Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
A		2419630

Based on Dynamic Models: The Interaction between Sea Lamprey Sex Ratios and Ecosystems

Summary

The sex ratios of species are shaped by environmental dynamics and also have significant influences on ecological equilibrium. The purpose of this paper is to establish a **Dynamics System Model** to simulate the interactions among lamprey populations, their food resources, predators, and environmental factors. Meanwhile, use **Agent-Based Model** to assess the impact of lampreys' sex ratio on ecosystem stability.

For the first problem, the sex ratio of lampreys alters depending on resource availability. This study clarifies the use of the food resources of sea lampreys to measure resource availability, namely the number of their hosts. Initially, a **Sex Determination Model** is used to establish the relation between resource availability and sex ratio. Subsequently, incorporating the sex ratio as a variable affecting the reproduction rate, a **Lotka-Volterra model** is constructed to study the relation between lampreys and their hosts. Finally, it is concluded that a decrease in the sex ratio of sea lampreys indicates a decrease in resource availability, which leads to an increase in future resource abundance, and vice versa.

For the second problem, analyzing from two aspects, one is the reproduction of the species, and the other is the utilization rate of resources. We build the **reproductive success model** and **resource utilization efficiency model** to assess the impact of the sex ratio. We can conclude that when the sex ratio is 1, both the reproductive success rate and resource utilization efficiency are at their highest, and any deviation from this balanced sex ratio will lead to a decline in both. The specific advantages and disadvantages are in the problem 2 conclusion.

For the third problem, this ecosystem stability is defined to be affected by three principles: resource acquisition, reproduction rule and sex ratio adjustment. We build **Agent-Based Model** to simulate it. By running the model with a set of specific parameters, the male sex ratio first rises from 50% then becomes relatively mild with a little decreasing trend around 72.5%. Consisting with the distribution of different genders, it indicates that the ecosystem stability will get worse as the male sex ratio increases.

For the fourth problem, this issue is analyzed from two perspectives: available resources and the state of being parasitized by parasites. For the first aspect, we established a **resource acquisition** model to observe how the amount of available resources changes with the sex ratio. For the other aspect, we developed a model for **infection chance** to explore the specific probabilities of being infected at different ratios. In conclusion, we find that when the sex ratio is around 66%, the maximum of resource acquisition and are least likely to be parasitized by parasites.

Keywords: Dynamic System Analysis, Lotka-Volterra Model, Sex Determination Model, Monte Carlo Simulation, Agent-Based Model

Contents

1 Introduction

Sea lamprey originally inhabited the Atlantic of Europe. In the 20th century, sea lamprey invaded the Great Lakes (see Fig. 1), posing a threat to the existed life. The sea lamprey's strong predatory nature and unique sex ratio can impact the ecosystem in the Great Lakes [1]. In this article, we explore the effect of the sex ratio on the sea lamprey itself and the whole ecosystem.

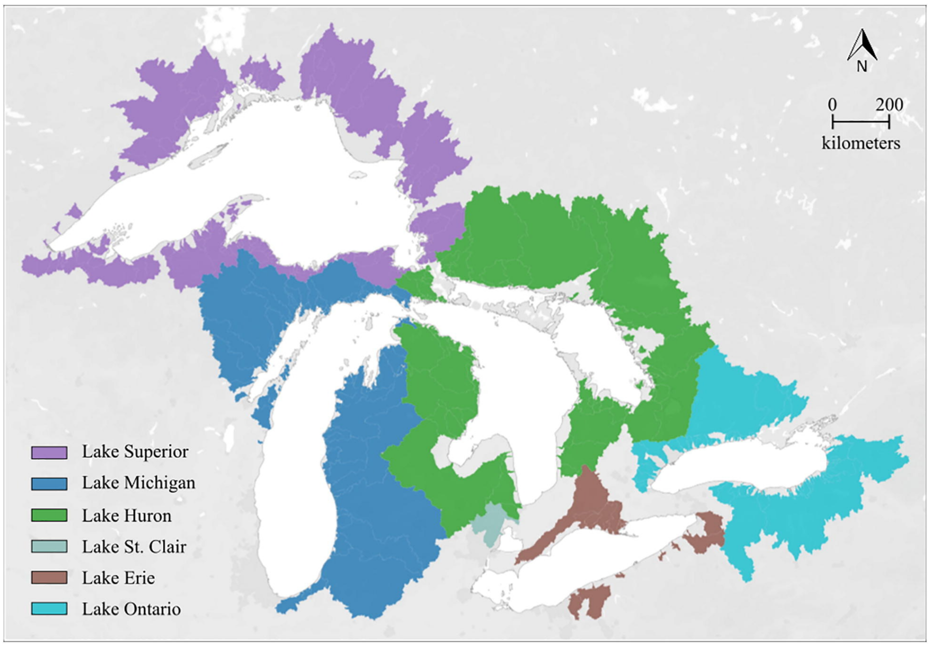

Fig. 1. Distribution in Great Lakes [1]

1.1 Problem Background

The focus of this topic is the variation in the sex ratio of animals, specifically the sex ratio changes in the lamprey. Lampreys are an ancient type of jawless fish that inhabit lakes or seas and ascend rivers to spawn. Actually their sex ratio changes in the period of transforming from larva to adult (see Fig. 2). That is because the sex ratio can vary with external environmental conditions, with the availability of food affecting the growth rate of their larvae, which in turn determines whether they become male or female. Moreover, lampreys are not just predators but also prey and as critical contributors to nutrient cycling upon their death post-spawning. The interplay between lampreys and their environment achieves a delicate balance essential for biodiversity and ecosystem sustainability.

Given that lampreys exhibit adaptive sex ratio variations and their populations have a significant impact on ecosystems, it is essential to develop a system dynamics model to further investigate and understand the interactions within ecosystems.

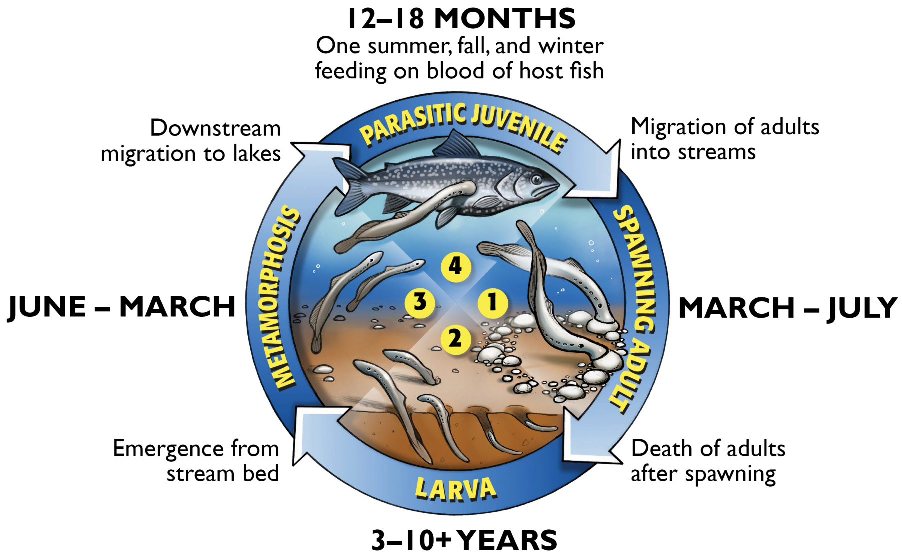

Fig. 2. Life Cycle of Lamprey [2]

1.2 Restatement of the problems

Considering the background information and constraints identified in the problem statement, we need to address the following problems:

- **Problem 1:** Take into account the dependency of sex ratio on resource availability and its effects on the ecosystem; build a dynamic model to depict the ecosystem's interrelations.
- **Problem 2:** Investigate the advantages and disadvantages of sex ratio variations in sea lamprey populations on their reproductive success and the changes in their environmental adaptability.
- **Problem 3:** Establish a system dynamics model to simulate the impact of sex ratio variations on ecosystem stability, encompassing resource competition, environmental conditions, and opportunities for reproduction.
- **Problem 4:** Consider how changes in sex ratio affect other species within the ecosystem, including predation and parasitic relations, and the resource acquisition in the ecosystem. Then analyze interactive advantages.

1.3 Our work

The topic requires us to create a model to measure the relation between sea lampreys' sex ratios and their dependence on the environment. Additionally, assesses the pros and cons of a species' capability to alter its gender ratio in response to variations in resource availability. Therefore, this model can help us to gain deeper insights into the interactions generated within the ecosystem as Fig. 3 shows.

- Gather sufficient data to understand the relevant behaviors and parasitic attributes of lampreys. Identify the variable of resource quantity and measure it primarily using the population of lamprey hosts to contextualize their impact on the ecosystem.
- Build an ecosystem model that encompasses lampreys and other key species. This model combines gender ratio modeling and resource dynamics modeling to simulate the dynamic changes in lamprey populations under different gender ratios and their impact on other components of the ecosystem.
- Establish Reproductive Success Model and Resource Utilization Efficiency Model. Meanwhile, using the Species Abundance Model combines the two above models. Then evaluate the advantages and disadvantages of the sex ratio.
- Establish an agent-based model through three criteria to simulation and set the limitations with real-world values. Then run it to study the influence of changing sex ratio on the ecosystem stability.
- Select the sex ratio as the characteristic of individuals randomly. Then develop a resource acquisition and infection chance. Finally, analyze the advantages given by different sex ratios in the ecosystem.

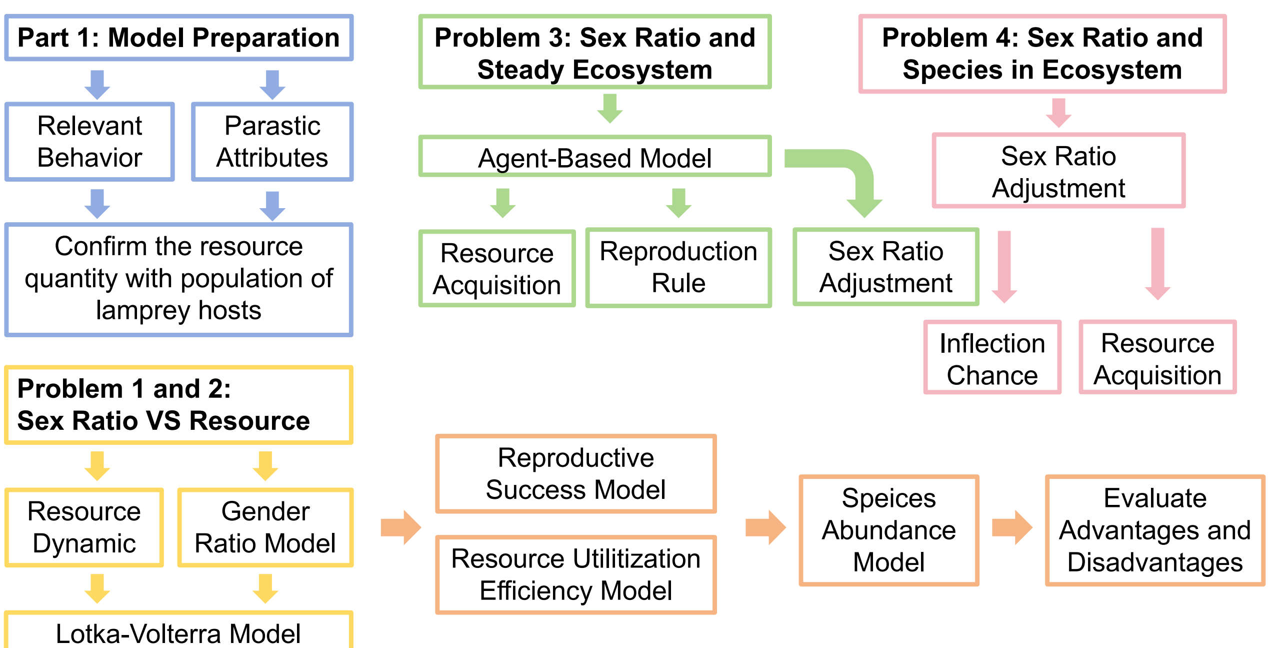

Fig. 3. Flowchart of Modeling

2 Assumptions

- **Assumption 1:** Assuming that the dynamics of resource change are only influenced by the regeneration rate of resources and the carrying capacity of the environment, without being affected by other factors such as the seasonal variation of resources, spatial heterogeneity, and human intervention measures.
- **Assumption 2:** Assuming the sex ratio of lampreys is influenced solely by the quantity of resources, without being affected by more complex factors such as genetics, behavior, and other environmental stresses.
- **Assumption 3:** It is reasonable for the sex ratio of lampreys to approach 1:1 when the amount of resources in an ecosystem equals the resource threshold level.
- **Assumption 4:** The relation between lampreys and others in the ecosystem is determined by whether they infect each other or not.
- **Assumption 5:** The frequency of inflection status is considered as the probability of inflection. For every male ratio, the total frequency of infected and uninfected is 100. Thus, if the frequency is 80, it implies that it has 80% to be infected.

3 Symbol Statement

Table 1
Symbols and Meanings

Symbol	Meaning	Units
R	the quantity of resource	1
R_0	the threshold of the resource quantity	1
K	Resource carrying capacity	1
r_R	the regeneration rate of resources	–
N	the number of lampreys	thousand
N_f	the number of female lampreys	thousand
N_m	the number of male lampreys	thousand
A	Sex ratio (female/male)	–
r_f	Female birth rate	–
r_m	Male birth rate	–
d_f	Female mortality	–
d_m	Male mortality	–

Table 1 – continued

Symbol	Meaning	Units
k	Sensitivity of gender transition influenced by resources	–
c_f	Female resource consumption rate	–
c_m	Male resource consumption rate	–
S	Reproductive success rate	–
E	Resource utilization efficiency	–
F	Relative abundance	–
R_a	Real resource acquisition	–
R_b	Basic resource acquisition	–
P_e	Environmental parameter	–
c_p	Competitive parameter	–
N_i	Neighbours of lamprey	–
R_s	Reproduction success rate	–
R_r	Resource access rate	–
P_{rc}	Reproduction competitive parameter	–
A_N	New male ratio	–
A_B	Base male ratio	–
G	Sex adjust factor	–
I_E	Environment availability	–
I_{Th}	Base threshold	–

4 Model and Solutions

The following are four problems and their models respectively.

4.1 Problem 1

For problem 1, taking account of the dependence of sex ratios on resource availability and the dynamic nature of resources, we mainly establish a dynamics model, which is combined with a Sex Determination Model.

The following is divided into three sections: the Sex Determination Model, the Resource Dynamic Model, and the Lotka-Volterra Model.

4.1.1 Sex Determination Model

Establishing a gender ratio adjustment function by integrating factors such as lamprey growth rate and food availability to predict changes in the gender ratio.

Assuming that the probability of a new individual becoming female, denoted as P_f, is correlated with the resource quantity R, it can be defined as:

$$P_f(R) = \frac{1}{1 + e^{-k(R-R_0)}} \tag{1}$$

This formula can be used to simulate the changes in the probability of female sea lampreys' production under different food supply conditions. The functional form of this formula is a **logistic function**, commonly used to describe smooth transitions from one state to another in the presence of a threshold.

Here, when the resource level $R < R_0$, the probability of producing females is lower than 50%. As the resource level R increases and exceeds the threshold R_0, the probability of producing females rapidly increases, approaching 1.

The characteristic of this function is that R_0 is set as the level of resources where the male-to-female sex ratio is 1:1, meaning that at $R = R_0$, the probability of female individuals being produced is 50%.

4.1.2 Resource Dynamic Model

The natural growth function $g(R)$ of the resource level can be described using a logistic growth model:

$$g(R) = r_R R(1 - \frac{R}{K}) \tag{2}$$

R represents the resource quantity; r_R is the maximum resource regeneration rate; K is the carrying capacity of the environment, which is the maximum possible value of the resource quantity.

4.1.3 Lotka-Volterra Model

Utilizing the Lotka-Volterra population dynamics model to depict the interaction between lamprey populations and their host (food) numbers. Literature review reveals that parasitic sea lampreys are considered top predators (due to their lifestyle, they are rarely preyed upon by other animals apart from humans, and human survival factors [3]), therefore, this model does not account for the interaction between lamprey populations and their predators [4][5].

By incorporating the previously defined sex determination function and resource dynamics into the dynamic model, the system of differential equations can be written as follows:

- Female Sea Lamprey Population Dynamics:

$$\frac{dN_f}{dt} = r_f P_f(R) N_f - d_f N_f \tag{3}$$

- Male Sea Lamprey Population Dynamics:

$$\frac{dN_m}{dt} = r_m(1 - P_f(R))N_f - d_m N_m \tag{4}$$

- Resource Dynamics:

$$\frac{dR}{dt} = r_R R(1 - \frac{R}{K}) - c_f N_f - c_m N_m \tag{5}$$

Fig. 4 shows the relation between the number of lampreys and the resource quantity.

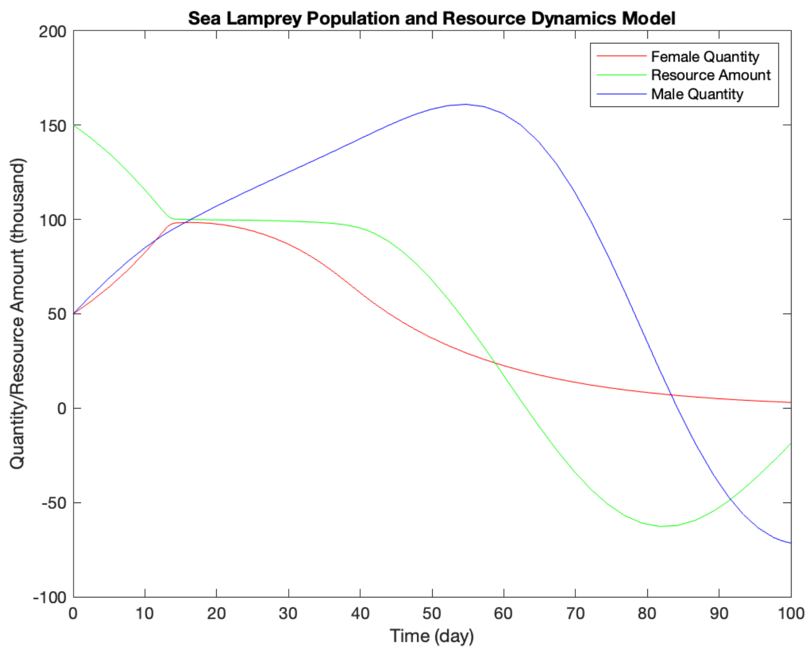

Fig. 4. Sex Ratio and Resource Quantity (Separate)

The following Fig. 5 was plotted by solving the above differential equations, successfully simulating the dynamic changes in sex ratio and resource amount over time.

Based on Fig. 5, we can conclude that a decrease in the sex ratio of sea lampreys signifies a reduction in population size, indicating a decrease in resource availability, which in turn leads to an increase in resource abundance in the future.

Conversely, an increase in the sex ratio suggests an expansion in population size, indicating a current increase in resource availability, but anticipates a decrease in resource abundance in the future.

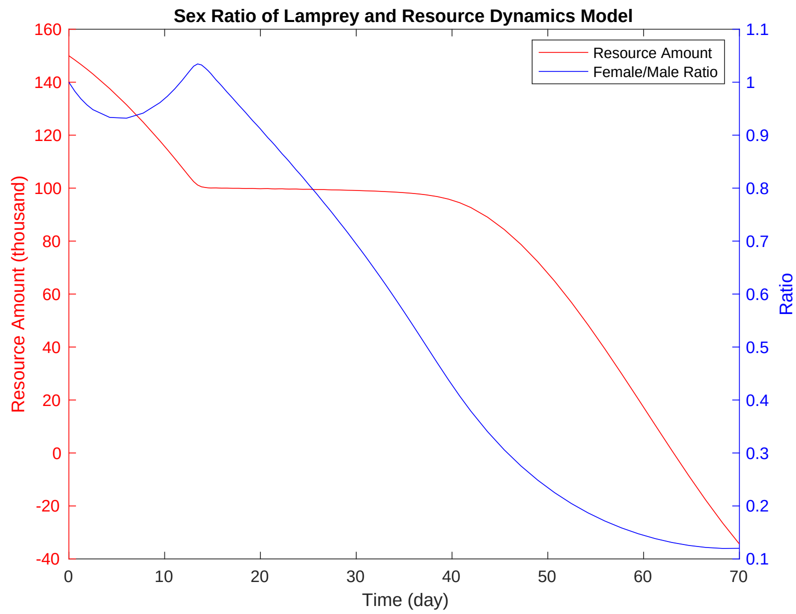

Fig. 5. Sex Ratio and Resource Quantity (Ratio)

4.2 Problem 2

For problem 2, to explore the advantages and disadvantages of lamprey populations changing their sex ratio, we need to consider what adjustments in the sex ratio can affect the species' survival and reproductive capacity. This involves analyzing the impact of changes in sex ratio on breeding success rate and resource utilization efficiency.

Therefore, we established two models following: the Reproductive Success Model and the Resource Utilization Efficiency Model.

4.2.1 Reproductive Success Model

Because the reproduction of sea lampreys requires mating, a lower number of individuals of one gender can lead to reduced mating opportunities, consequently lowering the reproductive success rate. Therefore, the gender ratio has an impact on the reproductive success rate.

Similarly, the abundance of resources affects the physical condition and health of individuals, which in turn affects their reproductive success rate.

More importantly, the availability of food has a deterministic influence on the gender of sea lampreys, further influencing the reproductive success rate (see Fig. 6).

Therefore, in this model, it is crucial to consider the complex interactions between gender ratios and resource quantity.

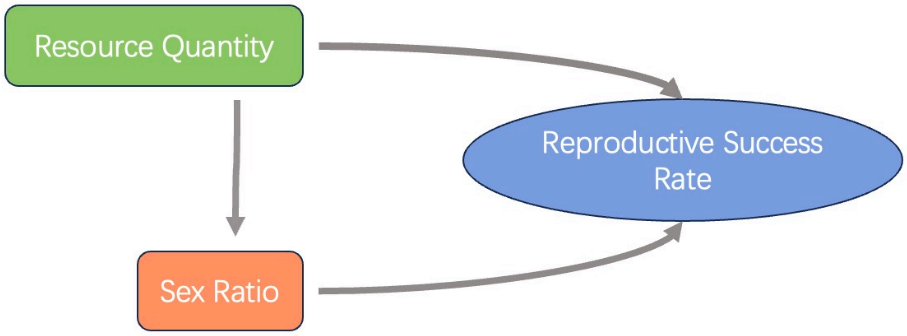

Fig. 6. Elements in Reproductive Success Model

- Assuming a steady state for the population, we can express N_f and N_m in terms of R and then find the gender ratio.

  At steady state:

$$r_f P_f(R) N_f - d_f N_f = 0 \tag{6}$$

$$r_m(1 - P_f(R))N_f - d_m N_m = 0 \tag{7}$$

- Solving the first equation for N_f given:

$$N_f = \frac{d_f N_f}{r_f P_f(R)} \tag{8}$$

- Using the second equation to express N_m in terms of N_f:

$$N_m = \frac{r_m(1 - P_f(R))N_f}{d_m} \tag{9}$$

- Hence, the sex ratio can be expressed as:

$$\frac{N_f}{N_m} = \frac{d_f d_m}{r_f r_m P_f(R)(1 - P_f(R))} \tag{10}$$

- Based on the coefficient choice and formulas established in Problem 1, $d_f = d_m = 0.05$, $r_f = r_m = 0.1$, $A = \frac{N_f}{N_m}$ and $P_f(R) = \frac{1}{1+e^{-k(R-R_0)}}$, we can derive the function:

$$R = R_0 - \ln \frac{2}{e^{(\frac{\ln A}{2} + \frac{3}{2})} - 2} \tag{11}$$

  This represents the relation between the sex ratio and resource quantity.

According to the derivations above, we can establish the reproductive success model with consideration of the sex ratio and resource quantity:

$$S(A,R) = \alpha \left(R_0 - \ln \frac{2}{e^{\left(\frac{\ln A}{2} + \frac{3}{2}\right)} - 2} \right) \frac{A}{(1+A)^2} \tag{12}$$

S represents the reproductive success rate, A represents the sex ratio of lampreys (i.e., female/male), and R represents the number of resources. This model combines the resource quantity and the proportion of females and males.

The curve in the Fig. 7 shows a pronounced peak, indicating that the reproductive success rate is highest when the sex ratio is close to 1:1. This means that any deviation from the equilibrium sex ratio in either direction will lead to a reduction in reproductive success.

This relation may be due to the increased opportunities for mating when the sex ratio is balanced, thus enhancing the chances of successful reproduction. An excess of one sex may lead to restrictions in mate selection, thereby reducing the reproductive success rate. Fig. 7 illustrates the impact of the sex ratio on the reproductive potential of populations.

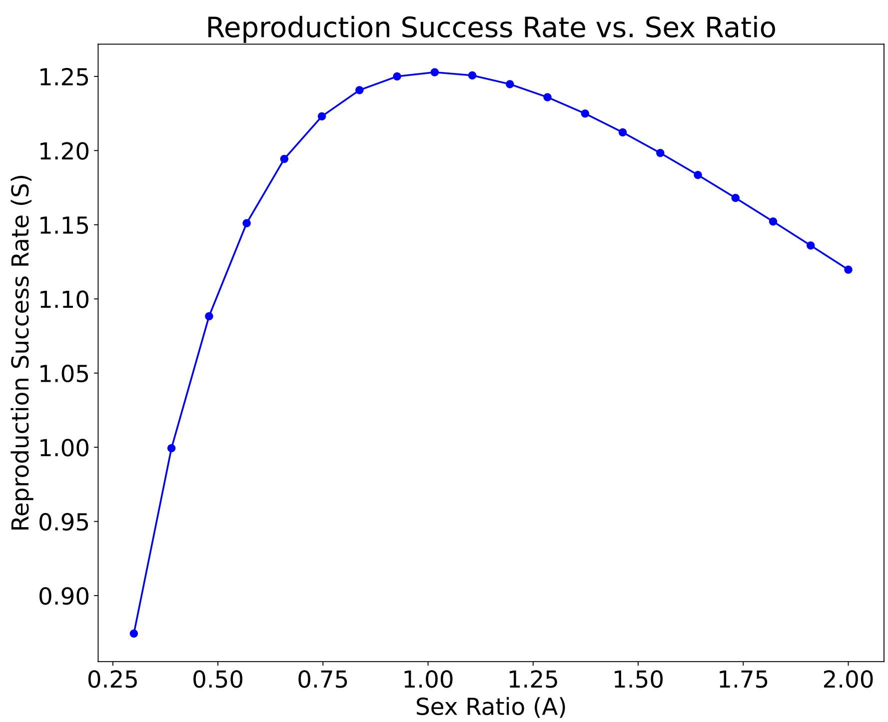

Fig. 7. Reproduction Success Rate

4.2.2 Resource Utilization Efficiency Model

The gender ratio of a species can influence the allocation and utilization of resources. The resource utilization efficiency model emphasizes the interactions between gender ratio, resource abundance, and resource utilization efficiency.

According to the previous analysis, we think that the resource utilization efficiency will reach its best when the sex ratio is balanced. In order to fit into the normal condition that the resource utilization efficiency should have maximum, we add $-\gamma(A-1)^2$ to the resource quantity R so that the function presents a trend of first increasing and then decreasing.

$$E(A) = \beta\left(R_0 - \ln\frac{2}{e^{\left(\frac{\ln A}{2}+\frac{3}{2}\right)} - 2} - \gamma(A-1)^2\right)A \tag{13}$$

E represents resource utilization efficiency, β is the efficiency coefficient, and γ is the influence coefficient of sex ratio on resource utilization efficiency. The model covers the ratio of gender and the condition of resource quantity.

From Fig. 8, we can find that the resource utilization efficiency will reach its highest when the sex ratio is in the range of $(1, 1.5)$ by sensitivity test.

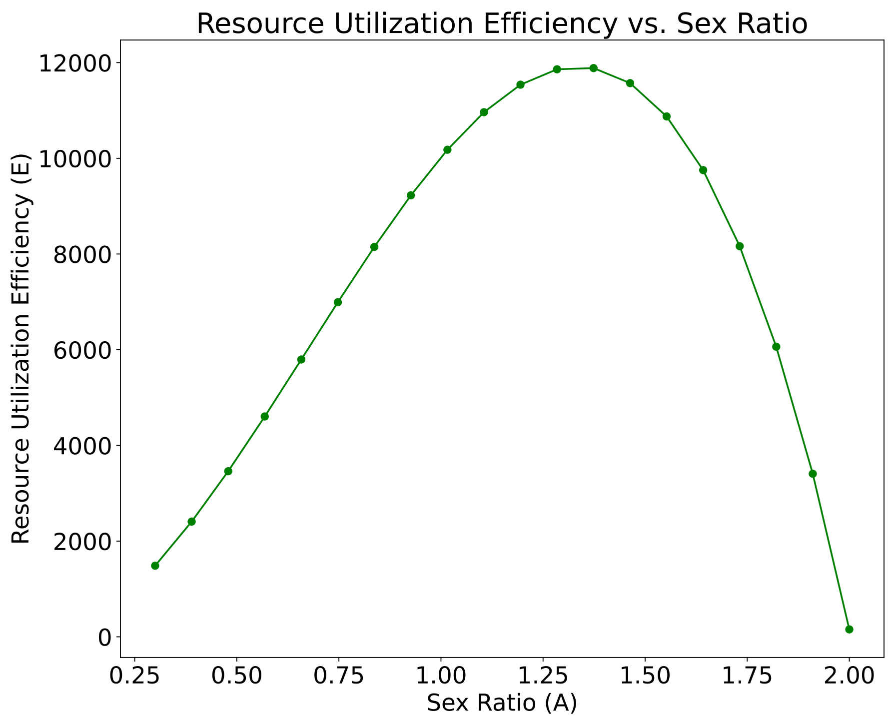

Fig. 8. Resource Utilization Efficiency

This suggests that a balanced sex ratio might be optimal for the utilization of resources. This may be because when the sex ratio is imbalanced, the opportunities for mating decrease, or more resources are required to find a mate, thus reducing the efficiency of resource utilization.

4.2.3 Species Abundance Model

To further illustrate the impact of the sea lamprey gender ratio on the ecosystem, we introduce the concept of species abundance to demonstrate that the gender ratio corresponding to the highest reproductive success rate of sea lampreys can result in higher species abundance in this ecosystem. This indicates that the ecosystem is more stable and healthy under such conditions.

Based on the information we found [6], it is known that when the sex ratio of a certain species is close to equal, the highest number of prey is captured. However, as the gender ratio becomes more skewed, the number of captured prey decreases (see Fig. 9), exhibiting a normal distribution.

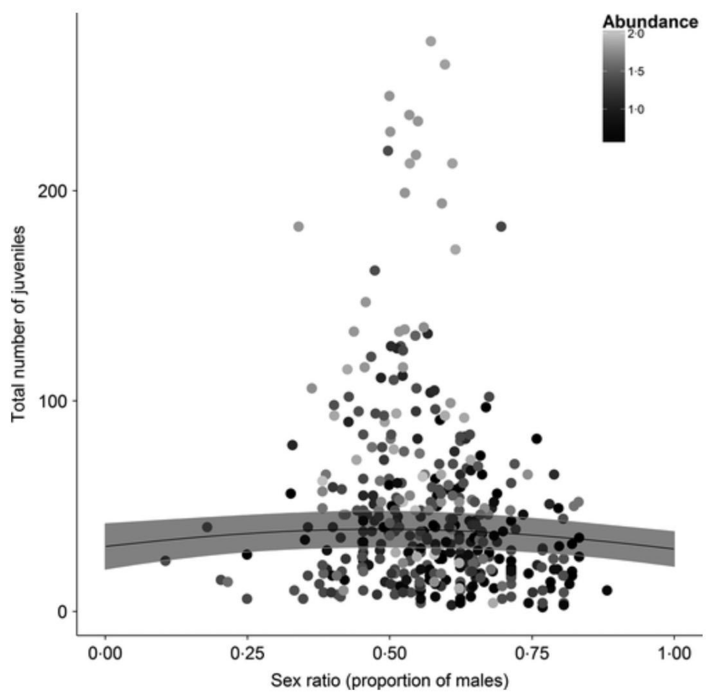

Fig. 9. Sex Ratio and Species Number [6]

From Fig. 10, it can be observed that at points where the sex ratio is imbalanced, the number of prey captured by predators is significantly reduced, and most of these points have very low population abundance. Conversely, as the sex ratio tends towards balance, the population abundance increases.

Therefore, combining Fig. 9 and Fig. 10, it can be inferred that the relation between gender ratio and abundance can also be fitted with a normal distribution approximately. From this,

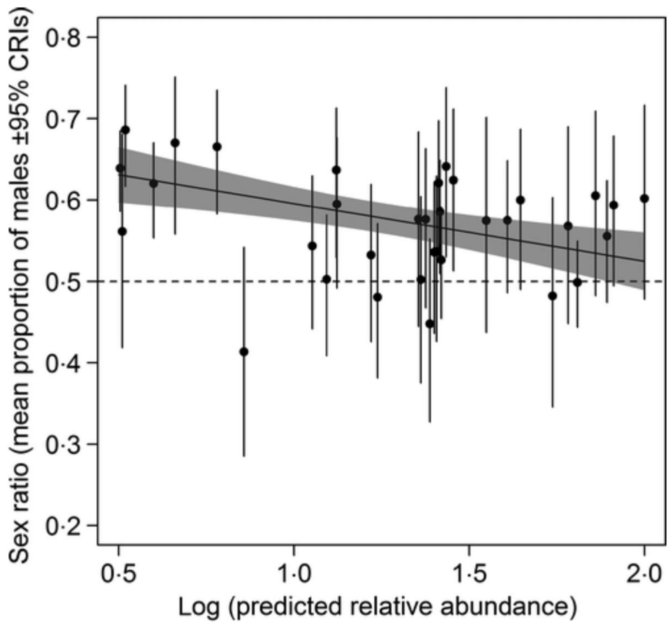

Fig. 10. Sex Ratio and Species Abundance [6]

we derive the following equation:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(A-\mu)^2}{2\sigma^2}} \tag{14}$$

σ is the standard deviation of the distribution. μ is the mean of the distribution. A is the sex ratio.

By observing Fig. 11, we find that the relative abundance is 0.5 when the sex ratio is 1:1. This means that higher abundance is not always better; the ecosystem is healthy and stable when the relative abundance is 0.5.

This reflects that a relatively balanced sex ratio is **conducive** to increasing the number of offspring and maximizing the use of resources; whereas an unbalanced sex ratio is **detrimental** to reproduction, affecting population growth and leading to the waste of resources.

4.3 Problem 3

Problem 3 aims to estimate the influence of changing sex ratio on ecosystem stability. The Agent-based model can be used to study the ecosystem and population changes over time under some specific variables. Thus, it is a good approach to simulate this circumstance.

4.3.1 Agent-Based Model

There are three main principles in this agent-based model.

- Resource Acquisition [7]:

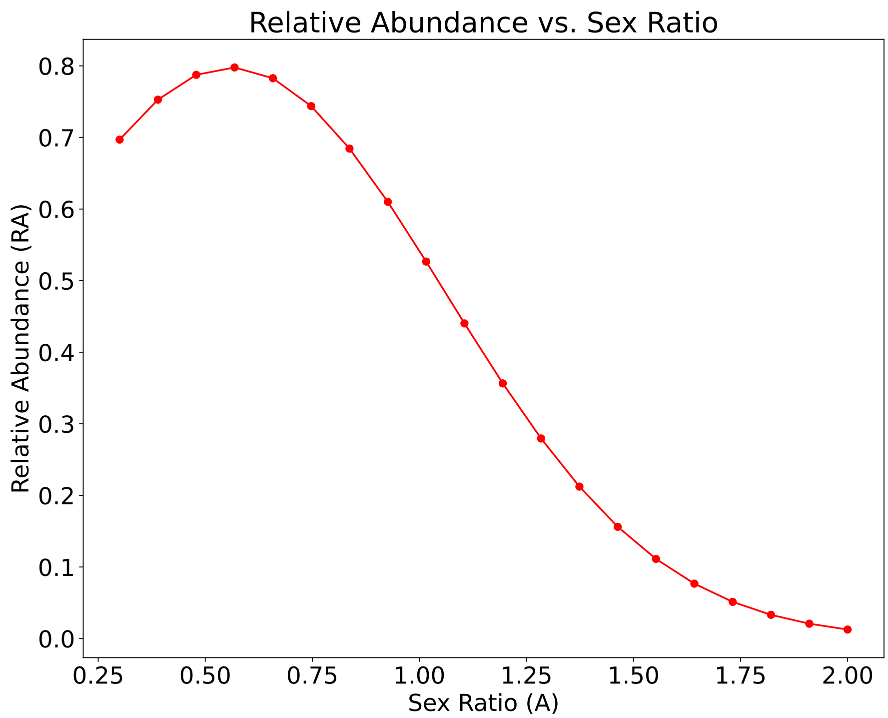

Fig. 11. Relative Abundance

The resource acquisition is represented by

$$R_a = R_b P_e - c_P \sum_{i=1}^{n} N_i \tag{15}$$

N_i comes from the neighbors of the lamprey. First randomly generate the resource need for every species in this ecosystem, then calculate the differences between lamprey and every other species. Finally, rearrange them and find the top 5 minimum species as neighbors, namely $n = 5$ in (15). The first term of (15) is the basic resource availability of lamprey and the second term represents the total resistance from the neighbors.

- Reproduction Rule:

  The reproduction rule is defined as

$$R_s = R_r P_{rc} R_a \tag{16}$$

  In (16), reproduction is defined as related to the following three main factors. R_r represents the accessible resource. P_{rc} represents all the other resistance factors during the reproduction process. R_a is resource acquisition in the previous step.

- Sex Ratio Adjustment:

The adjusted sex ratio is

$$A_N = A_B + G(I_E - I_{Th}) \tag{17}$$

The changing sex ratio is related to the environment, sex adjustment factor, and base threshold. When environment availability equals to base threshold, the base sex ratio will remain the same.

Based on these three principles, the ecosystem also needs some limitations.

- The lifespan of lamprey is usually between 5.5 to 8.5 years [8]. In this model, it is set to be 8 years.
- The lamprey begins to reproduce after 4 to 5 years [8]. In this model, it is set to be 5 years.

4.3.2 Simulation Result

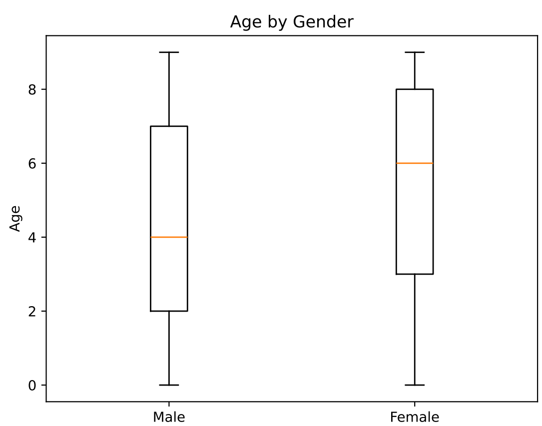

Fig. 12. Box plot of Gender

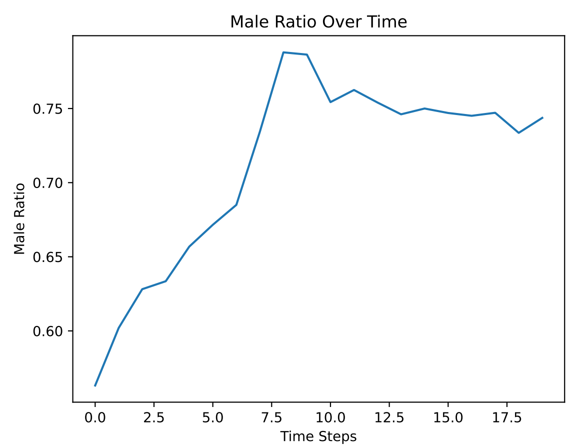

Fig. 13. Line Chart of Male Sex Ratio

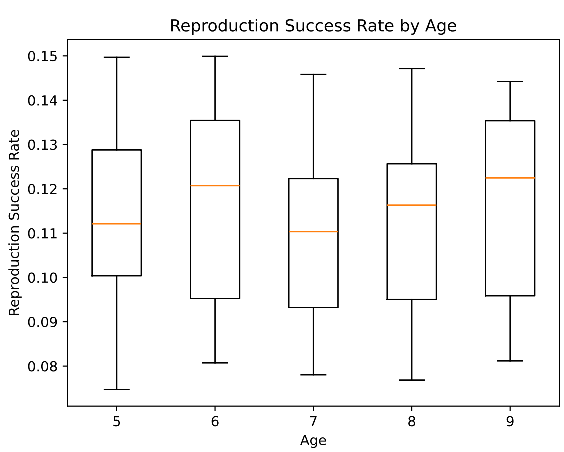

Fig. 14. Box plot of Different Age Groups

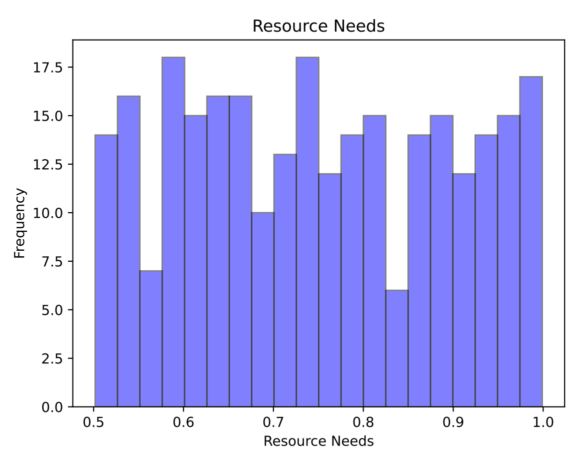

Fig. 15. Histogram of Resource Needs

Fig. 12 to Fig. 15 show the a specific simulation.

Fig. 12 shows the detailed composition of male lamprey and female lamprey. It shows that the average age of male lamprey is 4 years old and female lamprey is 6 years old. The maximum and minimum ages of males and females are nearly the same. The medium age of females is older than the males, which means there was a time when the female sex ratio was higher.

Fig. 13 shows the changing trend of the male sex ratio. The male sex ratio starts from a normal value of 50%. Due to the various influences, the ratio begins to increase, which means the available resources are decreasing, namely the the ecosystem stability becomes worse at a high speed. Then the changing of ratio becomes mild around 72.5%, which means the balance of the ecosystem stops declining fast, but not promising.

Fig. 14 shows the reproduction successful rate of different age groups. The average rate of each age group is quite equal and the middle half of them also distribute similarly. However, all the rates are quite low, which means the lamprey has a possibility of distinction.

Fig. 15 shows the frequency of the resource needs. Most of the number of needs appears randomly, which means there is no such extinct influence on ecosystem stability. However, the previous analysis concludes that the present environmental resources are not adequate enough. Low resource reserves with a continuous high resource need may lead to an ecosystem collapse.

According to this simulation, the male ratio is high, namely the environmental resources are not abundant. Thus this ecosystem is relatively fragile and may break down when facing some numerous external changes.

4.4 Problem 4

In problem 4, the purpose is to discuss the advantages for the ecosystem when lampreys have different gender ratios. First, we need to adjust different sex ratios. Then, the resource acquisition and the inflection chance are considered as the standard to judge the advantages.

4.4.1 Sex Ratio Adjustment

As we want to explore when the sex ratio is different, the influence on the ecosystem, the sex ratio should be adjusted to make sure that it is fixed with the target one.

- Target sex ratio: Generate sex ratio randomly.
- Calculate current sex ratio: The number of male lampreys.
- Calculate the number of adjustments.
- Choose the changeable number: If the number of males $<$ the number of females, randomly select some from female lampreys, the number should not exceed the smaller

number of male individuals needed to be increased and the current number of female individuals. If the number of males > the number of females, the process is just the opposite.

- Adjust the gender to fit with the target sex ratio.

4.4.2 Resource Acquisition

To quantify "the advantage to the ecosystem", we first select the actual resources acquired as the criterion for judgment [7].

- Initialize resource_adjust as the adjustment factor based on the gender of the lamprey.
- Set different adjustment factors.
- Express the competition effect:

$$\text{competition\_effect} = \text{competition\_factor} \times sum(\text{neighbor\_resources}) \tag{18}$$

  In the (18), neighbor_resources is the total resources needed by the species neighbored lampreys.

- Calculate the theoretical_resource:

$$\begin{aligned}\text{theoretical\_resource} =& \text{resource\_need} \times \text{environment\_availability} \\ & \times \text{resource\_adjust}\end{aligned} \tag{19}$$

- Let the max_competition effect be 0.5 times of theoretical resource, if not, the acquired resource will be less than 0 [9].
- The new competition effect can be expressed as the minimum of max_competition effect and the competition_effect.
- Acquired_resource is like:

$$\text{acquired\_resource} = max(0, \text{theoretical\_resource} - \text{competition\_effect}) \tag{20}$$

  To calculate the maximum confirms acquired_resource positive.

To conclude the advantages to the ecosystem by resource acquisition, we plot the figure in ten different sex ratios. According to the Fig. 16, it is readily apparent that variations in the male ratio tend to concentrate resource acquisition between 0.2 and 0.5, with only a few cases reaching above 0.7, and even no condition exceeding 0.9 or less than 0.1.

Then it is necessary to match the male ratio with the distribution of resource acquisition which is displayed in Fig. 17. Surprisingly, it is different from the common sense we are familiar with. Since the male ratio of most species is 0.5, like human beings, which is regarded as the most dominant that is, the situation that can make the ecosystem most stable. However, from the Fig. 17, we can actually find that it is easier to obtain resources when the male ratio is equal to 0.67 and 0.75, while when the male ratio = 0.53, it did not reach 0.5. At the same time, by comparing these ten graphs, it can be easily found that when the male ratio = 0.52, the amount of resources obtained can be relatively large, but when the male ratio increases to 0.53, it becomes very small. Therefore, it can be inferred that minor changes in the sex ratio can sometimes have significant impacts on the ecosystem.

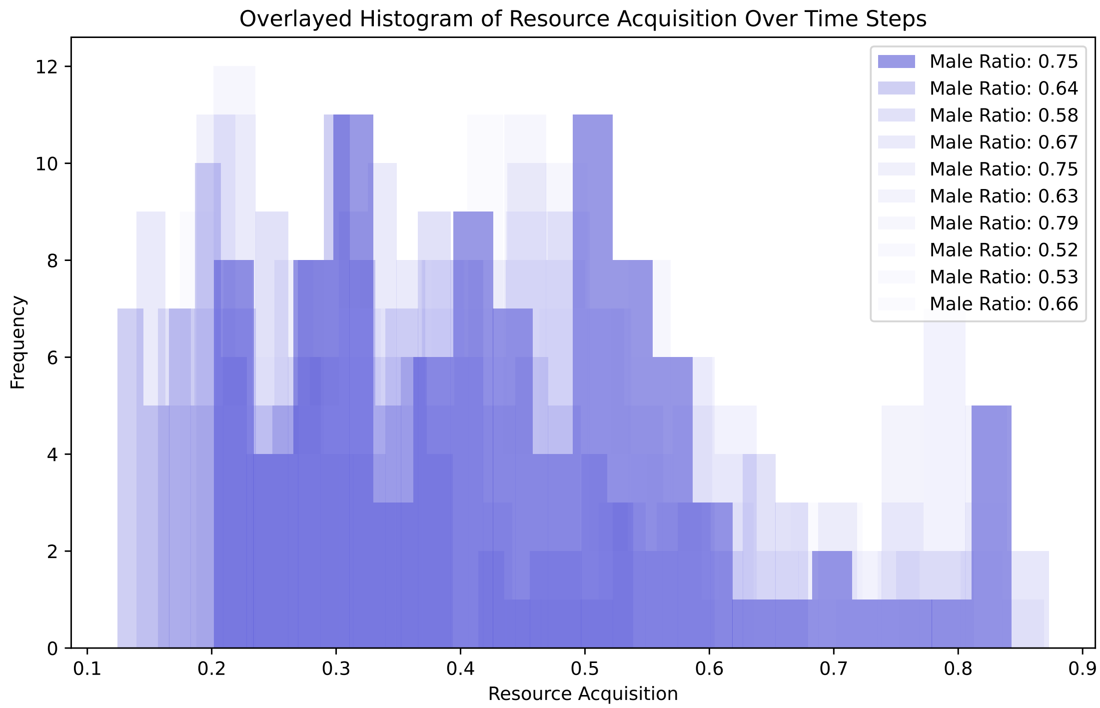

Fig. 16. Histogram of Resource Acquisition Over Time Steps

4.4.3 Infection Chance

Considering the impact of lampreys on other species in the ecosystem, such as parasites, in the following analysis, we use the success of parasite infestation as a standard of evaluation.

- Initialize the basic infection rate.
- Set the gender_adjust as the adjustment factor. If the lamprey is male, the inflection rate will increase because of the adjustment factor, for the female individuals, it is the opposite, meaning a decrease.

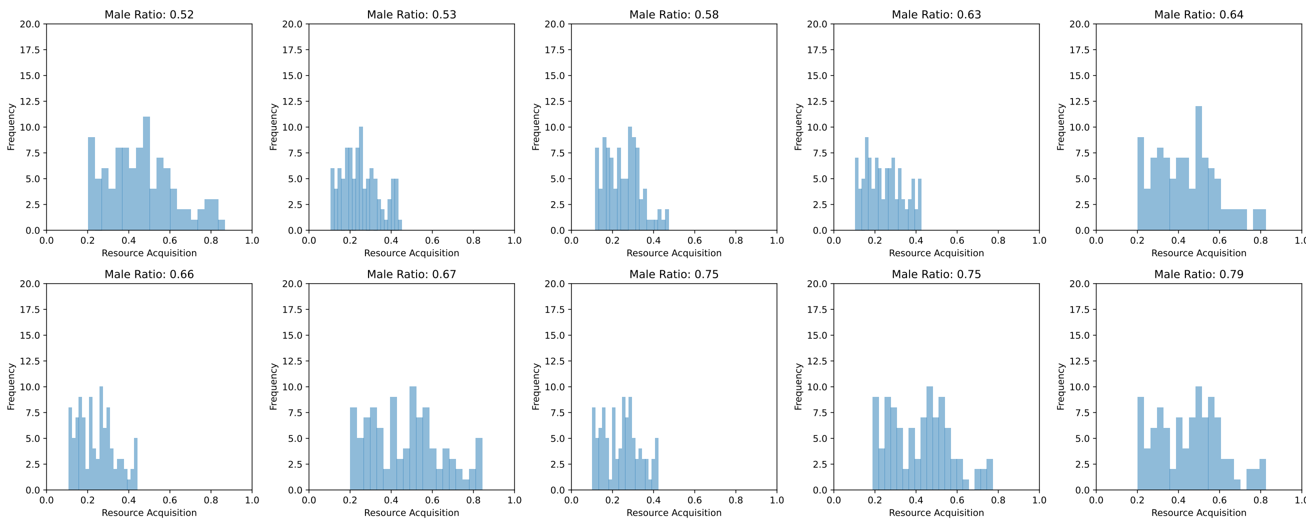

Fig. 17. Resource Distribution

- Consider require acquisition in the previous discussion,

$$\text{resource\_factor} = \frac{sum(\text{neighbor\_resources})}{\text{num\_neighbor\_resources}} \tag{21}$$

  If there is no neighbor resource, the resource_factor is defined as 1.

- Commonly, if resources obtained are higher, the infection rate will rise as well, so the following relation is established.

$$\text{new\_inflection\_rate} = \text{resource\_factor} \times \text{inflection\_rate} \tag{22}$$

- Judgement inflected or not inflected:

  With the help of the inflection rate, comparing it with a randomly generated number within the range [0,1), if this random number is less than the inflection rate, then set the individual's infection status to True (infected), otherwise, set it to False (not infected).

  From the perspective of lampreys, it does not wish to be parasitized, so we focus on the rectangles representing the uninfected conditions. From the Fig. 18, when the male ratio is 0.66, more than 80% individuals are not infested. If the parasites are harmful to the lampreys, the low inflection rate would enhance the survival rate of the lampreys. Similarly, compared with a male ratio close to 0.5, it is easy to find that the probability of being uninfected differs by more than 10% between those two.

  Conversely, parasites or other species in the ecosystem, prefer a high proportion of infection status. There are cases when the male ratio is 0.58 and 0.63, where the infection rate of lampreys can reach up to 35%. Meanwhile, it is advantageous for the survival of other organisms. Therefore, we can conclude: that when a minor change occurs in the sex ratio, the balance of the ecosystem changes a lot. For a specific species, its situation can instantly shift from an advantage to a disadvantage. For example, as the sex ratio of lampreys changes from 0.66 to 0.63, the inflection rate directly increases by 20%.

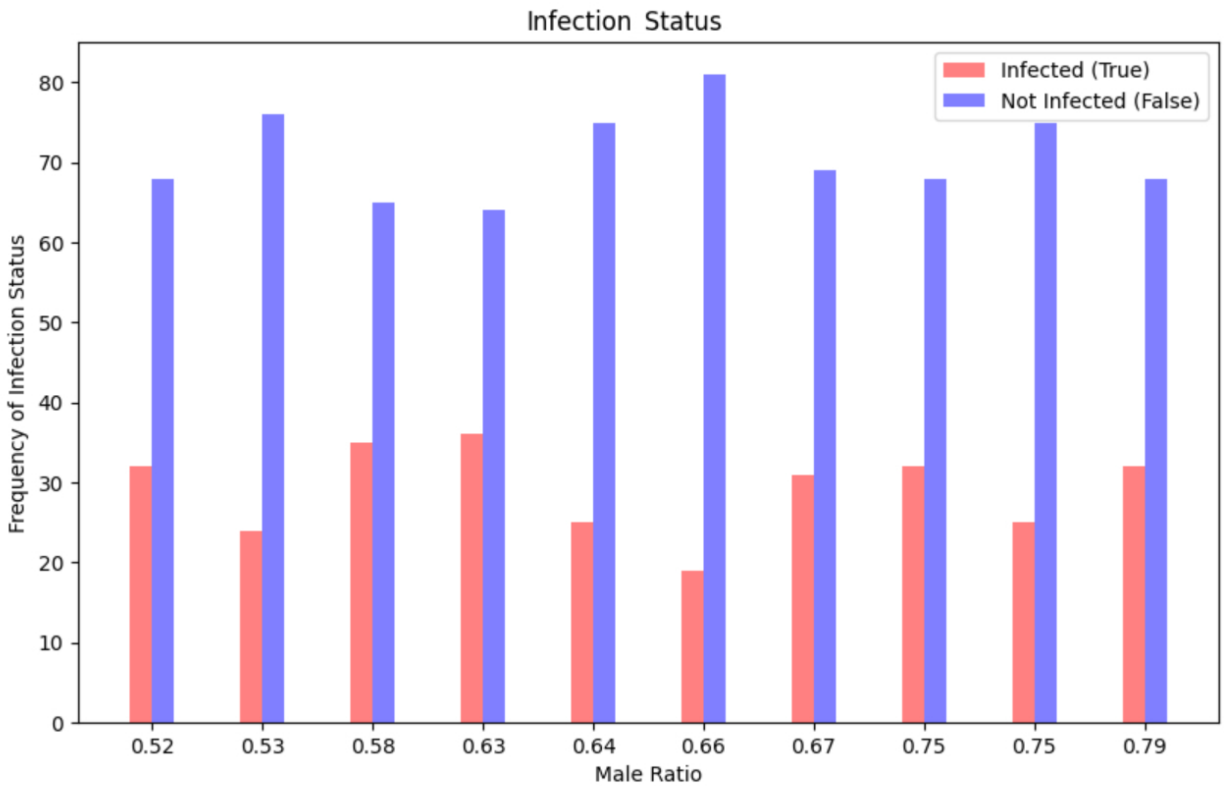

Fig. 18. Infection Status

Combining those two factors, we can find when the male ratio is 0.67, it is the optimal choice considering the resource acquisition. When the male ratio equals 0.66, the probability that lamprey is not inflected reaches the most frequency. So, it can be approximated that when the male ratio is 66%, we can achieve both obtaining more resources and reducing the risk of being parasitized by parasites.

5 Model Evaluation

To ensure an objective estimation of the model, as well as its reliability and scope for further improvement, the following discussion will consider both its strengths and weaknesses.

5.1 Strengths

- Consider every problem in many aspects. We considered not only the relation between the sex ratio and lampreys in isolation but also its connection with the entire ecosystem like it can be a predator or be parasitized as hosts.

- We visualized the assessment of advantages and disadvantages by using quantifiable characteristics such as reproductive success rate, resource utilization efficiency, resource acquisition and inflection chance for judgment and evaluation. This method provides a more persuasive and reasonable evaluation.
- The graphs in our conclusion for each problem corroborate each other. For instance, the graph showing relation Fig. 5 from our first question can be validated within the function of R derived in problem 2.

5.2 Weaknesses

- Some parameters and the functional relations are obtained empirically, lacking theoretical support.
- Compared to truth, the model is too simple to demonstrate real-world cases, while we choose the gender randomly and ignore some other factor in the environment.

6 Further Discussion

To solve the weaknesses of the model, we raise some methods to improve the model. Additionally, the management of sea lamprey in the Great Lakes and measures are shown.

6.1 Model Improvement

- Search more articles and data to support the assumption of parameters.
- In those equations, multiple conditions must be considered simultaneously, adding the combination of several equations to analyze the influence.

6.2 Managements and Measures

We have explored a lot about sea lamprey its reproduction success rate and its advantages in the ecosystem. However, in Great Lakes, sea lampreys have been considered an invasive species. Therefore, most ecologists are committed to addressing the issue of "the over-proliferation of sea lampreys in the Great Lakes and the significant threat to trout population in the original ecosystem". For example, the scientists use attractants and repellants to trap sea lampreys [1]. Additionally, some chemical substances like TFM mentioned is also practical in the management of sea lampreys [10].

7 Conclusions

In this study, we employed a dynamic model to explore the intricate relation between the sex ratio variations in lampreys and their impacts on ecosystem dynamics. The model integrated key ecological processes, including predation, competition, and resource availability, to elucidate the consequences of sex ratio changes on both lamprey populations and their broader ecological context. Through our simulations, we demonstrated how shifts in the sex ratio could significantly alter population structures, influence species interactions, and consequently affect the stability and resilience of ecosystems.

Our findings underscore the critical role of sex ratio management in the conservation of lamprey species and the maintenance of ecological balance. By highlighting the sensitivity of ecosystem dynamics to sex ratio variations, this research provides valuable insights into the complexities of ecological interactions and the potential for cascading effects stemming from alterations in fundamental biological characteristics.

Furthermore, the implications of this study extend beyond the academic interest, offering practical guidance for environmental management and conservation strategies. It suggests that careful consideration of sex ratios in population management can serve as a lever to mitigate adverse ecological impacts, promote biodiversity, and enhance ecosystem services. This research thereby contributes to the broader endeavor of ecological stewardship, advocating for informed, model-based approaches in environmental policy and management to safeguard our natural heritage for future generations.

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