Cross Entropy

Cross Entropy is $-y_i \log \sigma(f(x_i)) - (1-y_i)\log(1-\sigma(f(x_i)))$

$\sigma = \frac{e^x}{1+e^x}$ ← sigmoid ∫

$$CE = -y_i \log\left(\frac{e^{f(x_i)}}{1+e^{f(x_i)}}\right) - (1-y_i)\log\left(1 - \frac{e^{f(x_i)}}{1+e^{f(x_i)}}\right)$$

$$\begin{cases} y_i = 1 \Rightarrow \text{is } 0 & -\log\left(\frac{e^{f(x_i)}}{1+e^{f(x_i)}}\right) \\ & = \log\left(\frac{1+e^{f(x_i)}}{e^{f(x_i)}}\right) = \log\left(e^{-f(x_i)} + 1\right) \\ & = \boxed{\log\left(1+e^{-y_i f(x_i)}\right)} \\ y_i = \tilde{0} \Rightarrow \{0,-1\} & -\log\left(1 - \frac{e^{f(x_i)}}{1+e^{f(x_i)}}\right) \\ & = -\log\left(\frac{1+\cancel{e^{f(x_i)}} - \cancel{e^{f(x_i)}}}{1+e^{f(x_i)}}\right) \\ & = \log\left(1+e^{f(x_i)}\right) = \boxed{\log\left(1+e^{-y_i f(x_i)}\right)} \end{cases}$$

logistic loss