ECE 4960 Spring 2018: Computational and Software Engineering
Reading 2: Differentiation in Local Analysis

Deposit a pdf file of the two tables below to your Git directory before 11:59pm of 2/11

Programming environment: Dev-C++ 5.11
Language: C++
Operating system: Windows 10

Prob. 1. (Quadratic function to observe the tradeoffs between the truncation error and round-off error): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.

1.1 Use Eq. (1) below to estimate $f'(x=1)$ varying the value of h from 0.1 to 10^{-18} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.

1.2 Repeat your calculation with $f(x) = x^2 + 10^8$. Add your results to the same table.

1.3 Repeat the above two procedures by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \qquad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \qquad (2)$$

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}	**0.1**	**0.099991**	**4.44E-16**	**-1.91E-06**
10^{-2}	**0.01**	**0.009964**	**1.33E-15**	**-4.96E-05**
10^{-3}	**0.001**	**0.000809**	**-1.65E-13**	**-0.00014**
10^{-4}	**0.0001**	**0.002716**	**-7.76E-13**	**0.002716**
10^{-5}	**1.00E-05**	**-0.09265**	**2.00E-12**	**-0.09265**
10^{-6}	**1.00E-06**	**-0.09265**	**2.00E-12**	**-0.09265**
10^{-7}	**1.01E-07**	**-2**	**5.75E-11**	**-2**
10^{-8}	**-1.22E-08**	**-2**	**-6.60E-09**	**-2**
10^{-9}	**1.65E-07**	**-2**	**5.45E-08**	**-2**
10^{-10}	**1.65E-07**	**-2**	**1.65E-07**	**-2**
10^{-11}	**1.65E-07**	**-2**	**1.65E-07**	**-2**
10^{-12}	**0.000178**	**-2**	**6.68E-05**	**-2**
10^{-13}	**-0.0016**	**-2**	**-0.00049**	**-2**
10^{-14}	**-0.0016**	**-2**	**-0.0016**	**-2**
10^{-15}	**0.220446**	**-2**	**0.109424**	**-2**
10^{-16}	**-2**	**-2**	**-0.88978**	**-2**
10^{-17}	**-2**	**-2**	**-2**	**-2**

10^{-18}	**-2**	**-2**	**-2**	**-2**

Prob. 2. (Cubic function to observe the Richardson error estimation): For $f(x) = x^3$, we know the exact value of $f'(x=1) = 3$.

2.1 Use Eqs. (3) – (5) below to estimate $f'(x=1)$ varying the value of h from 2^{-4} to 2^{-40} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.

2.2 Estimate η from Eqs. (6) and (7) for each choice of h. Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \qquad E(h) = O(h) = \frac{1}{2} h f''(x) + O(h^2) \tag{3}$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \qquad E(2h) = O(h) = \frac{1}{2} 2h f''(x) + O(h^2) \tag{4}$$

$$f'(x) = \frac{-1}{2h} f(x+2h) - \frac{3}{2h} f(x) + \frac{2}{h} f(x+h) + O(h^2) \tag{5}$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \tag{6}$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \tag{7}$$

h	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	η βψ Eθ. (6)	η βψ Eθ. (7)
2^{-4}	0.199219	0.421875	0.1875	2.11765	2.11765
2^{-5}	0.09668	0.199219	0.09375	2.06061	2.06061
2^{-6}	0.047607	0.09668	0.046875	2.03077	2.03077
2^{-7}	0.023621	0.047607	0.023438	2.0155	2.0155
2^{-8}	0.011765	0.023621	0.011719	2.00778	2.00778
2^{-9}	0.005871	0.011765	0.005859	2.0039	2.0039
2^{-10}	0.002933	0.005871	0.00293	2.00195	2.00195
2^{-11}	0.001466	0.002933	0.001465	2.00098	2.00098
2^{-12}	0.000733	0.001466	0.000732	2.00049	2.00049
2^{-13}	0.000366	0.000733	0.000366	2.00024	2.00024
2^{-14}	0.000183	0.000366	0.000183	2.00012	2.00012
2^{-15}	9.16E-05	0.000183	9.16E-05	2.00006	2.00006
2^{-16}	4.58E-05	9.16E-05	4.58E-05	2.00003	2.00003
2^{-17}	2.29E-05	4.58E-05	2.29E-05	2.00002	2.00002
2^{-18}	1.14E-05	2.29E-05	1.14E-05	2.00001	2.00001
2^{-19}	5.72E-06	1.14E-05	5.72E-06	2.00001	2.00001
2^{-20}	2.86E-06	5.72E-06	2.86E-06	2	2
2^{-21}	1.43E-06	2.86E-06	1.43E-06	2	2

2^{-22}	7.15E-07	1.43E-06	7.15E-07	2	2
2^{-23}	3.58E-07	7.15E-07	3.58E-07	2	2
2^{-24}	1.79E-07	3.58E-07	1.79E-07	2	2
2^{-25}	8.94E-08	1.79E-07	8.94E-08	2	2
2^{-26}	4.47E-08	8.94E-08	4.47E-08	2	2
2^{-27}	4.47E-08	4.47E-08	7.45E-08	1	1
2^{-28}	0	4.47E-08	-2.98E-08	inf	inf
2^{-29}	0	0	0	nan	nan
2^{-30}	0	0	0	nan	nan
2^{-31}	0	0	0	nan	nan
2^{-32}	0	0	0	nan	nan
2^{-33}	0	0	0	nan	nan
2^{-34}	0	0	0	nan	nan
2^{-35}	0	0	0	nan	nan
2^{-36}	0	0	0	nan	nan
2^{-37}	0	0	0	nan	nan
2^{-38}	0	0	0	nan	nan
2^{-39}	0	0	0	nan	nan
2^{-40}	0	0	0	nan	nan