This note outlines potential analysis of the model developed by Daniel, Litterman, and Wagner (2016)

1. Suboptimal mitigation policies

The utility-based framework of the model allows to analyse suboptimal mitigation strategies and quantify the cost (in dollars) of not acting optimally. Examples of suboptimal strategies include the following

- mitigation is postponed, and instead of starting in 2015, mitigation starts in 2030. How much would this procrastination cost?
- the strategy is not fully implemented (i.e., instead of the optimal \$40 tax at time 0 it charges only \$20). How severe is this change in strategy?

This analysis is somewhat similar to the sensitivity analysis that we are currently doing (sensitivity to parameter estimates), but in this case we study welfare sensitivity to the *entire* strategy. This analysis is quite relevant after the US withdrew from the Paris Climate Agreement. Emissions remain the same but the number of countries willing to take actions is now reduced implying that it could be hard to implement the optimal strategy. The following approach will allow us to make the following type of statements: "Postponing mitigation until 2030 is equivalent to giving up 20% of today's consumption (world GDP)."

2. Clarifying example

For each strategy (optimal or abritrary) we can evaluate its utility and the utility for the optimal strategy is the largest (because the strategy is optimal). Assume that the state space is 2-dimensional, that is, expected lifetime utility of consumption C depends on 2 variables x_1 and x_2 and we explicitly write that it also depends on time-0 consumption c_0 as a parameter because c_0 is fixed. Thus, we write $\mathbb{E}[U(C)] = \mathbb{E}[U(C(x_1, x_2; c_0))]$. Let $x^* = (x_1^*, x_2^*)$ be the point that yields the maximum of expected lifetime utility (i.e., the optimal strategy), $x^{sub} = (x_1^{sub}, x_2^{sub})$ be any other strategy which will be referred to as a suboptimal strategy.[1] Now assume

$$\mathbb{E}[U(C(x^*; c_0))] = 10, \quad \mathbb{E}[U(C(x^{sub}; c_0))] = 9. \tag{2.1}$$

Since the utility function is unitless it is hard to say how bad the suboptimal strategy is. The only thing that we can say is that the startegy x^{sub} is worse than the optimal strategy because it yields lower expected utility. However, we do not know whether x^{sub} is nearly as good as the optimal strategy in some sense.

To determine how worse a given suboptimal strategy is, we somehow have to compare expected utilities in dollar terms. This task turns out to be relatively straightforward.

[1] A suboptimal strategy could be any strategy other than the optimal strategy. For example, $x^{sub} = x^* + (0, 1)$.

First, evaluate expected utility $\mathbb{E}[U(C(x^{sub}; c_0))]$ for a suboptimal strategy. By definition we have $\mathbb{E}[U(C(x^{sub}; c_0))] < \mathbb{E}[U(C(x^*; c_0))]$. Second, solve for z the following equation:

$$\mathbb{E}[U(C(x^{sub}; c_0))] = \mathbb{E}[U(C(x^*; c_0 - z))]. \tag{2.2}$$

Loosely speaking this equation says that acting suboptimally is equivaluent to giving up z dollars of consumption today and then act optimally. This gives us the cost z of following a suboptimal strategy. Obviously, for the above example (see assumption (2.1)), z could be any positive number and small z indicates that the suboptimal strategy is nearly as good as the optimal one. The conclusion depends on the underlying probability distribution and on how U and C depend on c_0 and on (x_1, x_2), i.e., the control variables.

It is imporant to emphasize that one has to maintain x at the optimal level (as appears on the right-hand side of (2.2)) while solving for z in (2.2). In other words, for each choice of z one has to re-evaluate the optimal strategy x^* because the strategy that is optimal for c_0 does not have to be optimal for $c_0 - z$. Therefore, one has to find x^* and z such that $\mathbb{E}[U(C(x^*; c_0 - z))]$ is equal to the expected utility achieved by following a suboptimal strategy. This can be achieved by solving, for example,

$$\min_z \left(\max_x \mathbb{E}[U(C(x; c_0 - z))] - \mathbb{E}[U(C(x^{sub}; c_0))] \right)^2 . \tag{2.3}$$

Notice that $\mathbb{E}[U(C(x^{sub}; c_0))]$ is just a constant. Based on economic reasoning it could be informative to notice that the function

$$J(c_0) := \max_x \mathbb{E}[U(C(x; c_0))] \tag{2.4}$$

is the maximum expected utility that one can achieve for a given level of today's consumption. Thus, $J'(c_0) > 0$ because one should expect higher expected utility if we start with higher consumption.

Although in a different context, but there have been a few studies on the optimal portfolio choice that surprisignly reported rather small losses for some strategies that are quite distinct from the optimal strategy (Larsen and Munk (2012)). This type of analysis is well-known in the literature, see for example, Das and Uppal (2004), Liu and Pan (2003), among others.

References

[1] Daniel, K., Litterman, R., Wagner, G.: Applying Asset Pricing Theory to Calibrate the Price of Climate Risk. Working Paper (2016)

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[3] Larsen, L., Munk, C.: The Cost of Suboptimal Dynamic Asset Allocation: General Results and Applications to Interest Rate Risk, Stock Volatility Risk, and Growth/Value Tilts. Journal of Economic Dynamics and Control 36(2), 266–293 (2012)

[4] Liu, J., Pan, J.: Dynamic Derivative Startegies. Journal of Financial Economics 69 (3), 401–430 (2003)