Programming assignment 4: SVM

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

from sklearn.datasets import make_blobs

from cvxopt import matrix, solvers
```

Your task

In this sheet we will implement a simple binary SVM classifier. Your task is to complete the functions where required. You are only allowed to use built-in Python functions, as well as any `numpy` functions. No other libraries / imports are allowed.

To solve optimization tasks we will use **`CVXOPT`** http://cvxopt.org/ - a Python library for convex optimization. If you use `Anaconda`, you can install it using

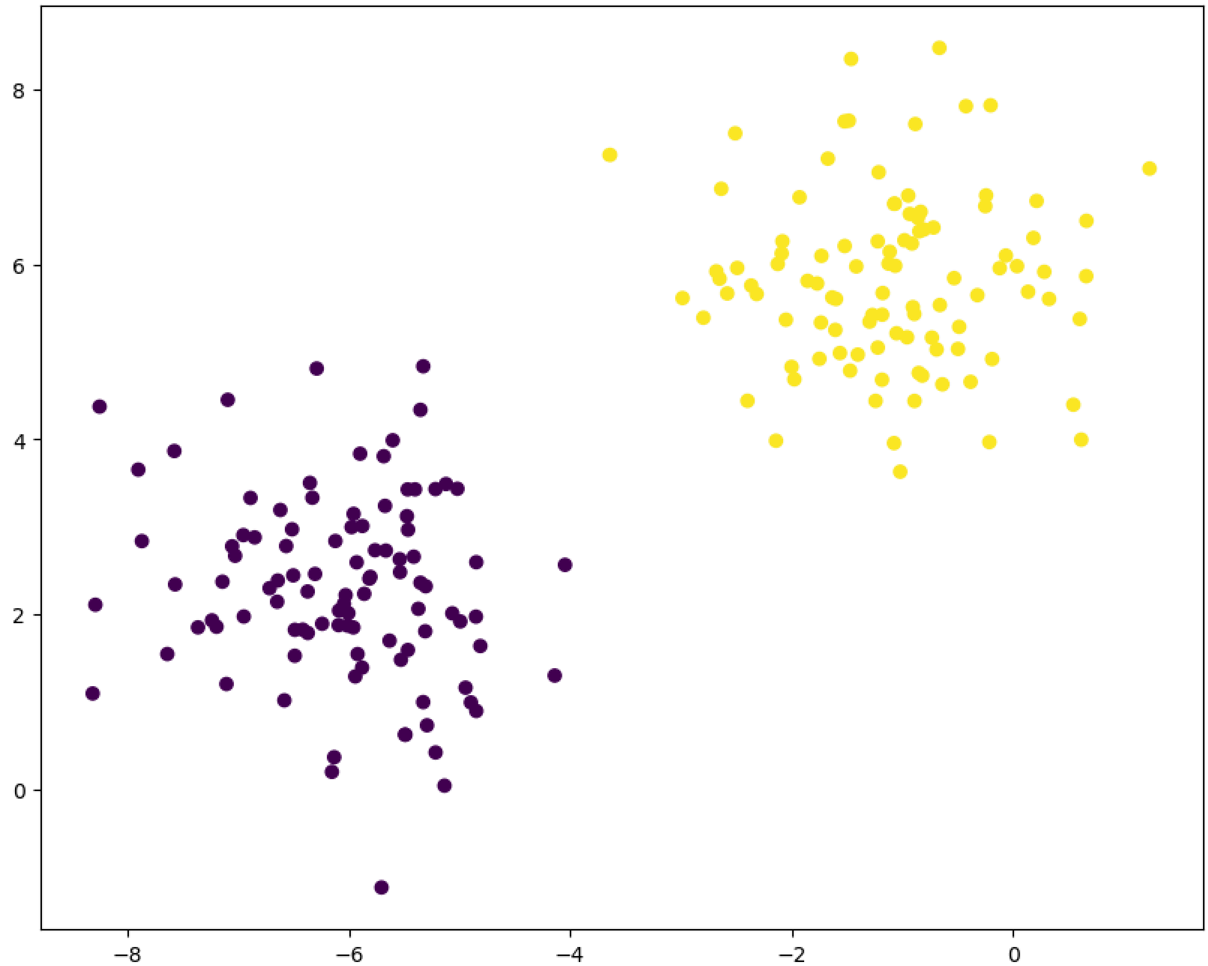
```
conda install -c conda-forge cvxopt
```

Generate and visualize the data

In [3]:
```
N = 200  # number of samples
D = 2  # number of dimensions
C = 2  # number of classes
seed = 1234  # for reproducible experiments

alpha_tol = 1e-4 # threshold for choosing support vectors

X, y = make_blobs(n_samples=N, n_features=D, centers=C, random_state=seed)
y[y == 0] = -1  # it is more convenient to have {-1, 1} as class labels (instead of
y = y.astype(float)
plt.figure(figsize=[10, 8])
plt.scatter(X[:, 0], X[:, 1], c=y)
plt.show()
```



Task 1: Solving the SVM dual problem

Remember, that the SVM dual problem can be formulated as a Quadratic programming (QP) problem. We will solve it using a QP solver from the `CVXOPT` library.

We use the following form of a QP problem:

$$\begin{aligned} \text{minimize}_{\mathbf{x}} \quad & \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \leq \mathbf{h} \text{ and } \mathbf{A}\mathbf{x} = \mathbf{b}\,. \end{aligned}$$

Your task is to formulate the SVM dual problems as a QP of this form and solve it using `CVXOPT`, i.e. specify the matrices $\mathbf{P}, \mathbf{G}, \mathbf{A}$ and vectors $\mathbf{q}, \mathbf{h}, \mathbf{b}$.

In [5]:
```
def solve_dual_svm(X, y):
    """Solve the dual formulation of the SVM problem.

    Parameters
    ----------
    X : array, shape [N, D]
        Input features.
    y : array, shape [N]
        Binary class labels (in {-1, 1} format).
```

```python
    Returns
    -------
    alphas : array, shape [N]
        Solution of the dual problem.
    """
    ### TODO: Your code below ###
    # These variables have to be of type cvxopt.matrix
    # Compute the kernel matrix
    N, D = X.shape
    P = matrix(np.einsum("i,j,ik,jk->ij", y, y, X, X))
    q = matrix(-np.ones([N, 1]))
    G = matrix(-np.eye(N))
    h = matrix(np.zeros(N))
    A = matrix(y.reshape(1, -1))
    b = matrix(np.zeros(1))
    solvers.options['show_progress'] = False
    solution = solvers.qp(P, q, G, h, A, b)
    alphas = np.array(solution['x'])
    return alphas.reshape(-1)
```

Task 2: Recovering the weights and the bias

In [7]:
```python
def compute_weights_and_bias(alpha, X, y):
    """Recover the weights w and the bias b using the dual solution alpha.

    Parameters
    ----------
    alpha : array, shape [N]
        Solution of the dual problem.
    X : array, shape [N, D]
        Input features.
    y : array, shape [N]
        Binary class labels (in {-1, 1} format).

    Returns
    -------
    w : array, shape [D]
        Weight vector.
    b : float
        Bias term.
    """
    ### TODO: Your code below ###
    # Compute weight vector w
    w = np.dot(X.T, alpha * y)

    # Identify support vectors
    support_vector_indices = alpha > alpha_tol
    support_vectors = X[support_vector_indices]
    support_labels = y[support_vector_indices]
    support_alphas = alpha[support_vector_indices]

    # Compute bias using support vectors
    biases = support_labels - np.dot(support_vectors, w)
```

```
    b = np.sum(support_alphas * biases) / np.sum(support_alphas)  # Weighted mean f
    return w, b
```

Visualize the result (nothing to do here)

In [8]:
```
def plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b):
    """Plot the data as a scatter plot together with the separating hyperplane.

    Parameters
    ----------
    X : array, shape [N, D]
        Input features.
    y : array, shape [N]
        Binary class labels (in {-1, 1} format).
    alpha : array, shape [N]
        Solution of the dual problem.
    w : array, shape [D]
        Weight vector.
    b : float
        Bias term.
    """
    plt.figure(figsize=[10, 8])
    # Plot the hyperplane
    slope = -w[0] / w[1]
    intercept = -b / w[1]
    x = np.linspace(X[:, 0].min(), X[:, 0].max())
    plt.plot(x, x * slope + intercept, 'k-', label='decision boundary')
    plt.plot(x, x * slope + intercept - 1/w[1], 'k--')
    plt.plot(x, x * slope + intercept + 1/w[1], 'k--')
    # Plot all the datapoints
    plt.scatter(X[:, 0], X[:, 1], c=y)
    # Mark the support vectors
    support_vecs = (alpha > alpha_tol)
    plt.scatter(X[support_vecs, 0], X[support_vecs, 1], c=y[support_vecs], s=250, m
    plt.xlabel('$x_1$')
    plt.ylabel('$x_2$')
    plt.legend(loc='upper left')
```

The reference solution is

```
w = array([0.73935606 0.41780426])

b = 0.919937145
```

Indices of the support vectors are

```
[ 78 134 158]
```

In [9]:
```
alpha = solve_dual_svm(X, y)
w, b = compute_weights_and_bias(alpha, X, y)
print("w =", w)
```

```python
print("b =", b)
print("support vectors:", np.arange(len(alpha))[alpha > alpha_tol])
```

```
w = [0.73935606 0.41780426]
b = 0.9199371344144434
support vectors: [ 78 134 158]
```

In [10]:
```python
plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b)
plt.show()
```