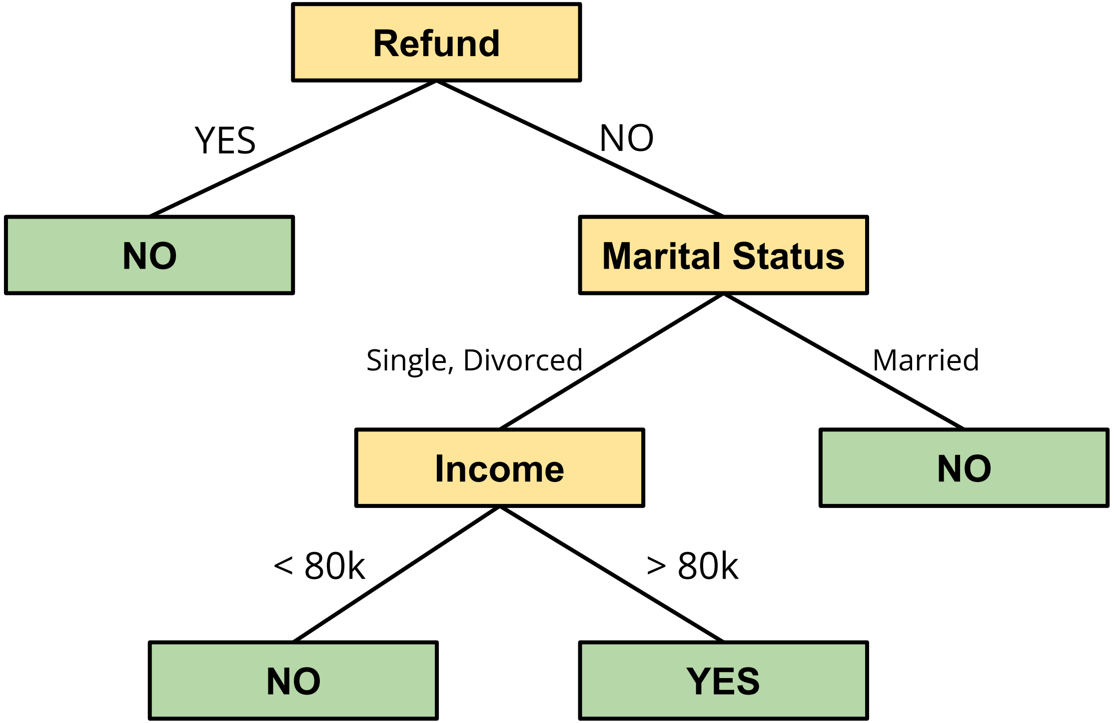
Decision Trees

Boston University CS 506 - Lance Galletti

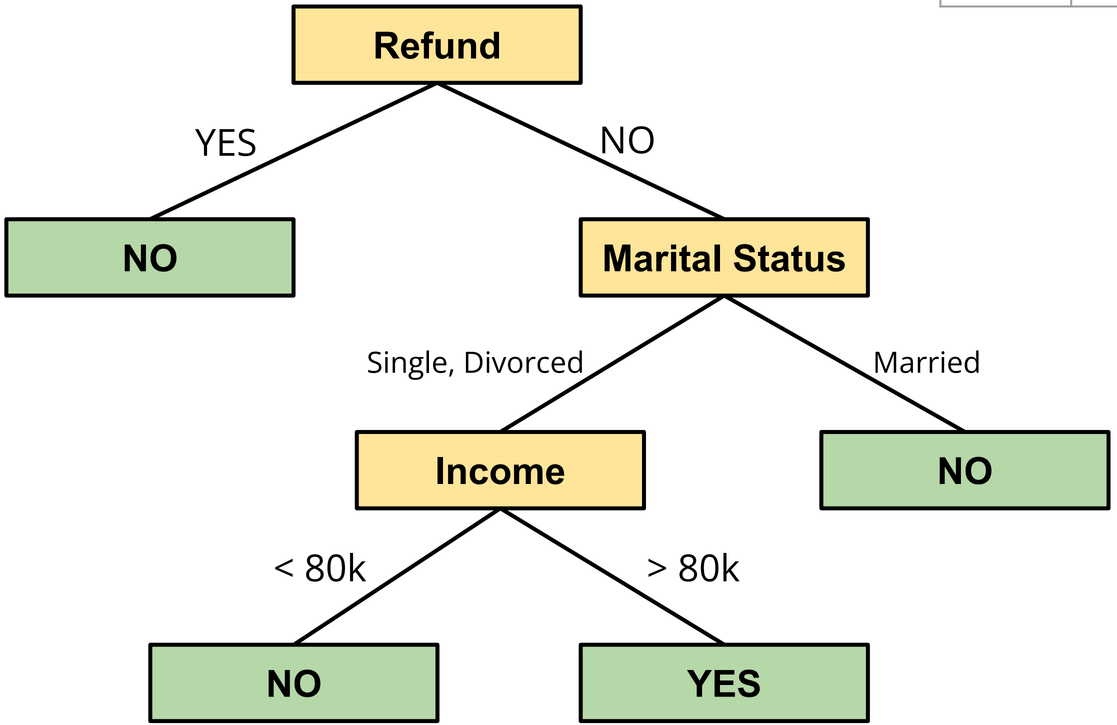
Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

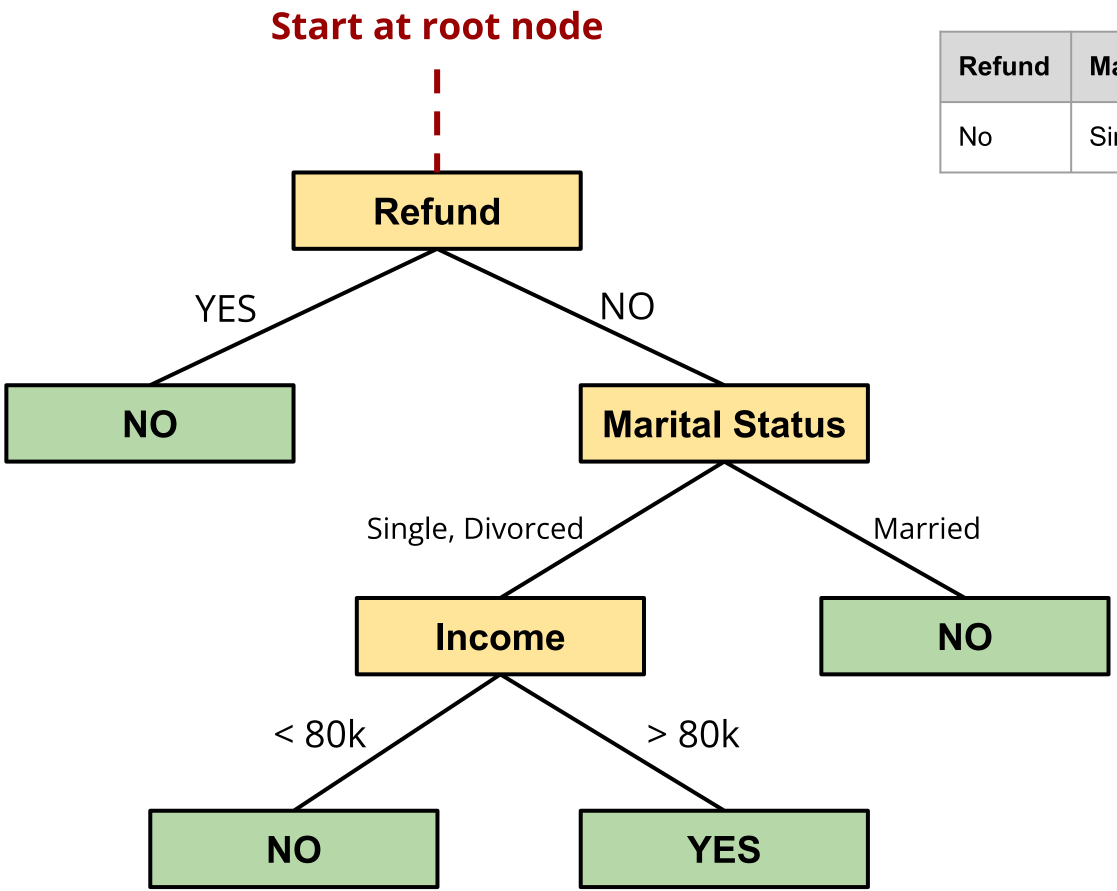
What a Decision Tree looks like



How it works

Refund	Marital Status	Income	Class
No	Single	70k	?





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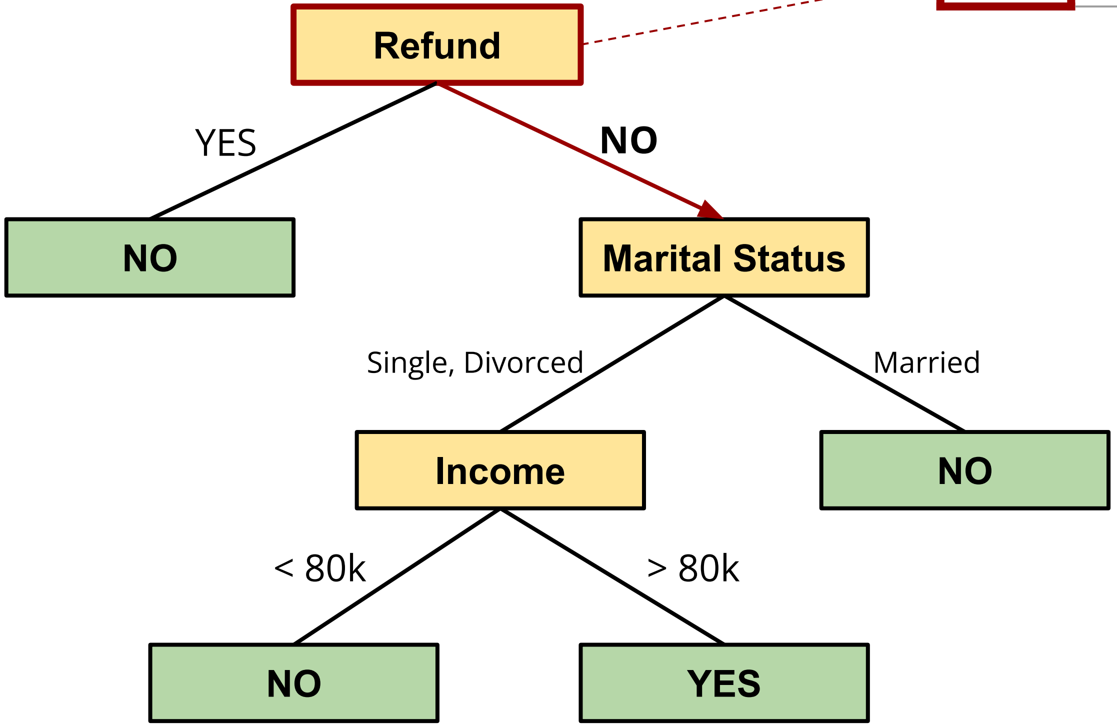
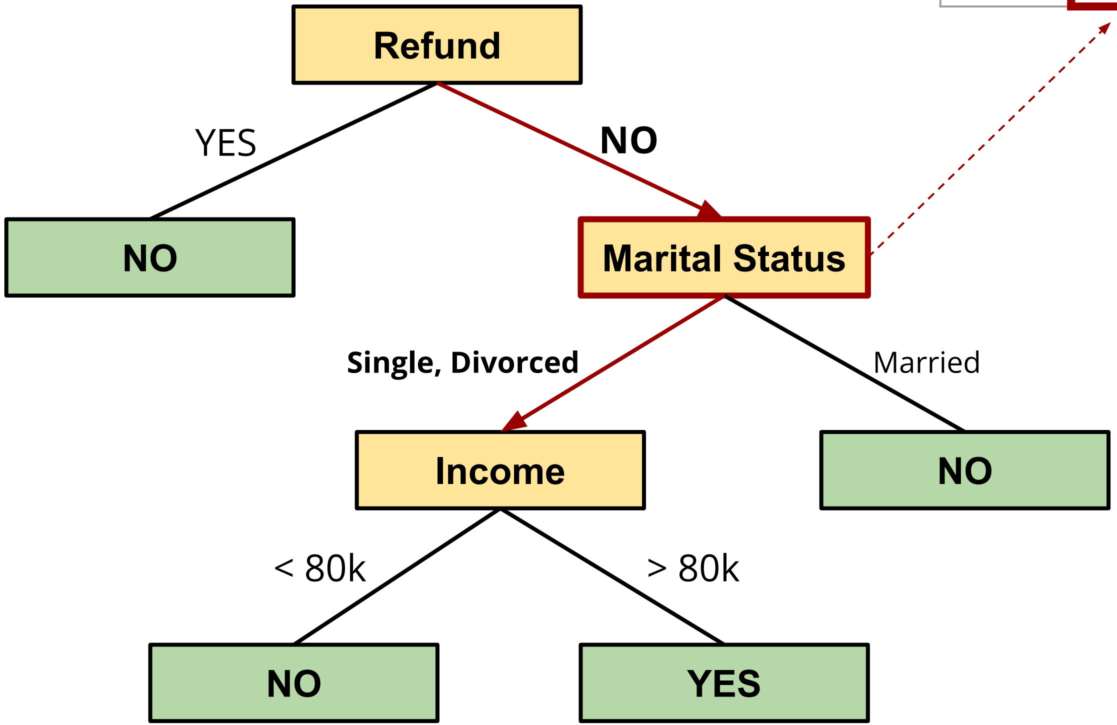


Refund	Marital Status	Income	Class
No	Single	70k	?



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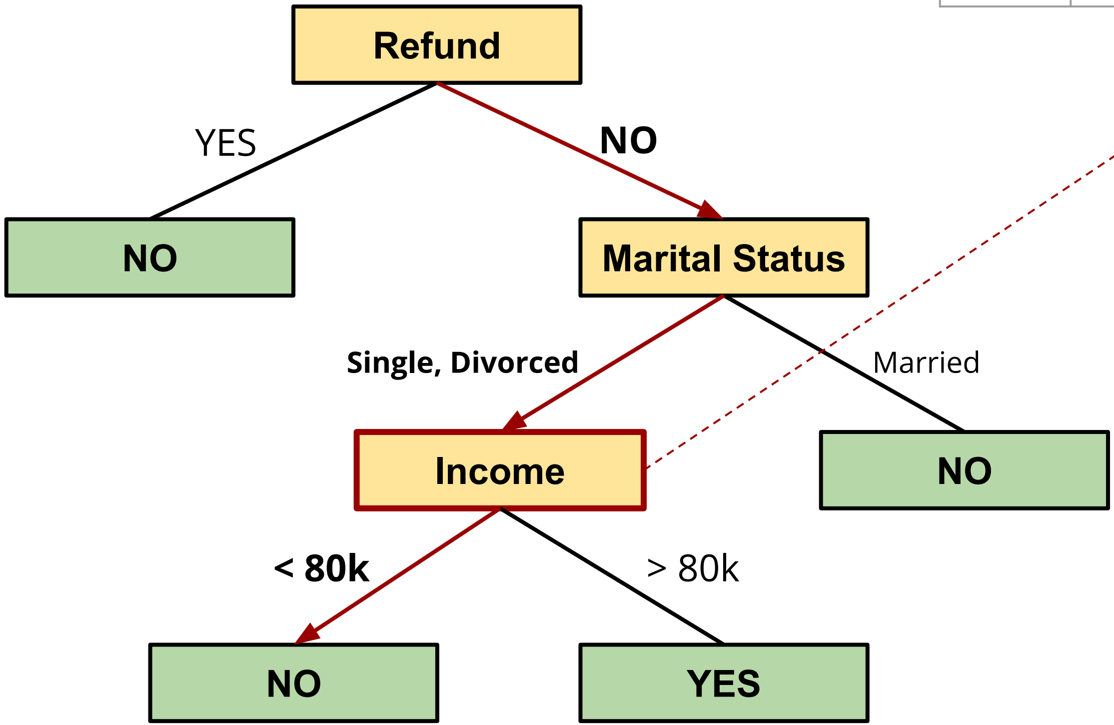

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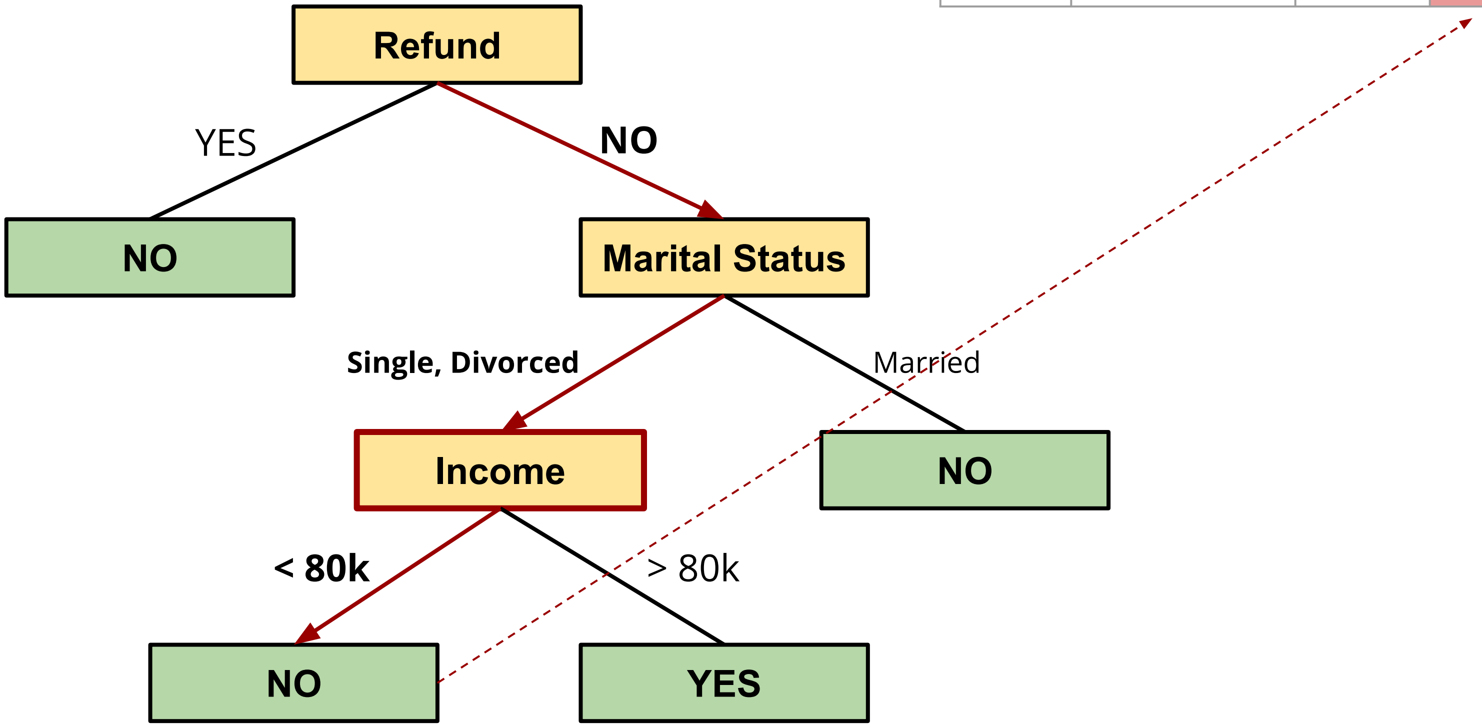

Part 1

How do we learn it?

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
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No	Single	90k	Yes

IF marital status == Married

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IF marital status == Married

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No	Married	60k	No
No	Married	75k	No

THEN class = NO

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No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

IF income < 60k

Refund	Marital Status	Income	Class
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No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

IF income < 60k

Refund	Marital Status	Income	Class

THEN ?

Hunt's Algorithm

- Recursive Algorithm
  - Repeatedly split the dataset based on attributes
- Base cases:
  - IF Split and all data points in the same class
    - Great! Predict that class
  - IF Split and no data points
    - No problem! Predict a reasonable default

Hunt’s Algorithm

The recursion (IF split and data points belong to more than one class)

- Find the attribute (and best way to split that attribute) that best splits the data

Example

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
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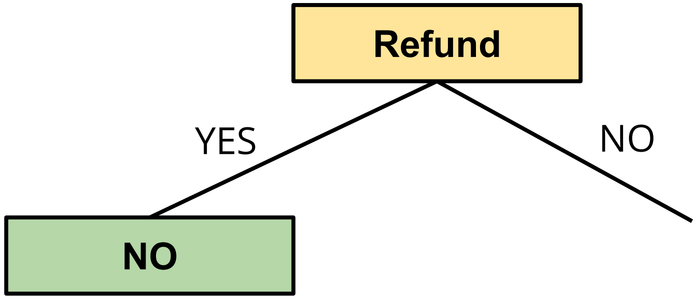

Refund	Marital Status	Income	Class
Yes	Single	125k	No
Yes	Married	120k	No
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Refund	Marital Status	Income	Class
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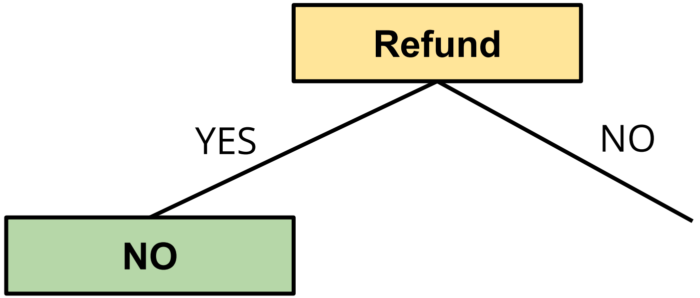

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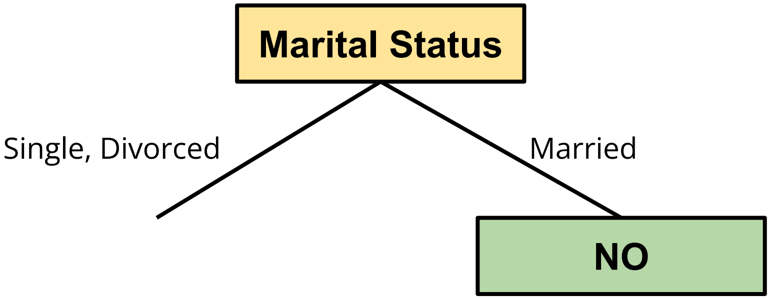

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No	Single	70k	No
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No	Single	85k	Yes
No	Single	90k	Yes

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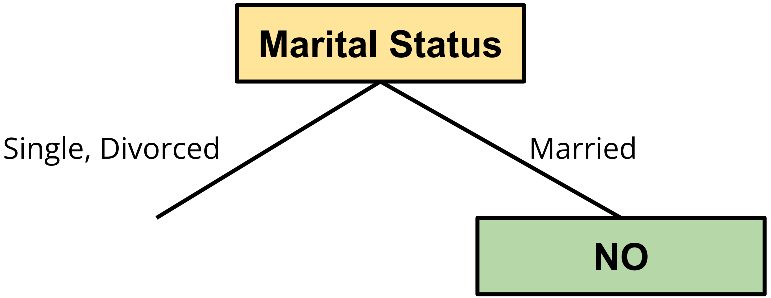

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Refund	Marital Status	Income	Class
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No	Married	60k	No
No	Married	75k	No

Refund	**Marital Status**	**Income**	**Class**
No	Single	70k	No
No	Divorced	90k	Yes
No	Single	85k	Yes
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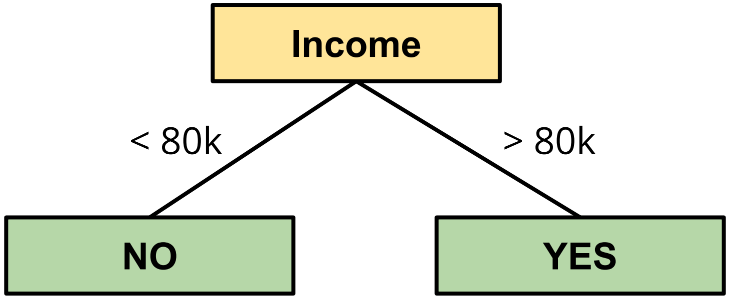

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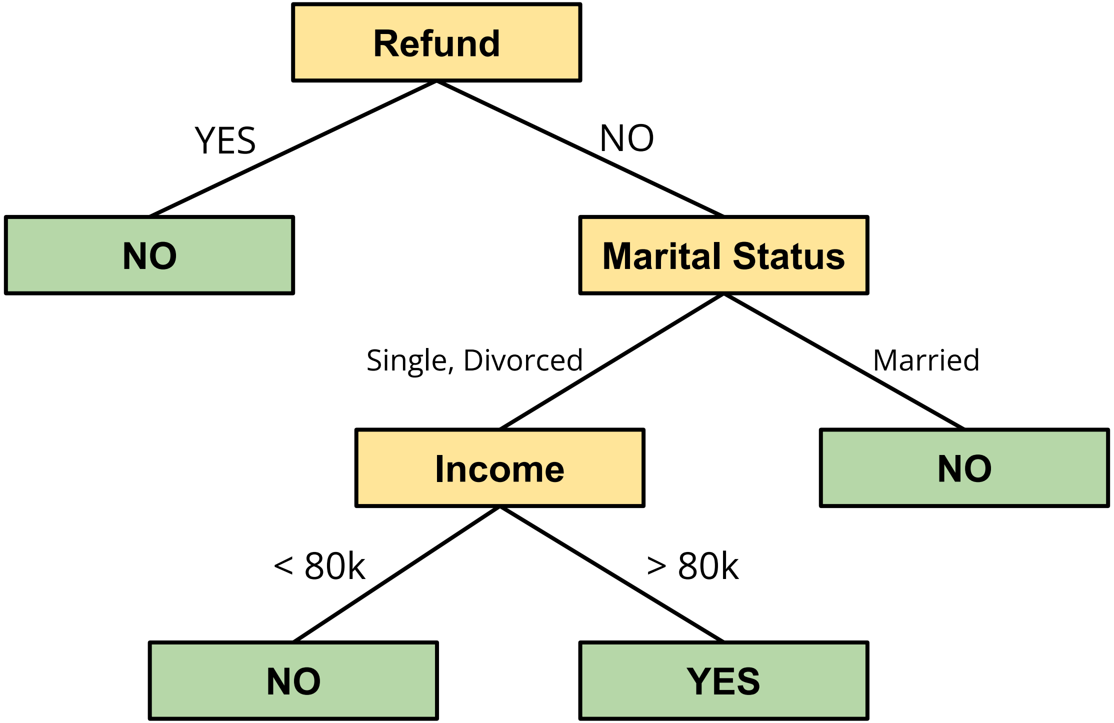
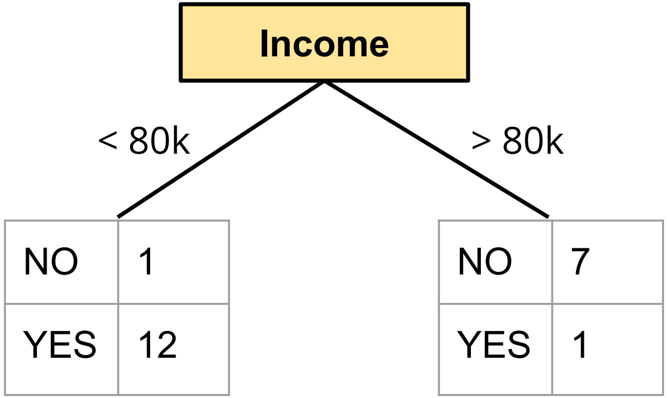

Refund
YES
NO
NO
Marital Status
Single, Divorced
Married
Income
NO
< 80k
> 80k
NO
YES

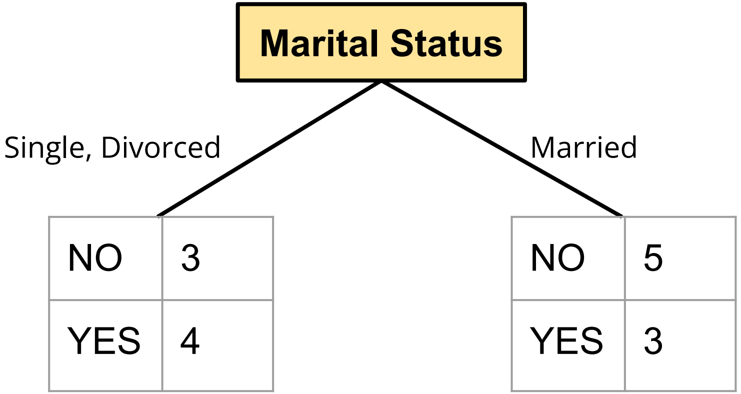
What do we mean by best split?

Many ways to split a given attribute

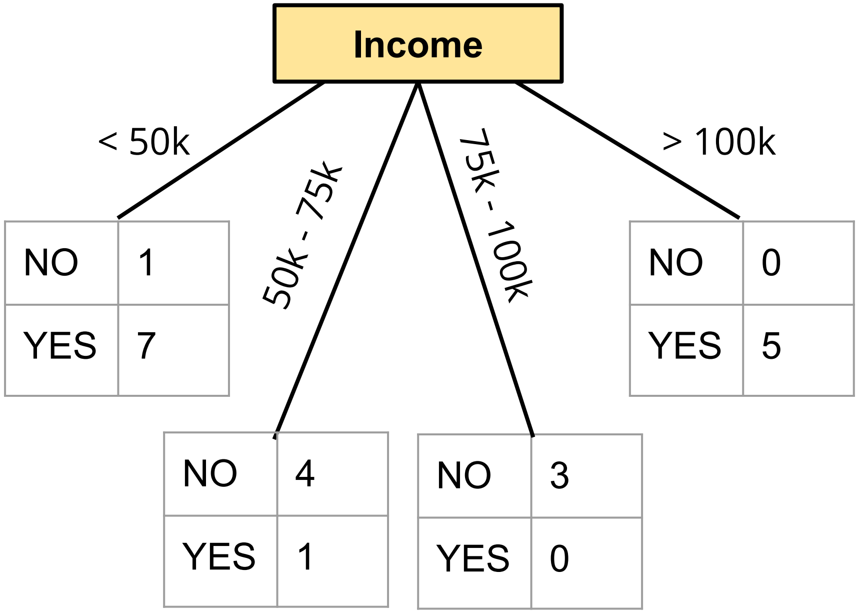
- Binary Split
- Multi-Way Split

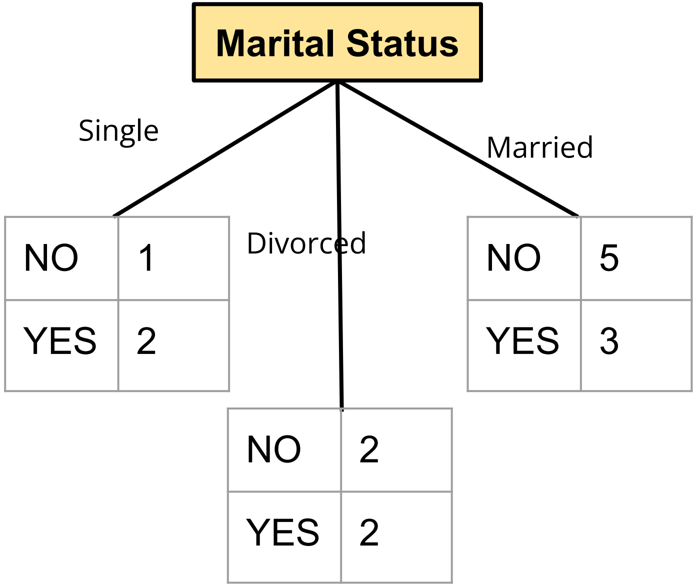
Binary Split





Multi-Way Split





Continuous Variables

- Use binning before running the decision tree
  - Can use clustering for that for example
- Compute a threshold while building the tree
  - $A > t$ vs $A < t$

Need a metric

That favors nodes like this:

NO	1
YES	7

Over nodes like this:

NO	4
YES	4

GINI index

Denote p(j | t) as the relative frequency of class j at node t.

NO	1
YES	7

p(NO | t) = ⅛
p(YES | t) = ⅞

NO	4
YES	3

p(NO | t) = 4/7
p(YES | t) = 3/7

GINI index

$$GINI(t) = 1 - \sum_j p(j|t)^2$$

NO	1
YES	7

p(NO | t) = ⅛
p(YES | t) = ⅞

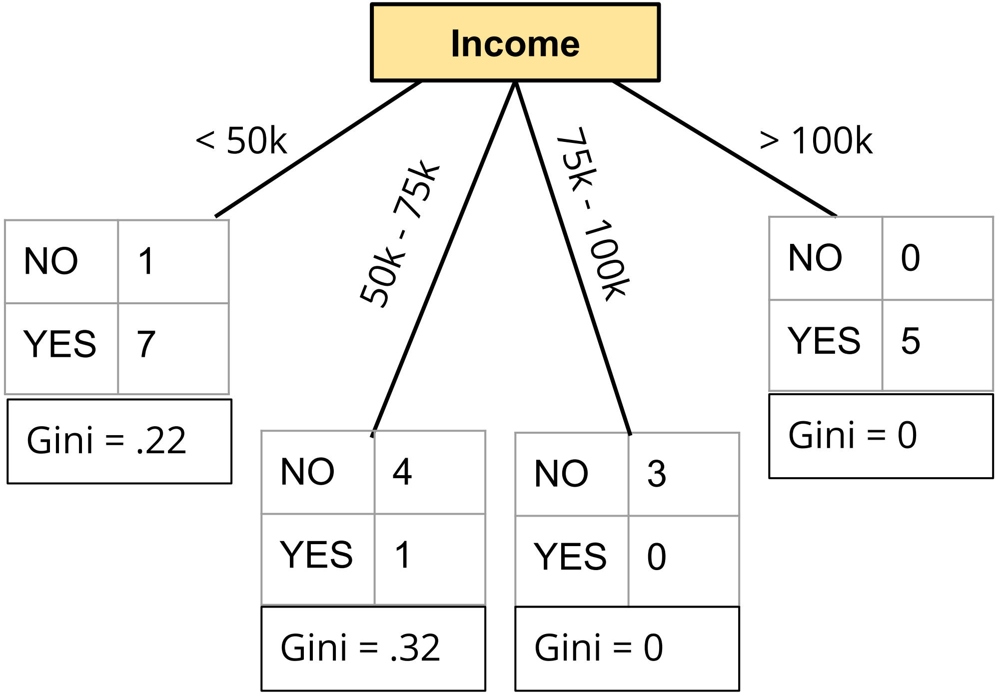
GINI(t) = 1 - 1/64 - 49/64 = 14/64

NO	4
YES	3

p(NO | t) = 4/7
p(YES | t) = 3/7

GINI(t) = 1 - 16/49 - 9/49 = 24/49

GINI of the Split



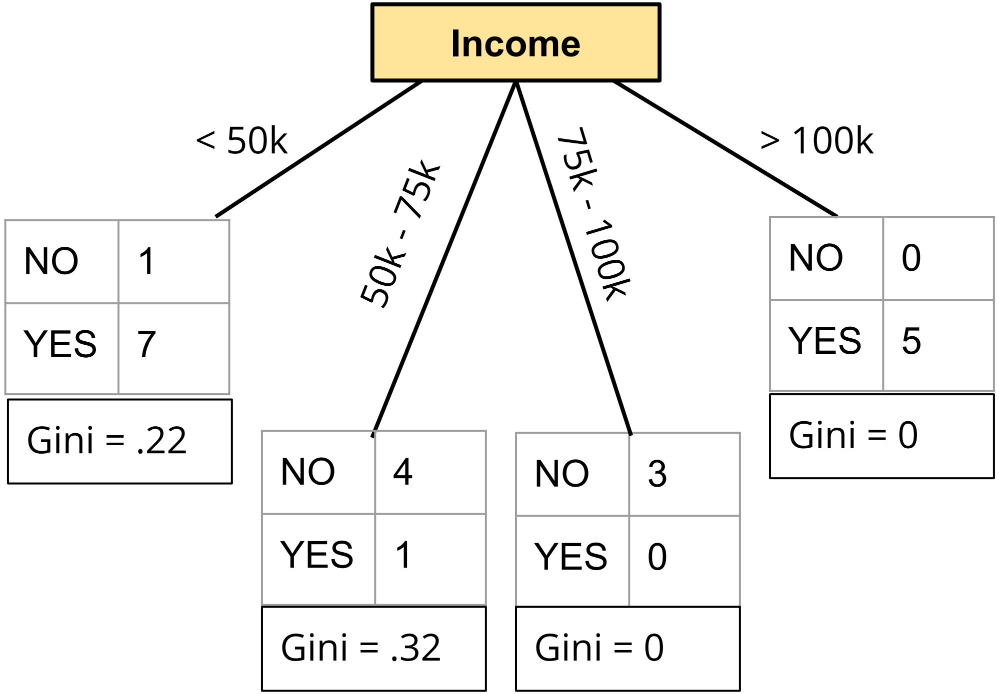
GINI of the split

$$GINI_{split} = \sum_{t=1}^{k} \frac{n_t}{n} GINI(t)$$

where:

n_t = number of data points at node t
n = number of data points before the split (parent node)

GINI of the split



$$GINI_{split} = \sum_{t=1}^{k} \frac{n_t}{n} GINI(t)$$

n = 21

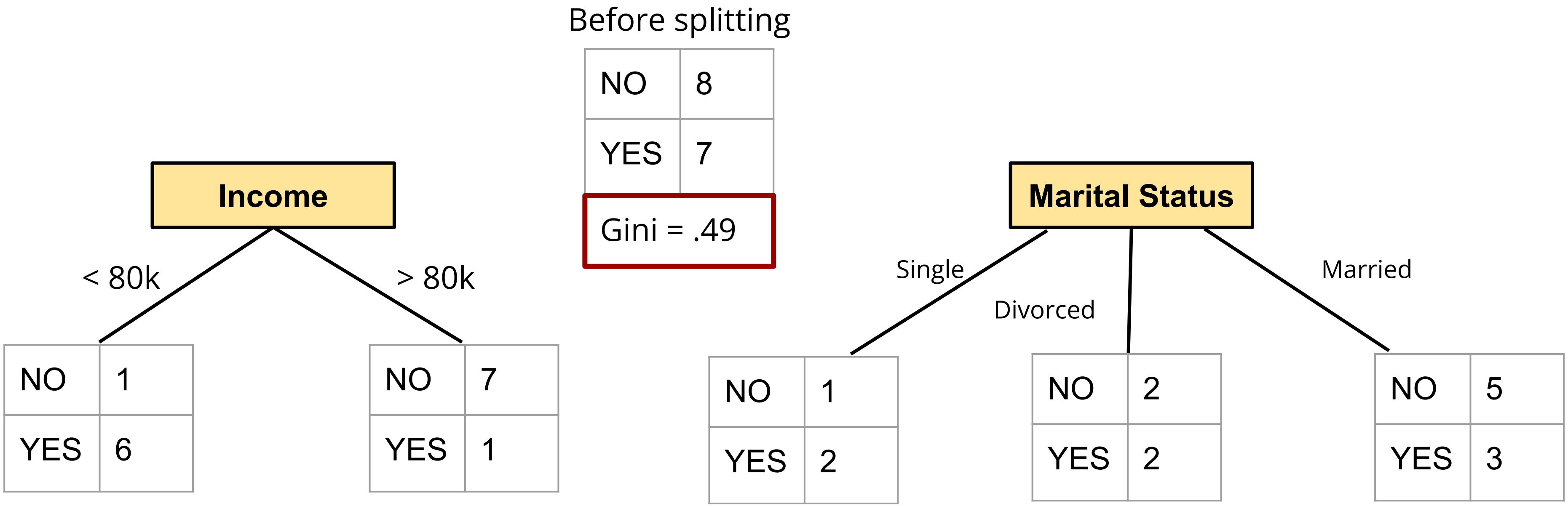
$$\begin{aligned} GINI_{split} &= .22 * 8/21 \\ &\quad + .32 * 5/21 \\ &\quad + 0 * 3/21 \\ &\quad + 0 * 5/21 \\ &= .16 \end{aligned}$$

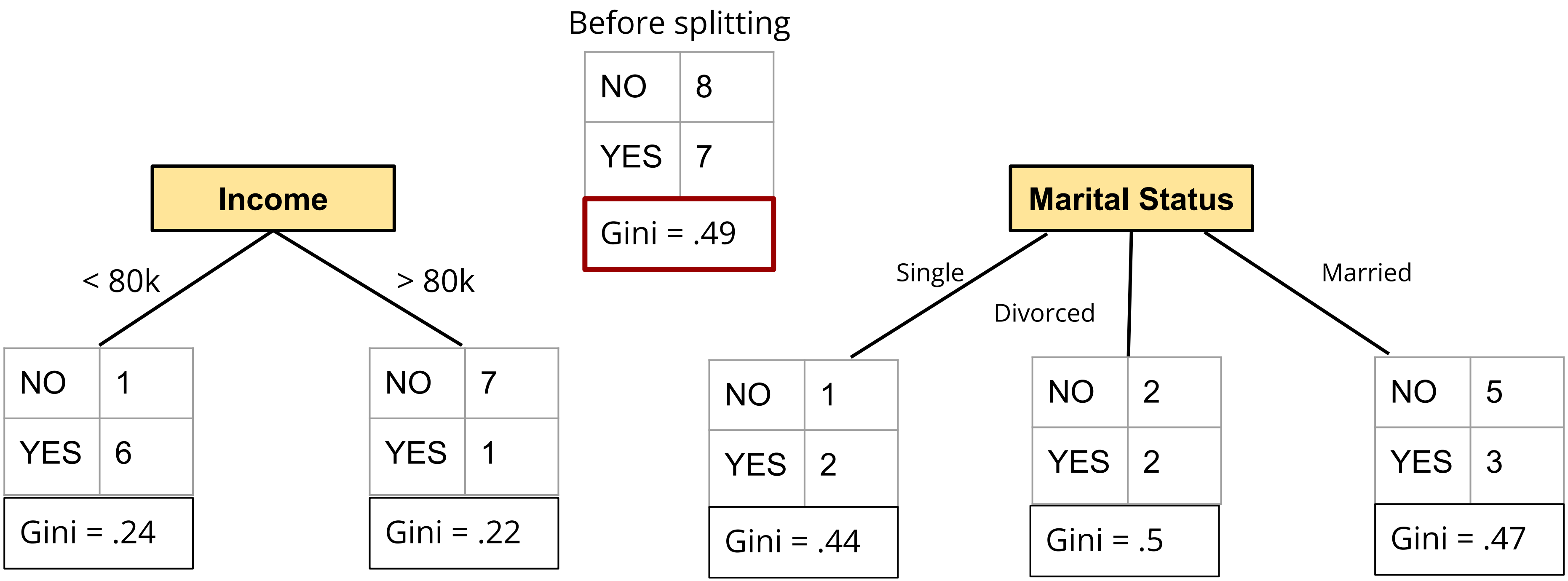
Part 2

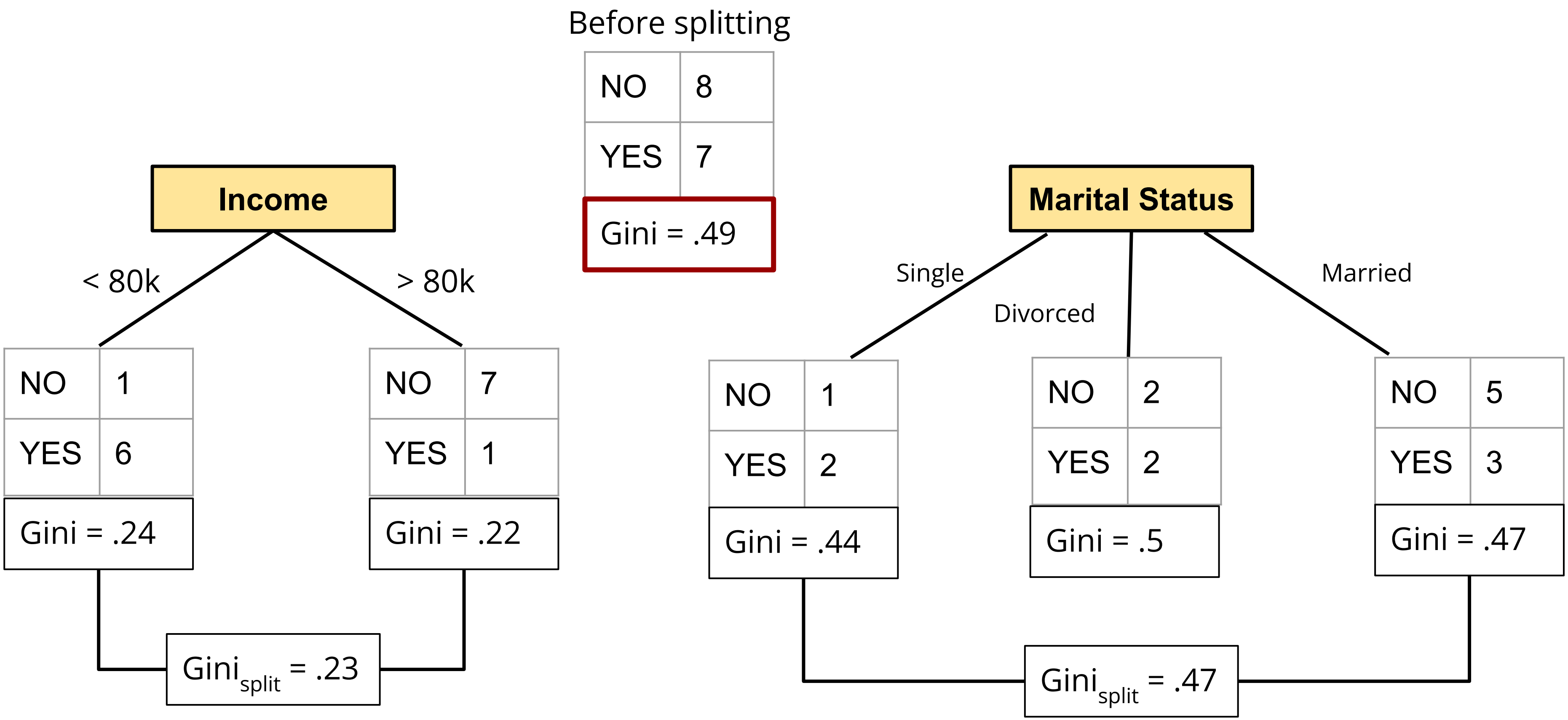
Putting it all together

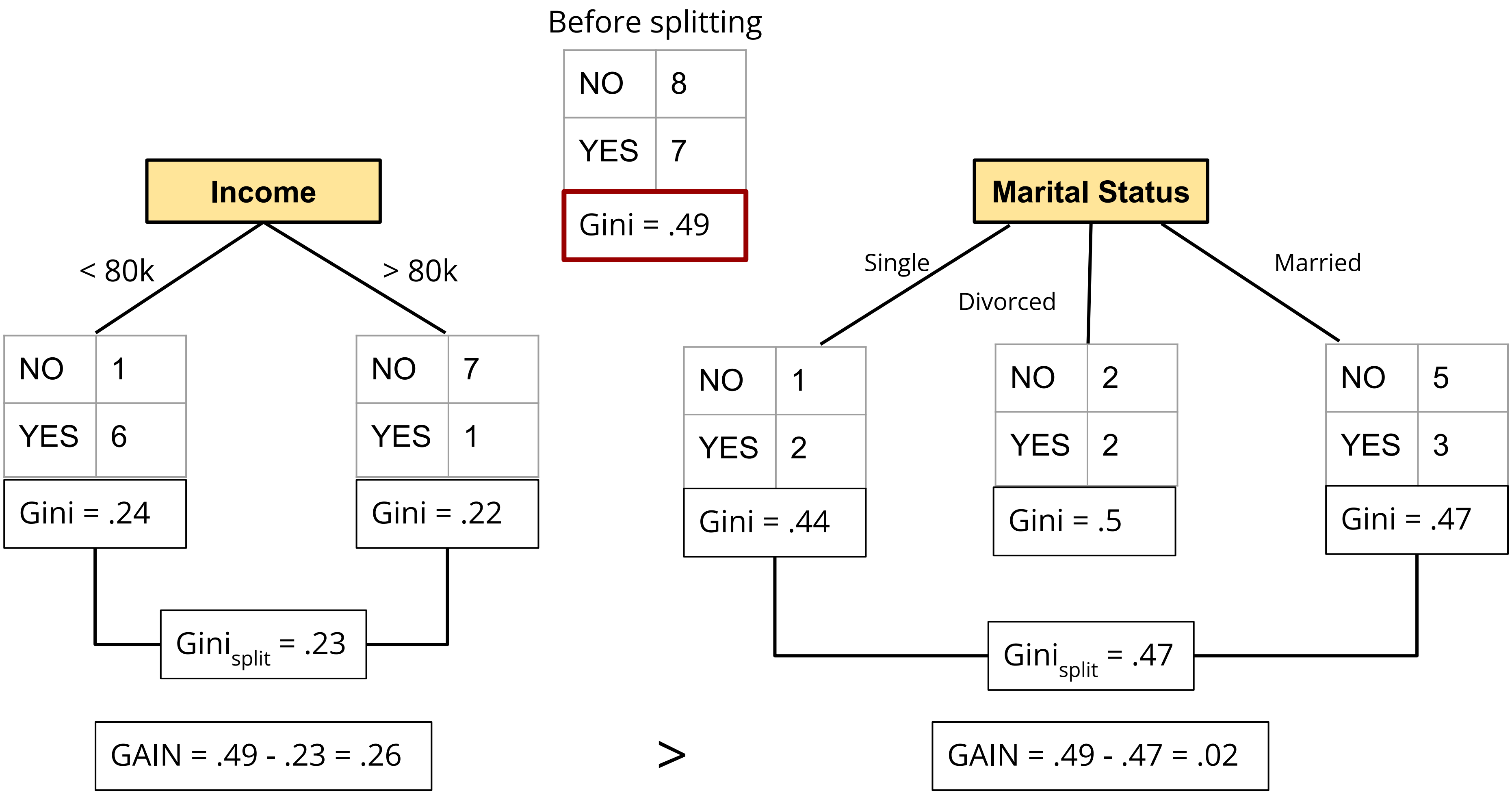
Before splitting

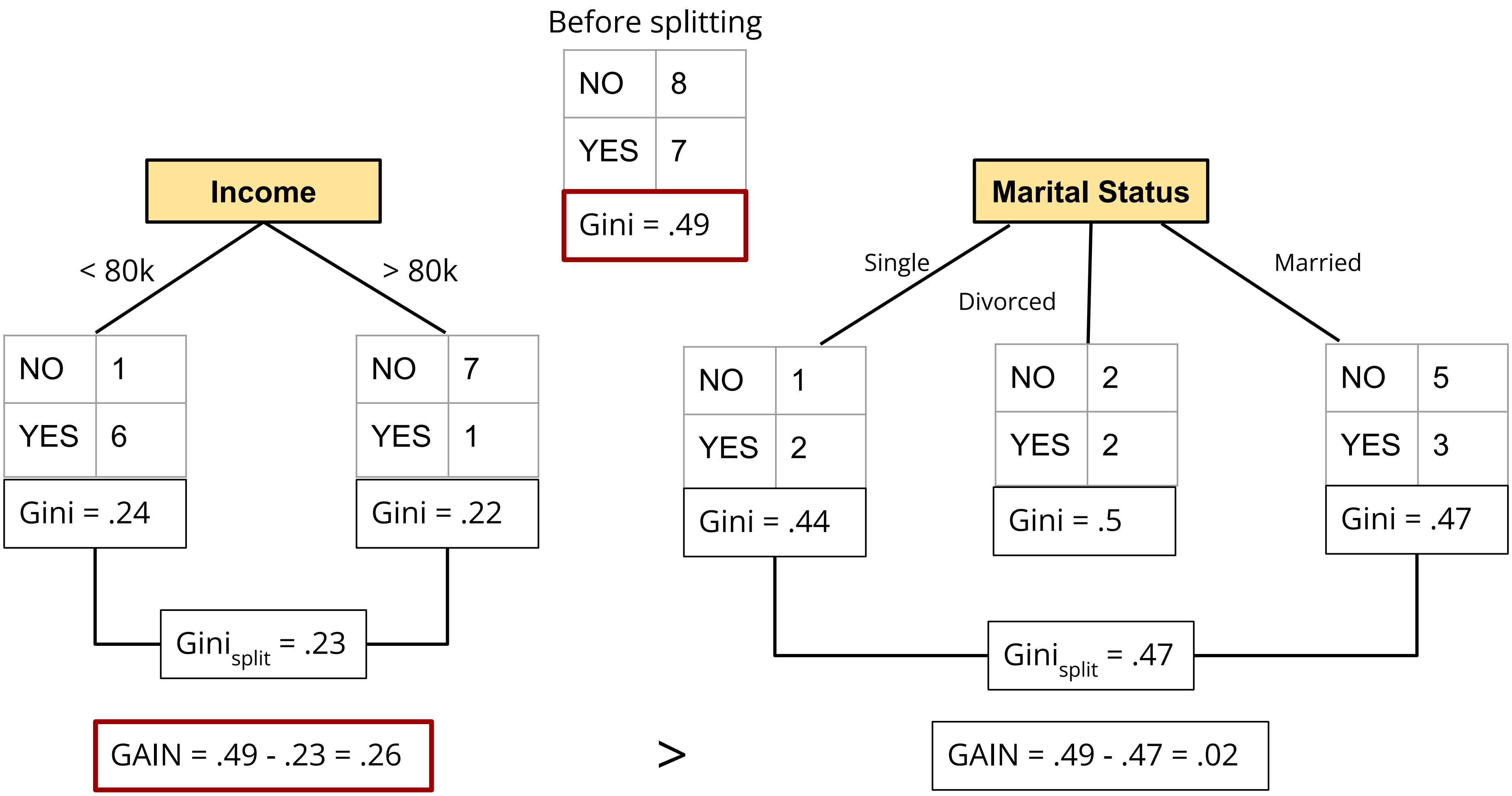
NO	8
YES	7
$\boxed{\text{Gini} = .49}$	


Before splitting
NO 8
YES 7
Gini = .49
Income
< 80k
NO 1
YES 6
> 80k
NO 7
YES 1
Marital Status
Single
NO 1
YES 2
Divorced
NO 2
YES 2
Married
NO 5
YES 3

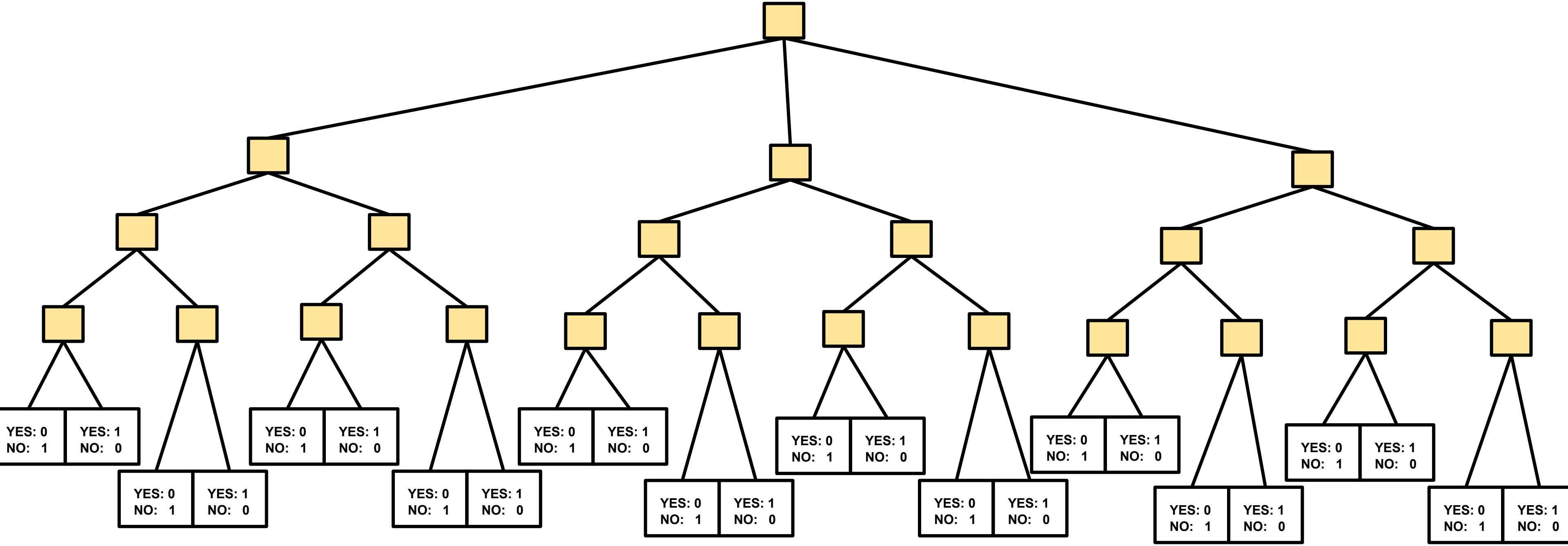

Before splitting
NO 8
YES 7
Gini = .49
Income
< 80k
> 80k
NO 1
YES 6
Gini = .24
NO 7
YES 1
Gini = .22
Marital Status
Single
Divorced
Married
NO 1
YES 2
Gini = .44
NO 2
YES 2
Gini = .5
NO 5
YES 3
Gini = .47


Before splitting
NO 8
YES 7
Gini = .49
Income
< 80k
> 80k
NO 1
YES 6
Gini = .24
NO 7
YES 1
Gini = .22
Gini$_{split}$ = .23
Marital Status
Single
Divorced
Married
NO 1
YES 2
Gini = .44
NO 2
YES 2
Gini = .5
NO 5
YES 3
Gini = .47
Gini$_{split}$ = .47


Before splitting
NO 8
YES 7
Gini = .49
Income
< 80k
> 80k
NO 1
YES 6
Gini = .24
NO 7
YES 1
Gini = .22
$Gini_{split}$ = .23
GAIN = .49 - .23 = .26
>
Marital Status
Single
Divorced
Married
NO 1
YES 2
Gini = .44
NO 2
YES 2
Gini = .5
NO 5
YES 3
Gini = .47
$Gini_{split}$ = .47
GAIN = .49 - .47 = .02


Before splitting
NO 8
YES 7
Gini = .49
Income
< 80k
> 80k
NO 1
YES 6
Gini = .24
NO 7
YES 1
Gini = .22
$Gini_{split}$ = .23
GAIN = .49 - .23 = .26
>
Marital Status
Single
Divorced
Married
NO 1
YES 2
Gini = .44
NO 2
YES 2
Gini = .5
NO 5
YES 3
Gini = .47
$Gini_{split}$ = .47
GAIN = .49 - .47 = .02

Limitations


YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0
YES: 0 NO: 1
YES: 1 NO: 0

Easy to construct a tree that is too complex and overfits the data.

Solutions:

- Early termination (stop before tree is fully grown - use majority vote at leaf node)
  - Stop at some specified depth
  - Stop if size of node is below some threshold
  - Stop if gini does not improve
- Pruning (create fully grown tree then trim)

Extensions

Other measures of node purity

- Entropy

$$\text{Entropy}(t) = -\sum_j p(j|t)\log(p(j|t))$$

- Misclassification Error

$$\text{Error}(t) = 1 - \max_j(p(j|t))$$

Part 3