

APPLYING ASSET PRICING THEORY TO CALIBRATE THE PRICE OF CLIMATE RISK

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Pricing greenhouse gas emissions involves making trade-offs between consumption today and unknown damages in the (distant) future. The optimal carbon dioxide (CO₂) price, thus, is based on society's willingness to substitute consumption across time and across uncertain states of nature. Standard constant relative risk aversion preference specifications, meanwhile, conflate the two. Moreover, they are inconsistent with observed asset valuations, based on a large body of work in macroeconomics and finance. This literature has developed a richer set of preferences that are more consistent with asset price behavior and separate risk across time and across states of nature.

In this paper, we explore the implications of these richer preference specifications for the optimal CO₂ price. We develop the EZ-Climate model, a simple discrete-time optimization model in which the representative agent has an Epstein-Zin preference specification, and in which uncertainty about the effect of CO₂ emissions on global temperature and on eventual damages is gradually resolved over time. We embed a number of features including potential tail risk, exogenous and endogenous technological change, and backstop technologies. In contrast to most modeled carbon price paths, the EZ-Climate model suggests a high optimal carbon price today that is expected to decline over time. It also points to the importance of backstop technologies and to potentially very large deadweight costs of delay. We decompose the optimal carbon price into two components: expected discounted damages and the risk premium. *JEL code:* D81, G11, Q54.

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I. INTRODUCTION

Each ton of carbon dioxide (CO₂) and other greenhouse gasses (GHGs) released into the atmosphere leads to global warming, ocean acidification, and other ecological degradation—all of which impacts societal well-being. The relationship between these damages and GHG emissions is uncertain. The problem will not solve itself without government intervention, as property rights to the atmosphere are poorly defined (Coase, 1960). However, following Pigou (1920), optimal usage of the atmosphere’s capacity to absorb GHGs can be obtained, in both theory and practice, when individuals are charged the full social cost of each ton they emit into the atmosphere, or conversely the benefits that accrue to society with the reduction of GHG emissions by one ton. The cost of putting an additional ton of CO₂ into the atmosphere at any given time t , assuming an optimal emissions reductions pathway in the future, is commonly known as the optimal CO₂ price.¹ This paper builds on a long literature in addressing the determination of that price path.

We approach climate change as a standard asset pricing problem. CO₂ in the atmosphere is an ‘asset’—albeit one with negative payoffs. The modern approach to asset pricing recognizes that the optimal CO₂ price is determined by appropriate discounting of the marginal benefits of reducing emissions by one ton at all future times and across all states of nature (Duffie, 2010; Hansen and Richard, 1987). In practice this can be done by discounting those future benefits by a stochastic discount factor appropriate to each possible outcome. This stands in contrast to standard carbon pricing approaches, often based on a constant-relative risk aversion (CRRA)² preference specification inconsistent with the evidence from financial economics.

The valuation assigned to different traded assets suggests that society is willing to pay only a small premium to substitute consumption across time, but a large premium to substitute across different states of nature. For example, between 1871 and 2012, a portfolio of U.S. bonds earned an average annual real return of 1.6 percent, and a diversified portfolio in U.S. stocks earned an average annual real return of 6.4 percent.

Conversely, society is willing to pay handsomely for the right pattern of cash flows across states: these numbers imply that a portfolio which was short US equities, providing

¹ The assumption of an optimal emissions reductions pathway beginning at $t = 0$ is an important assumption in this and similar modeling exercises (e.g., Nordhaus, 2013, 1992; Nordhaus and Sztorc, 2013). It is distinct from efforts that calculate the ‘Social Cost of Carbon’ (SCC), which typically assume no such path. Instead, the SCC focuses on pricing the marginal ton of emissions given the current trajectory (U.S. Government Interagency Working Group on Social Cost of Carbon, 2015). For a recent overview and assessment, see: National Academies of Sciences (2017). Also note that, throughout this paper, we calculate the optimal price of a ton of CO₂ as opposed to the price of a ton of carbon (C). Given the molecular weights of carbon and oxygen, \$100 per ton of CO₂ is equal to about \$27 per ton of C (= (12/44) \$100). Lastly, while we discuss GHGs more broadly, the calibration itself is, in fact, mostly applicable for long-lived climate forcers, primarily CO₂. Short-lived climate forcers necessitate their own damage function calibration, have their own marginal abatement cost curves, and will, thus, have different optimal pricing pathways (e.g., Marten and Newbold, 2012; Shindell et al., 2017).

² CRRA preferences are invariably known as constant-elasticity of substitution (CES) preferences or also as power utility, a special case of which is when utility is a natural-log function of consumption.

insurance against bad economic outcomes, earned an annual return of *negative* 4.8% per year over this long period. Society presumably discounts equity payoffs at a far higher discount rate because equities earn large returns in good economic times (when marginal utility is low) but often perform poorly precisely when economic growth is low (and marginal utility is high).³

The *high* historical equity premium, combined with the low historical volatility of consumption growth, suggests that society is unwilling to substitute consumption across states of nature at some future point in time. In contrast, the *low* risk-free rate in combination with the high average consumption growth rate over the past 150 years suggests that agents are far more willing to substitute consumption across time. These two empirical regularities are typically seen to be inconsistent with a CRRA preference specification.⁴

Our model uses a state-contingent discount rate, calibrated to the returns over time of financial assets. In contrast to the standard CRRA utility function used in most climate-economic studies, we use here an Epstein-Zin utility function (Epstein and Zin, 1991, 1989; Kreps and Porteus, 1978; Weil, 1990) used throughout the asset pricing literature. We henceforth call it the “EZ-Climate” model. It has CRRA utility as a special case and also allows for differences between the elasticity of intertemporal substitution (EIS)⁵ and risk aversion, which allows us to calibrate to standard financial returns, in particular the equity risk premium and risk-free interest rates.

The climate-economic literature has increasingly recognized the importance of exploring a richer set of preferences in calibrating climate risk. An earlier contribution by Ha-Duong and Treich (2004) explores the importance of Epstein-Zin preferences in general. Ackerman, Stanton, and Bueno (2013) extend the well-known dynamic integrated climate-economy (DICE) model to incorporate Epstein-Zin preferences and find a significant increase in the optimal CO₂ price as a result. Other important contributions include work by Christian Traeger, Derek Lemoine and co-authors⁶ as well as Cai, Lenton, and Lontzek (2016) and Belaia, Funke, and Glanemann (2017). One broad—perhaps somewhat unfair—conclusion from that work is that the added modeling sophistication and computational power needed compared to, for example, the standard DICE model, may not be justified. Both Crost and Traeger (2014) and Cai *et al.* (2016) find a small role for climate risk in the final optimal CO₂ price figure, which Crost and Traeger (2014) hypothesize might be due to a failure to properly account for climatic disasters. Belaia *et al.* (2017) focus on the effect of incorporating the Atlantic Thermohaline Circulation

³ Based on data collected by Shiller (2000) and since continuously updated: <http://www.econ.yale.edu/~shiller/data.htm>

⁴ These irregularities feature prominently in the equity premium, risk-free rate, and equity volatility puzzles (e.g., Bansal and Yaron, 2004; Mehra and Prescott, 1985; Weil, 1989). See Rietz (1988) for an early and oft-cited yet often overlooked contribution to the contrary, focusing on extreme events as the driver of the puzzle. For further explorations following this line of thought, see, e.g., Barro (2009, 2006), Barro and Jin (2011), Barro and Ursua (2008), and Weitzman (2007a).

⁵ Also invariably known as the intertemporal elasticity of substitution (IES) and the intertemporal marginal rate of substitution (IMRS).

⁶ See e.g. Crost and Traeger (2014), Jensen and Traeger (2014), Lemoine and Traeger (2014), Lemoine and Traeger (2016), Lemoine (2015), and especially Traeger (2015).

(THC) tipping point. They, too find that risk aversion is of less importance compared with other factors: “the assumption of higher risk aversion neither changes the near-term policy in a qualitatively meaningful way nor does it affect the THC dynamics to any great extent” (Belaia et al., 2017). Underplaying extreme climatic events may indeed be the root cause for the relatively low role of risk aversion, something that holds both in finance and with climatic events.⁷

DICE itself, meanwhile, does incorporate the costs of climatic disasters (Nordhaus, 2016, 2013). In particular, ever since the 1999 version of DICE, the willingness to avoid “catastrophic” damages has accounted for around two-thirds of total economic damages (Kopp et al., 2016). Nordhaus and Boyer (2000) incorporate risk based on an expert survey, asking scientists and economists to assess the probability of a “Great Depression”-size 25% loss of global income due to global average temperatures increasing by 3°C and 6°C (Nordhaus, 1994).⁸ That said, DICE, and integrated assessment models like it are simply not equipped to reflect the costs of distant tipping points like the THC. The tipping point occurs too far in the future to matter to today’s policy decisions, in part because of discounting, in part because of lack of inertia built into the model (Belaia et al., 2017).⁹

One important lesson to take from prior attempts to build on DICE or introduce alternative models—one also echoed by the National Academy of Sciences (2017)—is the importance of ‘keeping it simple’.¹⁰ DICE has long set the standard for climate-economy modeling because of its simplicity. The core of the model has fewer than 20 equations (Nordhaus and Sztorc, 2013). Implementations of Epstein-Zin utility functions typically do not lend themselves to this level of simplicity, ease of use, and transparency. It also means DICE needs to cut corners. In fact, Nordhaus and Sztorc (2013) cite Epstein and Zin (1991, 1989), but only while warning not to confuse a CRRA utility as representing risk aversion explicitly. The implementation of DICE reverts to the CRRA utility function.

In keeping with this missive for simplicity, we introduce an optimal control problem employing a discrete-time binomial tree model. This approach will be intensely familiar to financial economists, who employ such trees in many financial-economic modeling applications (see Cox, Ross and Rubinstein (1979) for an early example). We build on a related approach in Summers and Zeckhauser (2008), who employ a simple three-period

⁷ For the former, see references in footnote 4. For the latter, see, e.g., Barro (2015), Weitzman (2009, 2007b), and Wagner and Weitzman (2015).

⁸ See Kopp *et al.* (2016) for a discussion and critique of this methodology. See e.g. Brock and Hansen (2017), Burke et al. (2016), Convery and Wagner (2015), Pindyck (2013), Stern (2013), and Wagner and Weitzman (2015) for extensive discussions and critiques of prior treatments of risk and uncertainty in climate-economic models in general and U.S. SCC calculations in particular. See Weitzman (2015) for a direct response to Nordhaus’s (2013) book-length entry into this debate, and, in turn, Nordhaus’s (2015) review of Wagner and Weitzman (2015). Golosov *et al.* (2014), among others, pursues another extension of standard climate-economy models, employing a dynamic stochastic general-equilibrium (DSGE) model, while still relying on a CRRA utility function.

⁹ See Nordhaus (1991) for an early discussion of the effects of inertia in climate economics, Lemoine and Rudik (2014) for a theoretical exploration, and especially Mastrandrea and Schneider (2001) for a comprehensive discussion of the implications.

¹⁰ National Academy of Sciences (2017) emphasizes the importance of modularity of the modeling effort to ensure natural points of entry for multi-disciplinary collaborations providing crucial inputs into the core model.

model—sans Epstein-Zin preferences—to isolate the most important features of clearing up uncertainty over time.

While striving for simplicity in itself is an important goal, Figure I shows the profound implications of employing Epstein-Zin preferences: CRRA and Epstein-Zin preferences result in wildly divergent optimal CO₂ price pathways for different levels of risk calibration.¹¹

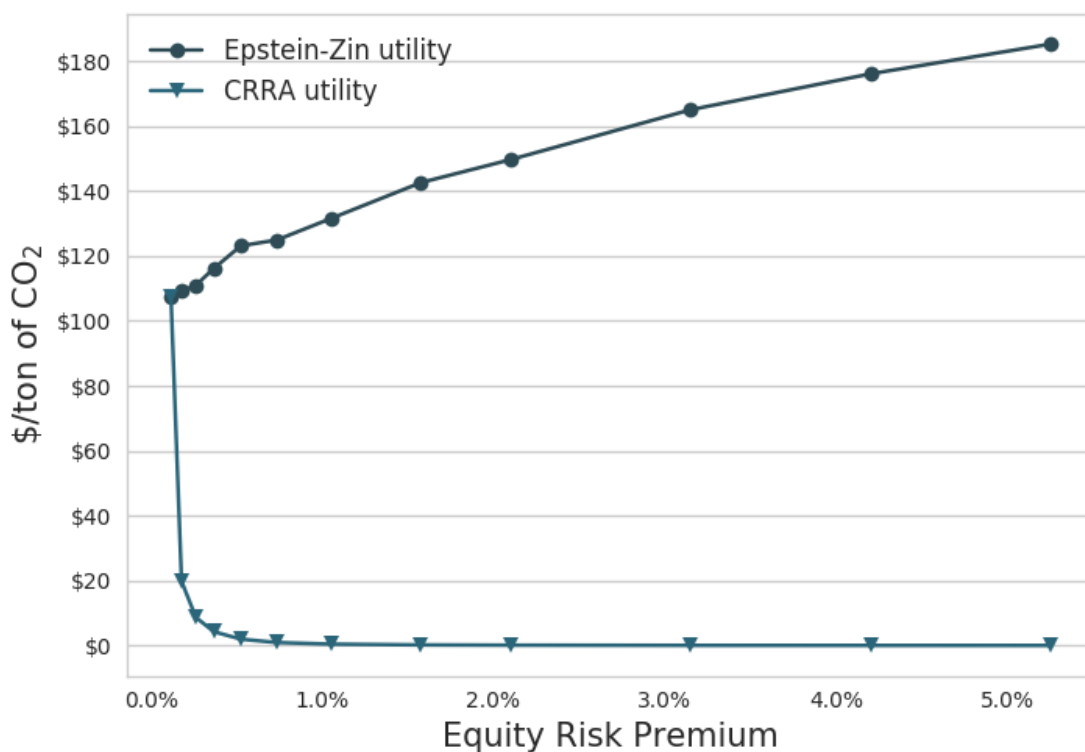


Figure I—Using Epstein-Zin utility functions results in increasing optimal 2015 CO₂ prices, in 2015 US\$, with increasing risk aversion, translated into the implied equity risk premium using Weil (1989)’s conversion, while holding implied market interest rates stable at 3.11%

Standard CRRA specifications embed the assumption that agents’ willingness to substitute consumption across states of nature is the same as their willingness substitute consumption over time. Thus, an increase in the coefficient of risk aversion (or, conversely, a decreased elasticity of substitution across states), is necessarily linked to a decreased EIS. Given the fact that consumption grows at a rate of about 1.5% per year, an unwillingness to substitute across time leads to a (counterfactually) high risk-free discount rate. Since consumption damages occur far into the future, a CRRA utility

¹¹ See part II for our EZ-Climate model setup and calibration. Our original calibration (Daniel et al., 2016), was anchored around the U.S. SCC of \$40 for a ton of CO₂ released in 2015, in 2015 US\$, the central value calculated by the U.S. Government Interagency Working Group on Social Cost of Carbon (2015). While further revising the paper, we introduced radiative forcing as a stock variable and added carbon cycle feedbacks explicitly (see section II.C). That step alone increased the 2015 number from \$40 to over \$100 in our base case. Even without any tail risks, the 2015 optimal CO₂ price never dips below \$60, well above the U.S. SCC figure. Note also that our optimal CO₂ price is distinct from the U.S. SCC (see footnote 1).

function with a high level of risk-aversion (and a reasonable rate of time preference) necessarily implies a high discount rate for these damages, and a low optimal CO₂ price—tending toward zero.

In contrast, Epstein-Zin utility allows for separation of the coefficient of risk-aversion and the EIS, consistent with the equity-premium/risk-free rate puzzle. With an Epstein-Zin specification, holding the EIS fixed at 0.9 and increasing the degree of risk aversion, the optimal CO₂ price increases, while the real interest rate remains at around 3.11%/year.¹² As the level of risk aversion is raised from a very low level to a level consistent with the historically observed equity-risk premium, the optimal CO₂ price increases by around 50% (Figure I).¹³

We first introduce the EZ-Climate model and its calibration (Section II) before presenting results, risk decomposition, and sensitivity analyses (Section III), and discussing future research and extensions (Section IV). We conclude with an analogy (Section V), grounded in the fundamental result presented by Figure I. Climate policy treated as an asset pricing problem is, after all, fundamentally about risk mitigation.

II. THE MODEL

Our representative agent solves the optimization problem of trading off the (known) costs of climate mitigation against the uncertain future benefits associated with mitigation. She maximizes lifetime utility at each time and for each state of nature by choosing the optimal path of mitigation, $x_t^*(\theta_t)$, dependent on Earth’s fragility, θ_t .

Mitigating emissions is costly. Hence, assuming no government action to price carbon, atomistic agents do zero mitigation. However, as GHGs build up in the atmosphere, temperatures rise. As a result, a fraction of the baseline consumption is lost to damages. The damages as a function of mitigation are not known ex-ante. They are, in turn, a function of θ_t . Each period of the model, agents learn more about the level of fragility, but they only know the actual fragility in the final two periods of the model.

These assumptions simplify reality in two important ways: As θ_t is the only unknown in EZ-Climate model, we do not allow for interactions of shocks to fragility with those to other state variables (e.g., productivity). The second simplification is the assumption of full knowledge of θ in period $T - 1$ (in the year 2300 in our base case). Important aspects of climate science are deeply and persistently uncertain, and science may not learn the

¹² The exact interest rate in our Epstein-Zin calibration is almost independent of the risk aversion coefficient. Bansal and Yaron (2004) are able to match the equity premium with a far lower coefficient of risk-aversion, owing to the presence of shocks to the long-term growth rate of consumption in their model, which are correlated with equity returns. Similarly, in the EZ-Climate model presented here, a link between higher climate fragility and lower consumption growth rates would lead to a higher optimal CO₂ price with a lower coefficient of risk-aversion. In general, climate damages hitting growth rates rather than levels of GDP can have a significant effect on the optimal CO₂ price (Bansal and Ochoa, 2011; Dell et al., 2012; Heal and Park, 2016; Moore and Diaz, 2015; Wagner and Weitzman, 2015).

¹³ Figure VIII in section III shows the optimal CO₂ price over time for our base case calibration of an EIS of 0.9 and $RRA = 7$.

true θ_t at a time scale relevant to policy (Wagner and Zeckhauser, 2016; Zeckhauser, 2006). We compromise by having the complete resolution of uncertainty delayed until 2300, when we might be able to expect to know the all-important climate sensitivity parameter, what happens to global average temperatures, in equilibrium, as concentrations of GHGs double from pre-industrial levels.¹⁴

The setting for the EZ-Climate model is a discrete time, endowment economy with a single representative agent. In each period $t \in \{0, 1, 2, \dots, T\}$, the representative agent is endowed with a certain amount of the consumption good, \bar{c}_t . However, she is not able to consume the full endowed consumption for two reasons: climate change and climate policy.

In periods $t \in \{1, 2, \dots, T\}$, some of the endowed consumption may be lost due to climate change damages. In periods $t \in \{0, 1, 2, \dots, T-1\}$, the agent may elect to spend some of the endowed consumption to reduce her impact on the climate. The resulting consumption c_t , after damages D_t and mitigation costs κ_t are taken into account, is given by:

$$\begin{aligned} (1) \quad & c_0 = \bar{c}_0 \cdot (1 - \kappa_0(x_0)), \\ (2) \quad & c_t = \bar{c}_t \cdot (1 - \kappa_t(x_t)) \cdot (1 - D_t(CRF_t, \theta_t)), \text{ for } t \in \{1, 2, \dots, T-1\}, \text{ and} \\ (3) \quad & c_T = \bar{c}_T \cdot (1 - D_T(CRF_T, \theta_T)). \end{aligned}$$

In equations (2) and (3), the climate damage function $D_t(CRF_t, \theta_t)$ captures the fraction of endowed consumption that is lost due to damages from climate change. If $D_t(CRF, \theta_t) = 0$, the agent would receive the full consumption endowment. However, damages from climate change can push D_t above zero. D_t , in turn, depends on two variables: CRF_t , which we define as the cumulative radiative forcing up to time t , which determines global average temperature, and θ_t , the Earth's fragility, a parameter that characterizes the uncertain relation between the global average temperature and consumption damages. Fragility θ_t evolves stochastically as described in section II.C.

Cumulative radiative forcing, CRF_t , in turn, depends on the level of mitigation in each period from 0 to t , as further discussed in Section II.C.iii. Cumulative mitigation is given by:

$$(4) \quad X_t = \frac{\sum_{s=0}^t g_s \cdot x_s}{\sum_{s=0}^t g_s},$$

where g_s is the flow of GHG emissions into the atmosphere in period s , for each period up to t , absent any mitigation.¹⁵ Cumulative mitigation, X_t , enters the determination of the rate of technological change, discussed in Section II.B.ii. The level of mitigation at any

¹⁴ The 'likely' range for climate sensitivity has been 1.5–4.5°C ever since Charney *et al.* (1979)—with one short exception. In its Fourth Assessment Report, IPCC (2007) narrowed the range to 2–4.5°C, only to be expanded back to the prior range in the Fifth Assessment Report (IPCC, 2013). The “equilibrium” climate sensitivity range is the so-called “fast” equilibrium, as distinct from Earth system sensitivity. The latter includes broader changes, which could, in turn, result in still larger changes. Previdi *et al.* (2013) present a range of 6–8°C for a doubling of atmospheric CO₂ concentrations. (See section II.C.ii.)

¹⁵ The cumulative GHG emissions that must be absorbed into the atmosphere or oceans is $G_t(1 - X_t)$, where $G_t = \sum_{s=0}^t g_s$ denotes the cumulative emissions under the BAU scenario.

time s is given by x_s , where $x_s = 0$ denotes no climate action at time s , and $x_s = 1$ denotes full mitigation, or equivalently that there is zero net flow of new GHG emissions into the atmosphere in period s .

Mitigation reduces the stock of GHGs in the atmosphere and leads to lower climate damages, and, hence, to higher future consumption. But mitigating GHG emissions is costly. Mitigating a fraction x_t of emissions costs a fraction $\kappa_t(x_t)$ of the endowed consumption. We describe the details of the cost function, and our calibration, in Section II.B.

Rather than restricting mitigation x_s to be below 1, however, in our baseline analysis we allow for the use of a backstop technology. This technology pulls CO₂ directly out of the atmosphere and is typically called carbon dioxide removal (CDR) or, confusingly, direct carbon removal (DCR). See the discussion in sections II.B.i on backstop technologies and II.C.iv on the resulting possibility of having concentrations fall below 280 ppm.

To make the solution tractable, the EZ-Climate model employs a binominal tree for the resolution of uncertainty about climate damage, discussed in detail in Section II.D. The baseline analysis uses a 7-period tree, beginning in 2015. An initial mitigation decision is made in 2015, and subsequent mitigation decisions are made after information is revealed about climate fragility and the resulting damages in years 2030, 2060, 2100, 2200, and 2300. The final period, in which consumption simply grows at a constant rate, begins in 2400 and lasts forever. At each node of the tree, more information about the consumption damage function is revealed (as reflected in the fragility parameter θ_t), but uncertainty is not fully resolved until the beginning of the next-to-last period in 2300.¹⁶ The agent's utility in each state is calculated based on interpolated consumption flows at five-year sub-periods. We solve for mitigation levels over time that maximize expected utility, looking forward, at the start of each period (except the final period), and in each fragility state θ_t . The resulting optimal CO₂ price in each period and state is the price that implements this level of mitigation.

In the next section, we describe the agent's preferences, and provide some additional motivation for the preferences specification we employ. In Sections II.B and II.C, we lay out the cost and damage functions, respectively, and describe their calibration. In keeping with the National Academy of Science's (2017) call for modularity in climate-economy modeling efforts, these calibrations are largely independent of each other and could easily be swapped for different, better calibrations. Section II.D describes EZ-Climate's tree structure in more detail. It, too, can be readily adjusted, in keeping with the modularity of EZ-Climate. Section 3 presents the results based on our calibrations. Section V concludes.

¹⁶ Possibly unique among climate-economy models, our setup creates the possibility of GHG concentrations going below 280 ppm—if, in the most optimistic draws in the final stage of our tree structure, fragility θ_t turns out to be much less bad than assumed in expectations. We, thus, include an increasing damage function for concentrations below a certain level in EZ-Climate, also perhaps unique among climate-economy models. See Section II.C for a more detailed description of climate damages.

II.A. PREFERENCES

As noted in the introduction, the CRRA specification typically used in climate-economy models like DICE embeds the assumption that agents' willingness to substitute consumption across states of nature is the same as their willingness to substitute consumption over time. This is inconsistent with the observed low risk-free rate and high equity premium (Mehra and Prescott, 1985; Weil, 1989). To resolve this puzzle, financial economists have begun to employ the preference specification suggested by Epstein and Zin (1991, 1989) and Weil (1990) that allows for different rates of substitution across time and states.¹⁷ This is the specification underlying EZ-Climate.

In an Epstein-Zin utility framework, the agent maximizes at each time t :

$$(5) \quad U_t = [(1 - \beta)c_t^\rho + \beta[\mu_t(\tilde{U}_{t+1})]^\rho]^{1/\rho},$$

where $\mu_t(\tilde{U}_{t+1})$ is the certainty-equivalent of future lifetime utility, based on the agent's information at time t , and is given by:

$$(6) \quad \mu_t(\tilde{U}_{t+1}) = (E_t[U_{t+1}^\alpha])^{1/\alpha}.$$

In this specification, $(1 - \beta)/\beta$ is the pure rate of time preference, commonly denoted by δ . The parameter ρ measures the agent's willingness to substitute consumption across time. The higher is ρ , the more willing the agent is to substitute consumption across time. The elasticity of intertemporal substitution is given by $\sigma = 1/(1 - \rho)$.

Finally, α captures the agent's willingness to substitute consumption across (uncertain) future consumption streams. The higher is α , the more willing the agent is to substitute consumption across states of nature at a given point in time. The coefficient of relative risk aversion at a given point in time is $\gamma = (1 - \alpha)$. This added flexibility allows for

¹⁷ See Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) for more detailed discussions. Bansal and Ochoa (2011, 2009) and Bansal, Ochoa and Kiku (2016) use this preference specification in combination with a framework in which temperature shocks affect future consumption growth. For example, Ackerman, Stanton, and Bueno (2013), Crost and Traeger (2014), and, most recently, Cai, Lenton, and Lontzek (2016) use this utility function in DICE. Making no further adjustments, doing so poses oft-significant computational challenges. Ha-Duong and Treich (2004) appear to have been the first to encounter those challenges. They explore the importance of Epstein-Zin preferences in calibrating climate risk in general and conclude, somewhat too cautiously, that setting $\rho = \alpha$ "may misinterpret the sensitivity of the climate policy to risk-aversion." Importantly, Ha-Duong and Treich (2004) explore a more general implementation of Epstein-Zin preferences, turning c_t also into a function equal to the certainty-equivalent of future lifetime income. Their equivalent of our equation (5) could have been written more generally as $U_t = [(1 - \beta)[\mu_t(\tilde{c}_t)]^\rho + \beta[\mu_t(\tilde{U}_{t+1})]^\rho]^{1/\rho}$, with the definition of $\mu_t(\tilde{c}_{t+1})$ mirroring our equation (6), for the current period t : $\mu_t(\tilde{c}_t) = (E_t[\tilde{c}_t^\alpha])^{1/\alpha}$. The difference is subtle but potentially important. Ha-Duong and Treich's (2004) extension allows for consumption to be uncertain within each period. Our and others' implementation of Epstein-Zin preferences in form of (5) and (6) implies full knowledge of each period's consumption at the time of the mitigation decision. This is a reasonable assumption for short time intervals. As those intervals become larger, within-period uncertainty might become more important.

calibration across states of nature and time. With $\rho = \alpha$, equations (5) and (6) are equivalent to the standard CRRA utility specification.

Plugging (6) into (5) results in EZ-Climate's utility specification:

$$(7) \quad U_0 = \left[(1 - \beta)c_0^\rho + \beta(E_0[\tilde{U}_1^\alpha])^{\rho/\alpha} \right]^{1/\rho}$$

$$(8) \quad U_t = \left[(1 - \beta)c_t^\rho + \beta(E_t[U_{t+1}^\alpha])^{\rho/\alpha} \right]^{1/\rho}, \text{ for } t \in \{1, 2, \dots, T - 1\}.$$

with c_0 and c_t , respectively, given by equations (1) and (2).

In the final period, which, in our base case, is the period starting in 2400, the agent receives the utility from all consumption from time T forward. Given our assumption that all uncertainty has been resolved at this point, consumption grows at a constant rate r from T through infinity:

$$(9) \quad c_t = c_T(1 + g)^{t-T} \text{ for } t \geq T.$$

The resulting final-period utility is:

$$(10) \quad U_T = \left[\frac{1 - \beta}{1 - \beta(1 + r)^\rho} \right]^{1/\rho} c_T,$$

with c_T given by equation (3).

II.B. MITIGATION COST

Calibrating the economic cost side of EZ-Climate requires specifying a relationship between the marginal cost of emissions reductions or per-ton tax rate, τ , the resulting flow of emissions in gigatonnes of CO₂-equivalent emissions per year (Gt CO₂e), $g(\tau)$, and the fraction of emissions reduced, $x(\tau)$.

Many modeling efforts have attempted to estimate the marginal abatement costs, often as part of integrated assessment models and based on a number of assumption.¹⁸ Perhaps the most influential, independent effort comes from McKinsey & Company in an attempt to estimate a bottom-up marginal abatement cost curve (MACC). McKinsey's MACCs are, to a large extent, based on bottom-up 'engineering' estimates. That makes them an easy target for critique by economists, which often focuses on the large abatement opportunities with 'negative' costs—positive net present value. McKinsey's report on energy efficiency in the United States identifies energy efficiency savings opportunities with positive net present value commensurate with emissions reductions of over 1 Gt CO₂e per year (McKinsey, 2009a). Lots of effort has gone into assessing (Gerarden et al., 2015)

¹⁸ See Stanford's Energy Modeling Forum as one such major effort, working with a number of different models: <https://emf.stanford.edu>. See Huntington (2011) as an overview of Stanford EMF's work focused on the 'energy efficiency gap'.

and helping to bridge (Gillingham and Palmer, 2014) that potential energy efficiency gap, without conclusive evidence (Allcott and Greenstone, 2012). These critiques notwithstanding, McKinsey’s effort stands as a unique, bottom-up, data-driven attempt at estimating abatement costs. We calibrate τ , $g(\tau)$, and $x(\tau)$ in EZ-Climate based on McKinsey’s global MACC effort (McKinsey, 2009b), with one crucial modification: We assume no mitigation ($x(\tau) = 0$) at $\tau \leq 0$; i.e. no net-negative or zero-cost mitigation. Table I shows the resulting calibration.¹⁹

Table I—Marginal abatement cost curve for 2030, based on McKinsey (2009b), modified to have $x(\tau)=0$ for $\tau \leq 0$.

GHG taxation rate τ	GHG emissions flow $g(\tau)$	Fractional GHG reduction $x(\tau)$
€0/ton	70 Gt CO ₂ e/year	0
€60/ton	32 Gt CO ₂ e/year	0.543
€100/ton	23 Gt CO ₂ e/year	0.671

Fitting McKinsey’s modified point estimates (in \$US using an exchange rate of 1.206 \$ per €) from Table I to a power function for $x(\tau)$ yields:

$$(11) \quad x(\tau) = 0.0923 \cdot \tau^{0.414}.$$

The corresponding inverse function, solving for the appropriate tax rate to achieve x is:

$$(12) \quad \tau(x) = 314.32 \cdot x^{2.413}.$$

Equation (12) shows the marginal cost of abatement. Ultimately, we are interested in the total cost to society, $\kappa(\tau)$, for each particular tax τ . We calculate $\kappa(\tau)$ using the envelope theorem. Intuitively, GHG emissions are an input to the production process that generates consumption goods. Assuming the agent chooses the level of GHG emissions $g(\tau)$ so as to maximize consumption c given τ , the marginal cost of increasing the tax rate must be the quantity of emissions at that tax rate, that is:

$$(13) \quad \frac{dc(\tau)}{d\tau} = -g(\tau),$$

Thus, to calculate the consumption associated with a particular tax rate of τ , we integrate (13), resulting in:

$$(14) \quad c(\tau) = \bar{c} - \int_0^\tau g(s) ds,$$

where \bar{c} is the endowed level of consumption (assuming zero climate damages). However, this equation is correct only if the GHG tax is purely dissipative—that is, if the government

¹⁹ We have emissions stabilize at 57% above current levels. In our ‘unmitigated’ baseline scenario, following the IEA’s New Policies Scenario (which does, in fact, have what we would describe as modest mitigation), GHG concentrations reach approximately 1,000 ppm by 2200.

were to collect the tax and then waste 100% of the proceeds. In our analysis, we make the opposite assumption: the proceeds of the tax ($g(\tau) \cdot \tau$) are refunded lump-sum.²⁰ That makes the decrease in consumption equal to the distortionary effect of the tax (in dollars):

$$(15) \quad K(\tau) = \int_0^\tau g(s) ds - g(\tau) \cdot \tau.$$

Writing $g(\tau) = g_0(1 - x(\tau))$, where g_0 is the baseline level of GHG emissions, we can rewrite (15) as:

$$(16) \quad \begin{aligned} K(\tau) &= g_0 \int_0^\tau (1 - x(s)) ds - \tau g_0(1 - x(\tau)) \\ &= g_0 \left[\tau - \int_0^\tau x(s) ds \right] - \tau g_0 + \tau g_0 x(\tau) \\ &= g_0 [\tau x(\tau) - \int_0^\tau x(s) ds] \end{aligned}$$

Substituting (11) into (16) and simplifying gives the total cost K as a function of the tax rate τ :

$$(17) \quad \begin{aligned} K(\tau) &= g_0 [0.09230 \cdot \tau^{1.414} - 0.06526 \cdot \tau^{1.414}] \\ &= g_0 \cdot 0.02704 \cdot \tau^{1.414}, \end{aligned}$$

Substituting (12) into (17) gives K as a function of fractional-mitigation x :

$$(18) \quad K(x) = g_0 92.08 \cdot x^{3.413},$$

where total cost $K(x)$ is expressed in dollars. Finally, we divide by current (2015) aggregate consumption to determine the cost as a fraction of baseline consumption:

$$(19) \quad \kappa(x) = \left(\frac{g_0 \cdot 92.08}{c_0} \right) \cdot x^{3.413},$$

where $g_0 = 52$ Gt CO₂e represents the current level of global annual emissions, and $c_0 = \$31$ trillion/year is current (2015) global consumption in 2015 dollars. Equation (19) expresses the societal cost of a given level of mitigation as a percentage of consumption. We assume that, absent technological change, the function $\kappa(x)$ is time invariant. See II.B.ii for our discussion of technological change. First, we explore the impact of backstop technologies.

²⁰ Note that were the proceeds from the (Pigouvian) GHG tax used to reduce other distortionary taxes, the effective cost of the carbon tax would be still lower than what we calculate here, and thus would justify a higher optimal τ . For a summary of this “double-dividend” argument, see Goulder (1995).

II.B.i. Backstop Technology

The McKinsey estimates on which our total cost function, $\kappa(x)$, are based reflect the cost of traditional abatement technologies. In addition, carbon dioxide removal (CDR) can pull CO₂, and potentially GHGs, directly from the atmosphere (National Research Council, 2015a). We label these *backstop technologies*.

We assume our backstop technology is available at a marginal cost of τ^* for the first ton of carbon that is removed from the atmosphere and that unlimited amounts of CO₂ can be removed as the marginal cost approaches $\tilde{\tau} \geq \tau^*$. In fitting the marginal cost curve to τ^* and $\tilde{\tau}$ we build a marginal cost function for the backstop technology of the form:

$$(20) \quad B(x) = \tilde{\tau} - \left(k/x\right)^{1/b}.$$

The upper bound of the cost function is, thus, $\tilde{\tau}$. We calibrate (20), such that:

$$(21) \quad B(x_0) = \tilde{\tau} - \left(k/x_0\right)^{1/b} = \tau^*,$$

which allows us to express:

$$(22) \quad k = x_0(\tilde{\tau} - \tau^*)^b,$$

where x_0 is the point at which the backstop technology begins to be used. We also impose a smooth-pasting condition at x_0 ; i.e. the derivative of the marginal cost curve is continuous at x_0 . This allows us to solve for parameter b :

$$(23) \quad b = \frac{\tilde{\tau} - \tau^*}{(\alpha - 1)\tau^*}.$$

Our base case assumes $\tau^* = \$2,000$ and $\tilde{\tau} = \$2,500$ in 2015 dollars. Under the most aggressive backstop scenario presented in the results section, we assume $\tau^* = \$300$ and $\tilde{\tau} = \$350$ in 2015 dollars. These aggressive values imply that the backstop technology kicks in at mitigation levels above around 100%, whereas our base case all but assures that backstop technologies do not get used for a considerable period of time. The \$350 is on the low end of possible assumptions, and the extent to which there can be a true backstop technology remains uncertain (Socolow et al., 2011). The need for one is clear, and much work remains to be done to demonstrate possible technologies and refine price estimates (Keith, 2000; National Research Council, 2015a).²¹ Figure II shows the marginal cost, $\tau(x)$, with both base-case and aggressive backstop technology assumptions.

²¹ Note that CDR as a backstop technology is entirely distinct from solar geoengineering, also known as ‘albedo modification’ or ‘solar radiation management’ (National Research Council, 2015b). The one commonality is that both ‘carbon geoengineering’, CDR, and the potential availability of solar geoengineering likely reduce the optimal CO₂ price. EZ-Climate models the former, not the latter.

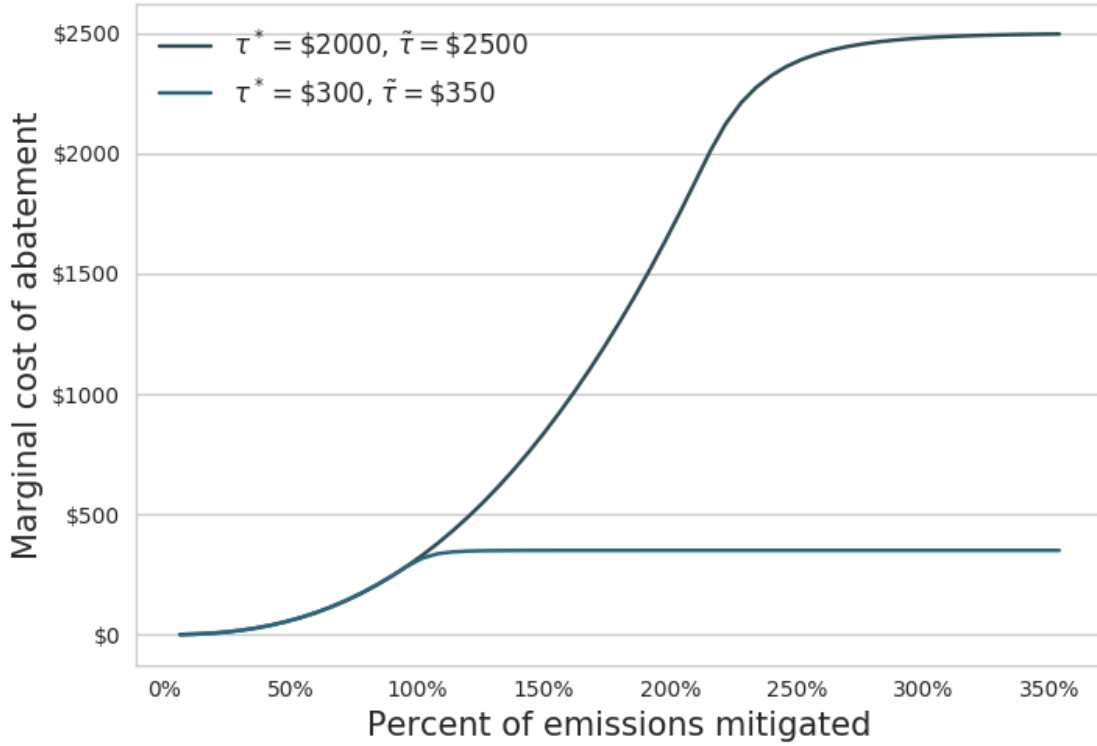


Figure II—Marginal cost of abatement, $\tau(x)$, in 2015 (in 2015 \$/ton) under base-case ($\tau^* = \$2,000$, $\tilde{\tau} = \$2,500$) and aggressive backstop technology ($\tau^* = \$300$, $\tilde{\tau} = \$350$) assumptions

II.B.ii. Technological Change

These cost curves are calibrated to $t = 0$. In subsequent periods, we allow the marginal cost curve to decrease at a rate determined by a set of technological change parameters: a constant component, φ_0 , and a component linked to mitigation efforts to date, $\varphi_1 X_t$, where X_t is the average mitigation up to time t , defined by equation (4). Thus, at time t , the total cost curve is given by:

$$(24) \quad \kappa_t(x) = \kappa(x)[1 - \varphi_0 - \varphi_1 X_t]^t.$$

This functional form allows for easy calibration. For example, if $\varphi_0 = 0.005$ and $\varphi_1 = 0.01$, and with average mitigation of 50%, marginal costs decrease as a percentage of consumption at a rate of 1% per year.

II.C. DAMAGE FUNCTION SPECIFICATION

We next specify the climate damage function $D_t(CRF_t, \theta_t)$.²² Damages are a function of temperature changes, which, in turn, are a function of cumulative solar radiative forcing

²² Our damage function calibration follows the basic logic of Pindyck (2012), with one crucial exception: Pindyck (2012) assumes gamma distributions for temperature levels given greenhouse gas concentrations,

CRF_t , which, in our setting, are determined by the mitigation path up to that point in time. We then compare CRF_t to three baseline emissions paths, g_t for which we have created associated damage simulations. The one way, then, to affect the level of damages is to change mitigation across time, x_t . The specification of damages has two components: a non-catastrophic component and an additional catastrophic component triggered by crossing a particular threshold. The hazard rate associated with hitting that threshold increases with temperature. If the threshold is crossed at any time, additional damages decrease consumption in all future periods.

We calculate the overall damage function $D_t(CRF_t, \theta_t)$ for the baseline emissions paths, g_t , using Monte-Carlo simulation. As we describe in detail below, we run a set of simulations for each of three constant mitigation levels X_t , which determine cumulative radiative forcing at each point in time. In each run of the simulation, we draw a set of random variables: [1] global average temperature change; [2] the parameter characterizing non-catastrophic damages as a function of temperature; [3] an indicator variable that determines whether or not the atmosphere hits a tipping point at any particular time and state, and [4] the tipping point damage parameter. The state variable θ_t indexes the distribution resulting from these sets of simulations, and interpolation across the three mitigation levels gives us a continuous function D_t across cumulative radiative forcing levels CRF_t .

II.C.i. Temperature as a Function of GHG Levels

The distribution of temperature outcomes as a function of mitigation strategies is calibrated to three carbon scenarios, indexed by a maximum level of CO_2 in the atmosphere. For the subsequent base case calibration, we follow Weitzman (2009) and Wagner and Weitzman (2015) in calibrating a log-normal distribution for equilibrium climate sensitivity—the eventual temperature rise as atmospheric concentrations of CO_2 double.²³ Specifically, Wagner and Weitzman (2015) calibrate a log-normal function assuming a 78% probability of climate sensitivity in the 1.5-4.5°C “likely” range.²⁴ Moreover, the Intergovernmental Panel on Climate Change (IPCC)’s Fifth Assessment Report (IPCC, 2013) judges climate sensitivity above 6°C to be “very unlikely,” giving it a 0-10% probability. We again follow Wagner and Weitzman (2015) in assigning it a roughly 5% chance.

Wagner and Weitzman (2015) then use this calibration to translate the International Energy Agency’s (IEA) projections for concentrations of CO_2 -equivalent tons into final temperature outcomes. Under the assumptions of the its “new policies scenario,” IEA (2013) projects that atmospheric concentrations will reach 700 ppm CO_2e by 2100. That

and for economic damages given temperature levels. We explore other functional forms for both. See sections II.C.i and II.C.ii.

²³ This log-normal calibration results in similar CO_2 price estimates as a distribution calibrated by Roe and Baker (2007). It results in higher CO_2 price estimates compared with Pindyck’s (2012) gamma distribution calibration.

²⁴ The IPCC says that range is “likely,” which it defines as having at least a 66% probability. The IPCC’s “very likely” designation implies at least a 90% probability. We follow Wagner and Weitzman (2015) in splitting the difference to arrive at 78%.

concentration would result in a projected, eventual median temperature increase of 3.6°C. Wagner and Weitzman (2015) present eventual median temperature outcomes for concentrations of between 400 and 800 ppm. We take their calibration and extrapolate to 1000 ppm, which we assume to be the zero-mitigation scenario, marking an upper bound of sorts. We similarly assume that 100% mitigation over time leads to a maximum GHG level of 400 ppm. Other fixed levels of mitigation are assumed to lead to damages associated with GHG levels linearly interpolated between those levels. Thus, mitigation of 50% through any point in time leads to the interpolated damages at that time along a path associated with a maximum GHG level of 700 ppm.

Table II gives the probability of different levels of ΔT_{100} —the temperature change over the next 100 years—for given maximum levels of GHGs in atmosphere. The 450 ppm, 650 ppm, and 1000 ppm maximum levels of CO₂ equivalents in the atmosphere reflect, respectively, a strict, a modest, and an ineffective mitigation scenario.

Table II—Probability of $\Delta T_{100} > T$

Maximum GHG Level (ppm of CO₂)			
T	450	650	1000
2°C	0.40	0.85	0.99
3°C	0.13	0.54	0.86
4°C	0.04	0.30	0.66
5°C	0.02	0.15	0.46
6°C	0.00	0.07	0.30

We then use assumptions akin to Pindyck (2012) to fit a displaced gamma distribution around final GHG concentrations, while setting levels of GHG 100 years in the future equal to equilibrium levels. Table III gives the parameters for these distributions, and the probabilities from the fitted displaced gamma distributions, which line up well with the numbers in Table II, especially for scenarios closer to 450 and 650 ppm than the 1,000 ppm zero-mitigation case.

Table III—Fitted values of $Prob(\Delta T_{100} > T)$ for three specified gamma distributions

Maximum GHG Level (ppm of CO₂)			
T	450	650	1,000
2°C	0.40	0.87	0.99
3°C	0.14	0.57	0.91
4°C	0.04	0.29	0.70
5°C	0.01	0.12	0.44
6°C	0.00	0.05	0.24
Gamma distribution parameters			
α	2.810	4.630	6.100
β	0.600	0.630	0.670
Displacement	-0.25	-0.5	-0.9

To obtain the temperature distribution at other times, we again follow Pindyck (2012), and specify that the time path for the temperature change at time t (in years) is given by:

$$(25) \quad \Delta T(t) = 2 \Delta T_{100} \left[1 - 0.5^{\frac{t}{100}} \right].$$

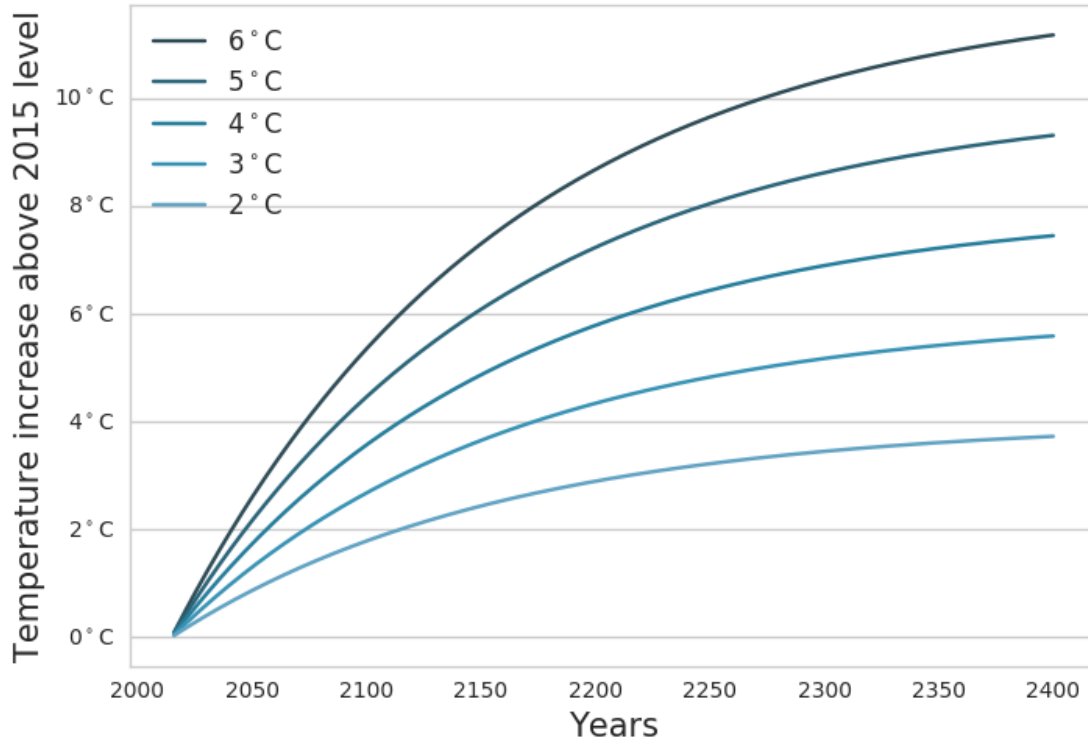


Figure III—Calibrated time path for temperature increases given assumed temperature increases within a century

Figure III plots temperature paths for different levels of ΔT_{100} . As time increases, the temperature change asymptotes to double the value of ΔT_{100} . Even though these calibrations are, by now, ‘established’ in the climate-economic literature, both the distribution of ΔT_{100} and the functional form for the path in equation (25) clearly merit further scientific scrutiny. Both are likely on the conservative side of actual projections.

II.C.ii. Damages as a Function of Temperature

The next step is to translate average global surface warming into global mean economic losses via the damage function D_t . There are two components to D_t : a *non-catastrophic* and a *catastrophic* one. The functional form of each component is known to the agent. However, as with the GHG- ΔT_{100} relationship discussed in the prior section, the functional form for each damage function component contains a parameter that characterizes the uncertainty in our present understanding of this relationship. In EZ-Climate the agent knows the form of the distribution of this parameter at the initial date,

and in each period she learns more about the distribution of the parameter. However, the final realization of the parameter is not known until the next-to-last period.

The non-catastrophic component of our damages follows Pindyck (2012), who fits a functional form to data from the IPCC’s Fourth Assessment Report (IPCC, 2007), and obtains a loss function of the form:

$$(26) \quad L(\Delta T(t)) = e^{-13.97 \cdot \gamma \cdot \Delta T(t)^2},$$

where γ is drawn from a displaced gamma distribution with parameters $r = 4.5$, $\lambda = 21341$, and $\theta = -0.0000746$.

Based on non-catastrophic damages, consumption at any time t is reduced as follows:

$$(27) \quad CD_t = \bar{c}_t \cdot L(\Delta T(t)).$$

A major concern with this damage function is that it effectively rules out catastrophic risks, even at high temperature changes. Take an 8°C temperature change, well outside the range typically assumed to be ‘safe’. If per capita consumption is assumed to grow in real terms by 2% annually, then such damage applied to consumption 50 years hence would reduce the average consumption from 2.7 times today’s value to 2.2 times, a significant reduction, but hardly a catastrophe of significant concern today. Even the 1% point in the outcome distribution conditional on an 8°C average temperature change is assumed here to be a reduction in consumption of only 32% which implies the representative agent is still 1.8 times wealthier than today. We hence augment Pindyck’s (2012) damage function with the possibility of catastrophic events after reaching a particular temperature threshold, which itself creates at least the potential for a much larger impact on consumption, once thus calibrated.

While the possibility of climate tipping elements is receiving considerable attention in the scientific community, there is no single right specification (Kopp et al., 2016). There is, however, seeming convergence around global average warming of 6°C representing something akin to an upper bound for what could conceivably be quantified. The figure is a recurring theme in the literature, from the EU’s High-End cLimate Impacts and eXtremes (HELIX) research project, which ends at 6°C, to Mark Lynas’s popular book *Six Degrees*, which does the same (Lynas, 2008).²⁵ Its sixth chapter, “Six Degrees,” begins with a vivid reference to Dante Alighieri’s Sixth Circle of Hell. We take 6°C as our base level calibration for a parameter we label “*peakT*,” above which we can expect to have hit a climatic ‘tipping point’ of sorts.

Specifically, we use $\text{Prob}(\text{TP})$ to denote the probability of hitting a ‘Tipping Point’ over a given interval of length “*period*” as a function of the global temperature change as of that time ($\Delta T(t)$), and of the parameter *peakT*:

²⁵ See Wagner and Weitzman (2015) for more context on the 6°C threshold. See helixclimate.eu for more on the EU’s HELIX project.

$$(28) \quad \text{Prob(TP)} = 1 - \left(1 - \left[\frac{\Delta T(t)}{\max[\Delta T(t), \text{peak}T]} \right]^2 \right)^{\frac{\text{period}}{30}}.$$

Figure IV plots Prob(TP) as a function of $\Delta T(t)$ for a 30-year *period* and a set of values of *peakT*. As *peakT* increases, the probability of reaching a climatic tipping point decreases for a given $\Delta T(t)$.

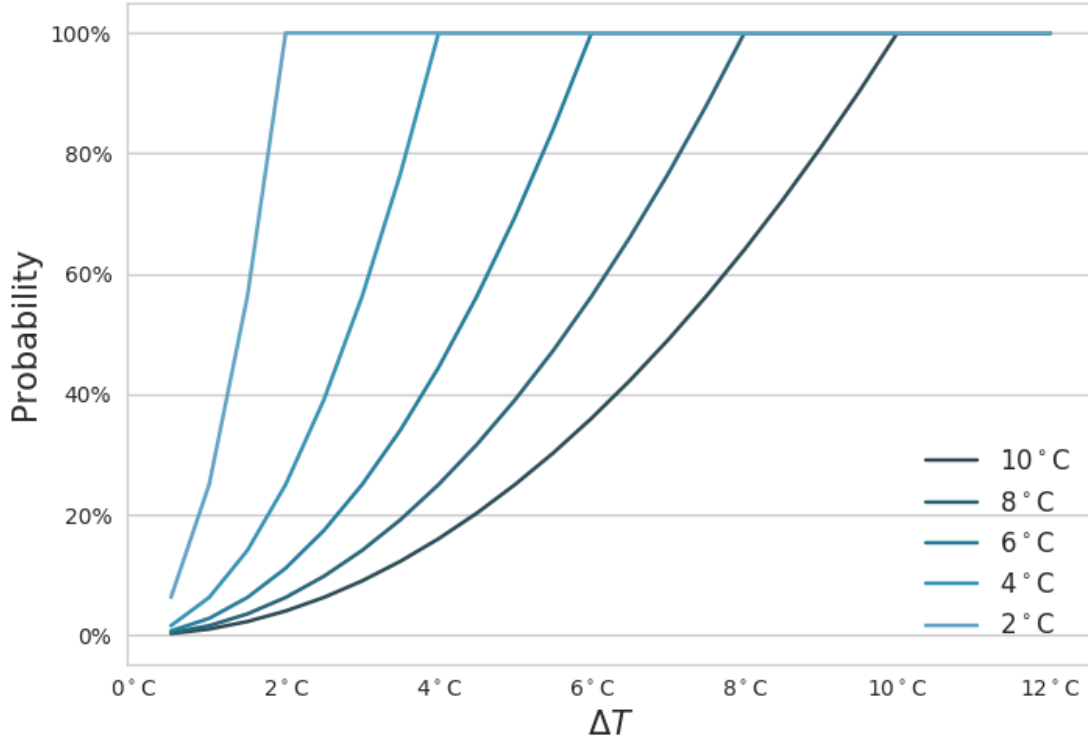


Figure IV—Probability of reaching a climatic tipping point as a function of *peakT*

In each period and for each state, there is a probability Prob(TP) that a tipping point will be hit, given $\Delta T(t)$ and *peakT*. Conditional on hitting a tipping point at time t^* , the level of consumption for each period $t \geq t^*$ is then at a level of:

$$(29) \quad \text{CDTP}_t = \text{CD}_t \cdot e^{-TP_damage} = \bar{c}_t \cdot L(\Delta T(t)) \cdot e^{-TP_damage} \text{ for } t \geq t^*,$$

where *TP_damage* is a random variable drawn from a gamma distribution with parameters $\alpha = 1$ and $\beta = \text{disaster_tail}$. Figure V shows the cumulative distribution for tipping point damage (i.e., $(1 - e^{-TP_damage})$) for values of *disaster_tail* ranging from 6 to 30. Our admittedly *ad hoc* base case calibration uses *disaster_tail* = 18.

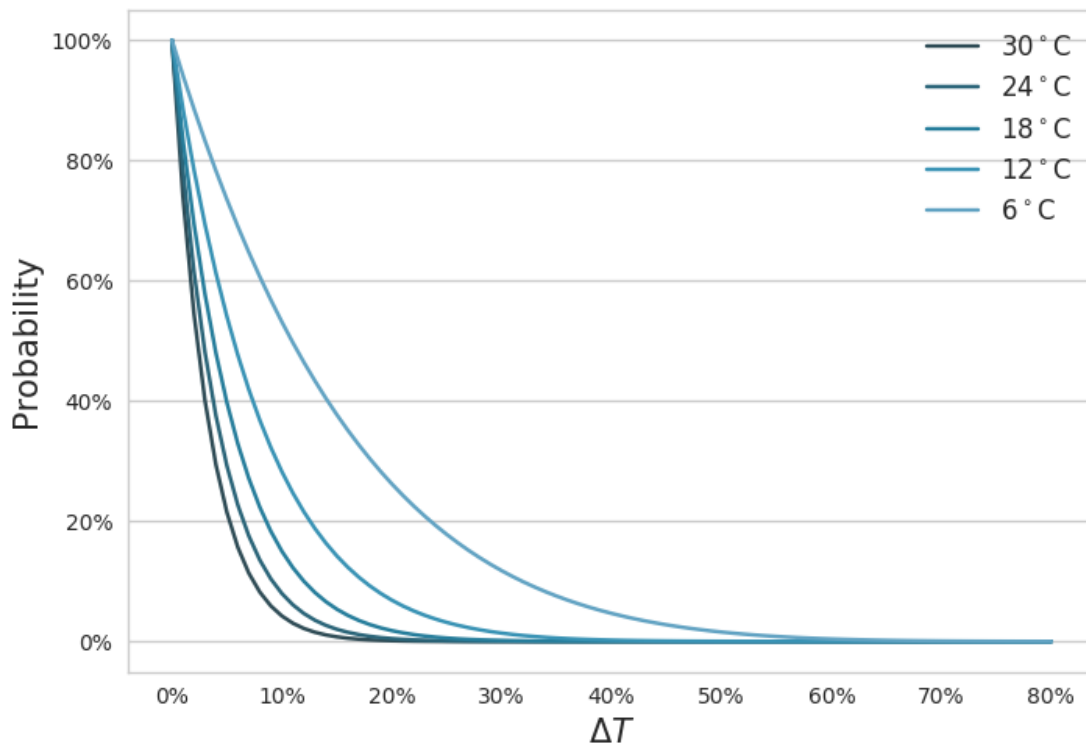


Figure V—Probability of damages greater than a particular percentage of output, given different levels of *disaster_tail*

II.C.iii. Damage Function Uncertainty

The mapping from mitigation policy to damages over time, D_t , goes via cumulative radiative forcing, which determines the excess energy created by GHGs in the atmosphere. The damage distribution associated with a given level of radiative forcing is interpolated, or extrapolated, relative to the radiative forcing of damage distributions estimated from the three baseline scenarios. The first is based on the IEA's (2013) reference New Policies Scenario and leads to eventual atmospheric CO₂ levels of around 1,000 ppm. The second assumes constant mitigation leading to eventual levels of 650 ppm, equivalent to reducing emissions by almost 60% relative to the 1000 ppm scenario. The third scenario assumes a constant mitigation of over 90%, leading to eventual CO₂ concentrations of 450 ppm.

For each of the three maximum GHG concentration levels—450, 650, and 1,000 ppm—we run a set of 6,000,000 random scenarios to generate a distribution of D_t for each period. We order the scenarios based on D_T , the damage to consumption in the final period. We then choose states of nature with specified probabilities to represent different percentiles of this distribution. For example, if the first state of nature is the worst 1% of outcomes, then we assume the damage coefficient at time t for the given level of mitigation is the average damage at time t for the worst 1% of values for D_t .

More generally, if the k^{th} state of nature represents the simulation outcomes in the range $[prob(k-1), prob(k)]$, then the damage coefficient for the k^{th} state of nature is the average damage in that range of scenarios in which the distribution for D_t lies within those percentiles.

The simulations are used to calculate damages in each period for any particular state of nature, θ_t , and any chosen time path for mitigation actions. We do this by first calculating the radiative forcing associated with each simulation at the end of each period, and then interpolating the damage smoothly between the three different simulations with respect to their levels of radiative forcing. Functional forms for both GHG levels and climate forcing as a function of GHG emissions are fitted to the Representative Concentration Pathway (RCP) scenarios adopted by the IPCC for its Fifth Assessment Report (IPCC, 2013). In the IPCC report emissions, GHG concentrations, and radiative forcing are given for each of three RCP scenarios. The radiative forcing is assumed to be given by a log-function fitted to these RCP scenarios.²⁶ The carbon absorption itself is similarly fit to the RCP scenarios, and is assumed to be proportional to the difference between the GHG level in the atmosphere and the cumulative carbon absorption up to that point in time, raised to a power.²⁷

Our task now is to calculate an interpolated damage function using our three simulations where we have damage coefficients (for a given state and period) to find a smooth function that gives damages for any particular level of radiative forcing up to each point in time. To do so, we assume a linear interpolation of damages between the 650 and 1,000 ppm scenarios, and a quadratic interpolation between 450 and 650 ppm. In addition, we impose a smooth pasting condition at 650 ppm, having the level and derivative of the interpolation below 650 ppm match the level and slope of the line above.

Below 450 ppm, we assume climate damages exponentially decay toward zero. Mathematically, we let $S = d \cdot p / (l \cdot \ln(0.5))$, where d is the derivative of the quadratic damage interpolation function at 450 ppm, $p = 0.91667$ is the average mitigation in the 450 ppm simulation, and the level of damages is l . Radiative forcing at any point below 450 ppm then is x percent below that of the 450 ppm simulation, with $x = \frac{R-r}{R}$, where R is the radiative forcing in the 450 ppm simulation and r is the radiative forcing given the mitigation policy. Letting $\sigma = 60$, the extension of the damage function for $x > 0$ is defined as: $Damage(x) = l \cdot 5^{(x \cdot S)} e^{-[(x \cdot p)^2 / \sigma]}$, which has the desired properties.

Figure VI shows the simulated distribution of the resulting damage functions in our base case, using $peakT = 6$ and $disaster_tail = 18$, and assuming constant mitigation.

²⁶ Radiative forcing in a ten-year interval is given by: $5.351 \cdot [\log(GHG) - \log(278.063)]$, where GHG is the average level of atmospheric CO₂. We estimated the constants from the three IPCC RCP scenarios.

²⁷ The carbon absorption in a ten-year interval is given by: $0.94835 \cdot |GHG - (285.6268 + 0.88414 \cdot \sum absorption)|^{0.741547}$, where the sum is over absorption in previous periods. We again estimated the constants from the three IPCC RCP scenarios.

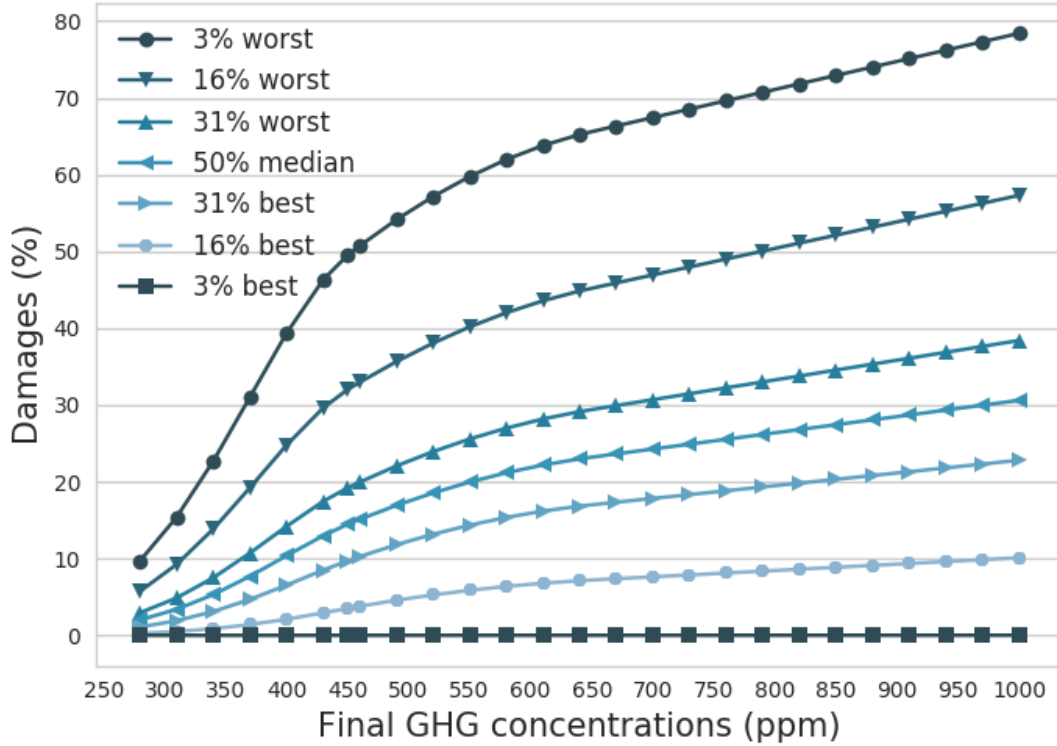


Figure VI—Interpolated final period damage functions

The climate sensitivity—summarized by state of nature θ_t —is not known prior to the final period ($t = T$). Rather, what the representative agent knows is the distribution of possible final states, θ_T . We specify that the damage in period t , given a cumulative radiative forcing, CRF_t , up to time t , is the probability weighted average of the interpolated damage function over all final states of nature reachable from that node. Specifically, the damage function at time t , for the node indexed by θ_t is assumed to be:

$$(30) \quad D_t(CRF_t, \theta_t) = \sum_{\theta_T} \Pr(\theta_T | \theta_t) \cdot D_t(CRF_t, \theta_T),$$

where the sum is taken over all states that are possible from the node indexed by θ_t (i.e., for which $\Pr(\theta_T | \theta_t) > 0$).

II.C.iv. Damages for Concentrations Below Pre-industrial Levels

Introducing carbon dioxide removal (CDR) backstop technologies, combined with stochastic fragility θ_t creates a unique possibility: that, in some states of the world, GHG concentrations may fall below pre-industrial levels of 280 ppm. There is nothing magical about 280 ppm—in an absolute sense, it may not be the ‘optimal’ climate to begin with—but it does serve as the baseline for damage calculations based on global warming above pre-industrial levels. It is clear that going (well) below 280 ppm would lead to climate damages, much like going (well) above 280 ppm does. We introduce a simple penalty function of the form:

$$(31) \quad f(x) = [1 + e^{k(x-m)}]^{-1},$$

where m is the level of GHG concentrations where calibrated at half the total penalty, and k is a simple scalar. For our base case calibration, we use $m = 200$ and $k = 0.05$. The benefit of a low k and, thus, a smooth penalty function is largely computational. More importantly, the calibration ensures that the penalty (31) at 280 ppm is close to zero.

II.D. TREE STRUCTURE

Figure VII illustrates the tree structure employed in EZ-Climate's baseline analysis. Beginning with the first node, in 2015, the agent is assumed to know the structure of the decision tree, the state probabilities, and the damage function in each future state of the world. Period zero runs from 2015 through 2030. In 2030, the agent learns whether the world is in state up ('u') or state down ('d'). There is a 50% probability of each of the two states. Similarly, at the end of period one (in year 2060) she learns whether the world is in state 'uu', 'ud', or 'dd', etc. Notice that at the end of period four, all uncertainty is resolved, in that the agent will learn which of the six final states the world she is in, and what the true damage function is. Following this point, in period five, she has one final period in which she can make a mitigation decision. In period six, which in our base case runs from 2400 on to infinity, the agent can no longer mitigate. Consumption continues to grow deterministically from this point forward at a rate g . Consumption, thus, is given by $c_t = c_T(1 + g)^{t-T}$ after $t = 2400$, and period six utility is given by equation (10).

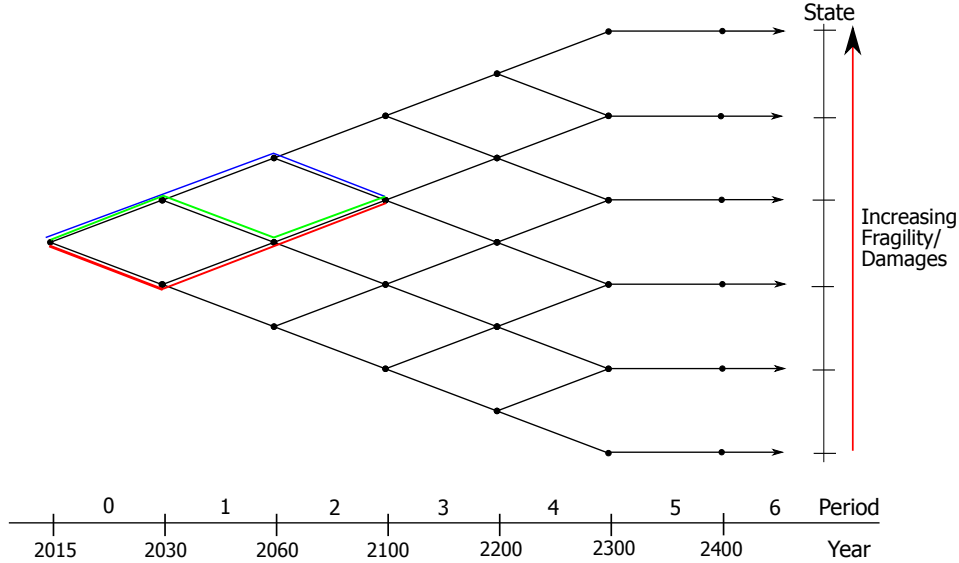


Figure VII—Diagram of tree structure used in solving the model for each state of nature across time

In the baseline model, where a move up or down in each period is equally likely, the probabilities of the final states are given by a binomial distribution, the simplest possible probability representation.

Another feature evident in Figure VII is particularly important, given our use of the Epstein-Zin preference specification: the recombining tree structure. This implies two features: For one, the damage function in each state (after period zero) is independent of the way in which information was revealed at the end of each period. For example, the damage function in state ‘*uud*’ (the blue path in Figure VII) is identical to that in ‘*udu*’ (green) and ‘*duu*’ (red).

Second, the agent’s utility is path-dependent. The history of mitigation depends on the process by which the agent learns the state. Thus, consumption, and mitigation, will depend upon the path. Consequently, in solving for the agent’s utility along each of these paths, we need to keep track of the path by which the agent learned about the damage function. Consumption decisions depend on it.

For example, following equation (2), the consumption flow at the start of period 1 is given by $c_1 = \bar{c}_0 \cdot e^{0.02 \times 15} (1 - D_1(CRF_1, \theta_1) - \kappa_1(x_1))$. That is, the consumption at the start of period one (in 2030), c_1 , is equal to endowed consumption (2015 consumption plus 1.5% growth for 15 years), minus the fractional cost of damages and of mitigation chosen at the beginning of period one. Mitigation is optimally chosen by the agent, and is therefore a function of the state—mitigation will be lower if the agent learns that the world is in state ‘*d*’ rather than state ‘*u*’.

This analysis gives us consumption levels c_0 and c_1 , for the two states in 2015 and in 2030, respectively. To interpolate between c_0 and c_1 , we fit an exponential growth function to consumption levels, using 5-year intervals. Note that this is equivalent to assuming that immediately after choosing the mitigation level in period zero, the agent’s consumption starts to reflect climate damages from the first revealed state (‘*u*’ or ‘*d*’). However, she is not allowed to change the period zero mitigation to reflect this knowledge until the next period.

This interpolation ensures that the agent’s consumption path is relatively smooth. It also introduces approximation errors. However, adding more periods at which the agent can choose a new level of mitigation would result in far higher computational costs. With T periods, we have a “ $2^{T+1} - 1$ ”-dimensional optimization problem: Each model run requires choosing $2^{T+1} - 1$ optimal mitigation levels. In the ‘easy’ spirit of EZ-Climate, simplifying makes the solution both tractable and doable in the first place. While similar attempts at integrating Epstein-Zin preferences into climate-economy models require supercomputers (see, e.g., Cai et al., 2016), EZ-Climate is solvable on a standard personal computer within minutes.

III. RESULTS

EZ-Climate’s main output is the optimal price of one ton of CO₂ today, in year 2015, and at the beginning of each of the subsequent five periods—years 2030, 2060, 2100, 2200, and 2300. These are the times in the model when mitigation decisions are made. Figure VIII shows the results for the CRRA model run and our base-case model calibration, using

$peakT = 6$ and $disaster_tail = 18$, as well as an EIS of 0.9 and $RRA = 7$, calibrated to observed financial asset prices.

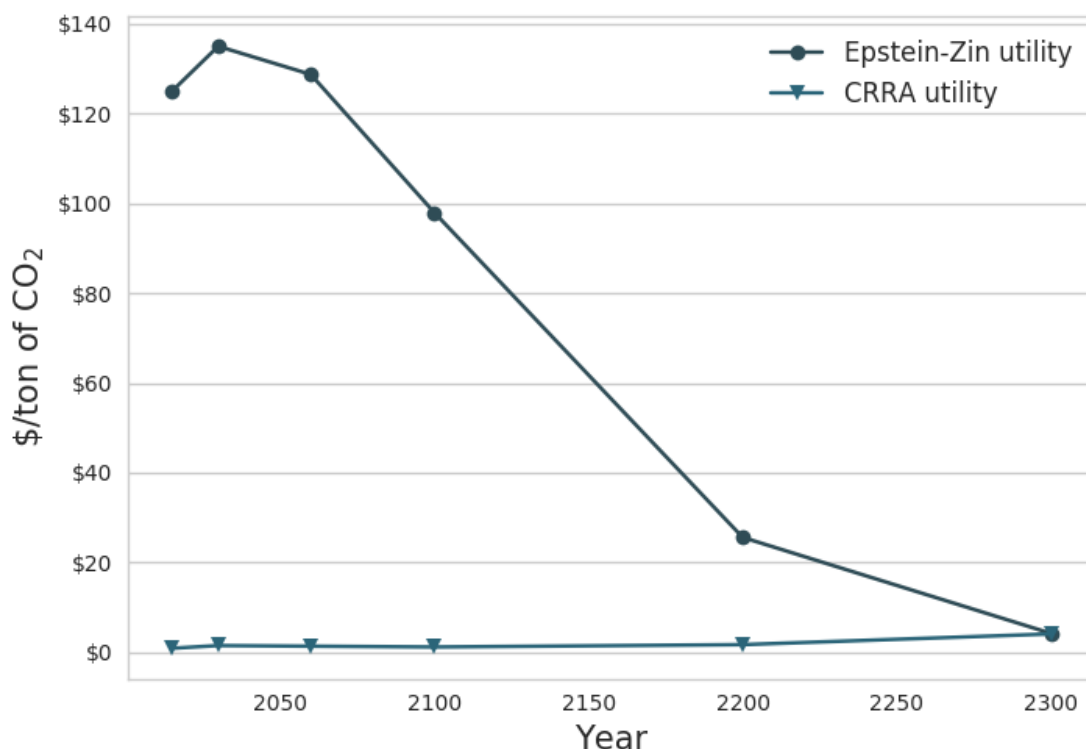


Figure VIII—Optimal price per ton of CO₂ under constant relative risk aversion (CRRA) and our base case, employing $RRA = 7$ in both cases, and $peakT = 6$, $disaster_tail = 18$, and $EIS = 0.9$ in the base case

The difference in optimal price paths is striking: Under the CRRA case, the expected optimal price of carbon increases smoothly over time; Under the Epstein-Zin base case, the price first increases slightly and then decreases to a value almost as low as in the CRRA case. Significantly, an $RRA=7$, calibrated to observed equity risk premia, decreases the CO₂ price under CRRA assumptions to below \$1 in 2015, while the price in EZ-Climate’s base case is well over \$100/ton CO₂.

It is difficult to overemphasize the probabilistic nature of this model. The precise 2015 price represented in Figure VIII for Epstein-Zin utility is \$124.94—around \$125. Even when leaving the damage simulation, explained in section II.C, constant optimal prices in a random sampling of 30 optimization runs range from \$124.89 to a high of \$126.50. The \$124.94, thus, is toward the lower, conservative end of this particular sample. Anything more precise than saying “around \$125,” however, would amount to false precision.

Note that the 2015 price comes from a single node in the tree. In each subsequent year, that price is set in expectation over all possible states of nature in that given year. The left panel of Figure IX shows optimal base-case CO₂ prices across time and states. All grouped nodes at a given time have the same degree of fragility and, thus, the same damage for a given amount of atmospheric greenhouse-gas concentrations. The lines connecting the

boxes indicate the paths that information about the Earth’s fragility, θ , has taken, a feature explored in Section II.D above.

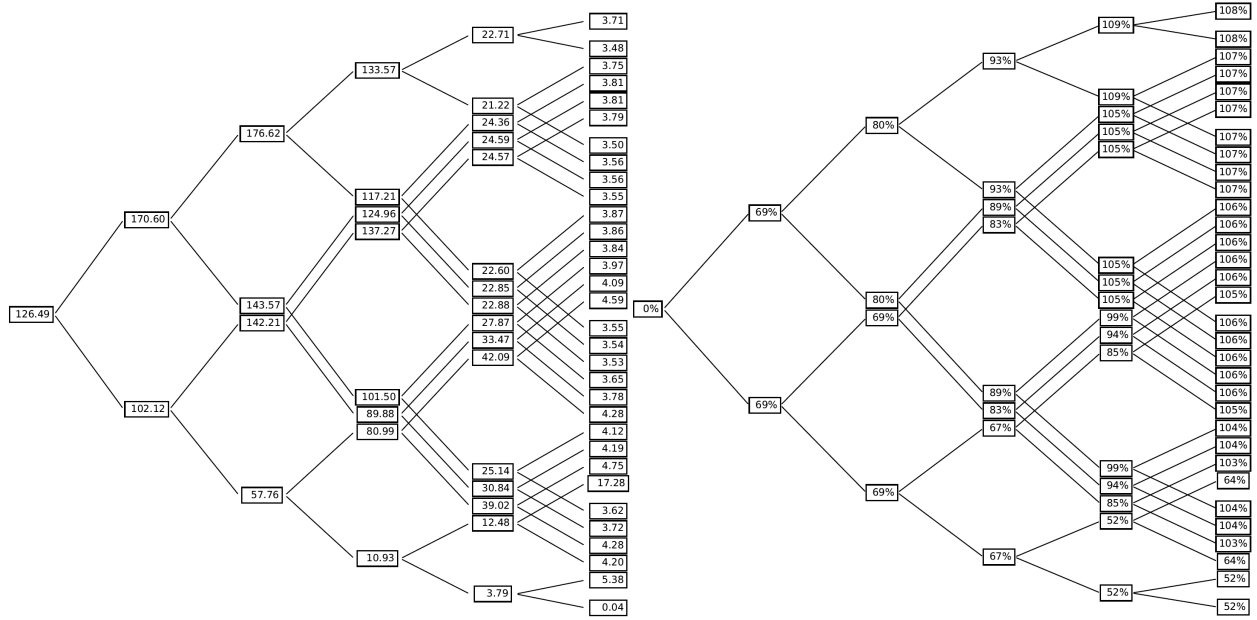


Figure IX—Optimal price per ton of CO₂ across time and states (left) and average mitigation up to a particular time and state (right) in the base-case calibration ($peakT = 6$, $disaster_tail = 18$, and $EIS = 0.9$)

The right panel of Figure IX shows the fractional, average mitigation up to each time and state. It reveals the reason for sometimes wildly different prices at the same node. In general, the greater is θ revealed to be, the higher is the optimal average mitigation effort, ranging from slightly over 50% to over 100% in the final period.

Optimal average mitigation levels, in turn, are closely linked to climate damages at each node, which similarly depend on the path chosen (Figure X, left panel). Both Figure IX and Figure X also show the large costs associated with negative θ draws in latter periods. Repeated good (d for down) draws in early periods, followed by a bad (u for up) draw in the final period results in half the amount of average mitigation up to that point (52%) compared with reaching the same node via an early bad (u) draw followed by good draws (104%). Associated climate damages range from 5.5% ($ddddu$) to 1.4% ($u d d d d$), respectively (Figure X, left panel).

Lastly, the left panel in Figure X plots GHG concentrations along the optimal base-line pathway. The relatively small changes along most paths across time reveal the inherent inertia in the climate system. The large differences across nodes in the final period, from close to pre-industrial levels of 280 ppm to well over 800 ppm, reveal the enormous costs of bad θ draws. Looking at GHG levels also confirms our prior conclusions around the importance of path dependency: the $ddddu$ path results in ultimate GHG levels above 800 ppm, whereas the $u d d d d$ path results in levels close to 300 ppm. Bad news is costly. Bad news received late is extremely costly.

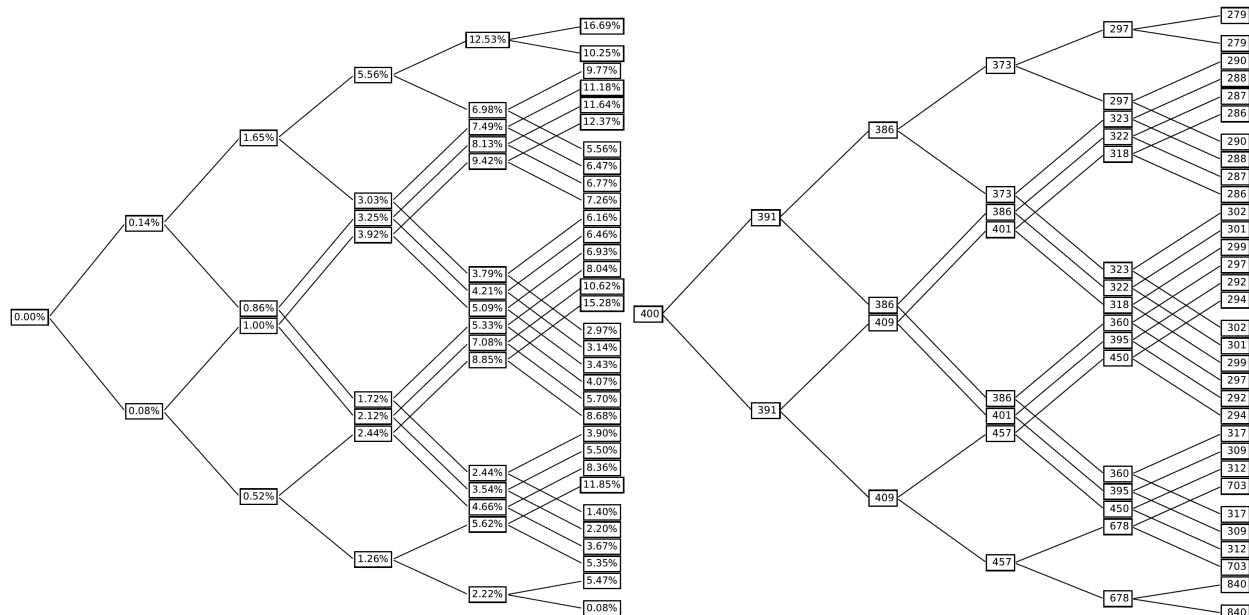


Figure X—Climate damages (left) and GHG concentrations (in parts per million, ppm, right) along the optimal CO₂ price path across time and states in the base-case calibration

Following GHG levels closely across time, especially in early periods, also demonstrates the positive environmental impact of moving onto an optimal CO₂ price path. Despite inertia, from 2015 to 2030 alone, optimal GHG concentrations decline from 400 to around 390 ppm.²⁸

EZ-Climate’s optimal carbon price depends on a number of inputs. Figure I reveals the importance of *RRA* for calibrating economic variables to capture observed equity risk premia, and its influence on the optimal CO₂ price.

Given the apparent importance of the use of Epstein-Zin preferences to capture climate risk, it might then be surprising to see how little of the overall optimal CO₂ price is explained by risk aversion as opposed to expected climate damages (Figure XI). There, too, the importance of moving to Epstein-Zin preferences in the first place is apparent, but once done, most of the impact comes from the expected damage component of the damage distribution being discounted at lower rates, rather than the higher curvature of the utility function across states of nature.

Crost and Traeger (2014) and Cai *et al.* (2016) support this conclusion, though their explanations differ. Crost and Traeger (2014) suggests it is because of a failure to account for disasters. Cai *et al.* (2016) attempt to model such climate disasters and still find a similarly small role for risk. One explanation might be that even the type of disasters modeled by Cai *et al.* (2016) and represented here in EZ-Climate do not yet capture true uncertainty (Brock and Hansen, 2017; Wagner and Zeckhauser, 2016). For example, although we include tipping points in the simulation of our damage function, these events are averaged with others to create an average loss in a given state. Moreover, although the

²⁸ See Section III.E on the large social costs of delayed implementation of the optimal CO₂ price path.

state of nature is not known in advance, the probability of each state and the average loss in any given state and for any given degree of mitigation is known in advance. That information is used optimally to mitigate against those potential damages. Our tipping points do not in any sense catch our agent by surprise. All this ascribes yet more importance to the calibration of the damage function itself, and to considerations of model mis-specification (Brock and Hansen, 2017).

III.A. RISK DECOMPOSITION

Figure I presents the optimal CO₂ price as a function of the assumed equity risk premium for both Epstein-Zin and CRRA utility and points to the importance of using the former. We further decompose the optimal CO₂ price into a risk aversion and an expected damages component.

Let $D_{s,t}$ denote the marginal damage, that is the loss of consumption in state s in future period t that results from putting one more ton of carbon into the atmosphere today (at time 0). The optimal CO₂ price then is given by:

$$(1) \sum_{t=1}^T \sum_{s=1}^{S(t)} \pi_{s,t} m_{s,t} D_{s,t} \left(= \sum_{t=1}^T E_0 [\tilde{m}_t \tilde{D}_t] \right),$$

where $m_{s,t}$ is the pricing kernel in state s at time t , which is the marginal value today of one additional unit of consumption in state s at time t ,²⁹ $\pi_{s,t}$ denotes the probability of state s at time t , and $S(t)$ denotes the number of states at time t . That is, to calculate the cost to the representative agent of an additional ton of carbon emissions, we sum over all consumption damages, in every state of nature at every future time, multiplied by the value of an additional unit of consumption in that state at that time. Equation (1) can be decomposed to equal:

$$(2) \underbrace{\sum_{t=1}^T E_0 [\tilde{m}_t] \cdot E_0 [\tilde{D}_t]}_{ED} + \underbrace{\sum_{t=1}^T cov_0(\tilde{m}_t, \tilde{D}_t)}_{RP}.$$

Note that $E_0[\tilde{m}_t] = \frac{1}{R^f(0,t)}$, where $R^f(0,t)$ is the payoff, at time t , to a \$1 investment in a risk-free bond at time 0. Alternatively, $E_0[\tilde{m}_t]$ is the risk-free discount factor between today and t . We can, thus, rewrite the first component of (2) as the sum of the marginal damages, discounted back to the present at the risk-free rate and label it expected damages, $ED = \sum_{t=1}^T \frac{E_0[\tilde{D}_t]}{R^f(0,t)}$. The second component is the risk premium (RP) over the expected damages that society is willing to pay, defined as the covariance of the marginal damages with marginal utility.

²⁹ Equivalently, $m_{s,t}$ is defined as the ratio of marginal utility with respect to current consumption in that state to the marginal utility today, that is $m_{s,t} = \left(\frac{\partial U}{\partial c_{s,t}} \right) / \left(\frac{\partial U}{\partial c_0} \right)$, where $c_{s,t}$ denotes the agent's consumption in state s at time t .

Rewriting (2) then gives the risk premium as the difference between the optimal CO₂ price and the expected-damages:

$$(3) RP = \text{optimal CO}_2 \text{ price} - ED,$$

both of which are readily calculated by EZ-Climate. Figure XI shows the result for our base-case calibration ($peakT = 6$ and $disaster_tail = 18$).

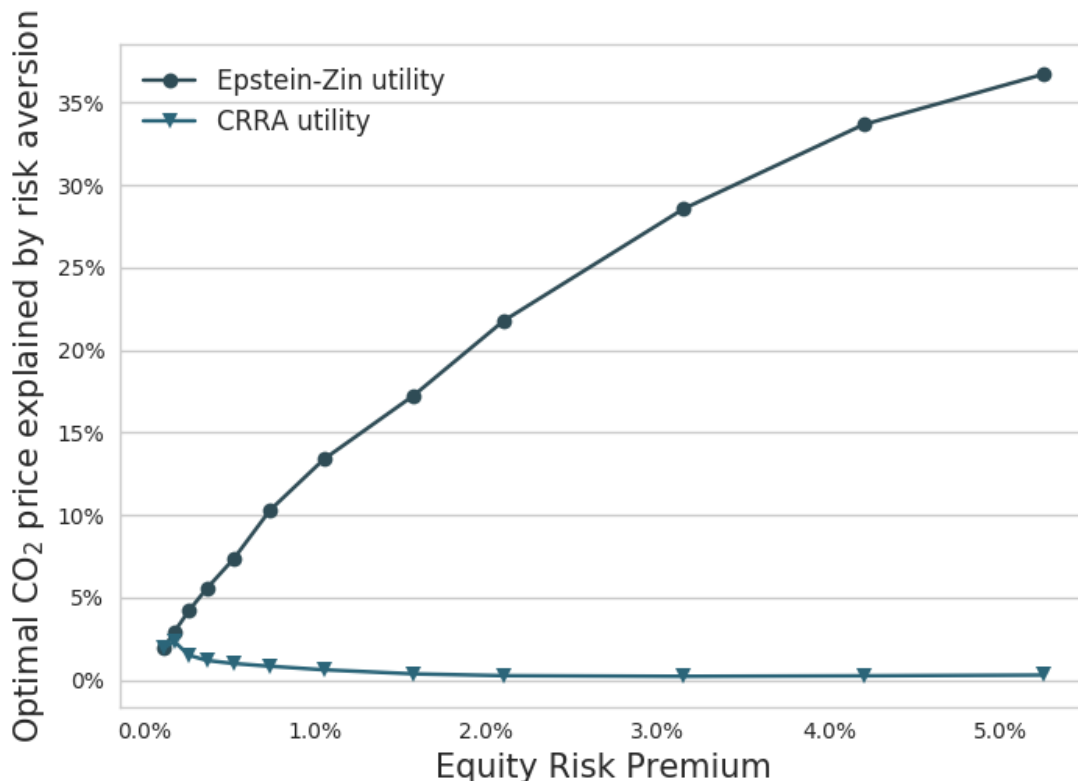


Figure XI—Percentage of the optimal 2015 CO₂ price explained by risk aversion, as opposed to expected damages, under Epstein-Zin and CRRA preferences, translated into the implied equity risk premium using Weil (1989)’s conversion, while holding implied market interest rates stable at 3.11%

Like the optimal CO₂ price in Figure I, this decomposition varies widely with the assumed equity risk premium—and it crucially depends on the distinction between using Epstein-Zin versus CRRA preferences. For low risk premia, less than 5% of the optimal CO₂ price is explained by risk aversion. For our base case calibration with an EIS of 0.9 and $RRA = 7$, around 10% is explained by risk aversion.

III.B. SENSITIVITY TO CLIMATE DAMAGE PARAMETERS

Two key damage-function parameters are $peakT$ and $disaster_tail$, assumed to equal 6 and 18, respectively, in our base-case calibration. Figure XII shows the sensitivity of the

optimal CO₂ price to both parameters: the optimal CO₂ price increases with decreasing *peakT* and *disaster_tail*.

There is no single ‘right’ combination of *peakT* and *disaster_tail*, at least given our current state of knowledge of the underlying climate science. The base-case calibration in EZ-Climate employs *peakT* = 6 and *disaster_tail* = 18. Lower values of each increases the optimal CO₂ price along their respective dimension.³⁰

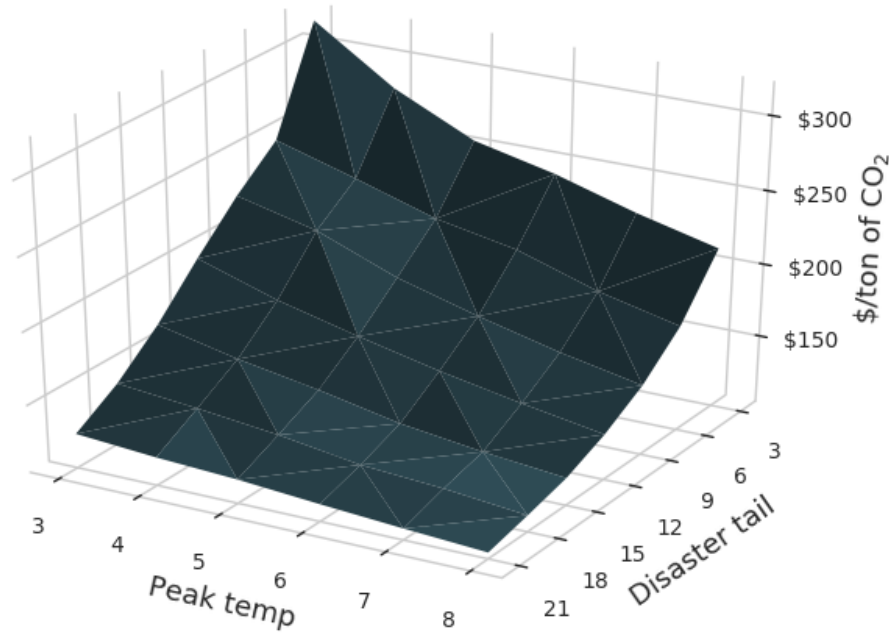


Figure XII—Optimal 2015 CO₂ price per ton of carbon increases with decreasing *peakT* and *disaster_tail*

III.C. SENSITIVITY TO MITIGATION COST PARAMETERS

On the mitigation cost side, two important factors are the rate of technological change and the potential for an outright backstop technology. Climate-EZ includes parameters capturing both exogenous and endogenous technical change, represented by coefficients φ_0 and φ_1 , respectively, in equation (24).

Figure XIII shows how the optimal CO₂ price in early periods first increases with φ_0 moving from 0 to 1.5% before decreasing again with increased exogenous technical change. The relatively low optimal price of CO₂ in early years with low φ_0 is explained by large GHG concentrations, leading to large damages in bad states, significantly decreasing the marginal benefit of mitigation today.

³⁰ See Section II.C.ii, in particular Figure IV and Figure V, on more on the damage-function calibration, incorporating *peakT* and *disaster_tail*.

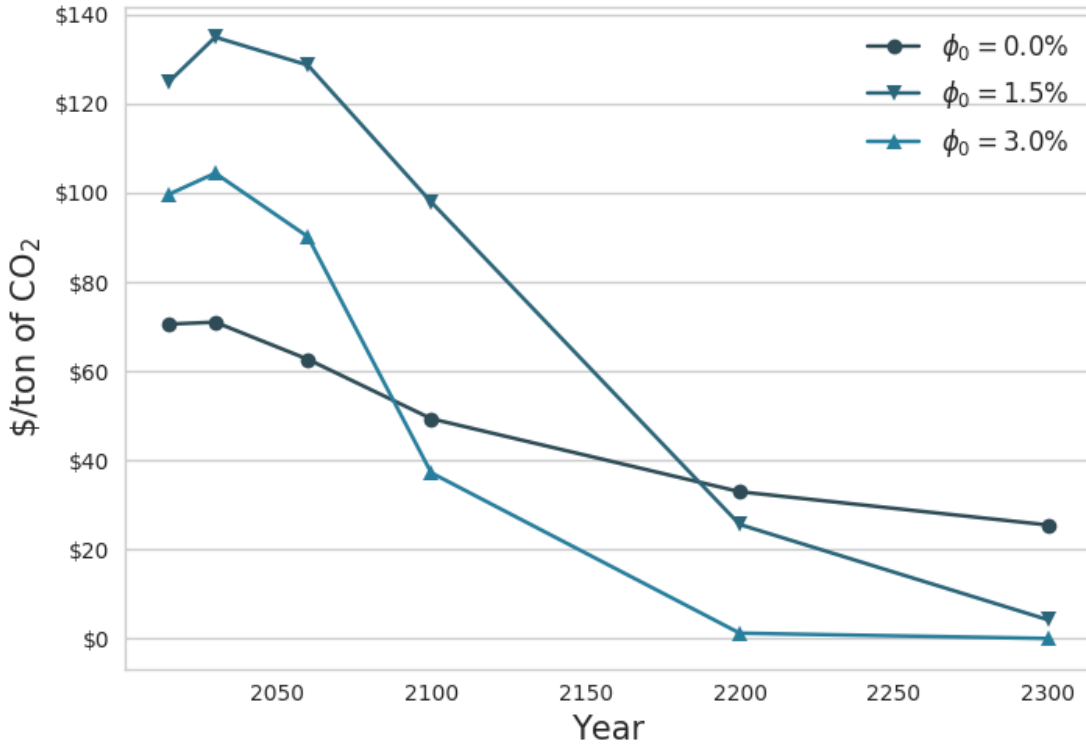


Figure XIII—Optimal CO₂ price in early years first increases then decreases with higher exogenous technical change, ϕ_0

Endogenous technical change is assumed to be proportional to average mitigation up to time t . Figure XIV shows the sensitivity of the optimal CO₂ price to ϕ_1 ranging from 0 to 1.5%, which can best be seen as multiples of average mitigation affecting learning-by-doing.³¹ The optimal CO₂ price increases at first slightly and then decreases with increased endogenous technical change in later years.

³¹ In all cases, exogenous technical change, ϕ_0 , is 1.5% per year, plus the multiple ϕ_1 of average mitigation to date (see Section II.B.ii). For example, with $\phi_1 = 1$ and average mitigation for the first 15 years equal to 0.50, then the rate of technical change would be ϕ_0 , or 1.5%, plus $\phi_1 \cdot X_{15}$, or $1 \cdot 0.50\%$, summing to 2.0% per year. Costs after 15 years, $\kappa_{15}(x)$, will have declined to $(1 - 2\%)^{15} = 73.8\%$ of today's costs, given by equation (24). For simplicity, our base case assumes $\phi_1 = 0\%$, in addition to $\phi_0 = 1.5\%$.

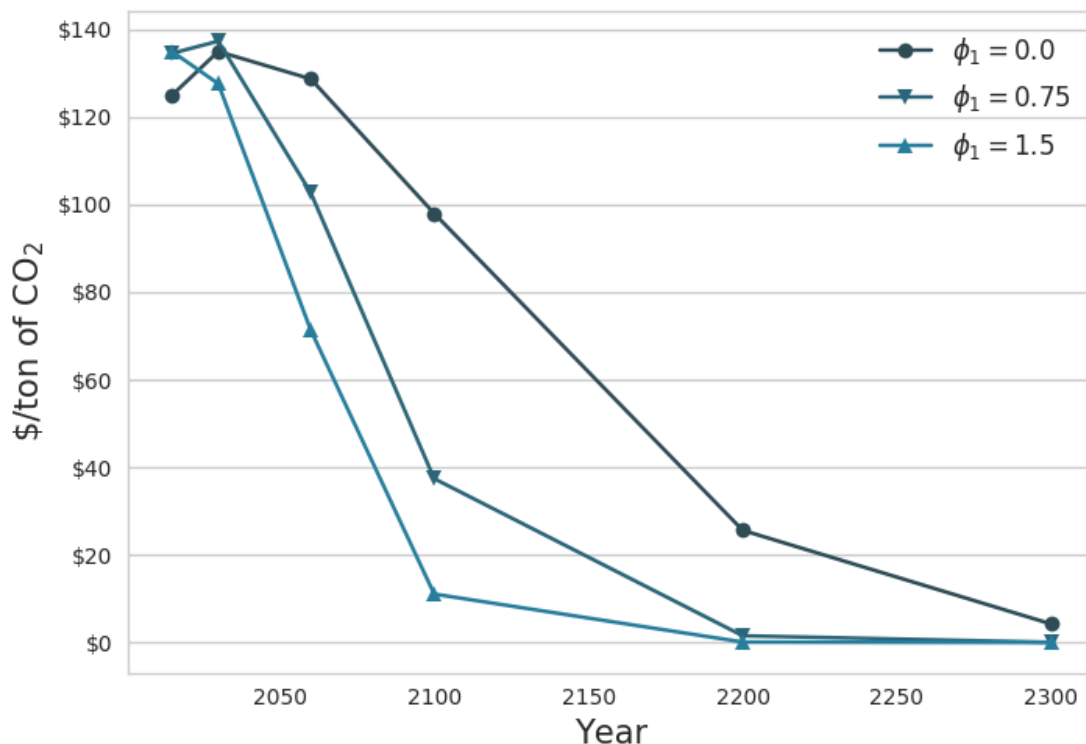


Figure XIV—Optimal CO₂ price decreases with increased endogenous technical change in later years

Endogenous technical change, in turn, interacts with the availability of a backstop technology (Figure XV). The EZ-Climate base case assumes essentially no backstop technology—i.e. a ‘join’ price of $\tau^* = \$2,000$ and a full-on backstop at $\tilde{\tau} = \$2,500$.³² The “\$350 backstop” path assumes $\tau^* = \$300$ and $\tilde{\tau} = \$350$, with no endogenous technical progress, $\phi_1 = 0$. The third scenario assumes $\tau^* = \$300$, $\tilde{\tau} = \$350$, and $\phi_1 = 0.67$. This \$350 backstop assumption plus endogenous technical change at first leads to a slightly higher optimal CO₂ price, though over time endogenous technical change helps decrease the optimal CO₂ price as well.

³² See Section II.B.i for an explanation of the integration of backstop technologies. Figure II shows this baseline backstop assumption, and the more aggressive scenario assuming $\tau^* = \$300$ and $\tilde{\tau} = \$350$.

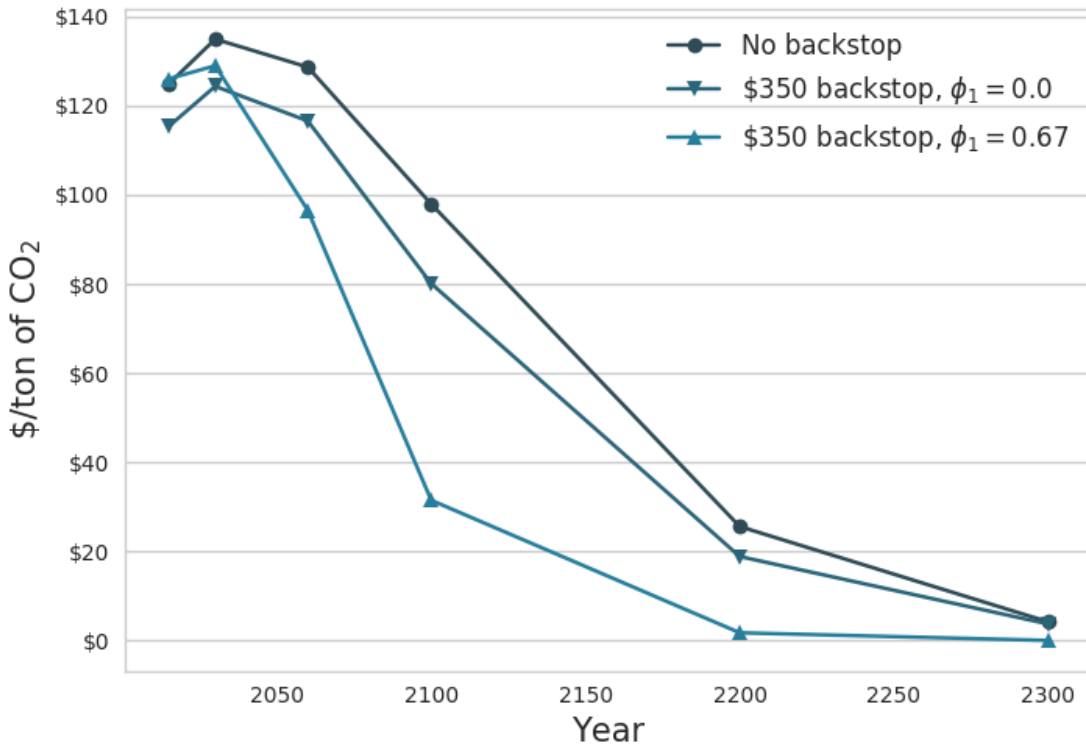


Figure XV—Optimal CO₂ price decreases with backstop, with or without endogenous technological change

III.D. SENSITIVITY TO ECONOMIC AND PREFERENCE PARAMETERS

As important as climate risk and mitigation cost parameters are in EZ-Climate, a proper calibration of fundamental economic parameters has at least as much influence on the optimal CO₂ price. One key input is the assumed rate of economic growth.

A large fraction of climate damages is likely to occur in the distant future. Even in the worst states of nature modeled here, the economy in 2100 or 2300 will be a multiple of the size of today's. As a result of this large disparity in current and future consumption levels, future damages are discounted at a high rate³³, posing significant ethical challenges that can cut both ways.³⁴ On the one hand, a much richer society will be better equipped to deal with the challenges of climate change. On the other, a much richer society will be willing to pay comparatively more for a healthy planet—and, in the case of oft-irreversible damages brought about by climate change, would have wanted earlier generations to mitigate more (Sterner and Persson, 2008).

³³ In the EZ-Climate base case, with a growth rate of 1.5%/year, the risk-free rate is around 3.11%/year.

³⁴ The broader ethical and discounting debate has spawned hundreds of contributions, often with well-founded opposing conclusions (see, e.g., Arrow et al., 2013; Broome, 2008; Dasgupta, 2008; Gollier, 2012; Summers and Zeckhauser, 2008).

EZ-Climate, like most other climate-economy models such as, for example, DICE, takes economic growth as exogenous. Our base case assumption, before climate damages and mitigation costs, is an exogenous rate of growth of 1.5% per year. This assumption is consistent with empirically observed growth rates over the past century. However, a number of scholars now argue that we should anticipate far lower growth going forward than we have observed in the past century (**e.g., Gordon, 2016, 2012**). At the same time, we cannot exclude the possibility of higher growth rates.

Figure XVI shows the effect of changing growth rates, while also adjusting the EIS, to hold real interest rates constant at the assumed rate of around 3.11%.³⁵ Optimal CO₂ prices in early periods are highly sensitive to these simultaneous changes in economic growth rates and EIS.

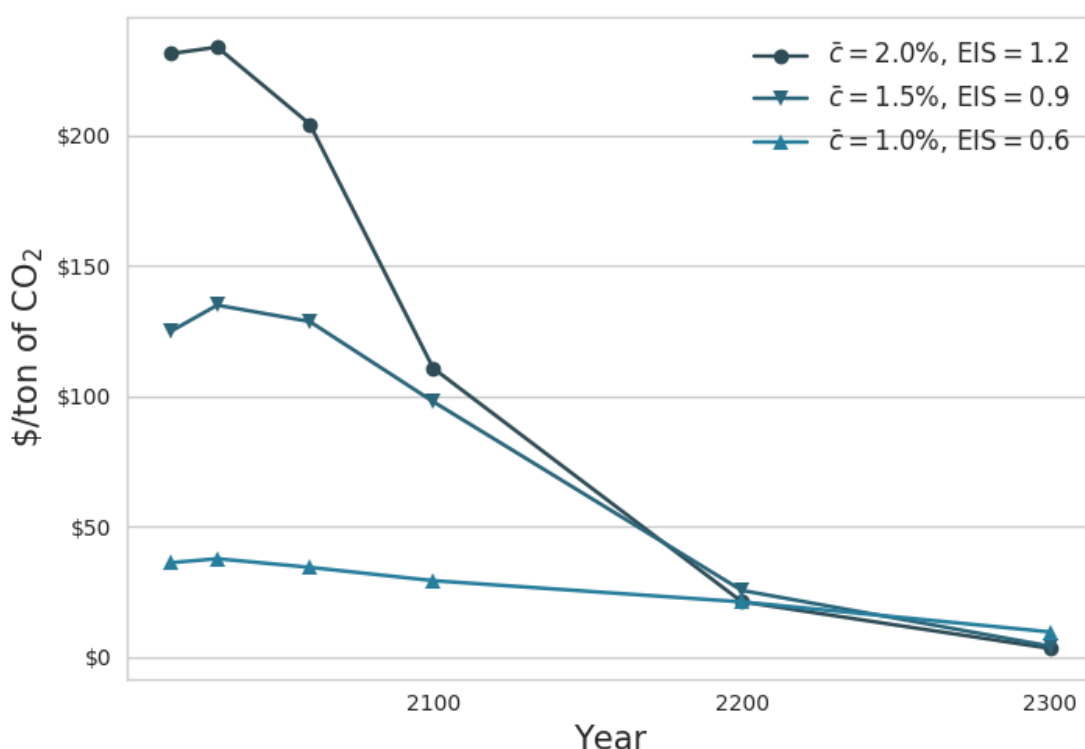


Figure XVI—Increasing (decreasing) economic growth rates, while changing EIS to keep real interest rates constant at 3.11% per year, increases (decreases) optimal CO₂ prices dramatically in early periods

Figure XVII, by contrast, shows the impact of changing growth rates while holding the EIS constant. The impact on optimal CO₂ prices is comparatively small.

³⁵ See footnote 12 for a discussion of real interest rates in our base case.

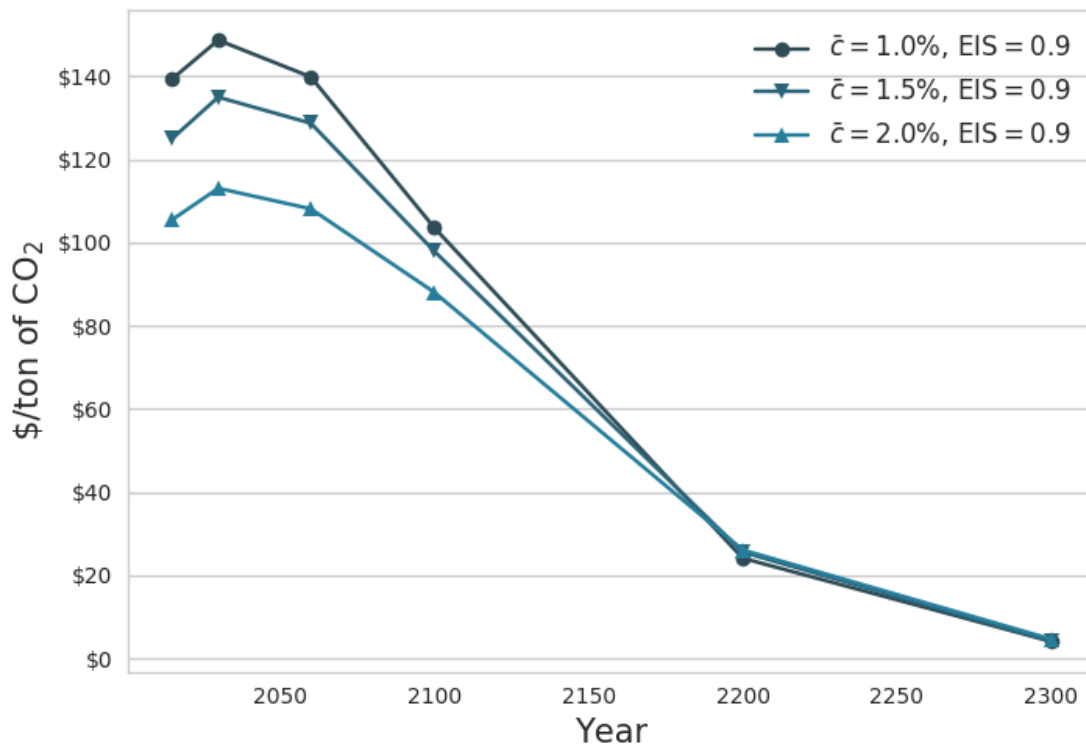


Figure XVII—Changing economic growth rates, while keeping EIS constant at 0.9, has little impact on optimal CO₂ prices (note the y-axis scale compared with Figure XVI)

Given the considerably smaller discount rate, why is it that the SCC does not increase more in response to this change? The answer lies, in part, in our damage specification in equation (2), where damages, for a given level of fragility and cumulative mitigation, are proportional to the baseline level of consumption, given any specific economic growth rate r : $\bar{c}_t = \bar{c}_0 \cdot (1 + r)^t$. Thus, in an alternative specification with a 1% as opposed to 1.5% annual growth rate, and consumption in year 2300 that is lower by a factor of 4, we are also implicitly specifying that damages are lower by a factor of 4. Whether it is reasonable to assume that, if people are poorer in the future, they will be hurt proportionally less by climate change is subject to vigorous debate, and worthy of future research (see, e.g., Convery and Wagner, 2015; Heal and Park, 2016).

The comparison of Figure XVI and Figure XVII also makes the importance of the EIS clear. Ours is calibrated to a risk-free bond yield rate of around 3.11%, implying an EIS of 0.9.³⁶ Figure XVIII shows the optimal CO₂ price path for various EIS levels. Note that this figure does not hold real interest rates constant. That connection with bond yields implies the EIS is not a free parameter to be chosen but one that needs to be calibrated based on

³⁶ Note that calibrations of EIS have changed widely over time. Earlier estimates, most notably by Hall (1988), are much closer to zero. Bansal and Yaron (2004), in a model with Epstein-Zin preferences and consumption shocks, estimate an EIS of 1.5. Other estimates vary further (Thimme, 2017), while Epstein *et al.* (2014) offers a potent critique by pointing to the sizeable magnitude of implied time premia by standard calibrations of Epstein-Zin utility specifications. Note that our calibrated EIS of 0.9 is below Bansal and Yaron's (2004), largely because their model assumes persistent shocks to consumption growth rates. The only shocks to consumption on Climate-EZ stem from climate risk.

assumed bond yields. Our base-case EIS of 0.9 corresponds with a bond yield of 3.11%. The higher EIS of 1.2 corresponds with a yield of 2.74%, and the lower EIS of 0.6 a yield of 3.77%.

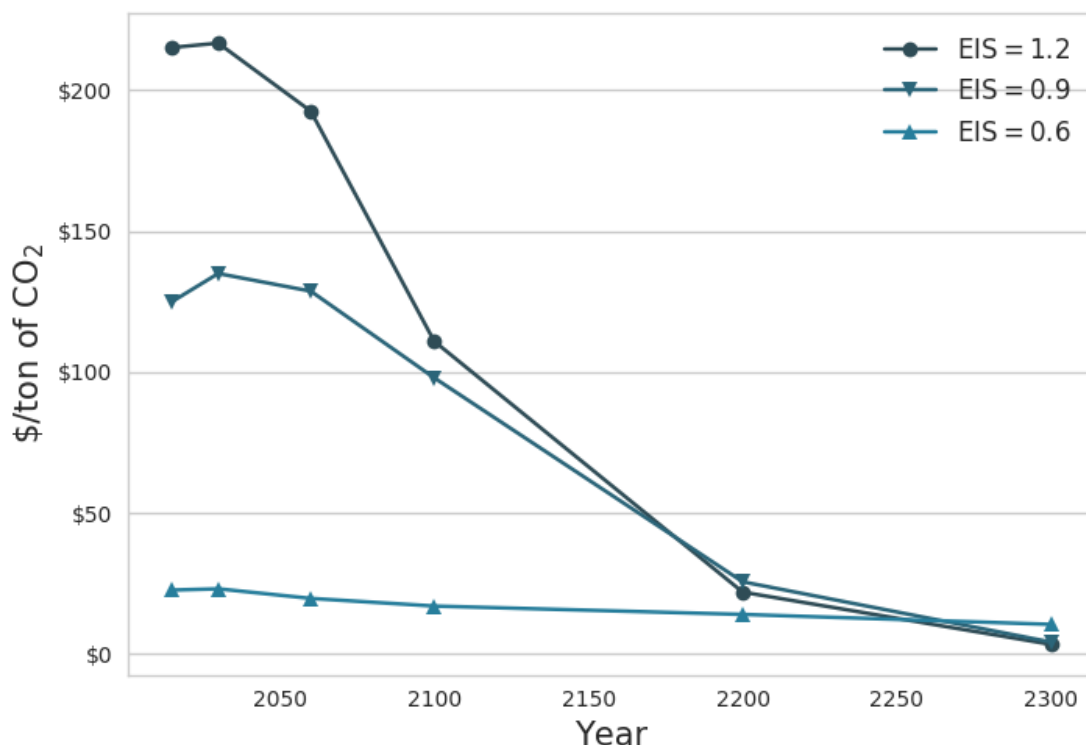


Figure XVIII—A higher (lower) EIS goes hand-in-hand with a higher (lower) optimal CO₂ price in early years

Lastly, we explore the impact of the pure rate of time preference, $\delta = (1 - \beta)/\beta$. First, we hold the EIS fixed at 0.9, while δ varies from 0.25% to 0.75% (Figure XIX). Holding EIS fixed, a higher δ implies a higher real interest rate and, thus, lower optimal CO₂ prices, and vice versa. Corresponding bond yields vary from 2.89% to 3.32% for δ from 0.25% to 0.75%, respectively. Note the large changes in early optimal CO₂ prices, despite holding EIS constant at 0.9.

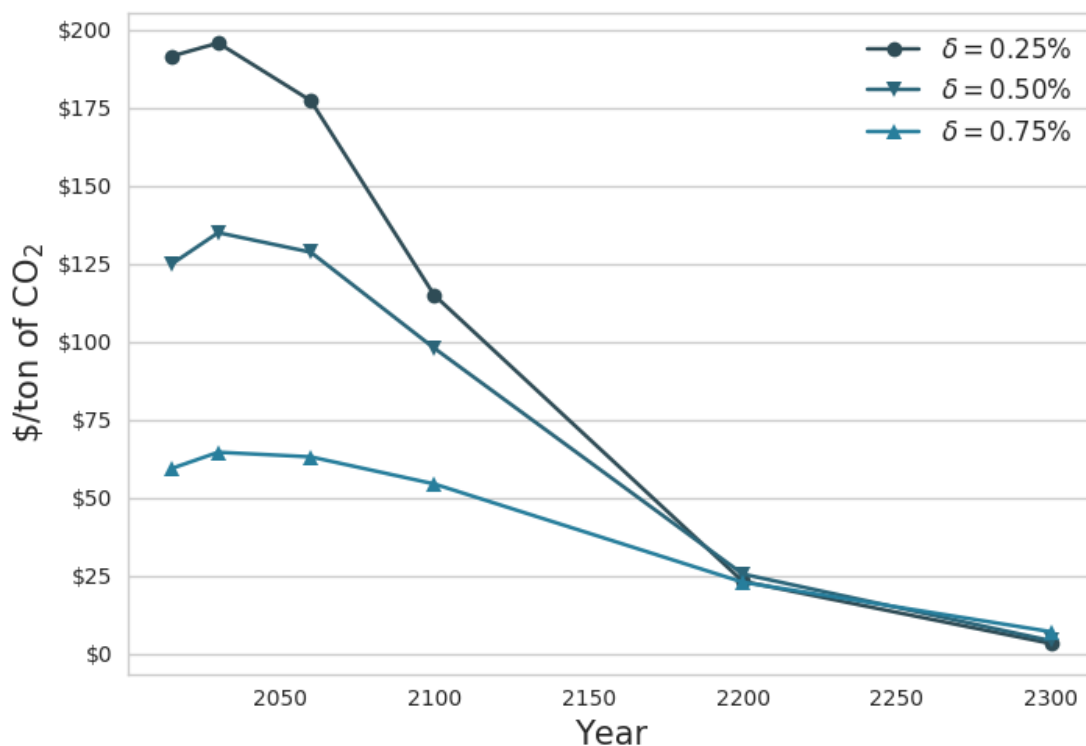


Figure XIX—Optimal CO₂ prices increase (in early years) with decreasing pure rate of time preference, δ , holding EIS fixed at 0.90

Second, we vary EIS while holding real interest rates fixed at around 3.11% by adjusting δ (Figure XX). The two effects from EIS and δ counteract each other. On net, a lower EIS, corresponding with a lower δ , implies a higher optimal CO₂ prices.

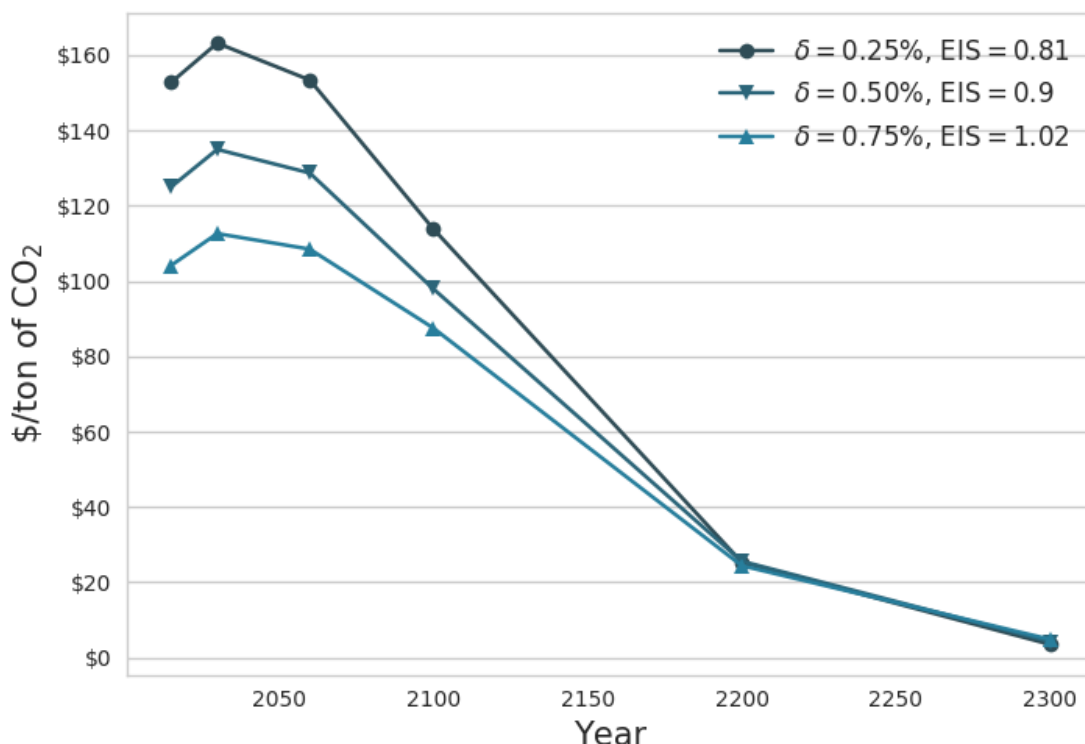


Figure XX—Optimal CO₂ prices increase with decreasing pure rate of time preference, δ , holding real interest rates fixed at close to 3.11%, while adjusting EIS accordingly

Note that as EZ-Climate is calibrated to observed real interest rates, δ is a less important parameter than, for example, in a Ramsey-style model like DICE. The most significant economic parameters in EZ-Climate are: the real interest rate, which, in turn, allows us to calibrate EIS; and future economic growth rates.

III.E. SOCIAL COST OF DELAY

Up to this point in the paper we have focused on analyzing the optimal CO₂ price, given that climate policy is assumed to be optimal over time. It is not. What then is the cost of a delay in pricing emissions at their optimal level?

We are hardly the first to point to the potential costs of delay (e.g., Nordhaus, 2016; Stern, 2015).³⁷ Nordhaus (2017a) suggests that the large increases in the optimal price in his preferred calibration of the DICE model, from around \$5 per ton of CO₂ emitted in 2015 in early iterations published in the 1990s to around \$30 per ton in the latest iteration, is largely a reflection of those costs of delay in implementing an optimal CO₂ price path.³⁸ However, this argument also points to the limits of models like DICE and the need for an

³⁷ This discussion of the cost of delay is distinct from debates about the proper incorporation of climate risk, reflected, for example, in debates among Nordhaus (2015, 2013), Pindyck (2013, 2012), Stern (2013), and Weitzman (2011, 2009). See the discussion in the introductory section and references in footnotes 6 and 8.

³⁸ Nordhaus (2017b), meanwhile, decomposes the change further and counts “revisions in the treatment of the carbon cycle” as the largest single contributor, followed by revisions to the damage and utility functions.

optimal control model like EZ-Climate. In fact, looking at the price response to delay is positively misleading. The optimal CO₂ price in the second period, after it has been constrained to be \$0 in the first period, is lower than in the unconstrained case. That merely reflects adjustment costs of going from a CO₂ price of \$0 to one reflecting the marginal benefits of emissions reductions. But the price alone does not show the enormous costs to society of not following the optimal path. That necessitates a look at the deadweight cost of delay.

We find that the cost of delay in pricing emissions is shockingly high. Employing EZ-Climate, we quantify the deadweight cost of delay by constraining mitigation to zero in the first period and asking how much additional consumption would be required during that period in order to bring the utility of the representative agent to the level of utility of the unconstrained optimal solution. With our 15-year-long first period in the base case, consumption throughout the first period would have to increase by almost 70% or \$20 trillion, given current global consumption of around \$30 trillion.

That deadweight cost increases at a slightly faster rate than with the square of time. For example, decreasing the first-period length to 5 years, and, thus, having a 5-year instead of 15-year delay results in a total utility loss equivalent to increased consumption of 32% over the 5 years. For a 10-year delay, the equivalent consumption loss over the period is almost 70%. Each year of delay causes the equivalent consumption loss to increase by roughly 2%, and it also increases the time interval of the loss, thus leading to a more than quadratic rate of increase in deadweight loss of utility over time (Table IV). Given global consumption of around \$30 trillion, a one-year delay in pricing emissions in our base case would cost the equivalent of about 2% or approximately \$580 billion. A five-year delay on the other hand would cost the equivalent of \$14 trillion, and a 15 year delay would cost \$146 trillion, about five times annual global consumption.

Table IV—Social cost of delay by first-period length

First-period length	Unconstrained optimal 2015 CO ₂ price	Marginal benefit of emissions reductions (in 2015\$)	Per-year consumption impact
<i>Unconstrained base-case calibration</i>			
15 years	\$125	\$125	n/a
<i>Calibration with first period constrained to \$0</i>			
5 years	\$225	\$222	<u>6.4%</u>
10 years	\$229	\$356	<u>6.9%</u>
15 years	\$125	\$500	<u>4.6%</u>

The marginal damage of emissions is much greater if mitigation is delayed. In our base case, the marginal benefit of reductions equals the carbon price of around \$125 per ton of CO₂ in 2015. If mitigation is constrained to zero in the first 15-year period, the marginal benefit of each ton of CO₂ reduced increases around four-fold to over \$500 per ton. That marginal benefit of emissions reductions increases dramatically with the length of delay (Table IV).

IV. FUTURE RESEARCH AND EXTENSIONS

We have emphasized throughout the discussion here that the EZ-Climate model and its parameterization are *ad hoc* and merit considerable future scrutiny by the scientific community. What we believe is unique and innovative is the methodology and solution approach outlined here. The EZ-Climate model better takes account of climatic risks, without putting undue computational burden.

There are many possible avenues for future work and research. First and foremost is a better calibration of some of the key parameters, perhaps led by climate damages in general and the probability and impact of tail risks in particular. The modular format of EZ-Climate allows for other, more sophisticated climate damages efforts to replace our current calibrations, while maintaining the fundamental model structure. Hsiang *et al.* (2017) is but the latest and perhaps most sophisticated entry into this literature, in this case focused on climate impacts in the United States. Our base case climate damages calibration assumes a *peakT* of 6 and *disaster_tail* of 18. While we explore various specifications, none is linked to actual probabilities and impacts of low-probability, high-consequence climatic events. The impacts of both parameters, and their interplay, is potentially large (Figure XII).

A starting point for further work might be some of the guidance provided by Kopp *et al.* (2016): start with candidate ‘climatic tipping elements’ and take the growing body of empirical, econometric analyses to estimate their effects. Second, employ experiments with empirical, process-based impact models to assess the relative importance of different tipping elements. Third, look at social tipping elements such as civil conflict and their associated costs. Projects like the Climate Impacts Lab are beginning to quantify the impacts of some of these climatic and social tipping elements (Houser *et al.*, 2015; Hsiang *et al.*, 2017). Finally, where data are scarce, conduct structured expert elicitations to generate probability functions around tipping elements and their possible economic impacts (e.g., Cooke, 2013).³⁹ These calibrations are much needed to derive defensible estimates of the optimal CO₂ price.

A further extension is a reconsideration of how climatic risk and its interplay with economic variables is modeled. Climate damages, right now, are modeled to affect levels of consumption. What if they were to affect factor productivity and, thus, economic growth rates directly? While EZ-Climate, in its current iteration, relies on deterministic economic growth assumptions, a move to a stochastic economic growth framework would allow for exploring the interplay between climate and economic shocks.

Another possible extension is around climate impacts on poor versus rich. EZ-Climate’s representative agent framework models the average, globally representative consumer. One could easily imagine different climate impacts based on wealth and income. While the rich may have more capital at risk, it would likely be the poor who will be hurt proportionally more, as they are less able to adapt.

³⁹ See Mach *et al.* (2017) on the broader need to consider expert judgement in environmental assessment, and how to do so.

Lastly, any model like EZ-Climate needs to be seen in the correct context. Brock and Hansen (2017), among others, emphasize how climate-economic assessments are beset with at least three different types of uncertainty: climate risk in the traditional, Knightian sense of the word (Knight, 1921); ambiguity around model selection; and misspecification of any one particular model. Further improvements of EZ-Climate can hope to address the third kind of uncertainty. Addressing the first two will require an informed observer to draw the right conclusions. Some observers might point to the importance of calculating the optimal CO₂ price in the first place. Some others might conclude that any such model conveys false precision to policy-makers, who may make too much of any one single number. The optimal CO₂ price, in the end, is an important guide to policy-makers, but it is just that. Good climate policy goes much beyond simply getting the price right. It considers limits to rational climate policy making (Wagner and Zeckhauser, 2011) and takes into account political economy considerations of implementing optimal climate policy (Keohane et al., 1998; Meckling et al., 2015; Wagner et al., 2015).

V. CONCLUSION

An oft-told analogy in climate economics represents the climate system as a hard-to-navigate ocean liner. This image is often used to argue for early action through a slow and gradually increasing carbon price. Too strong a policy early on would be overly costly; a small course correction now will save us from hitting the far-off proverbial iceberg. There are indeed real costs of action. Tradeoffs abound. But as EZ-Climate's base case optimal CO₂ price paths show, once we include a proper accounting of risk aversion and extreme events, this standard logic gets turned on its head: The optimal carbon price may, in fact, be high today, declining over time (Figure VIII).⁴⁰

In EZ-Climate, that decline in optimal CO₂ prices over time reflects the rate at which information is revealed going forward, the degree of risk aversion, and the potential for technological progress and backstop technologies. Either way, the initial 'ocean liner' logic doesn't hold. Or perhaps it gets completed: for turning a large ship long down the line takes bearing off decisively and early, especially in a world of uncertain obstacles. The less certain we are about the risks facing us in future states of the world, the higher the optimal price on carbon today.

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⁴⁰ Ulph and Ulph (1994) similarly derive conditions for a declining carbon price, though, in their case, driven by optimal resource extraction in the context of climate policy.

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