3333 & P(\text{ACC} = \textit{'rent'} \mid \neg fr) & = & 0.1429 \\
P(\text{ACC} = \textit{'free'} \mid fr) & = & 0 & P(\text{ACC} = \textit{'free'} \mid \neg fr) & = & 0.0714
\end{array}
$$

Credit History	Guarantor/CoApplicant	Accommodation	Fraudulent
paid	guarantor	free	?

$P(fr)$	$=$	0.3	$P(\neg fr)$	$=$	0.7
$P(CH = paid \mid fr)$	$=$	0.1666	$P(CH = paid \mid \neg fr)$	$=$	0.2857
$P(GC = guarantor \mid fr)$	$=$	0.1666	$P(GC = guarantor \mid \neg fr)$	$=$	0
$P(ACC = free \mid fr)$	$=$	0	$P(ACC = free \mid \neg fr)$	$=$	0.0714

$$\left(\prod_{k=1}^{m} P(\mathbf{q}[k] \mid fr)\right) \times P(fr) = 0.0$$

$$\left(\prod_{k=1}^{m} P(\mathbf{q}[k] \mid \neg fr)\right) \times P(\neg fr) = 0.0$$

Credit History	Guarantor/CoApplicant	Accommodation	Fraudulent
paid	guarantor	free	?

- The standard way to avoid this issue is to use **smoothing**.
- Smoothing takes some of the probability from the events with lots of the probability share and gives it to the other probabilities in the set.
- There are several different ways to smooth probabilities, we will use **Laplacian smoothing**.

Laplacian Smoothing (conditional probabilities)

$$P(f = v|t) = \frac{count(f = v|t) + k}{count(f|t) + (k \times |Domain(f)|)}$$

Raw	$P(GC = none \mid \neg fr)$	$=$	0.8571
Probabilities	$P(GC = guarantor \mid \neg fr)$	$=$	0
	$P(GC = coapplicant \mid \neg fr)$	$=$	0.1429
Smoothing	k	$=$	3
Parameters	$count(GC \mid \neg fr)$	$=$	14
	$count(GC = none \mid \neg fr)$	$=$	12
	$count(GC = guarantor \mid \neg fr)$	$=$	0
	$count(GC = coapplicant \mid \neg fr)$	$=$	2
	$\lvert Domain(GC) \rvert$	$=$	3
Smoothed	$P(GC = none \mid \neg fr) = \frac{12+3}{14+(3\times 3)}$	$=$	0.6522
Probabilities	$P(GC = guarantor \mid \neg fr) = \frac{0+3}{14+(3\times 3)}$	$=$	0.1304
	$P(GC = coapplicant \mid \neg fr) = \frac{2+3}{14+(3\times 3)}$	$=$	0.2174

Table: Smoothing the posterior probabilities for the GUARANTOR/COAPPLICANT feature conditioned on FRAUDULENT being False.

$P(fr)$	$=$	0.3	$P(\neg fr)$	$=$	0.7
$P(CH = none \| fr)$	$=$	0.2222	$P(CH = none \| \neg fr)$	$=$	0.1154
$P(CH = paid \| fr)$	$=$	0.2222	$P(CH = paid \| \neg fr)$	$=$	0.2692
$P(CH = current \| fr)$	$=$	0.3333	$P(CH = current \| \neg fr)$	$=$	0.2692
$P(CH = arrears \| fr)$	$=$	0.2222	$P(CH = arrears \| \neg fr)$	$=$	0.3462
$P(GC = none \| fr)$	$=$	0.5333	$P(GC = none \| \neg fr)$	$=$	0.6522
$P(GC = guarantor \| fr)$	$=$	0.2667	$P(GC = guarantor \| \neg fr)$	$=$	0.1304
$P(GC = coapplicant \| fr)$	$=$	0.2	$P(GC = coapplicant \| \neg fr)$	$=$	0.2174
$P(ACC = own \| fr)$	$=$	0.4667	$P(ACC = own \| \neg fr)$	$=$	0.6087
$P(ACC = rent \| fr)$	$=$	0.3333	$P(ACC = rent \| \neg fr)$	$=$	0.2174
$P(ACC = Free \| fr)$	$=$	0.2	$P(ACC = Free \| \neg fr)$	$=$	0.1739

Table: The Laplacian smoothed, with $k = 3$, probabilities needed by a Naive Bayes prediction model calculated from the fraud detection dataset. Notation key: FR=FRAUDULENT, CH=CREDIT HISTORY, GC = GUARANTOR/COAPPLICANT, ACC = ACCOMODATION, T=*'True'*, F=*'False'*.

CREDIT HISTORY	GUARANTOR/COAPPLICANT	ACCOMMODATION	FRAUDULENT
paid	guarantor	free	?

$P(fr)$	$=$	0.3	$P(\neg fr)$	$=$	0.7
$P(CH = paid \mid fr)$	$=$	0.2222	$P(CH = paid \mid \neg fr)$	$=$	0.2692
$P(GC = guarantor \mid fr)$	$=$	0.2667	$P(GC = guarantor \mid \neg fr)$	$=$	0.1304
$P(ACC = Free \mid fr)$	$=$	0.2	$P(ACC = Free \mid \neg fr)$	$=$	0.1739

$$\left(\prod_{k=1}^{m} P(\mathbf{q}[m] \mid fr)\right) \times P(fr) = 0.0036$$

$$\left(\prod_{k=1}^{m} P(\mathbf{q}[m] \mid \neg fr)\right) \times P(\neg fr) = 0.0043$$

Table: The relevant smoothed probabilities, from Table 4 [15], needed by the Naive Bayes prediction model in order to classify the query from the previous slide and the calculation of the scores for each candidate classification.

$$P(t|\mathbf{d}) = \frac{P(\mathbf{d}|t) \times P(t)}{P(\mathbf{d})} \tag{1}$$

- A Naive Bayes' classifier naively assumes that each of the descriptive features in a domain is conditionally independent of all of the other descriptive features, given the state of the target feature.
- This assumption, although often wrong, enables the Naive Bayes' model to maximally factorise the representation that it uses of the domain.
- Surprisingly, given the naivety and strength of the assumption it depends upon, a Naive Bayes' model often performs reasonably well.