Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		2202396

Everything That Kills Me Makes Me Feel Alive: Seeking for Properly Thinning

Carbon sequestration, enhancing stocks of CO_2 sequestered out of the atmosphere by the biosphere or by mechanical means, is a significant process for mitigating climate change. Trees, especially large trees, soils, and water environments in forests can sequester carbon. However, this does not mean that the forest should be completely protected. As trees reach a certain age and continue to get older, the efficiency of carbon sequestration decreases. We can get the best overall carbon sequestration efficiency by cutting down a portion of the aging trees, converting them into products, and replacing the vacant spaces with seedlings. In addition, ecological conservation, economic efficiency, and other factors must be taken into account in practice. To this end, we have designed a set of models with strong generalization capabilities to guide forest managers around the world on how to manage their forests.

We accomplished 3 Tasks and established several models:

For Task1: Our analysis of the forest begins with the calculation of a single tree. We used general multiplicative inverse proportion function to establish **Diameter Growth Model** and Logistic function to establish **Height Growth Model**. By regarding the tree as a cylinder, we obtained a tree's **Volume Growth Model** and **Weight Growth Model**. We proved that CO_2 sequestered per year **is proportional to** this tree's weight increment this year. By assuming the age of trees in the forest is distributed uniformly, we obtained the mass of CO_2 sequestered by the forest in a year. We got the age when the average CO_2 sequestered per year in a tree's lifetime reaches its maximum. By **harvesting all trees that reaches this age** and planting new trees periodically, we maximized the forest's carbon sequestration ability. We applied our model to Huashan pine and Quercus aliena and found that they should be harvested when they are **84** and **109** years old, respectively. We obtained that the weight of CO_2 sequestered by Huashan pine forest in Shaohua Mountain is **177,927** tons per year, which is **2.83** times higher than natural situation. Then we did a sensitivity analysis and proved our model stable.

For Task2: We established **Total Value Model** versus the area **Proportion p** implementing harvesting strategies described above. Forests have different kinds of utilities, such as carbon sequestration as mentioned in the previous section, as well as positive impacts on ecosystems and potential commercial benefits after harvesting. Since ecological value is a holistic concept, some community characteristics need to be used as independent variables. Our model chooses the proportion of land p that performs the harvesting strategy as the indicator. For the purpose of quantifying them, we set up equations for **Carbon Sequestration Value**, **Ecological Value** and **Product Value** and sum them to obtain the total value. By solving for the extreme value point we derive the optimal solution for p and obtain dividing strategies. For maximal Total Value, 67% of this forest should be under harvesting strategy and 37% should be protected. Adopting the strategy , the total value of the Huashan pine forest reaches \$57,435,569, increasing by **1.08** times. Sensitivity analysis proved our model robust.

For Task3: We applied our model to the **Huashan pine forest** in Shaohua Mountain in Task 1, 2 and to a **Quercus aliena forest** in Taibai Mountain in Task 3. We found that Quercus aliena has lower Ecological Value and higher Total Value comparing to Huashan pine. Then we found that the weight of CO_2 sequestered over 100 years by the Huashan pine forest using three different strategies is **9,722,800** tons, **27,515,500** tons and **19,959,900** tons respectively. To determine the policy for harvesting interval transition, we decomposed the Total Loss caused by transition as Economic Loss, Carbon Sequestration Loss and Policy Loss. For Huashan pine, **we made the policy that** the interval should be increased by 0.507 year per year, which means it will take 19.71 years to finish this process.

Ultimately, we analyzed models' strengths and weaknesses, and wrote a newspaper article.

Keywords: Carbon Sequestration, Harvesting & Dividing Strategies, Optimization, Progressive Delay

Contents

1 Introduction

1.1 Problem Background

"Global warming is too serious for the world any longer to ignore its danger or split into opposing factions on it." said by Tony Blair. In recent years, carbon emissions have been increasing due to human burning of fossil fuels, which will not only melt glaciers and permafrost, induce sea level rise and threaten the balance of natural ecosystems, but also affect the survival of human beings. Clean energy is not yet widespread and thus pollution has not been effectively controlled. We can find the following graph to see the carbon dioxide production in each country and the global greenhouse effect:

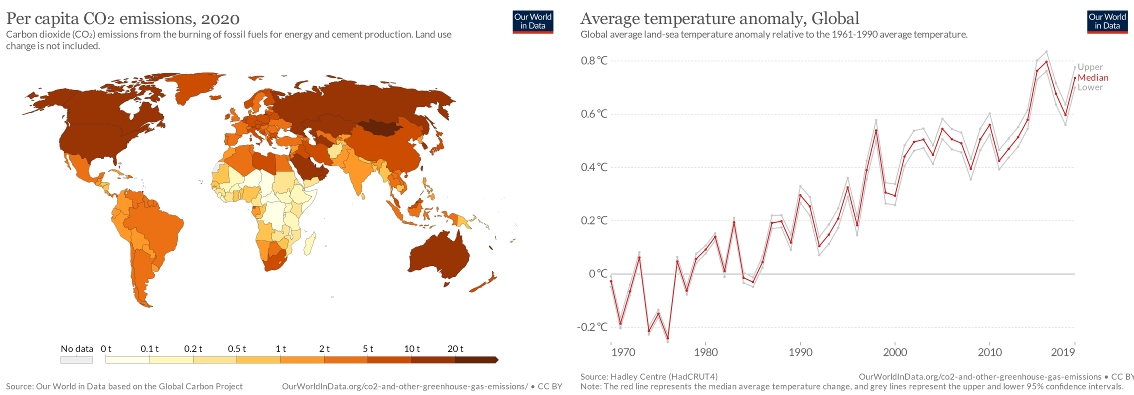


Figure 1: Greenhouse Gases and Climate Change [OurWorldInData, 2020]

The data above comes from the website Our World in Data, one of the largest scientific online publication that focuses on large global problems. As can seen above: both developing and developed countries emit large amounts of CO_2 every year, and as a result, temperatures have been rising steadily for the last fifty years, by almost 1 degree Celsius. In order to respond to the current situation, it is not enough to simply save energy and reduce emissions, we need to continuously consolidate the results of carbon sequestration.

1.2 Restatement of the Problem

1. Develop a model to determine the amount of carbon dioxide that a forest and its products can sequester, which can determine the most effective forest management plan on carbon sequestration.

2. Develop a model to determine the best forest management plan balancing carbon sequestration and other aspects like biodiversity or recreational uses. Some factors are taken into consideration to better understand the model:

   - The spectrum of management plans made by the model.
   - Conditions that would result in the forest being uncut.
   - Conditions that would result in the forest being wholly harvested periodically.

- Differences between various tree species that lead to changes in dividing strategy.

3. Apply models to various forests and identify one forest whose management plan includes harvesting.
   - Over 100 years, how much carbon dioxide will the forest and its products sequester?
   - What plan should be used for this forest? Why is it the best?
   - Suppose the interval between harvests in the best plan is ten years longer than current practices. Identify a strategy to transition from the current timeline to the new timeline.

1.3 Our Work

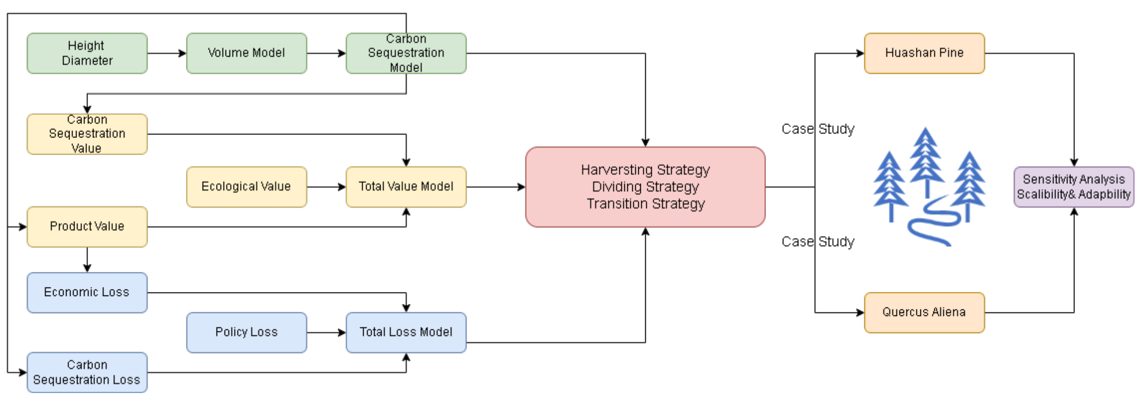


Figure 2: Our Work

2 Assumptions and Justifications

Considering that real-world problems often contain many complex factors, we first need to make some reasonable assumptions to simplify the model, and each assumption is supported by a corresponding justification.

1. **The age of trees in forests is generally discretely uniformly distributed and the overall number of trees remains stable. The age of a tree is measured in years as the minimum unit.**
   Justification: The number of trees in the world's forests has a large variance, but generally follows a symmetric probability distribution. Since the central limit theorem is not necessarily followed, we use the uniform distribution to estimate.

2. **Competition between trees in forest is not considered.**
   Justification: In close to nature forestry, more nutrients are available than in planted forests, and tree growth is more limited by its own species than by competition.

3. **The wet-to-dry ratio of each tree in the same forest remains consistent and does not change during the complete growth cycle.**

Justification: The wet-to-dry ratio of a tree is more often a characteristic of the tree itself or a response to the external climatic environment, and does not change with age. Therefore, the only factors that affect the wet-to-dry ratio of trees are the tree species and the climatic environment in which they are located.

4. **Any forest management strategy has little effect on the amount of carbon sequestered by other organisms in the forest.**
Justification: Trees sequester a high proportion of the carbon in forest ecosystems. Therefore, when trees are cut down and made into wood products, the environmental ecological impact on other microorganisms and fungi is slight, and thus changes in their carbon sequestration capacity are not taken into account.

3 Notations

Symbol	**Description**
t	Age
D	Diameter
H	Height
V	Volume
L	Lifespan
L_H	Harvesting age
ρ_S	Plant density
$\overline{\Delta M_{CO_2}}$	Annual carbon sequestration of a tree under natural conditions
$\overline{\Delta m_{CO_2}}$	Annual carbon sequestration of a tree using harvesting strategies
p	Percentage of forests using harvesting strategies
v_c	Carbon sequestration value
v_e	Ecological value
v_p	Product value
v_t	Total value

4 Task 1: Carbon Sequestration Model

In order to determine the amount of carbon dioxide that a forest sequesters over a period of time, we consider calculating the weight increment of the trees during this time. Thus, we need to identify the **Volume Growth Model** so that we can get the volume and further the weight of a tree as long as we know its age. We established **Diameter Growth Model** and **Height Growth Model** to get the Volume Growth Model.

Then we calculated one after another the tree's total mass, dry matter mass, carbon content mass, and **carbon dioxide sequestered**. During this process, we mainly used the wood's density, dry matter ratio, biomass carbon ratio, carbon weight ratio in CO_2, and get the final result.

After that we discussed the management strategy. Whenever there is a tree older than the threshold t_0, it should be harvested and a new tree should be planted, where t_0 varies with tree species. To find the threshold t_0, we need to find the tangent line of carbon sequestration curve by dichotomous.

After each of the above models was established, we used the data of Huashan pine and Quercus aliena from [Chen, 2007] to **validate our work**, and calculated the precise value of arguments and variables. We also calculated Mean Square Error (MSE), R-Square (R^2) and other values, which shows that our model fits well.

At last, we implemented a sensitivity analysis to show the effect of some special variables on the model.

The whole process can be summarized as the following graph:

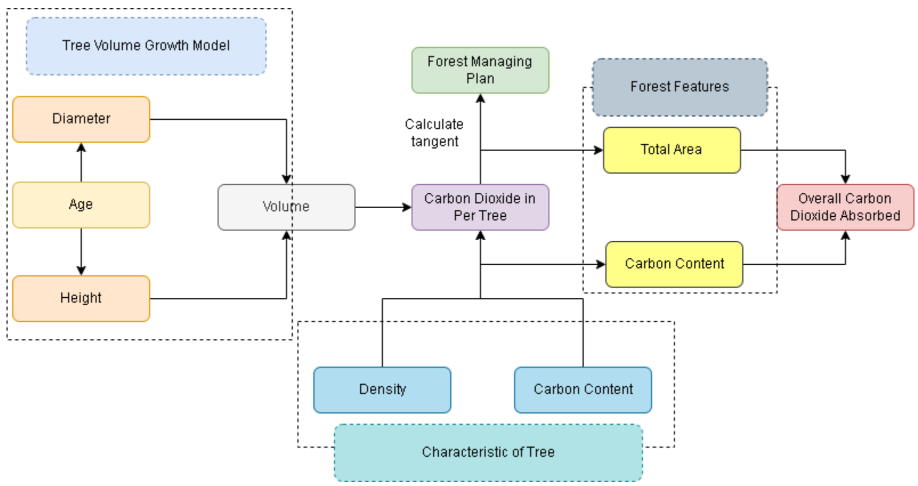


Figure 3: Strategy Making Process

4.1 Tree's Volume Growth Model

On the one hand, there is little data about the tree's volumes with ages, which may lead to an unconvincing Volume Growth Model. On the other hand, it tends to be hard to measure the tree's volume in practice, making it hard to use the model in real life. Consequently, we establish models to fit the tree's diameter and height growth model, and then use diameter and height to get the volume model.

4.1.1 Diameter Growth Model

To fit the **diameter** growth of a tree, we choose the **general multiplicative inverse proportion function**, which is consistent with the fast growth in the tree's young ages and slow growth in old ages. We denote D meter as the tree's diameter, t year as the tree's age. Then we established the following model:

$$D = a + \frac{b}{t + c} \tag{1}$$

To show that the relationship between tree diameter and age fits this type of curve, we fitted the data for Huashan pine and Quercus aliena with reference [Chen, 2007]. The data for the subsequent prediction of tree height and volume are also taken from this article. All these constants vary with tree species. The graphs of diameter variation with age are as follows:

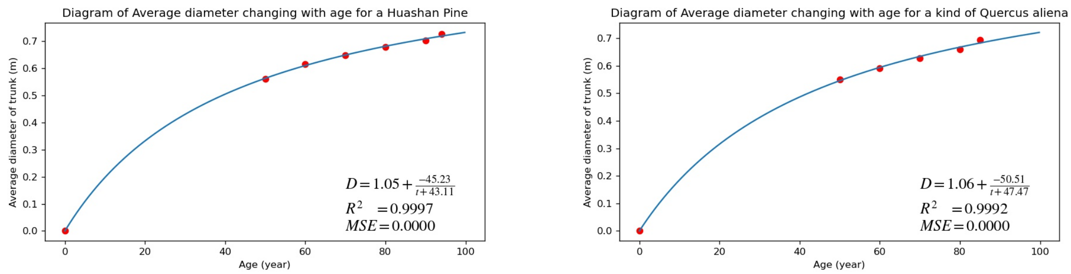


Figure 4: Tree Diameter Multiplicative Inverse Proportion Growth Curve

From the figure above, we can figure out that the R^2 is close to 1.0 and the MSE is quite low, so it shows that it is reasonable to use this multiplicative inverse function to estimate the diameters of different tree species.

4.1.2 Height Growth Model

It is known that the **height** increases fast in a tree's early ages, and then increases slower and slower. By the **Logistic** retarded growth principle, we recognize that the height growth rate should retard due to the trees height limitation and the environmental carrying capacity. We denote H meter as the tree's height, t year as the tree's age, H_m meter as the statistically maximum height of specified tree, and r as a factor to control the growth rate. Under this situation, the height growth rate should meet the following equation:

$$\frac{dH}{dt} = rH(1 - \frac{H}{H_m}) \tag{2}$$

Let C be the constant of integration. By solving this equation, we can get:

$$H = \frac{H_m}{1 + e^{-(rt+C)}} \tag{3}$$

The graph of height variation with age is as follows:

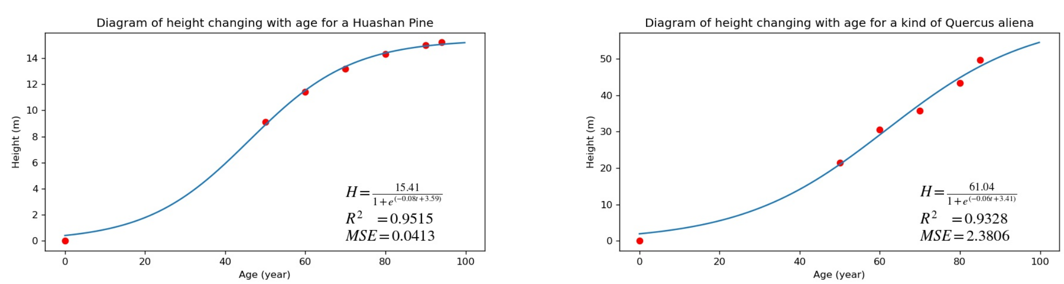


Figure 5: Tree Height Logistic Regression Growth Curve

From the figures above, we can find that this model fits the height growth well, so they show that it is reasonable to use the logistic function to estimate the height of different tree species. The figures shows that Huashan pine may grow up to **15** meters high and Quercus aliena may grow up to **60** meters high.

4.1.3 Volume Growth Model

To simply this problem, we regard the tree as a cylinder with height H meter and diameter D meter. Thus, we can use Equation (2) and Equation (3) to get the tree's **Volume Growth Model**:

$$V = \frac{\pi}{4} D^2 H \tag{4}$$

$$= \frac{\pi}{4}(a + \frac{b}{t+c})^2 \frac{H_m}{1 + e^{-(rt+C)}} \tag{5}$$

The graph of volume variation with age is as follows:

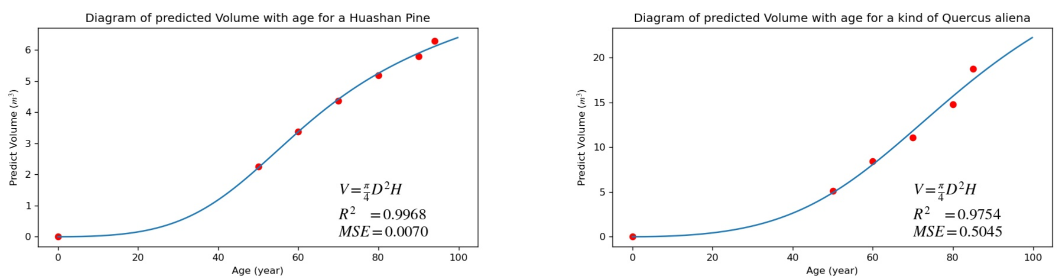


Figure 6: Tree Volume Nonlinear Growth Curve

From the figure above, we can figure out that the R^2 is close to 1.0 and the MSE is relatively low, so it shows that it is reasonable to use this function to estimate the volume of different tree species. The figure intuitively shows that the growth rate of Huashan pine starts to slow down at age **80** and the growth rate of Quercus aliena starts to slow down at age **100**.

4.2 Carbon Sequestration Model

4.2.1 Model Establishment

Having obtained the volume of the tree at a certain age, we can use the density of the tree to calculate the mass at this age. Let M be the mass and ρ be the density, then we will get:

$$M = \rho V = \frac{\pi}{4}\rho(a + \frac{b}{t+c})^2 \frac{H_m}{1 + e^{-(rt+C)}} \tag{6}$$

We referred to [Bartelink, 1998] and found that tree consists of roughly 72.5% dry matter and 27.5% moisture on average. To determine the dry weight of the tree, we just need to multiply the total green weight of the tree by 72.5%.

$$M_{dry} = 0.725M = 0.725\frac{\pi}{4}\rho(a + \frac{b}{t+c})^2 \frac{H_m}{1 + e^{-(rt+C)}} \tag{7}$$

We referred to [Thomas and Martin, 2012] and learned that the biomass carbon ratio may vary from 40% to 60% depending on different species or climates, with an average of about 50%. Here we choose 50% to cover most situations for a better generalization capability.

$$M_{carbon} = 0.5M_{dry} = 0.3625\frac{\pi}{4}\rho(a + \frac{b}{t+c})^2\frac{H_m}{1+e^{-(rt+C)}} \tag{8}$$

We all know that 1 molecule Carbon Dioxide (CO_2) consists of 1 molecule Carbon (C) and 2 molecule Oxygen (O), and the atomic weight of Carbon is 12 (u) and the atomic weight of Oxygen is 16 (u), Since 12 grams carbon in tree comes from 44 grams carbon dioxide, we need to multiply the weight of carbon content by $44/16 = 3.67$ to get the weight of carbon dioxide sequestered.

$$M_{CO_2} = 3.67M_{carbon} = 1.33\frac{\pi}{4}\rho(a + \frac{b}{t+c})^2\frac{H_m}{1+e^{-(rt+C)}} \tag{9}$$

In order to generalize and model the carbon sequestration of trees in a forest, we need to first determine the number of trees. The number of trees N is specified by the whole area S, forest coverage rate R and plant density ρ_S.

$$N = \frac{SR}{\rho_S} \tag{10}$$

Since each tree is independent from each other, the amount of **Total Forest Carbon Sequestration** by the forest is the sum of the amount of carbon sequestered by the trees.

$$M_{TFCS} = \sum_{i=1}^{N} M_{CO_2} = \sum_{i=1}^{N} 1.33\frac{\pi}{4}\rho(a + \frac{b}{t_i+c})^2\frac{H_m}{1+e^{-(rt_i+C)}} \tag{11}$$

For the sake of calculating the amount of carbon sequestered by the forest per year, the function needs to be differentiated. Before doing so, we need to introduce a constant, the life span of the tree species L, as a vertical asymptote. Combined with the assumption of discrete distribution, a more accurate model under the effective constraint of the independent variable is built.

$$t \le L, t \in N^* \tag{12}$$

After integrating the derivative, the sum of **Annual Forest Carbon Sequestration** of all trees in the forest is obtained. Because of the uniform distribution of tree ages, the **Newton-Leibniz formula** can be used to derive it. When the number of trees is k times the life span of the tree species, the annual carbon sequestration is the same as the carbon sequestration during the complete growth cycle of k trees. Thus, if only the amount of carbon sequestered is considered, kL trees growing for one year and k trees growing for a lifetime are equivalent.

$$M_{AFCS} = \sum_{i=1}^{N} \Delta M_{CO_2} = \frac{SR}{L\rho_S}\int_0^L M'_{CO_2}\Delta t = \frac{SRM_{CO_2}(L)}{L\rho_S} \tag{13}$$

4.2.2 Model Application

In the previous section, we have derived the equations for the tree production curve and the amount of carbon sequestered by the forest per year. In the next step, we will apply the existing model to calculate the values from both a single Huashan pine and a forest of Huashan pines and compare with the data in the real world to verify the model.

We calculated the average annual carbon sequestration of a single tree according to the following formula:

$$\overline{\Delta M_{CO_2}} = \frac{1.33\rho V(L)}{L} \tag{14}$$

Hint: The above value of the constant 1.33 was calculated by combining the average data of multiple kinds of trees. It is really difficult to obtain precise data for a specific tree species. However, you can refine this constant through precise scientific experiments to get a more accuracy result.

Referring to the trend in the above Figure 6, we can assume that the maximum volume can be reached by Huashan pine near the end of its life is $7m^3$.

Then we obtained the densities of different tree species from [heirloomkayak, 2022], collected **the densities** of various pine trees and took the average value $510kg/m^3$ as the density ρ of Huashan pine. Also, we found the maximum lifespan of various pine trees from [VirginiaBigTrees, 2020] and took the average value as the maximum lifespan of Huashan pine as 300 years.

Next, we calculated that **the average carbon sequestration** of Huashan pine is $15.827kg$ per year, which is not significantly different from the generally considered [EnvironmentalQualityProtectionFoundation, 2020] $12kg$ per year, and we think that the model is fairly reasonable for the carbon sequestration of a single tree.

After this, we estimate the **annual carbon sequestration** for Shaohua Mountain Forest, which is located in Xi'an, China with Mount Hua. According to [Pan, 2020], we acquired the number of trees per hectare of pine forest ρ_S as the value 1084. With a forest area of 6300 ha and a forest cover of about 90%, from equation (13) we calculate that $97,277t$ of carbon can be sequestered annually.

Since administrators didn't provide direct public data to verify the model, we chose data for the entire province of Shaanxi for comparison. With reference to [ShaanxiProvinceGovernment, 2019] and [ShaanxiForestryAdministration, 2019], back to 2018, forest coverage in Shaanxi was $80,262km^2$ and annual carbon sequestration was $102,965,000t$, which was approximately $12.829t$ per ha. And according to the model we built, for Huashan pine, the annual carbon sequestration was calculated to be $17.156t$ per ha. Considering that the plant density in centralized forest is higher, the model is basically considered to be reasonable and has application potential.

4.3 Harvesting Strategy

Replace old trees with new trees Based on the assumptions presented earlier, the ages of trees in forests are generally uniformly distributed and the overall number of trees remains stable. The carbon sequestration ability of one specified kind of tree only depends on its age, and trees won't affect each other. To maximize the whole carbon sequestration ability of a forest, we just need to replace the less capable trees with better ones, which means harvesting older trees and plant new trees. Now the problem is at what age the trees should be harvested.

Carbon-Mass is solved indirectly by tree-volume To determine the carbon sequestration ability of a tree, we can use the ratio of the total mass of CO_2 sequestered to its age. Let $ACSR$ be the **Average Carbon Sequestration Ratio**, and we will get $ACSR = M_{CO_2}/t$. To consider from the figure of the M_{CO_2} function, we just need to maximize the slope of the line crossing the origin and the point on the curve. When a line crosses the origin is tangent to the curve, $ACSR$ meets its maximum value.

Assuming that this line has a slope k and is **tangent** to the curve at $t = t_0$, we can establish the following system of equations and another similar system of equations:

$$\begin{cases} M_{CO_2}(t) = kt \\ M'_{CO_2}(t) = k \end{cases} \quad \begin{cases} 2M_{CO_2}(t) = kt \\ 2M'_{CO_2}(t) = k \end{cases}$$

If the solution of the left system is $k = k_1$ and $t = t_0$, then the solution of the right system will be $k = 2k_1$ and $t = t_0$. That's to say, if M_{CO_2} multiplies by a factor a, it will result in k multiplying the same factor a and t_0 remaining unchanged. From equation (9) we can see that M_{CO_2} is proportional to V, which means M_{CO_2} is equal to V multiplied by a factor. Thus, we can use **the curve of** V to find t_0. Once we get t_0, the carbon sequestered can be obtained by equation (9).

Numerical solution by dichotomous instead of symbolic solution Trying to find this tangent line by symbolic computation is very challenging because the volume formula of the cylinder will combine the negative exponential powers with the logistic function in a very complicated way. Therefore, we consider the use of the limit method and the dichotomous method to solve the numerical solution.

The limit method is mainly based on the formula $f'(x) = \frac{(x+\Delta x)-(x-\Delta x)}{2\Delta x}$, which can obtain the derivatives in a none-formula way with computer. While the dichotomy is based on the fact that both functions that make up the volume formula have a horizontal asymptote, and when the asymptote remains after the product, it can be assumed that at infinity the volume curve will have a horizontal tangent line greater than zero. Furthermore, it is easy to prove that for a concave function with x slightly larger than 0, the value of the tangent line that passes the Y-Axis must be less than zero, that is, the volume curve must have a tangent line with an intercept less than zero near zero. The volume curve is continuous, so the tangent intercept is continuous, and the tangent point at the boundary of $[0, \infty)$ there is a negative value and a positive value on each side. As the result, we can use the dichotomous method to get the parameters of the tangent line $y = kx$ which pass the origin .

A single-tree example of the cutting threshold age As is explained above, this system of equations is so complex that no symbolic solution can be obtained. However, arguments like H_m, r can be obtained by curve fitting, so it is easy to get the numeric solution of maximum $ACSR$ k and cutting threshold age t_0 and use them to determine the management strategy.

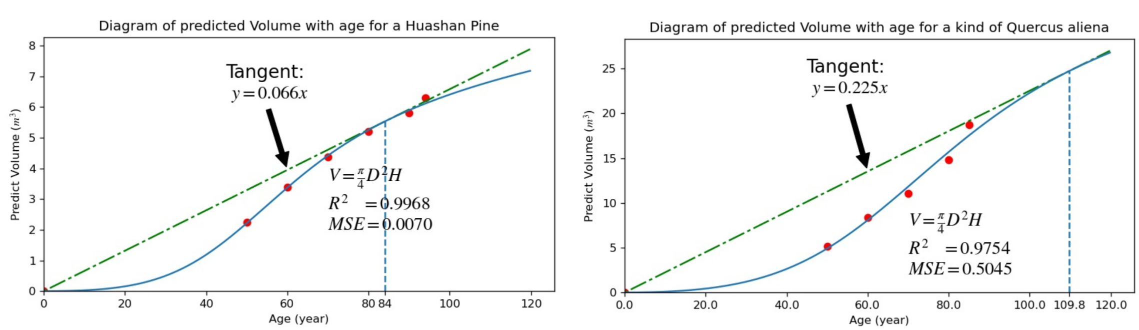


Figure 7: Tree Volume Nonlinear Growth Curve Tangent Derivation

By using the method mentioned above, we can get the cutting threshold age of Huashan pine and

Quercus aliena. Figure 7 shows that Huashan pine equal to or greater than **84** years old should be harvested and Quercus aliena equal to or greater than about **110** years old should be harvested.

Strategy When the cutting threshold age for the tree is t_0, the management strategy can be described as follows: in a forest where the ages of trees distribute uniformly, **all trees older than or equal to t_0 should be harvested and the same amount of tree seeds should be planted**. It is worth noting that this strategy is equivalent to the situation that all trees in a forest are at the same age and are harvested every t_0 year. That's to say, the time interval between two harvests is equal to the cutting threshold t_0.

When using several strategies mentioned above, the annual forest carbon sequestration is still different even if the tree harvest interval has been kept the same, so we consider the average annual forest carbon sequestration.Setting L_H as the age of harvested trees, the average annual tree carbon sequestration $\overline{\Delta m_{CO_2}}$under the support of forest management plan was obtained.

$$\overline{\Delta m_{CO_2}} = \frac{1.33\rho V_{CO_2}(L_H)}{L_H} \tag{15}$$

Efficiency After calculation, the average annual carbon sequestration per Huashan pine when the forest management plan is implemented as $44.77kg$, while $15.83kg$ per year if no method is used. To put it differently, the average carbon sequestration efficiency of our strategy is **2.83 times** higher than the original plan.

To implement the model in the real world, we compute the annual carbon sequestration increment in the Shaohua Mountain Forest (mainly Huashan pine), which turns out to be $177,927.05t$, which means our plan can makes a great improvement by harvesting timber periodically.

4.4 Sensitivity Analysis

Review We have demonstrated that in Section 4.3 when calculating the age of harvesting, only the volume curve has an effect on this process, while the factors of mass have no effect. Therefore we separate the sensitivity analysis into 2 parts, one for Volume model(cutting year prediction), one for the final CO_2 mass model.

Volume Model Because our model uses curve fitting to simulate real tree growth, here we faked some very similar data to see how sensitive the model is to trees with similar traits.

We used the data of Huashan pine as an example, for each measurement point of the original information, after providing a perturbation factor of 0.95-1.05 randomly in the longitudinal direction, and then inputting it into the system to calculate the cutting year. The results obtained are within a 5% deviation from the originally predicted year and sometimes provide a moderate offset of 3%. This indicates that our model is sensitive enough but not over-sensitive. In other words, the harvest year prediction model passes the sensitivity analysis.

Table 1: Sensitive test datas and results for the Volume model

ID	year 50	year 60	year70	year 80	year 90	year 94	Predict Year	Deviation
Raw	9.1	11.4	13.2	14.3	14.98	15.2	84.04	**0.000%**
Fake1	9.066	11.969	12.588	14.930	14.877	15.694	85.05	**1.202%**
Fake2	9.252	11.530	13.796	13.783	14.841	15.156	81.03	**-3.582%**
Fake3	8.972	11.754	13.754	13.935	15.436	15.761	84.13	**0.107%**
Fake4	9.041	11.375	13.443	14.006	14.748	15.743	84.770	**0.869%**
Fake5	9.107	11.330	13.507	14.180	14.726	14.728	81.19	**-3.391%**

CO_2 **Mass computing Model** For a certain Volume, three less well-determined constants, the dry weight ratio to green weight, the biomass carbon ratio, and plant density ρ_s are used when computing the Annual Forest Carbon Sequestration, which is referred to as Equation (7), Equation (8) and Equation (10). Fortunately, these variables are multiplied directly, which means that these variables always proportional to the final result. Slight changes only cause slight influences. As the result, this part of our model can easily pass the sensitivity analysis.

5 Task 2: Forest Value Evaluation Model and Decision Guide

In the previous section, we have calculated the harvesting age and made a prediction for Huashan pine harvest. For forests, different species composition brings variability, but in fact it only affects the parameters of the strategy development.

After solving the problem about when to cut the trees, we now turn our attention to **how many trees should under such special management**. Because when harvesting all the years, workers may harm the environment in the forest and affect other equally significant values. We do not want to disturb the whole forest just for more carbon sequestration, but keep a balance with environment friendliness.

In addition to the carbon sequestration value discussed earlier, there are other values of forests, such as product value and ecological value. For carbon sequestration value and product value, they can be expressed as a certain function of the tree mass, which in turn can be estimated by the age of the tree. However, ecological value is a holistic concept, and it is difficult to characterize the ecological value of a tree by the age of that tree.

In the real world, the problem is somewhat complex: a significant portion of montane forests are somewhat more difficult to thine. After incorporating multiple value considerations, we argue that we should model the proportion of forest area freely available for thinning as the independent variable p and estimate the value v.

This forest with a proportion of p is cut in year t_0 using the harvesting strategy described in Task 1, called **harvesting area** in the following parts , while the remaining part of the forest is kept in its natural growth condition without human intervention, and we would call it **protecting area**.

5.1 Forest Value Evaluation Model

Since we want to estimate the proportion p of the thinning portion of the total forest and maximize the total value v, we need to establish a function for it as a function of that proportion. The total value is divided into carbon sequestration value v_c, ecological value v_e and product value v_p. Therefore, we need to find out the function of these values about this proportion separately.

5.1.1 Carbon Sequestration Value

To facilitate comparison with real data and to verify the validity of the model, all values in this section are calculated for one hectare of forest. Since the amount of carbon sequestration has been calculated above, the value of carbon sequestration for one hectare of Huashan pine can be estimated on this basis.

The value of carbon sequestration per hectare of forest is obtained by setting E to the number of dollars paid per ton of CO_2 emitted.

$$v_C = (1-p) \times E \times \rho_S \times \overline{\Delta M_{CO_2}} + p \times E \times \rho_S \times \overline{\Delta m_{CO_2}} \tag{16}$$

In the graph below, obtained from ICAP, you can see that different countries in the world pay different amounts for the same amount of CO_2 emissions. We took the average value of \$46.27 and combined it with the Huashan pine data to estimate the value of carbon sequestration per hectare at \$793.81. Since most of the forests in the country are not normally harvested, it is treated as $p = 0$ here.

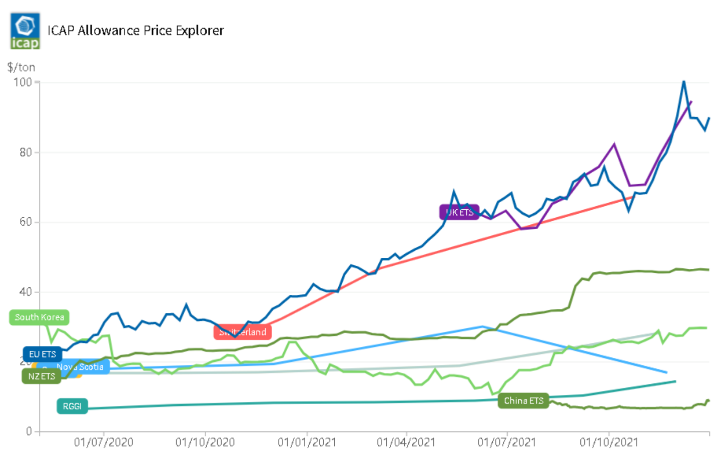


Figure 8: The allowance price of carbon emission [InternationalCarbonActionPartnership, 2022]

Referring to [Wang, 2016], the value of carbon sequestration and oxygen release per hectare of forest ecosystem service function in China is 9800 CNY, which is converted to \$1548.4. Considering that only the value of carbon sequestration was calculated above, and there are differences in carbon sequestration capacity among various species of trees, the error is considered to be within the acceptable range and the model is considered to be reasonable.

5.1.2 Ecological Value

As we know, the longer a tree grows, the stronger its root system is. While the denser crown is and the taller tree is , the better it can conserve water and prevent wind and sand. Therefore, cutting down trees and planting new ones will **reduce the total ecological value v_e of the forest**, i.e., the overall growth of trees in the forest is positively related to their v_e.

For example, an extended harvesting area can lead to more severe damage in some places by the tree cutting process, and forests with generally lower tree age can hardly be combined into wind and sand control fronts. To construct a equation between ecological value and the harvesting ratio, we considered analyzing the ecological impact of the cutting process and constructed a model based on it.

To simplify the question, **consider a fan-shaped forest** and that the exit gate for this forest area is at the lower-left corner. When it is necessary to expand the harvesting area, it is expanded outward in a circular arc from the center of the circle, i.e., the harvesting area is the green part and the protecting area is the outer white part. If **a polar coordinate system** is established for this forest with the lower left corner as the origin, the point P under this coordinate system can be represented by (r, θ), where r is the distance of the point from the origin and θ is the angle of deflection of the line connecting the point and the origin. Let R be the radius of the harvesting area and R_0 be the radius of the protecting area. This fan-shaped forest and the polar coordinate is shown in Figure 9 below.

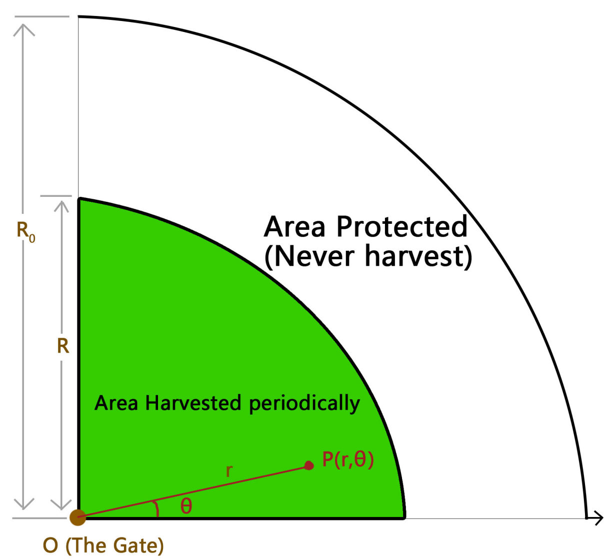


Figure 9: The harvest area and the protected area

The first thing to note is that our model is simple but efficient enough to cover most circumstances. The only thing you need to do is to change the angle of the origin to 180 degrees to fit in a gate on a straight side of a forest, other angles are available as well. However, no matter the angle you choose to fit your forest, **it won't affect the final result of our model**. Here we just use the quarter circle for demonstration.

Subsequently, we will focus on the harvesting area by assuming that the trees will be transported in a straight line to the exit gate at the origin after they are cut down. In this case, the land in the inner circle will suffer damage due to the transport of trees in the outer circle, and the more inward the land is, the more transport damage it will suffer. This process can be represented by the Figure 10. There are three diagrams to explain the assumption from the real world to mathematics. (**Only the harvesting area is shown**)

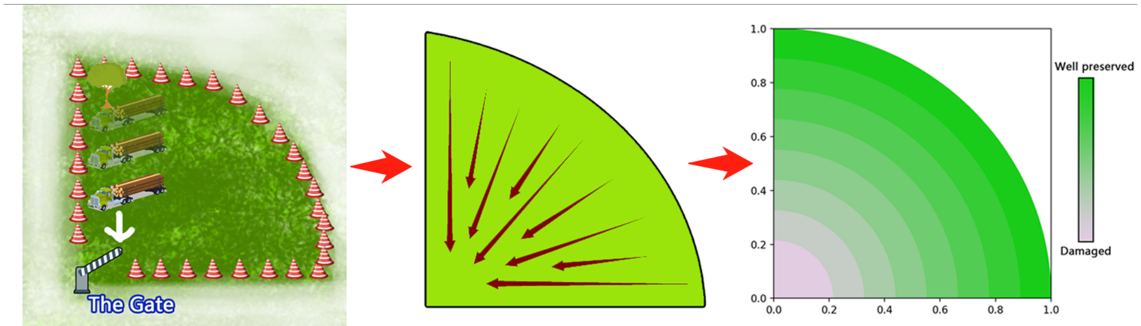


Figure 10: diagrammatic sketch of our assumption for environmental disruption by cutting woods

The environment at this point is damaged not only by harvesting, but also by transporting when trees at $(r' > r, \theta' = \theta)$ are harvested. This damage caused by transporting should be proportional to the distance from that point to the circumference of the circle. Suppose this ratio is b and the damage at the circumference is a, then the damage to the point can be expressed as $a + b(R - r)$. Integrating over the entire region, the total damage can be obtained as:

$$Harm = \int_0^{\pi/2} \int_0^R [a + b(R - r)]rdrd\theta = \frac{\pi R^2}{4}a + \frac{\pi R^3}{12}b \tag{17}$$

We assume that $Harm_0$ is the damage that every point in the forest takes the most damage $(a+bR_0)$, then $Harm0 = \pi/4aR_0^2 + \pi/4bR_0^3$. For computational simplicity, we can let a=0. Then We assume that the ecological value of the forest is reduced by an amount proportional to the amount of damage it suffers. Therefore, if the ecological value of the forest in its natural state is v_{E_0}, then we can obtain the ecological value of the forest with our strategy:

$$v_E = v_{E_0} - v_{E_0}\frac{Harm}{Harm_0} = v_{E_0} - \frac{1}{3}\frac{R^3}{R_0^3} \tag{18}$$

Noting that $p = \frac{\pi R^2}{\pi R_0^2}$, so $\frac{R^3}{R_0^3} = p^{3/2}$. Then we can obtain:

$$v_E = v_{E_0} - \frac{1}{3}p^{\frac{3}{2}}v_{E_0} \tag{19}$$

Using this equation, we can get the ecological value of the forest with our strategy.

5.1.3 Product Value

Trees that die in the natural environment often have problems such as being troubled by pests and diseases, and most species also decay quickly after death, so they are not considered here to have commercial value for forming products. Thus, when considering the value of the product, we only calculate the average profit for area that are harvested periodically.

There are large differences in timber prices between tree species because of variations in tree rarity and engineering quality. To measure this capacity, C_1 and C_2 are set to denote the product value per kg and per m^3 of tree, respectively.

$$v_P = 0.75 \times C_1 \times p \times \rho_S \times \overline{\Delta m_{CO_2}} \tag{20}$$
$$= \frac{p \times \rho_S \times C_2 \times V(L_H)}{L_H} \tag{21}$$

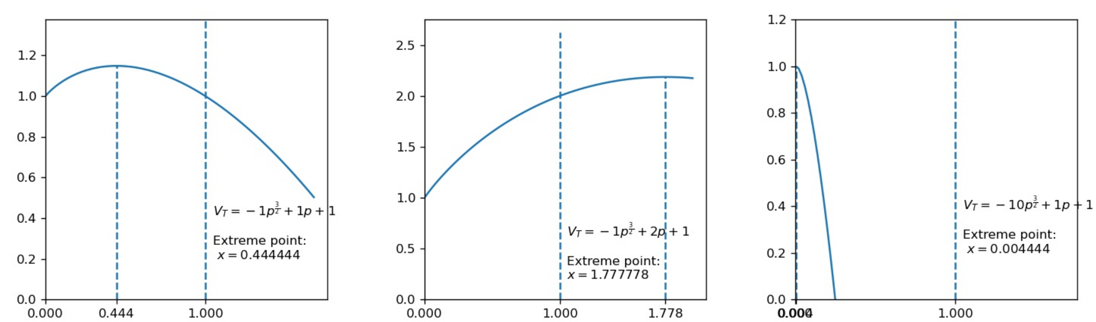
Referring to [TimberUpdate, 2022], the average price of pine timber is \$55 per MBF, which in turn calculates a timber value of \$1971.20 per hectare of Huashan pine, which is generally in accordance with the estimated product value of \$1883 per hectare of pine timber value stated in [FOREST2MARKET, 2019].

5.1.4 Total Value

As discussed earlier, the Carbon Sequestration Value, Ecological Value and Products Value can all be measured in dollars per hectare per year. By adding them up, we can get the Total Value of the forest, which is a non-integer power polynomial with the highest number of 3/2.

$$\begin{aligned} v_T =& v_C + v_E + v_P \\ =& -\frac{1}{3} v_{E_0} p^{\frac{3}{2}} + (E \times \rho_S \times \overline{\Delta M_{CO_2}} + v_{E_0}) \\ &+ (E \times \rho_S \times \overline{\Delta m_{CO_2}} - E \times \rho_S \times \overline{\Delta M_{CO_2}} + C_2 \times \rho_S \times V(L_H)/L_H) p \\ =& A p^{\frac{3}{2}} + Bp + C \end{aligned}$$

Once the species of the tree is determined, the values of the parameters are also uniquely determined. It is clear that $A < 0$, $B > 0$ and $C > 0$. It is easy to see that the curve of this function is similar to the curve of the quadratic function. Here we draw the image of this function in three cases.



(a) (b) (c)

Figure 11: Three cases of the Total Value

It is easy to obtain that $v_T' = \frac{3}{2} A \sqrt{p} + B$, and v_T meets its maximum at $p = (-\frac{2}{3}\frac{B}{A})^2 > 0$. The figure above shows three different cases. In case (a), the extreme value point is between 0 and 1, which means some area of the forest should be harvested periodically; in case (b), the extreme value point is greater than 1, which means all trees in the forest should be harvested periodically; and in case (c), the extreme value point is very close to 0, which means almost no trees should be harvested and almost all trees should grow naturally (though there will still be a very small proportion of the forest should be harvested, this small proportion can be ignored).

5.2 Decision Guidance

There are some noteworthy aspects of this composite value, which we will explain below.

Q_1**: What is the possible spectrum of management plans suggested by this model?**

The range of p is $[0, 1]$ where 0 means leaving the whole forest protected and 1 means treating all the trees in the forest with the harvesting strategy described in Task 1. Therefore, the management plan may vary from protecting all the forest area to applying harvesting strategies to all the forest area.

Q_2**: Are there any conditions which would result in a forest that should be left uncut?**

This situation almost never happens, because the abscissa of the extreme value point is always greater than 0. However, when p is very small and close to 0, the strategy is similar to leaving the whole forest protected. This happens when the Ecological Value of the tree species is very high and the Carbon Sequestration Value and Product Value are low.

Q_3**: Are there any conditions which would result in a forest that should be wholly harvested periodically?**

This situation exists. When the abscissa of the extreme value point of the v_T curve is equal to or greater than 1, the v_T reaches its maximum value at $p = 1$ in the domain of definition. It means that whole forest is under the harvesting strategy. This happens when the Ecological Value of the tree species is very low and the Carbon Sequestration Value and Product Value are high.

It should be noted that, though the above two problems are seemingly similar, they do not lead to the same conclusions. According to the data we collected, the Total Value v_T and the proportion p of most tree species show a left skewed distribution, so the probability that Q_3 happens is **much larger** than Q_2.

Q_4**: What differences between various tree species lead to changes in dividing strategy?**

The proportion between the three values of trees mainly leads to the change of dividing strategy, and the factors that influence it may be the V-t curve of trees, density, wet-to-dry ratio, timber price, lifespan, and influence on ecosystem.

5.3 Strategy Example

Here we took the Huashan pine as an example. We managed to get its Total Value Model and make the corresponding forest management strategy.

The parameters of Huashan pine have been explained and calculated earlier, so the specific values are given here directly without giving the units. $E = 46.27, \rho_S = 1084, \overline{\Delta M_{CO_2}} = 15.827, \overline{\Delta m_{CO_2}} = 44.77, C_2 = 65, V = 2.35, L_H = 84$. To obtain the Ecological Value in its natural state, we used the data by [Wang, 2016] and added up watershed value, soil protecting value, biodiversity value and etc. We can get $v_{E_0} = 8616$.

Using the parameters above, we can obtain its Total Value Model:

$$v_T = -2872p^{\frac{3}{2}} + 3423p + 9410 \tag{22}$$

Here we can draw its image and solve its maximum point. This function achieves a maximum value of 10129.73 at $p = 0.63$. Therefore, the management strategy of this forest is: 37% of the forest should grow naturally and 63% of the forest should be harvested periodically (trees older than 84 should be harvested).Thus, the total value of Huashan forest is \$57,435,569. If harvesting strategy is not used, the total value is \$53,351,459. Adopting the strategy increases the total value of the forest by 1.08 times.

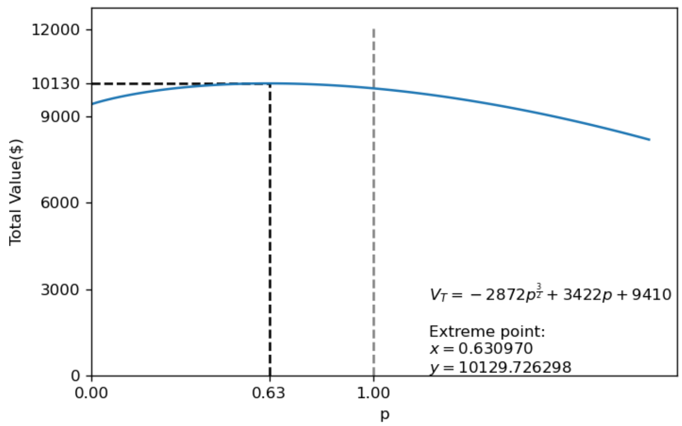


Figure 12: Total Value Curve of Huashan pine

5.4 Sensitivity Analysis

Timber price In this model, the timber price C_2 in Equation (21) is subject to large fluctuations due to market and other factors, which in turn affect the decision. In the Huashan pine sample, we use to validate the model, the unit price of wood for the season is \$65, and we observe the effect of a slight perturbation in C_2 ($\pm 5\%$) on the harvesting/protection ratio p and the final total value v_T. The impact on p is confirmed to be between $-5.845\% \sim 5.671\%$, and on v_T between $-0.632\% \sim 0.597\%$.

This result indicates that the division ratio p has a reasonable sensitivity to the price of wood, while the effect of p on the total value exists, but is very slight, because there are many other factors.

Natural ecological value of forest V_{E_0} in Equation (18) is another variable for which precise values are difficult to obtain directly and can fluctuate widely depending on the method used to evaluate it. Here we perform a sensitivity analysis on it. When V_{E_0} fluctuates in the $\pm 5\%$ interval, it has a **negative** correlation from $+9\% \sim -10\%$ to the choice of the p point, while the effect on v_T is only between $+0.6\% \sim -0.7\%$

It shows that when solving for the ratio p, the model is quite sensitive to the V_{E_0}, but still acceptable. For this reason, model users must be careful to estimate the ecological value of an undeveloped forest to help them make optimal decisions. Different forest managers may have different scores of ecological value for the same forest. We think this is reasonable; after all, you are the boss, and you call the shots. As long as you can measure the ecological value in monetary terms with a scientific way, our model can always help you to make the decision.

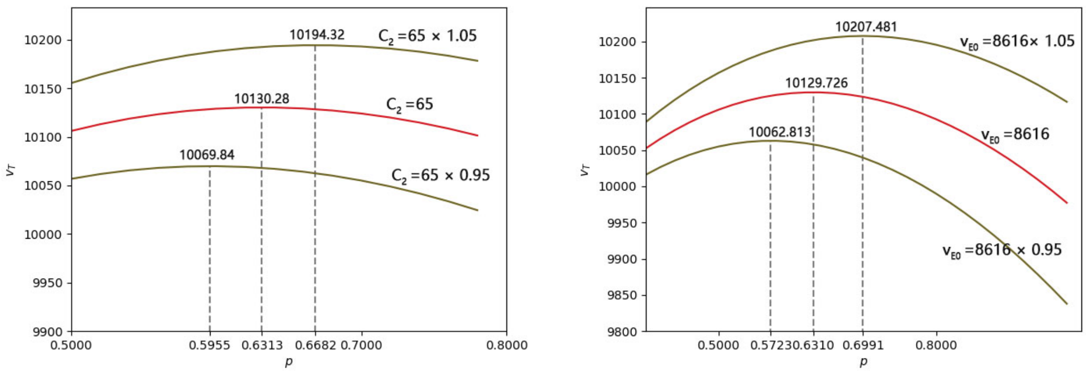


Figure 13: Details for the sensitivity analysis on C_2 and V_{E_0}

6 Task 3: Model Application and Case Study

6.1 Model Application on various forests

Here we apply the model to a variety of different forests, not only for testing the extensibility of our model but also for verifying the rationality of our harvesting and dividing strategy. Previously, we have applied the model to a **forest planted with Huashan pine**. Here we selected **Quercus aliena in the Taibai Mountain Forest** for our study.

Quercus aliena is a typical type of white oak. From [Pan, 2020] and [VirginiaBigTrees, 2020], we obtained the density and lifespan of it as $770kg/m^3$ and $600years$. Following on from **Section 4.3**, $\overline{\Delta M_{CO_2}}$ and $\overline{\Delta m_{CO_2}}$ are respectively $51.205kg$ and $230.4225kg$ per year. According to [administration of China, 2012], ρ_s is 167 trees per ha.

With a forest area of 2949 ha and a forest cover of about 94.3%, we calculate that $23704.54t$ of carbon dioxide can be sequestered annually. When using the harvesting strategy, the annual carbon sequestration of the Taibai Mountain forest increased to $106,670.43t$, which is **4.5** times more than the original result.

Referring to [PennStateExtension, 2021], the average price of Quercus aliena is \$601 per MBF. We use equation (16) and (20) to obtain \$394.48 and \$9569.2 per pa, separately.

Since differences in environmental impacts between tree species are not significant, v_{E_0} is considered to remain constant as 8616 per ha, therefore the equation for the total value is as follows:

$$v_T = -2872p^{\frac{3}{2}} + 10950p + 1775 \tag{23}$$

It is not difficult to find that the total value curve exhibits a single increase in the range of values of the independent variable p due to the exceeding value of the product of Quercus aliena, with the extreme point taken at $p > 1$. The forest management plan is to implement a harvesting and dividing strategy for the entire area, leaving no forest protected. This conclusion verifies Q_3 in **Section 5.2**, while the differences between tree species are well represented.

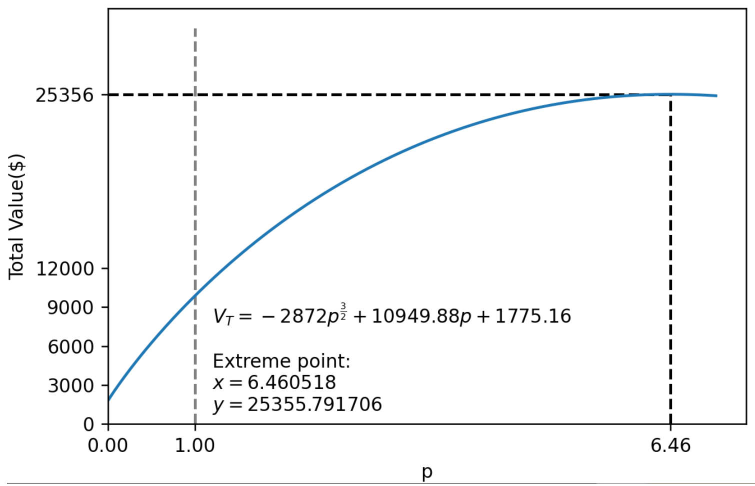


Figure 14: Total Value Curve of Quercus aliena

What can be found in the above figure is that when both harvesting and dividing strategies are implemented, the carbon sequestration and product values of Quercus aliena are greater than Huashan pine and the ecological value is less than it for the same forest area.

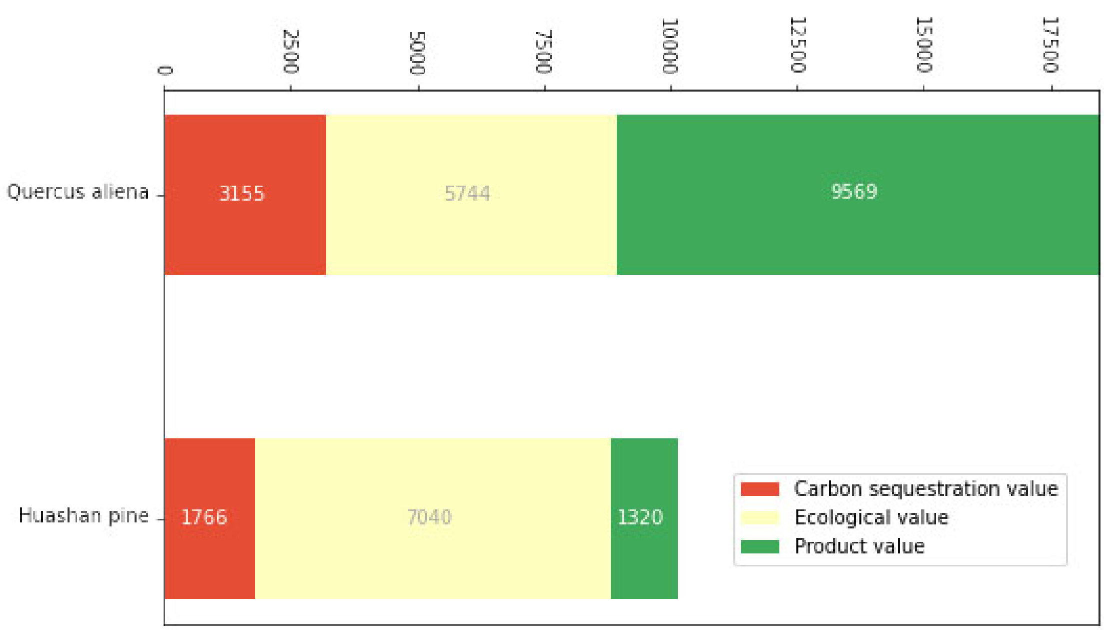


Figure 15: Value Comparison between Huashan Pine and Quercus aliena

6.2 Case Study: Huashan Pine

As is estimated earlier, the forest of Huashan pine should be partly harvested periodically for a higher total value. As a result, we choose Shaohua Mountain Forest (mainly Huashan pine) to discuss the following questions. We calculated the amount of carbon dioxide sequestered, described the forest management plan and its reason, and discussed the transition strategy from the current timeline to the new timeline.

6.2.1 Carbon Sequestration over 100 Years

The amount of carbon dioxide sequestered by the forest in its natural state is $97,228t$ per year. If all the trees that are older than 84 years old are harvested periodically to maximize carbon sequestration ability, the carbon dioxide sequestered is $275,155t$ per year. After taking Ecological Value and Products Value into consideration, 63% of the trees should be harvested periodically and 37% of the trees should grow naturally, and the carbon dioxide sequestered is $199,599t$ per year. As a result, the amount of carbon dioxide sequestered by the forest over 100 years is **9,722,800t**, **27,515,500t** and **19,959,900t** respectively.

6.2.2 Management Plan and Analysis

The management plan and how we obtained have been discussed previously.

To maximize the carbon sequestration ability, all the trees that are older than 84 years old should be harvested periodically and the same amount of new trees should be planted. Since we measure this ability by the ratio of carbon sequestered in tree's lifetime and its lifespan, we need to maximize $\frac{M_{CO_2(t)}}{t}$. As a result, by harvesting a tree when it is 84 years, the average carbon sequestered per year in its lifetime meets its maximum. This strategy is reasonable because the tree is more capable of sequestering carbon in its young ages. So replacing older trees (older than 84) with younger trees is beneficial.

To maximize the Total Value, 63% of the trees should be harvested periodically (when they are 84 years old) and 37% of the trees should grow naturally. We use p to denote the proportion of forest that should be harvested periodically and express the forest's Carbon Sequestration Value, Ecological Value and Products Value with p. By maximizing the Total Value, we obtained $p = 63\%$. Though harvesting trees may increase carbon sequestration value or products value, it will also decrease ecological value. So the value of proportion p that we find successfully balances these values and maximizes the total value.

6.2.3 Timeline Transition Strategy

Here we try to find a strategy for transitioning from the existing timeline to the new timeline if the harvest interval in the best management plan is 10 years longer than that in current practice. We note that the harvest threshold age for one tree is also the harvest interval for this tree. Because the harvest interval tree is 84 years in the best plan, the current interval is 74 years. If we change the harvest threshold from 74 to 84 directly, it will be detrimental to the interests of forest managers and users.

Therefore, we want to design a **Progressive Delay Harvesting Strategy** inspired by the Progressive Delay Retirement Strategy which means **increasing the harvest interval by x year per year**. We consider estimating the loss caused by the transition process, which is related to x. The process of implementing this policy will last $10/x$ years. Here we divide the **Total Loss** into Economic Loss, Carbon Sequestration Loss and Policy Loss. By minimizing the Total Loss, we can obtain the best x and the transition strategy.

To calculate Economic Loss, we assume that the loss is b dollars per hectare and we could harvest a hectares trees. Because of the policy being implemented, only $(1-x)a$ hectares trees can be harvested every year. As a result, the Economic Loss is abx dollars every year.

To calculate Carbon Sequestration Loss, we consider t-th years after the policy starts. We can sequester $M_{CO_2}(84)$ kg CO_2 in the future, but now we can only sequester $M_{CO_2}(74+xt)$ kg CO_2. Assuming that it worth E dollars every kg CO_2 sequestered, we can obtain that the Carbon Sequestration Loss is $E(M(84) - M(74+xt))$ dollars at the t-th year.

To calculate Policy Loss, considering if the transition process is fast , the value of x is large. It will do great harm to the forest manager and other people, so we regard Policy Loss as a penalty term to avoid x being too big. Here we can simply let Policy Loss be $10000x$.

Therefore, we can obtain the Total Loss:

$$Loss(x) = \int_0^{10/x} [abx + E(M(84) - M(74+xt))]dt + 10000x \tag{24}$$

The derivative of this Total Loss is:

$$Loss'(x) = -\frac{10ab}{x} + 10000 \tag{25}$$

So when $x = 10000/ab$, the Total Value achieves its minimum value. We know that $ab = 1971$ for Huashan pine, so $x = 0.507$. Therefore, **t**he transition policy is: increasing the harvest interval by 0.507 year every year until the interval meets 84 years, which means it will take 19.71 years to finish the implementation of this policy.

7 Strengths and Weaknesses

7.1 Strengths

$\mathscr{P}$ **Simple but universal**: Although the model looks simple, it can be applied to different forests by introducing comprehensive parameters. Therefore, our model has a strong generalization ability.
$\mathscr{P}$ **Statistical tests**: In the case study, the estimated values calculated using our model are generally similar to the real data.
$\mathscr{P}$ **Significant progress**: With the harvesting and dividing strategy, our model shows significant improvements for several indicators(annual carbon sequestration, total value), indicating that our model is effective.
$\mathscr{P}$ **Fairly robust**: Sensitivity analyses show that the model is robust when varying the parameters.

7.2 Weaknesses

$\mathscr{N}$ **Strong assumptions**: We make a strong assumption in the tree competition relationship, which doesn't always hold in reality. This relationship determines the finite density of the forest.
$\mathscr{N}$ **Data relied**: Most statistics are acquired from reports and papers, and the data collected are not as complete as desired. Estimations are depended on data.
$\mathscr{N}$ **Disasters not considered**: Exigence events are not taken into account, such as natural disasters or sudden man-made damage.

You may be surprised at this title on the first sight. With global warming getting worse and forests being destroyed, almost all the media and environmental orgnizations are emphasizing the importance of protecting forests. Aren't forests good at absorbing carbon dioxide? Doesn't cutting down trees mean making the environment worse?Recently, a mathematical modeling team studied the Shaohua Forest in China and discussed forest management strategies, trying to maximize the forest's ecological values, economic values and carbon sequestration capacity. With approprite harvesting, the carbon sequestration capacity of the forest is expected to be increased by 2.83 times.

Everything will age

Even the strong sprinter Usain Bolt, will have to retire because of aging and functional degeneration. Getting aged, sickness and death are the rules of life on earth, and trees are no exception.

In fact, trees can also be divided into teenagers, adults and seniors. The body functions gradually become stronger over time, and then slowly age. The ability to sequester carbon comes from the tree's photosynthesis, and as the tree gets older, the rate of carbon sequestration is affected by aging and gradually slows down. When this rate slows down to a certain point, it is better to cut down the old trees and replace them with younger ones that are more efficient at sequestering carbon. In this way, cutting down trees may help to increase the forest's carbon sequestration rate.

By cutting down trees over 84 years old in Shaohua Forest and planting new ones, the high carbon sequestration capacity of the younger trees can be utilized to the maximum, allowing the entire forest to sequester as much carbon as possible. With such a management strategy, this forest could absorb an additional amount of 177,927 tons of carbon dioxide every year. Since the annual CO2 emissions per person in the United States is about 16 tons, this forest can absorb the CO2 emitted by 11,120 people aditionally..

Choosing approprate ratio

In addition to carbon sequestration value, forests also have great ecological and economic values. We cannot ignore the ecological value to increase the carbon sequestration; we must find a balance between these values.

Unlike the strategy stated earlier, about 60% of the forest should be harvested periodically to maximize the total value of Shaohua forest, or the damage to the environment will overwhelm the benefit. Under this strategy, the extra profit made by this forest is about 4,084,110 dollars.

With the development of science technology, it's easier and easier for people to use natural resources more precisely with the help of accurate calculation and practice. The modeling team stated that their research was based on existing statistics. They hope to communicate with more teams and find a better strategy.

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