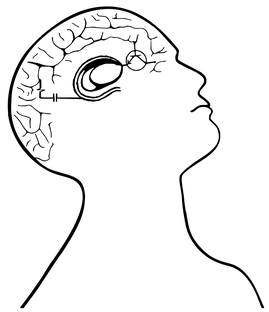

Hierarchical Bayesian estimation of the drift diffusion model: quantitative comparison with other estimation methods

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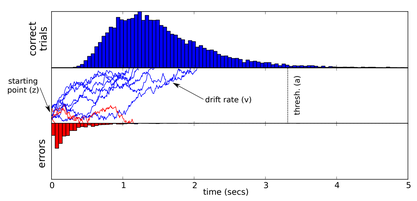
Highlights

- Drift-diffusion model widely used to study decision making.
- Takes complete correct and error RT distribution into account.
- Common estimation method is maximum likelihood or chi-square using quantiles (Ratcliff).
- We show by simulation:
  - With little trials, ML does poor, quantiles completely fail.
  - Inclusion of inter-trial variability exacerbates this.
- Hierarchical Bayesian estimation solves these problems.
- New open-source toolbox called **HDDM**.

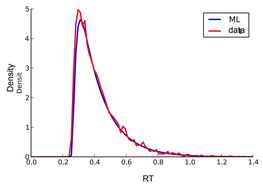
Drift-diffusion model



- Noisy evidence accumulation recovers RT distribution.
- Matches neuroscience findings (Yang, Shadlen 2007).
- Successfully used to link psychological processing to neuronal computation (e.g. Forstmann et al 2009; Cavanagh et al 2011)

Maximum-Likelihood estimation

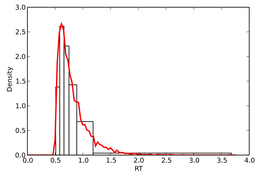
- Maximize log-likelihood via optimization.
- Can only fit single subject.
- No confidence intervals (bootstrapping required).
- Fast.



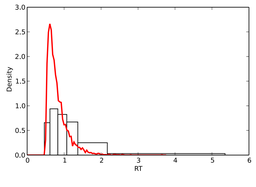
Quantile based estimation via Chi²

- Minimize chi² of theoretical and empirical quantiles via optimization.
- Can fit either subject or group parameters (i.e. subjects share no similarity, or are equivent).
- Group parameters are fit by quantile averaging.
- No confidence intervals (bootstrapping required).
- Robust to outliers.
- Fast.

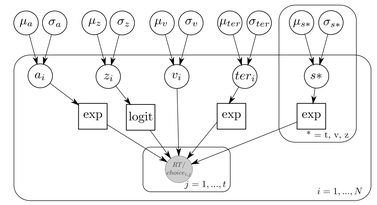
Good fit -- low Chi²



Bad fit -- high Chi²



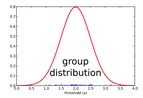
Hierarchical Bayesian Estimation



$$\mu_a \sim \mathcal{N}(0,1) \quad \sigma_a \sim \mathcal{U}(1e^{-10},100) \quad a_i \sim \mathcal{N}(\mu_a,\sigma_a)$$
$$\mu_z \sim \mathcal{N}(0,1) \quad \sigma_z \sim \mathcal{U}(1e^{-10},100) \quad z_i \sim \mathcal{N}(\mu_z,\sigma_z)$$
$$\mu_v \sim \mathcal{N}(0,1) \quad \sigma_v \sim \mathcal{U}(1e^{-10},100) \quad v_i \sim \mathcal{N}(\mu_v,\sigma_v)$$
$$\mu_{ter} \sim \mathcal{N}(0,1) \quad \sigma_{ter} \sim \mathcal{U}(1e^{-10},100) \quad ter_i \sim \mathcal{N}(\mu_{ter},\sigma_{ter})$$
$$\mu_{sv} \sim \mathcal{N}(0,1) \quad \sigma_{sv} \sim \mathcal{U}(1e^{-10},100) \quad sv_i \sim \mathcal{N}(\mu_{sv},\sigma_{sv})$$
$$\mu_{sz} \sim \mathcal{N}(0,1) \quad \sigma_{sz} \sim \mathcal{U}(1e^{-10},100) \quad sz_i \sim \mathcal{N}(\mu_{sz},\sigma_{sz})$$
$$\mu_{ster} \sim \mathcal{N}(0,1) \quad \sigma_{ster} \sim \mathcal{U}(1e^{-10},100) \quad ster_i \sim \mathcal{N}(\mu_{ster},\sigma_{ster})$$

$$wfpt_{i,j} \sim wfpt(x_{i,j}|a_i, z_i, v_i, ter_i, sv_i, sz_i, ster_i)$$

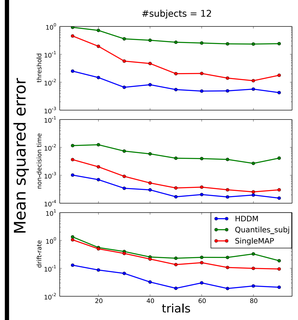
- Markov-Chain Monte-Carlo to estimate posteriors.
- Group and subject parameters estimated simultaneously. (Assumes subject parameters are normally distributed according to a group distribution).



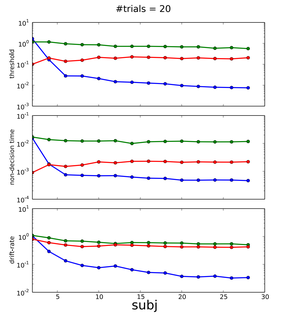
- Estimation of joint posterior (direct measure of confidence).
- Requires sampling (comparably slow):
  - Group-mean posteriors are Gibbs sampled.
  - Group-variance posterios are slice sampled.
  - Individual subject posteriors are Metropolis-Hastings sampled.

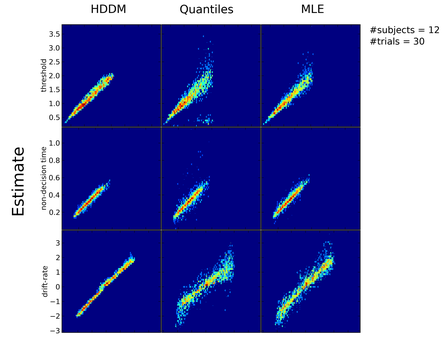
Simple DDM
Subject parameter recovery

HDDM needs fewer trials

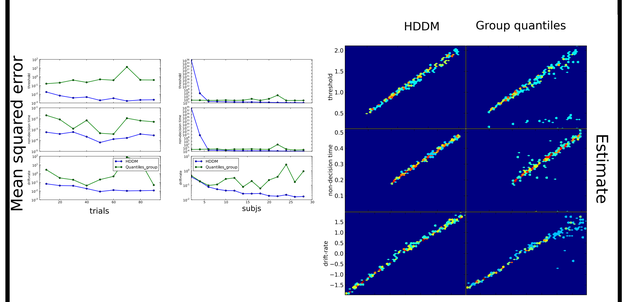


More subjects improve HDDM fit

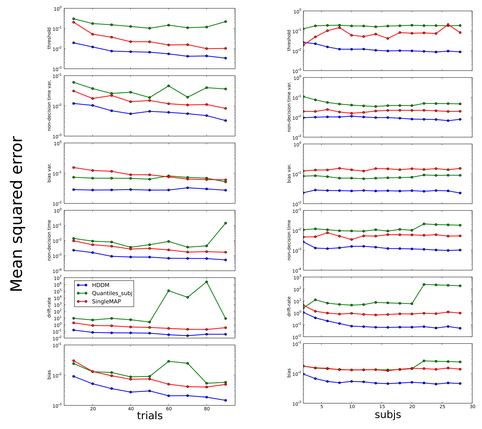


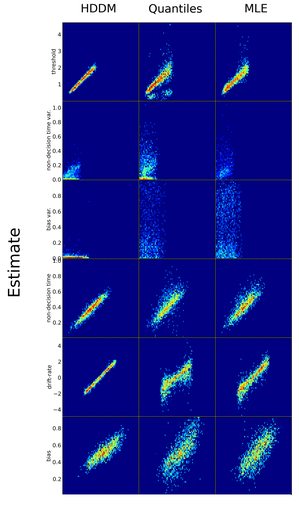


Simple DDM
Subject parameter recovery

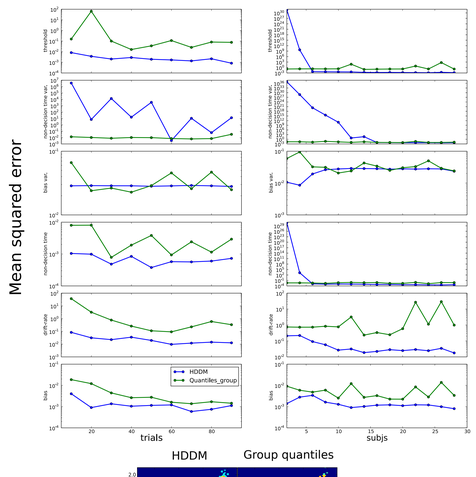


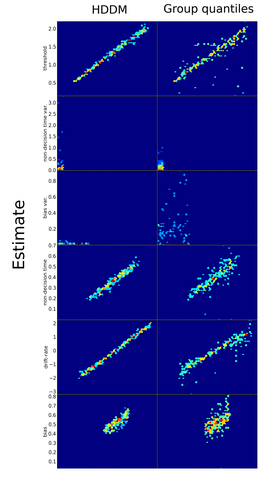
Full DDM
Subject parameter recovery





Full DDM
Group parameter recovery





Software

- All simulations perfomed with HDDM:

  `http://ski.clps.brown.edu/hddm_docs/`

- Code to run simulations:

  `https://github.com/hddm-devs/HDDM-paper`

Conclusions

- Hierarchical Bayesian estimation
  - Highest accuracy (unless very few subjects).
  - Fully Bayesian.
  - Subject and group parameters simultaneously.
  - Comparably slow.
- Maximum likelihood seems consistently better than the quantile method. To do: effect of outliers.
- Variability parameters basically unrecoverable with any estimation method.