Documentation for Analysis.py

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1 Introduction

This code file is used to do sensitivity analysis about the EZ-climate model.

2 Methods

additional_ghg_emission: additional ghg caused by emission is the mitigation level now times the ghg level change w.r.t the emission in this period.

```
def additional_ghg_emission(m, utility):
	"""Calculate the emission added by every node.

	Parameters
	----------
	m : ndarray or list
		array of mitigation
	utility : `Utility` object
		object of utility class

	Returns
	-------
	ndarray
		additional emission in nodes

	"""
	additional_emission = np.zeros(len(m))
	cache = set()
	for node in range(utility.tree.num_final_states, len(m)): # the number of final
		path = utility.tree.get_path(node)
		for i in range(len(path)):
			if path[i] not in cache:
```

```
                        additional_emission[path[i]] = (1.0 - m[path[i]]) *  uti
                        cache.add(path[i])
    return additional_emission
```

store_trees: save the values in BaseStorageTree to a csv file.

```
def store_trees(prefix=None, start_year=2015, **kwargs):
    """Saves values of `BaseStorageTree` objects. The file is saved into the 'data
    in the current working directory. If there is no 'data' directory, one is crea

    Parameters
    ----------
    prefix : str, optional
        prefix to be added to file_name
    start_year : int, optional
        start year of analysis
    **kwargs
        arbitrary keyword arguments of `BaseStorageTree` objects

    """
    if prefix is None:
        prefix = ""
    for name, tree in kwargs.items():
        tree.write_columns(prefix + "trees", name, start_year)
```

delta_consumption: Calculate the changes in consumption and the mitigation cost component of consumption when increasing period 0 mitigation with ‘delta$_m$‘$.returnsatuplecontains$:

storage tree of changes in consumption per delta m

ndarray of costs in first periods

new utility at the start point

```
def delta_consumption(m, utility, cons_tree, cost_tree, delta_m):
    """Calculate the changes in consumption and the mitigation cost component
    of consumption when increaseing period 0 mitigiation with `delta_m`.

    Parameters
    ----------
    m : ndarray or list
        array of mitigation
    utility : `Utility` object
        object of utility class
    cons_tree : `BigStorageTree` object
        consumption storage tree of consumption values
        from optimal mitigation values
    cost_tree : `SmallStorageTree` object
```

```
                cost storage tree of cost values from optimal mitigation values
        delta_m : float
                value to increase period 0 mitigation by

        Returns
        -------
        tuple
                (storage tree of changes in consumption per delta m, ndarray of co

        """
        m_copy = m.copy()
        m_copy[0] += delta_m

        new_utility_tree, new_cons_tree, new_cost_tree, new_ce_tree = utility.utili

        for period in new_cons_tree.periods:
                new_cons_tree.tree[period] = (new_cons_tree.tree[period]-cons_tree.t

        first_period_intervals = new_cons_tree.first_period_intervals
        cost_array = np.zeros((first_period_intervals, 2))
        for i in range(first_period_intervals):
                potential_consumption = (1.0 + utility.cons_growth)**(new_cons_tree
                cost_array[i, 0] = potential_consumption * cost_tree[0]
                cost_array[i, 1] = (potential_consumption * new_cost_tree[0] - cost

        return new_cons_tree, cost_array, new_utility_tree[0]
```

constraint_first_period: Calculate the changes in consumption, the mitigation cost component of consumption, and new mitigation values when constraining the first period mitigation to ‘first_node‘.

```
def constraint_first_period(utility, first_node, m_size):
        """Calculate the changes in consumption, the mitigation cost component of
        and new mitigation values when constraining the first period mitigation to

        Parameters
        ----------
        m : ndarray or list
                array of mitigation
        utility : `Utility` object
                object of utility class
        first_node : float
                value to constrain first period to

        Returns
        -------
```

```
    tuple
            (new mitigation array, storage tree of changes in consumption, nd

    """
    #fix the first period
    fixed_values = np.array([first_node])
    fixed_indicies = np.array([0])
    ga_model = GeneticAlgorithm(pop_amount=150, num_generations=100, cx_prob=0.
                                                        num_feature=m_size,
                                                        fixed_indicies=fixe

    gs_model = GradientSearch(var_nums=m_size, utility=utility, accuracy=1e-7,
                                                  iterations=250, fixed_val
                                                  print_progress=True)
    #run opt again
    final_pop, fitness = ga_model.run()
    sort_pop = final_pop[np.argsort(fitness)][::-1]
    new_m, new_utility = gs_model.run(initial_point_list=sort_pop, topk=1)
    return new_m
```

find_ir: Find the price of a bond that creates equal utility at time 0 as adding ‘payment‘ to the value of consumption in the final period. The purpose of this function is to find the interest rate. embedded in the ‘EZUtility‘ model. The first variable here is the utility with the final payment and the second variable is the utility with the initial payment.
find_term_structure: Find the price of a bond that creates equal utility at time 0 as adding ‘payment‘ to the value of consumption just before the final period. The purpose of this function is to find the interest rate. embedded in the ‘EZUtility‘ model. The first variable here is the utility with fix payment just before the final period and the second variable is the utility with the initial payment.
find_bec: Used to find a value for consumption that equalizes utility at time 0 in two different solutions. The first variable here is utility with one init consumption and the second variable is the utility with another init consumption
perpetuity_yield: Find the yield of a perpetuity starting at year ‘start_date‘. The first variable here is the final price and the second is $\frac{\frac{100}{(perp\_yield+100.)^{start\_date}}*(perp\_yield+100)}{perp\_yield}$
All of the function above is finding a point on the axis where the first variable equals the second using brentq method from scipy package.

```
def find_ir(m, utility, payment, a=0.0, b=1.0):
        """Find the price of a bond that creates equal utility at time 0 as adding
        consumption in the final period. The purpose of this function is to find
        embedded in the `EZUtility` model.

        Parameters
        ----------
```

```
        m : ndarray or list
                array of mitigation
        utility : `Utility` object
                object of utility class
        payment : float
                value added to consumption in the final period
        a : float, optional
                initial guess
        b : float, optional
                initial guess - f(b) needs to give different sign than f(a)

        Returns
        -------
        tuple
                result of optimization

        Note
        ----
        requires the 'scipy' package

        """

        def min_func(price):
                utility_with_final_payment = utility.adjusted_utility(m, final_cons
                first_period_eps = payment * price
                utility_with_initial_payment = utility.adjusted_utility(m, first_pe
                return utility_with_final_payment - utility_with_initial_payment

        return brentq(min_func, a, b)

def find_term_structure(m, utility, payment, a=0.0, b=1.5):
        """Find the price of a bond that creates equal utility at time 0 as adding
        consumption just before the final period. The purpose of this function is
        embedded in the `EZUtility` model.

        Parameters
        ----------
        m : ndarray or list
                array of mitigation
        utility : `Utility` object
                object of utility class
        payment : float
                value added to consumption in the final period
        a : float, optional
                initial guess
```

```
        b : float, optional
                initial guess - f(b) needs to give different sign than f(a)

        Returns
        -------
        tuple
                result of optimization

        Note
        ----
        requires the 'scipy' package

        """

        def min_func(price):
                period_cons_eps = np.zeros(int(utility.decision_times[-1]/utility.p
                period_cons_eps[-2] = payment
                utility_with_payment = utility.adjusted_utility(m, period_cons_eps=
                first_period_eps = payment * price
                utility_with_initial_payment = utility.adjusted_utility(m, first_pe
                return  utility_with_payment - utility_with_initial_payment

        return brentq(min_func, a, b)

def find_bec(m, utility, constraint_cost, a=-0.1, b=1.5):
        """Used to find a value for consumption that equalizes utility at time 0 

        Parameters
        ----------
        m : ndarray or list
                array of mitigation
        utility : `Utility` object
                object of utility class
        constraint_cost : float
                utility cost of constraining period 0 to zero
        a : float, optional
                initial guess
        b : float, optional
                initial guess - f(b) needs to give different sign than f(a)

        Returns
        -------
        tuple
                result of optimization
```

```
        Note
        ----
        requires the 'scipy' package

        """

        def min_func(delta_con):
                base_utility = utility.utility(m)
                new_utility = utility.adjusted_utility(m, first_period_consadj=delt
                print(base_utility, new_utility, constraint_cost)
                return new_utility - base_utility - constraint_cost

        return brentq(min_func, a, b)

def perpetuity_yield(price, start_date, a=0.1, b=10.0):
        """Find the yield of a perpetuity starting at year `start_date`.

        Parameters
        ----------
        price : float
                price of bond ending at `start_date`
        start_date : int
                start year of perpetuity
        a : float, optional
                initial guess
        b : float, optional
                initial guess - f(b) needs to give different sign than f(a)

        Returns
        -------
        tuple
                result of optimization

        Note
        ----
        requires the 'scipy' package

        """

        def min_func(perp_yield):
                return price - (100. / (perp_yield+100.))**start_date * (perp_yield

        return brentq(min_func, a, b)
```

2.1 Climate Output Class

Calculate and save output from the EZ-Climate model

2.1.1 Inputs and Outputs

Inputs:

- **utility** : ('Utility' object) object of utility class

Outputs: Calculated values based on optimal mitigation. For every **node** the function calculates and saves:

- average mitigation
- average emission
- GHG level
- SCC

For every **period** the function also calculates and saves:

- expected SCC/price
- expected mitigation
- expected emission

2.1.2 Attributes

- **utility** : ('Utility' object) object of utility class
- **prices** : ndarray SCC prices
- **ave_mitigations** : (ndarray) average mitigations
- **ave_emissions** : (ndarray) average emissions
- **expected_period_price** : (ndarray) expected SCC for the period
- **expected_period_mitigation** : (ndarray) expected mitigation for the period
- **expected_period_emissions** : (ndarray) expected emission for the period

```
def calculate_output(self, m):
        """Calculated values based on optimal mitigation. For every **node** the f

                * average mitigation
                * average emission
                * GHG level
                * SCC
```

```
as attributes.

For every **period** the function also calculates and saves

        * expected SCC/price
        * expected mitigation
        * expected emission

as attributes.

Parameters
----------
m : ndarray or list
        array of mitigation

"""

bau = self.utility.damage.bau
tree = self.utility.tree
periods = tree.num_periods

self.prices = np.zeros(len(m))
self.ave_mitigations = np.zeros(len(m))
self.ave_emissions = np.zeros(len(m))
self.expected_period_price = np.zeros(periods)
self.expected_period_mitigation = np.zeros(periods)
self.expected_period_emissions = np.zeros(periods)
additional_emissions = additional_ghg_emission(m, self.utility)
self.ghg_levels = self.utility.damage.ghg_level(m)

for period in range(0, periods):
        years = tree.decision_times[period]
        period_years = tree.decision_times[period+1] - tree.decision_times[
        nodes = tree.get_nodes_in_period(period)
        num_nodes_period = 1 + nodes[1] - nodes[0]
        period_lens = tree.decision_times[:period+1]

        for node in range(nodes[0], nodes[1]+1):
                path = np.array(tree.get_path(node, period))
                new_m = m[path]
                mean_mitigation = np.dot(new_m, period_lens) / years
                price = self.utility.cost.price(years, m[node], mean_mitiga
                self.prices[node] = price
                self.ave_mitigations[node] = self.utility.damage.average_mi
```

```
                self.ave_emissions[node] = additional_emissions[node] / (pe

            probs = tree.get_probs_in_period(period)
            self.expected_period_price[period] = np.dot(self.prices[nodes[0]:no
            self.expected_period_mitigation[period] = np.dot(self.ave_mitigatio
            self.expected_period_emissions[period] = np.dot(self.ave_emissions[

    def save_output(self, m, prefix=None):
        """Function to save calculated values in `calculate_output` in the file `p
        in the 'data' directory in the current working directory.

        The function also saves the values calculated in the utility function in t
        `prefix` + 'tree' in the 'data' directory in the current working director

        If there is no 'data' directory, one is created.

        Parameters
        ----------
        m : ndarray or list
                array of mitigation
        prefix : str, optional
                prefix to be added to file_name

        """
        utility_tree, cons_tree, cost_tree, ce_tree = self.utility.utility(m, retur

        if prefix is not None:
            prefix += "_"
        else:
            prefix = ""

        write_columns_csv([m, self.prices, self.ave_mitigations, self.ave_emissions
                        prefix+"node_period_output", ["Node", "Mitigation", "Pri
                        "Average Emission", "GHG Level"], [range(len(m))])

        append_to_existing([self.expected_period_price, self.expected_period_mitiga
                                        prefix+"node_period_output", header
                                        "Expected Emission"], index=[range(

        store_trees(prefix=prefix, Utility=utility_tree, Consumption=cons_tree,
                Cost=cost_tree, CertainEquivalence=ce_tree)
```

2.2 Risk Decomposition Class

new risk decomposition method, need the new paper to document it.

```
class RiskDecomposition(object):
        """Calculate and save analysis of output from the EZ-Climate model.

        Parameters
        ----------
        utility : `Utility` object
                object of utility class

        Attributes
        ----------
        utility : `Utility` object
                object of utility class
        sdf_tree : `BaseStorageTree` object
                SDF for each node
        expected_damages : ndarray
                expected damages in each period
        risk_premium : ndarray
                risk premium in each period
        expected_sdf : ndarray
                expected SDF in each period
        cross_sdf_damages : ndarray
                cross term between the SDF and damages
        discounted_expected_damages : ndarray
                expected discounted damages for each period
        net_discount_damages : ndarray
                net discount damage, i.e. when cost is also accounted for
        cov_term : ndarray
                covariance between SDF and damages

        """

        def __init__(self, utility):
                self.utility = utility
                self.sdf_tree = BigStorageTree(utility.period_len, utility.decision
                self.sdf_tree.set_value(0, np.array([1.0]))

                n = len(self.sdf_tree)
                self.expected_damages = np.zeros(n)
                self.risk_premiums = np.zeros(n)
                self.expected_sdf = np.zeros(n)
                self.cross_sdf_damages = np.zeros(n)
```

```
        self.discounted_expected_damages = np.zeros(n)
        self.net_discount_damages = np.zeros(n)
        self.cov_term = np.zeros(n)

        self.expected_sdf[0] = 1.0


    def sensitivity_analysis(self, m):
        """Calculate sensitivity analysis based on the optimal mitigation.
        periods given by the utility calculations, the function calculate

                * discount prices
                * net expected damages
                * expected damages
                * discounted expected damages
                * risk premium
                * cross SDF & damages
                * covariance between SDF and damages

        as attributes.

        Parameters
        ----------
        m : ndarray or list
                array of mitigation
        utility : `Utility` object
                object of utility class
        prefix : str, optional
                prefix to be added to file_name

        """

        utility_tree, cons_tree, cost_tree, ce_tree = self.utility.utility(
        cost_sum = 0
        # Calculate the changes in consumption and the mitigation cost com
        self.delta_cons_tree, self.delta_cost_array, delta_utility = delta_
        # Calculate the marginal utilities
        mu_0, mu_1, mu_2 = self.utility.marginal_utility(m, utility_tree, c
        sub_len = self.sdf_tree.subinterval_len
        i = 1
        # for every period in sdf_tree (except the init point),
        for period in self.sdf_tree.periods[1:]:
                node_period = self.sdf_tree.decision_interval(period)
                period_probs = self.utility.tree.get_probs_in_period(node_p
                expected_damage = np.dot(self.delta_cons_tree[period], peri
```

```
                self.expected_damages[i] = expected_damage # calculate the

                if self.sdf_tree.is_information_period(period-self.sdf_tree
                        total_probs = period_probs[::2] + period_probs[1::2
                        mu_temp = np.zeros(2*len(mu_1[period-sub_len]))
                        mu_temp[::2] = mu_1[period-sub_len]
                        mu_temp[1::2] = mu_2[period-sub_len]
                        sdf = (np.repeat(total_probs, 2) / period_probs) *
                        period_sdf = np.repeat(self.sdf_tree.tree[period-su
                else:
                        sdf = mu_1[period-sub_len]/mu_0[period-sub_len]
                        period_sdf = self.sdf_tree[period-sub_len]*sdf

                self.expected_sdf[i] = np.dot(period_sdf, period_probs)
                self.cross_sdf_damages[i] = np.dot(period_sdf, self.delta_c
                self.cov_term[i] = self.cross_sdf_damages[i] - self.expecte

                self.sdf_tree.set_value(period, period_sdf)

                if i < len(self.delta_cost_array):
                        self.net_discount_damages[i] = -(expected_damage +
                        cost_sum += -self.delta_cost_array[i, 1] * self.exp
                else:
                        self.net_discount_damages[i] = -expected_damage * s

                self.risk_premiums[i] = -self.cov_term[i]/self.delta_cons_t
                self.discounted_expected_damages[i] = -expected_damage * se
                i += 1

def save_output(self, m, prefix=None):
        """Save attributes calculated in `sensitivity_analysis` into the f
        in the `data` directory in the current working directory.

        Furthermore, the perpetuity yield, the discount factor for the las
        expected damage and risk premium for the first period is calculat
        prefix + `tree` in the `data` directory in the current working di
        one is created.

        Parameters
        ----------
        m : ndarray or list
                array of mitigation
        prefix : str, optional
                prefix to be added to file_name
```

```python
"""
end_price = find_term_structure(m, self.utility, 0.01)
perp_yield = perpetuity_yield(end_price, self.sdf_tree.periods[-2])

damage_scale = self.utility.cost.price(0, m[0], 0) / (self.net_disc
scaled_discounted_ed = self.net_discount_damages * damage_scale
scaled_risk_premiums = self.risk_premiums * damage_scale

if prefix is not None:
        prefix += "_"
else:
        prefix = ""

write_columns_csv([self.expected_sdf, self.net_discount_damages, se
                       self.cross_sdf_damages, self.discounted_expe
                       scaled_discounted_ed, scaled_risk_premiums],
                                   ["Year", "Discount Prices", "Net
                                    "Cross SDF & Damages", "Discoun
                                    "Scaled Risk Premiums"], [self.

append_to_existing([[end_price], [perp_yield], [scaled_discounted_e
                        [self.utility.cost.price(0, m[0], 0)]], pre
                        header=["Zero Bound Price", "Perp Yield", "
                                        "SCC"], start_char='\n')

store_trees(prefix=prefix, SDF=self.sdf_tree, DeltaConsumption=self
```

2.3 Constraint Analysis Class

2.4 Output

- Constraint Cost (optimum cost - cost with constraint on first period)
- Delta Consumption (value for consumption that equalizes utility at time 0 in two different solutions)
- Delta Consumption $b (Delta Consumption * consumption per ton constant / emit level on init point
- Delta Emission Gton (optimum emit level on the init point)
- Deadweight Cost (Delta Consumption * consumption per ton constant / optimum mitigation level on init point)
- Marginal Impact Utility (change of utility when add 0.01 on the init consumption)

- Marginal Benefit Emissions Reduction (change when change mitigation level on init point by 0.01 / change when change consumption level on init point by 0.01 * consumption per ton constant
- Marginal Cost Emission Reduction (change on price when add constraint on the first period)

where consumption per ton constant is equal to $\frac{cons\_at\_0}{emit\_at\_0}$

```
class ConstraintAnalysis(object):
        def __init__(self, run_name, utility, const_value, opt_m=None):
                self.run_name = run_name
                self.utility = utility
                self.cfp_m = constraint_first_period(utility, const_value, utility.
                self.opt_m = opt_m
                if self.opt_m is None:
                        self.opt_m = self._get_optimal_m()

                self.con_cost = self._constraint_cost()
                self.delta_u = self._first_period_delta_udiff()

                self.delta_c = self._delta_consumption()
                self.delta_c_billions = self.delta_c * self.utility.cost.cons_per_t
                                                                        * self.utility.dama
                self.delta_emission_gton = self.opt_m[0]*self.utility.damage.bau.em
                self.deadweight = self.delta_c*self.utility.cost.cons_per_ton / sel

                # adjusted benefit when +0.01 to the mitigation level at time zero
                self.delta_u2 = self._first_period_delta_udiff2()
                self.marginal_benefit = (self.delta_u2 / self.delta_u) * self.utili
                self.marginal_cost = self.utility.cost.price(0, self.cfp_m[0], 0)

        def _get_optimal_m(self):
                try:
                        header, index, data = import_csv(self.run_name+"_node_perio
                except:
                        print("No such file for the optimal mitigation..")
                return data[:, 0]

        def _constraint_cost(self):
                opt_u = self.utility.utility(self.opt_m)
                cfp_u = self.utility.utility(self.cfp_m)
                return opt_u - cfp_u

        def _delta_consumption(self):
                return find_bec(self.cfp_m, self.utility, self.con_cost) # value fo
```

```
def _first_period_delta_udiff(self):
        u_given_delta_con = self.utility.adjusted_utility(self.cfp_m, first
        cfp_u = self.utility.utility(self.cfp_m)
        return u_given_delta_con - cfp_u

def _first_period_delta_udiff2(self):
        m = self.cfp_m.copy()
        m[0] += 0.01 # adjusted with a fixed number
        u = self.utility.utility(m)
        cfp_u = self.utility.utility(self.cfp_m)
        return u - cfp_u

def save_output(self, prefix=None):
        if prefix is not None:
                prefix += "_"
        else:
                prefix = ""

        write_columns_csv([self.con_cost, [self.delta_c], [self.delta_c_bil
                                   [self.deadweight], self.delta_u,
                                   prefix + self.run_name + "_const
                                  ["Constraint Cost", "Delta Consum
                                   "Delta Emission Gton", "Deadweig
                                   "Marginal Benefit Emissions Redu
```