Ordinary deformation at p

- K: finite extension of $\mathbb{Q}_p$
  $\chi: G_F \longrightarrow \mathbb{Z}_p^*$: cyclotomic character
  $K^{ur} := \bar{K}^{I_K}$, $\Gamma_K = Gal(K^{ur}/K)$
  M: discrete (possibly infinite) Γ_K-module over $\mathbb{Z}_p$ on which p is nilpotent

For any finite sub-rep $M' \subset M$, consider ~~the twist~~ $M'(1) = M \otimes_{\mathbb{Z}_p} \mathbb{Z}_p(1)$

$$\left[\begin{array}{l} \mathbb{Z}_p(1) = \varprojlim_k \mu_{p^k} \\ \text{along } \mu_{p^{k+1}} \to \mu_{p^k},\ a \mapsto a^p \end{array} \right]$$

$\mathbb{Z}_p(1)$ arises from a p-divisible group (cf. Definition 4.6.1)
$+$ M' is unramified $\xrightarrow{\text{Coro 3.9.14}}$ $M'(1)$ arises from a finite flat group scheme $/\mathcal{O}_K$

- $H^1_f(G_K, M'(1))$ (Definition and so on)

(1) Suppose M' is a finite Γ_K-module

Inflation – Restriction exact sequence

$$\rightsquigarrow 0 \longrightarrow \boxed{H^1(\Gamma_K, M'(1)^{I_K})} \xrightarrow{\text{Inf}} H^1(G_K, M'(1)) \xrightarrow{\text{Res}} H^1(I_K, M'(1))^{\Gamma_K} \longrightarrow \boxed{H^2(\Gamma_K, M'(1)^{I_K})} \to \cdots$$

$$M'(1)^{I_K} = 0 \Rightarrow H^1(G_K, M'(1)) \xrightarrow{\cong} H^1(I_K, M'(1))^{\Gamma_K}$$

$\Updownarrow$

($\chi \pmod p$ is non-trivial on I_K)

$\Uparrow \longrightarrow$ Hilbert 90

M' is free $\mathbb{Z}_p$-module

Suppose p^n annihilates M'. Then

$$H^1(I_K, M'(1)) \cong H^1(I_K, \mu_{p^n}) \otimes_{\mathbb{Z}} M' \cong \varinjlim_{L/K^{ur}} H^1(G_L, \mu_{p^n}) \otimes_{\mathbb{Z}} M$$