Problem 4:

a. False, assuming $f(x) = x^2$, $g(z) = (z-1)^2$

$h(x) = g(f(x)) = (x^2-1)^2$

$$\frac{d^2 h(x)}{dx^2} = \frac{d\, 2(x^2-1)\cdot 2x}{dx} = \frac{d(4x^3-2x)}{dx} = 12x^2 - 2$$

if $x = 0$, $\frac{d^2 h(x)}{dx^2} = -2 < 0$

$h(x)$ is not convex

b. True. $f(x)$ is convex

$\Rightarrow \lambda f(x_0) + (1-\lambda) f(x_1) \geq f(\lambda x_0 + (1-\lambda) x_1) \quad \lambda \in [0,1]$

$g(x)$ is nondecreasing

$\Rightarrow g(\lambda f(x_0) + (1-\lambda) f(x_1)) \geq g(f(\lambda x_0 + (1-\lambda) x_1))$

$g(x)$ is convex

$\Rightarrow g(\lambda f(x_0) + (1-\lambda) f(x_1)) \leq \lambda g(f(x_0)) + (1-\lambda) g(f(x_1))$

$\Rightarrow g(f(\lambda x_0 + (1-\lambda) x_1)) \leq \lambda g(f(x_0)) + (1-\lambda) g(f(x_1))$

$h(\lambda x_0 + (1-\lambda) x_1) \leq \lambda h(x_0) + (1-\lambda) h(x_1)$

$\Rightarrow h(x)$ is convex

Problem 5:

a). $f(x_1, x_2)$ is sum of convex function, so $f(x_1, x_2)$ is convex

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = x_1 + 2 = 0$$

$\Rightarrow x_1 = -2$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = 2x_2 + 1 = 0$$

$\Rightarrow x_2 = -0.5 \qquad x^* = [-2, -0.5]$

b). $X^{(0)} = (0,0) \Rightarrow x_1^{(0)} = 0$, $x_2^{(0)} = 0$

$$\frac{\partial f(x_1,x_2)}{\partial x_1}\Big|_{x_1=0} = 2$$

$$\frac{\partial f(x_1,x_2)}{\partial x_2}\Big|_{x_2=0} = 1$$

$$\begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \end{pmatrix} - \tau \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 1 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\frac{\partial f(x_1,x_2)}{\partial x_1}\Big|_{x_1=-2} = 0$$

$$\frac{\partial f(x_1,x_2)}{\partial x_2}\Big|_{x_2=-1} = -1$$

$$\begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \end{pmatrix} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} - \tau \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} - 1 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

c).

$$\frac{\partial f(x_1,x_2)}{\partial x_1}\Big|_{x_1=-2} = 0$$

$$\frac{\partial f(x_1,x_2)}{\partial x_2}\Big|_{x_2=0} = 1$$

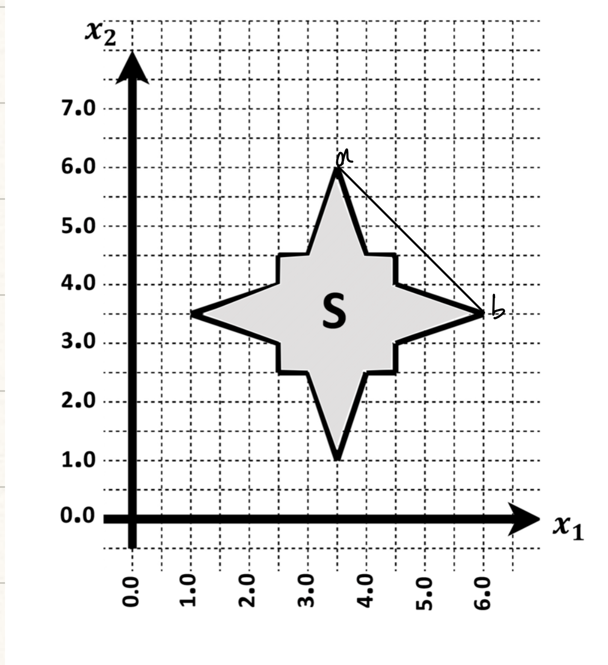
$$\begin{pmatrix} x_1^{(3)} \\ x_2^{(3)} \end{pmatrix} = \begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \end{pmatrix} - \tau \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} - 1 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix}$$

so it can't arrive the global minimum $\begin{pmatrix} -2 \\ -0.5 \end{pmatrix}$.

we can't decrease the learning rate τ to solve this problem.

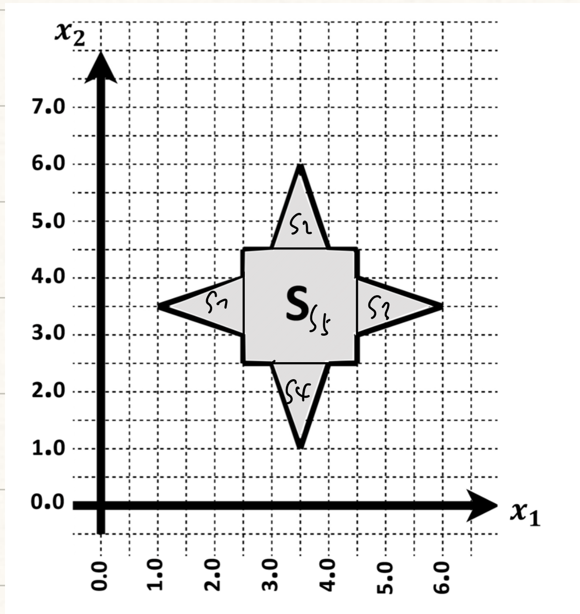
Problem 7:

a) .



there is line ab that is not in the region S. so this region is not convex

b).



seperate the region S to $S_1 \sim S_5$

every sub region S_i is convex

global minimum of f over S is

$opt = \min_{S_i} \{ CnvOpt(f, S_i) \}$