Positive Definiteness of Hessian Matrices

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In order to look into the curvatures of our objective function ($U(0)$) near the optimal solution, we investigate the hessian at the optimal solution point as well as some random points around it. It turns out that the hessian matrix at the optimal solution point is positive definite. However, all the hessian matrices for random point are not positive definite.

Table 1: Positive Definiteness at Optimal Point

Positive Definiteness	✓
Condition Number	6.1240×10^4
Number of Negative Eigenvalues	0
Magnitude of Negative Eigenvalues	0

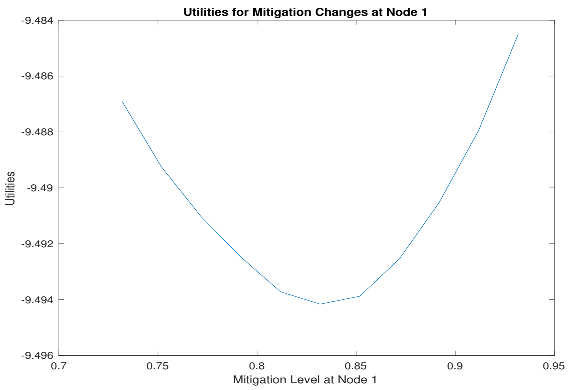
Table 2: Positive Definiteness at Random Points around the Optimal Solution

Sample	1	2	3	4
Positive Definiteness	✗	✗	✗	✗
Condition Number	1.8021×10^4	2.8887×10^6	1.6641×10^4	2.0296×10^4
Number of Negative Eigenvalues	28	28	21	25
Magnitude of Negative Eigenvalues	0.7815	1.1802	0.0070	0.1447
Sample	5	6	7	8
Positive Definiteness	✗	✗	✗	✗
Condition Number	3.3926×10^4	9.7412×10^3	7.0411×10^3	1.0419×10^4
Number of Negative Eigenvalues	28	21	24	24
Magnitude of Negative Eigenvalues	1.2618	0.0144	0.5292	0.1025

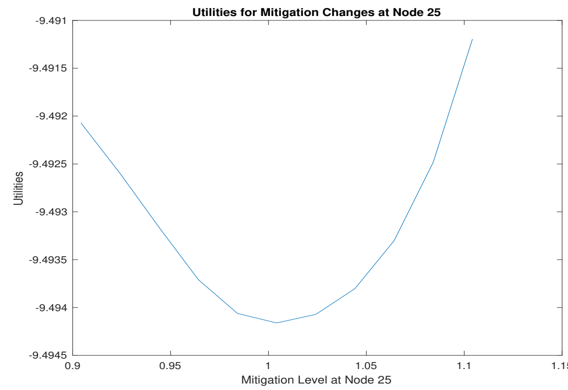
Based on our observation of the results above, we look into utilities with mitigation change in 1 and 2 dimensions. Each time we calculate the utilities with 10 change of size 0.02 in 1 dimension. For example, suppose the optimal mitigation solution is $\mathbf{x}$ with mitigation level for node 1 being x_1. When we investigate the influence of mitigation change at node 1, we take 11 mitigation levels $x_1 - 0.02 \times 5$, $x_1 - 0.02 \times 4$, $\ldots$, $x_1 + 0.02 \times 5$ and fix the other mitigation levels of $\mathbf{x}$. Mitigation level changes like this give us different utility values and generate the plots below.

When we check 1-dimension changes on the optimal solution point, the utility curves are convex at all nodes except the last one. The utility curve is not convex at node 63 because of the default precision level. Here we give the pictures for changes at node 1, 25, 50, and 63. Please refer to them for more information.

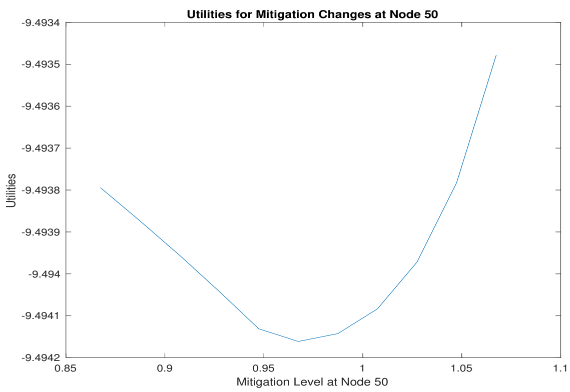
Figure 1: Utilities with 1-dimension Mitigation Change



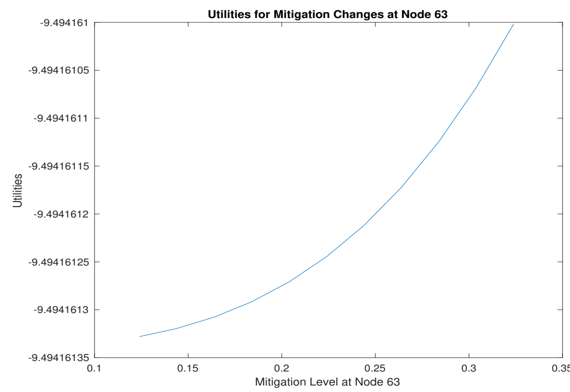
(a) Node 1



(b) Node 25



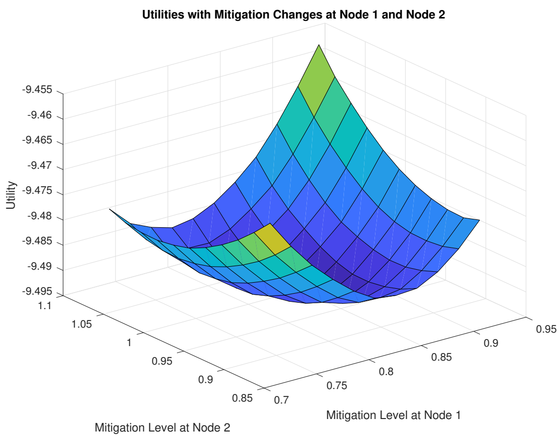
(c) Node 50



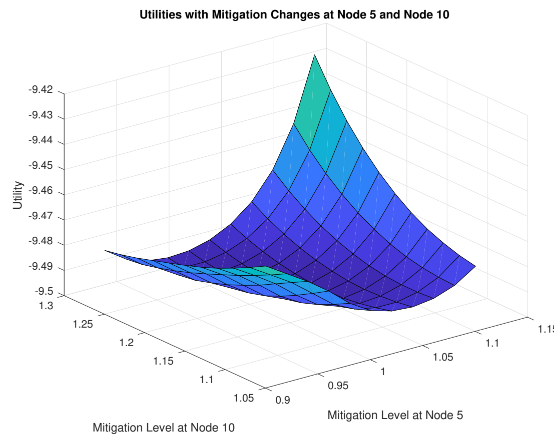
(d) Node 63

Some 2-dimension changes give the plots below.

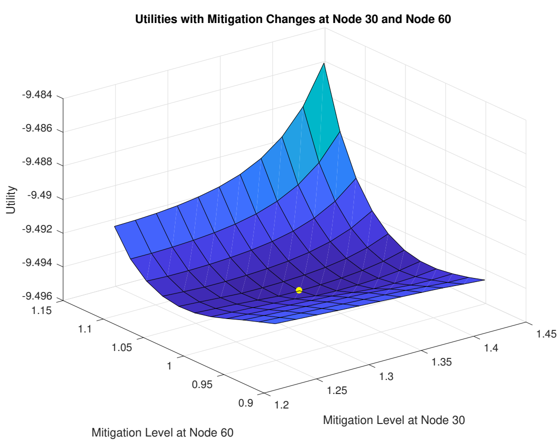
Figure 2: Utilities with 2-dimension Mitigation Changes



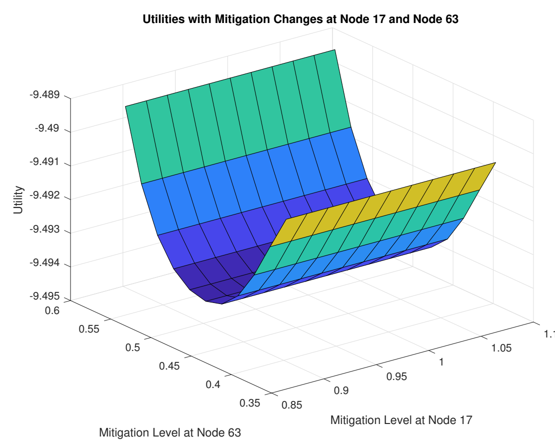
(a) Node 1 and 2



(b) Node 5 and 10



(c) Node 30 and 60



(d) Node 17 and 63