1.

Tasks	input	output
ASR	acoustic signal	sentence (string of written words)
Machine Translation	word sequence	word sequence of another Language.
Pattern recognition	image of handwriting text.	word sequence

2. a) $p(\text{economy}) = 0.576 + 0.144 + \cancel{0.064}^{\,0.008} + \cancel{0.016}^{\,0.072}$

$= 0.8$

b) $p(\text{oil}) = 0.576 + 0.144 + 0.064 + 0.016 = 0.8$

$p(\neg\text{oil}) = 0.008 + 0.072 + 0.012 + 0.108 = 0.2$

c) $p(\text{economy} \mid \text{oil}) = \dfrac{p(\text{economy}, \text{oil})}{p(\text{oil})} = \dfrac{0.576 + 0.144}{0.8} = 0.9$

$p(\neg\text{economy} \mid \text{oil}) = \dfrac{p(\neg\text{economy}, \text{oil})}{p(\text{oil})} = \dfrac{0.064 + 0.016}{0.8} = 0.1$

d) $p(\text{oil} \mid \text{economy}, \text{crisis}) = \dfrac{p(\text{oil}, \text{economy}, \text{crisis})}{p(\text{economy}, \text{crisis})} = \dfrac{0.576}{0.576 + 0.008} = 0.986$

$p(\neg\text{oil} \mid \text{economy}, \text{crisis}) = \dfrac{p(\neg\text{oil}, \text{economy}, \text{crisis})}{p(\text{economy}, \text{crisis})} = \dfrac{0.008}{0.576 + 0.008} = 0.014.$

3. a) $p(\text{fake}) = \dfrac{1}{n}$ $\quad p(\neg\text{fake}) = p(\text{normal}) = \dfrac{n-1}{n}$

$p(\text{fake} \mid \text{head}) = \dfrac{p(\text{head} \mid \text{fake}) \cdot p(\text{fake})}{p(\text{head})} = \dfrac{1 \cdot \frac{1}{n}}{p(\text{head} \mid \text{fake}) \cdot p(f) + p(h \mid \text{nor}) \cdot p(\text{nor})}$

$= \dfrac{1 \cdot \frac{1}{n}}{1 \cdot \frac{1}{n} + \frac{1}{2} \cdot \frac{n-1}{n}}$

$* \; p(\text{head}) = p(\text{head} \mid \text{fake}) + p(\text{head} \mid \text{nor}) \cdot p(\text{nor})$

$= \dfrac{2}{n+1}$

b) $p(\text{fake} \mid h_1^k) = \dfrac{p(h_1^k \mid \text{fake}) \cdot p(\text{fake})}{p(h_1^k \mid \text{fake}) \cdot p(\text{fake}) \cdot p(h_1^k \mid \text{nor}) \cdot p(\text{nor})} = \dfrac{\prod_{n=1}^{k} p(h_n \mid h_1^{n-1}, \text{fake}) \cdot p(\text{fake})}{\prod_{n=1}^{k} p(h_n \mid \text{fake}) \cdot p(\text{fake}) + \prod_{n=1}^{k} p(h_n \mid \text{nor}) \cdot p(\text{nor})}$

with $p(h \mid \text{nor}) = \dfrac{1}{2}$

$= \dfrac{1 \cdot \frac{1}{n}}{1 \cdot \frac{1}{n} + \left(\frac{1}{2}\right)^k \cdot \frac{n-1}{n}}$

$= \dfrac{1}{1 + \left(\frac{1}{2}\right)^k (n-1)}$

c) probability of return normal, but all k flips come up heads.

$$P(\text{normal}, h_1^k) = P(\cancel{\text{normal}}\ h^k \mid \text{normal}) \cdot P(\text{normal})$$
$$= \left(\frac{1}{2}\right)^k \cdot \frac{n-1}{n}$$

4.

a) $$P_{k+1} = \frac{\lambda}{k+1} \cdot \frac{\lambda}{k} \cdot \frac{\lambda}{k-1} \cdots \frac{\lambda}{1} P_0$$
$$P_k = \frac{\lambda^k}{k!} P_0.$$

b) $$\sum_{k=0}^{\infty} P_k = 1 = \sum_k \frac{\lambda^k}{k!} P_0 = 1$$ $0 \leq P_k \leq 1$ fullfilled.
$$e^\lambda P_0 = 1, \text{ with } \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^\lambda$$
$$P_0 = e^{-\lambda}$$

c) $$P_k = \frac{\lambda^{k-1}}{\cancel{(k-1)!}\ k!} \cdot P_1 \qquad \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{k!} P_1 = 1.$$
$$\Rightarrow \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} P_1 \cdot \frac{1}{\lambda} = \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} P_1 \cdot \frac{1}{\lambda} + P_1 \cdot \frac{1}{\lambda} - P_1 \cdot \frac{1}{\lambda}$$
$$= \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} P_1 \cdot \frac{1}{\lambda} - P_1 \cdot \frac{1}{\lambda}$$
$$= e^\lambda \cdot \frac{P_1}{\lambda} - P_1 \cdot \frac{1}{\lambda} = 1 = P_1\left(\frac{e^\lambda}{\lambda} - \frac{1}{\lambda}\right)$$
$$\Rightarrow P_1 = \frac{\lambda}{e^\lambda - 1}$$

5.

a) see "explanation.pdf"

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