\documentclass[12pt]{article}

\usepackage[utf8]{inputenc}

\usepackage{latexsym}

\usepackage{float}

\usepackage{parskip}

\usepackage{amsfonts}

\usepackage{caption}

\usepackage{commath}

\usepackage{amssymb,amsmath}

\usepackage{graphicx}

\usepackage[backend=bibtex,style=numeric,sorting=none]{biblatex}

\usepackage[export]{adjustbox}

\usepackage{subcaption}

\usepackage[top=1in, bottom=1in,left=1in, right=1in]{geometry}

\usepackage{minted}

\usepackage{color}

\newcommand{\tc}{\textcolor{red}}

\newcommand{\bs}{\boldsymbol}

\newenvironment{alphafootnotes}

{\par\edef\savedfootnotenumber{\number\value{footnote}}

\renewcommand{\thefootnote}{\alph{footnote}}

\setcounter{footnote}{0}}

{\par\setcounter{footnote}{\savedfootnotenumber}}

\begin{document}

\begin{center}

\Large\bf{Documentation for Cost.py}

\end{center}

\normalsize

\pagenumbering{arabic}

\section{Introduction}

The cost.py file has 2 classes \textbf{Cost} and \textbf{DLWCost}.

\begin{itemize}

\item \textbf{Cost} is an abstract cost class that defines abstract methods for cost and price.

\item \textbf{DLWCost} is a class to evaluate the cost curve for the EZ-Climate model, based on the functions we give in the following section.

\end{itemize}

\section{Model of Mitigation Cost Function}

\subsection{Traditional Mitigation Cost: GHG Tax Rate}

The traditional method to mitigate emission is implementing GHG tax rate. In this model, we consider

\begin{itemize}

\item $\tau$: the tax rate per ton of emission.

\item $g$: resulting flow of emissions in gigatonnes of CO$\_2$-equivalent emissions per year, i.e. GT CO$\_2$e.

\item $x$: fraction of emission reduced.

\end{itemize}

The paper gives the marginal abatement cost curve from McKensey's estimates that

\begin{equation}

x(\tau)=0.0923 \tau ^{0.414}

\end{equation}

whose inverse function gives the appropriate tax rate to achieve the mitigation level $x$:

\begin{equation}

\tau(x)=314.32 x^{2.413}

\end{equation}

Essentially, this is the marginal cost with GHG tax rate. We are interested in $\kappa (x)$, the cost to the society when a GHG tax rate of $\tau$ is imposed. We can calculate $\kappa (x)$ using the envelope theorem. Intuitively, GHG emissions are an input to the production process that generates consumption goods. At any tax rate $\tau$, assuming agents choose the level of GHG emissions $g(\tau)$ so as to maximize consumption given $\tau$, then the marginal cost of increasing the tax rate must be the quantity of emissions at that tax rate, that is:

\begin{equation}

\frac{d c(\tau)}{d \tau} = -g(\tau)

\end{equation}

Thus, to calculate the consumption associated with a GHG tax rate of $\tau$ we integrate this expression and get

\begin{equation}

c(\tau) = \bar{c} -\int\_{0}^{\tau} g(s)ds

\end{equation}

where $\bar{c}$ is the endowed level of consumption (assuming zero damages). However, this equation is correct only if the GHG tax is purely dissipative - that is, if the government were to collect the tax and then waste 100\% of the proceeds. In our analysis, we instead assume that the tax is non-dissipative, meaning that the proceeds of the tax $(g(\tau)\cdot \tau)$ would be refunded lump-sum, making the decrease in consumption just equal to the distortionary effect of the tax (in dollars) which is:

\begin{equation}

K(\tau)=\int\_{0}^{\tau} g(s) ds-g(\tau)\cdot\tau

\end{equation}

Writing $g(\tau)=g\_0 (1-x(\tau))$, where $g\_0$ is the baseline level of GHG emissions, we can rewrite $K(\tau)$ as:

\begin{align}

K(\tau) & =g\_0 \int\_{0}^{\tau} (1-x(s))ds -\tau g\_0(1-x(\tau))\\

& =g\_0 \left[\tau - \int\_{0}^{\tau} x(s) ds\right] -\tau g\_0 +\tau g\_0 x(\tau)\\

& = g\_0\left[\tau x(\tau) -\int\_{0}^{\tau} x(s) ds\right]

\end{align}

Then by substituting and simplifying we can get the total cost $K$ as a function of the tax rate $\tau$:

\begin{equation}

K(\tau) = g\_0 \left[0.09230\cdot \tau ^{1.414}-0.06526\cdot\tau^{1.414}\right]= g\_0\cdot 0.02704\cdot \tau^{1.414}

\end{equation}

Substituting gives $K$

as a function of fractional-mitigation $x$: \tc{the code takes $m$ as an input and set it as 92.08, and I do not know exactly why it is not taken as a fixed coefficient}

\begin{equation}\label{K}

K(x)=m \cdot g\_0 \cdot x^{\alpha} =92.08 \cdot g\_0 \cdot x^{3.413}

\end{equation}

where total cost $K(x)$ is expressed in dollars. Finally, we divide by 2015 aggregate consumption $c\_0$ to determine the average cost as a fraction of baseline consumption:

\begin{equation}\label{alpha}

\kappa(x)=\left(\frac{m\cdot g\_0}{c\_0}\right)x^{\alpha}=\left(\frac{92.08\cdot g\_0}{c\_0}\right)x^{3.413}

\end{equation}

where $g\_0=52$ Gt CO$\_2$ represents the level of global annual emissions and $c\_0=\$31$ trillion/year is global consumption in 2015. The equation $\kappa(x)$ expresses the total cost of an arbitrary fractional-mitigation level as a percentage of consumption, and we hold that fixed in all periods except for the impact of technological changes. We assume that, absent technological change, the function is time-invariant.

\subsection{Backstop Technology}

In addition to standard mitigation, modern technologies are available for pulling CO$\_2$ directly out of the atmosphere, namely \textit{backstop technologies}. Some examples are carbon dioxide removal (CDR) or direct carbon removal (DCR) (National Research Council, 2015). Here we need to consider the cost of $CO\_2$ removal.

The backstop technology does not kick in until the mitigation level achieves $x^\*$. We assume the backstop technology is available at a marginal cost of $\tau^\*$ for the first ton of carbon that is removed from the atmosphere. The marginal cost increases as extraction increases, but it has an upper bound $\widetilde{\tau}$ , so unlimited amounts of $CO\_2$ can be removed as the marginal cost approaches $\widetilde{\tau} \geq \tau^\*$. \tc{The paper assumes a price of \$350 per ton for $\tau^\*$ and a price of \$400 per ton for $\widetilde{\tau}$. However, $\tau^\*$ and $\widetilde{\tau}$ equal to 2000 and 2500 respectively in the example. The underlying cost curve for emissions mitigation imply that the backstop technology kicks in at mitigation levels above 104\%, but we try to determine this threshold, namely $x^\*$, in the code. We use $x^\*$ here because the $x\_0$ in the original paper is a little bit confusing.}

Fitting the marginal cost curve to $\tau^\*$ and $\widetilde{\tau}$ gives us

\begin{align}

B(x) & =\widetilde{\tau}-\left(\frac{k}{x}\right)^{\frac{1}{b}}\\

B(x^\*) & = \widetilde{\tau}-\left(\frac{k}{x^\*}\right)^{\frac{1}{b}}=\tau^\*

\end{align}

since $\widetilde{\tau}$ is the upper bound. If we impose a smooth-pasting condition, in which the derivative of the marginal cost curve is continuous at $x^\*$, we can get:

\begin{equation}

b=\frac{\widetilde{\tau}-\tau^\*}{(\alpha-1)\tau^\*}

\end{equation}

\begin{equation}

k=x^\* (\widetilde{\tau}-\tau^\*)^b

\end{equation}

By equalizing the marginal costs for the benchmark mitigation level $x^\*$ under traditional tax rate $\tau(x^\*)$ and backstop technology $B(x^\*)$, we find

\begin{equation}\label{ma}\label{xstar}

x^\* = \left(\frac{\tau^\*}{m \cdot \alpha}\right)^{\frac{1}{\alpha-1}}=\left(\frac{\tau^\*}{92.08 \cdot 3.413}\right)^{\frac{1}{2.413}}

\end{equation}

Hence, when we have mitigation level above $x^\*$, the cost from the backstop technology is given by

\begin{equation}

\int\_{x^\*}^{x} B(s) ds = \int\_{x^\*}^{x} \widetilde{\tau}-\left(\frac{k}{s}\right)^{\frac{1}{b}} ds = \widetilde{\tau} x - \widetilde{\tau} x^\* -\frac{bx(\frac{k}{x})^{\frac{1}{b}}}{b-1}+\frac{bx^\*(\frac{k}{x^\*})^{\frac{1}{b}}}{b-1}

\end{equation}

\subsection{Technological Change}

Since the first period, the marginal cost curve is allowed to decrease at a rate determined by a set of technological change parameters, $\phi\_0$ and $\phi\_1X\_t$:

\begin{itemize}

\item $\phi\_0$: a constant component.

\item $\phi\_1$: a component linked to mitigation efforts. It has to do with the average mitigation up to time $t$:

$$X\_t=\frac{\sum\_{s=0}^t g\_s\cdot x\_s}{\sum\_{s=0}^t g\_s} \text{, where }g\_s \text{ is the flow of GHG emissions into the atmosphere in period $s$}$$

\end{itemize}

At time $t$, the total cost curve is given by:

$$\kappa(x,t)=\kappa(x)[1-\phi\_0-\phi\_1 X\_t]^t$$

This functional form allows for easy calibration. For example, if $\phi\_0 = 0.005$ and $\phi\_1=0.01$, then with average mitigation of 50\%, marginal costs decrease as a percentage of consumption at a rate of 1\% per year.

\section{Inputs}

The python file uses many parameters that can be derived from the cost model described above. Here, we use the \textbf{TreeModel} from tree.py and all the other parameters are floats.

\begin{itemize}

\item $\mathbf{a}$: $\alpha$, curvature of the cost function. In our model, $a=3.413$, as in equation \ref{K}.

\item $\mathbf{g}$: $m$, coefficient of the total traditional cost function. In our model, its value is 92.08, see equation \ref{K}.

\item \textbf{cons\\_at\\_0}: $c\_0$, current global consumption. The default value is $\$$ 30460 billion based on US 2010 values. \tc{cannot find the source of this number}

\item \textbf{emit\\_at\\_0}: $g\_0$, current GHG emission level, derived from bau.py.

\item \textbf{join\\_price}: $\tau^\*$, the lower bound for the marginal cost regarding the backstop technology when it kicks in. The example uses 2000.0.

\item \textbf{max\\_price}: $\widetilde{\tau}$, the upper bound for the marginal cost regarding the backstop technology. The example uses 2500.0.

\item \textbf{tech\\_const}: $\phi\_0$, the degree of exogenous technological improvement over time in percentage. For example, a value of 1.0 implies that the mitigation cost decreases by 1 percent per year. The example uses 1.5.

\item \textbf{tech\\_scale}: $\phi\_1$, the sensitivity, in percentage, of technological change to previous mitigation efforts. The example uses 0.0.

\end{itemize}

\section{Python: Cost}

Define the abstract class Cost.

\begin{minted}{python}

class Cost(object):

"""Abstract Cost class for the EZ-Climate model."""

\_\_metaclass\_\_ = ABCMeta

@abstractmethod

def cost(self):

pass

@abstractmethod

def price(self):

pass

\end{minted}

\subsection{Attributes}

The class \textbf{DLWCost} has some attributes that stand for important parameters in our cost functions. We have seen some of them in the Section \textbf{Inputs}.

\begin{itemize}

\item $\mathbf{a}$: $\alpha$, curvature of the cost function. In our model, $a=3.413$. (see equation \ref{K})

\item $\mathbf{g}$: $m$, coefficient of the total traditional cost function. In our model, its value is 92.08. (see equation \ref{K})

\item \textbf{max\\_price}: $\widetilde{\tau}$, the upper bound for the marginal cost regarding the backstop technology when it kicks in. The example uses 2000.0.

\item \textbf{cbs\\_level}: $x^\*$, the fractional-mitigation level at which the backstop technology kicks in.

\item \textbf{cbs\\_b}: $b=\frac{\widetilde{\tau}-\tau^\*}{(\alpha-1)\tau^\*}$

\item \textbf{cbs\\_k}: $k=x^\* (\widetilde{\tau}-\tau^\*)^b$

\item \textbf{cons\\_per\\_ton}: $\text{cons\\_per\\_ton}=\frac{\text{cons\\_at\\_0}}{\text{emit\\_at\\_0}}=\frac{c\_0}{g\_0}$ is the denominator of \textbf{cbs\\_level} that finally gives us $\kappa(x)$. See equation \ref{xstar}.

\item \textbf{tech\\_const}: $\phi\_0$, the degree of exogenous technological improvement over time in percentage. For example, a value of 1.0 implies that the mitigation cost decreases by 1 percent per year. The example uses 1.5.

\item \textbf{tech\\_scale}: $\phi\_1$, the sensitivity, in percentage, of technological change to previous mitigation efforts. The example uses 0.0.

\end{itemize}

Define the class DLWCost.

\begin{minted}{python}

class DLWCost(Cost):

"""Class to evaluate the cost curve for the EZ-Climate model.

Parameters

----------

tree : `TreeModel` object

tree structure used

emit\_at\_0 : float

initial GHG emission level

g : float --> const of k = gx^a

initial scale of the cost function

a : float --> alpha

curvature of the cost function

join\_price : float -->tau\_\*

price at which the cost curve is extended

max\_price : float --> tau\_tilda

price at which carbon dioxide can be removed from atmosphere in unlimited scale

tech\_const : float --> alpha\_0

determines the degree of exogenous technological improvement over time. A number

of 1.0 implies 1 percent per year lower cost

tech\_scale : float --> alpha\_1

determines the sensitivity of technological change to previous mitigation

cons\_at\_0 : float --> c\_bar

initial consumption. Default $30460bn based on US 2010 values.

Attributes cbs: cost as a fraction of baseline consumption

----------

tree : `TreeModel` object

tree structure used

g : float

initial scale of the cost function

a : float

curvature of the cost function

max\_price : float

price at which carbon dioxide can be removed from atmosphere in unlimited scale

tech\_const : float

determines the degree of exogenous technological improvement over time. A number

of 1.0 implies 1 percent per year lower cost

tech\_scale : float

determines the sensitivity of technological change to previous mitigation

cons\_at\_0 : float

initial consumption. Default $30460 billion based on US 2010 values.

cbs\_level : float

constant

cbs\_deriv : float

constant

cbs\_b : float

constant

cbs\_k : float

constant

cons\_per\_ton : float

constant

"""

\end{minted}

\begin{minted}{python}

def \_\_init\_\_(self, tree, emit\_at\_0, g, a, join\_price, max\_price,

tech\_const, tech\_scale, cons\_at\_0):

self.tree = tree

self.g = g

self.a = a

self.max\_price = max\_price

self.tech\_const = tech\_const

self.tech\_scale = tech\_scale

self.cbs\_level = (join\_price / (g \* a))\*\*(1.0 / (a - 1.0)) #after which the backstop tech kicks in

self.cbs\_deriv = self.cbs\_level / (join\_price \* (a - 1.0))

self.cbs\_b = self.cbs\_deriv \* (max\_price - join\_price) / self.cbs\_level

self.cbs\_k = self.cbs\_level \* (max\_price - join\_price)\*\*self.cbs\_b

self.cons\_per\_ton = cons\_at\_0 / emit\_at\_0

\end{minted}

\subsection{Methods}

The DLWCost has two methods \textbf{cost} and \textbf{price}, corresponding to the abstract methods in class \textbf{Cost}. They give us the aggregate social cost and the marginal social cost concerning the given mitigation levels.

\textbf{cost} gives the total social mitigation cost, given a period, corresponding fractional-mitigation, and average mitigation level. The underlying equations are

$$

\text{total social mitigation cost}=[1-\phi\_0-\phi\_1 X\_t]^t\cdot\begin{cases}

\frac{g\_0}{c\_0}\cdot m \cdot x^{\alpha}, & x\leq x^\*\\

\frac{g\_0}{c\_0}\cdot m \cdot (x^\*)^{\alpha}+ \widetilde{\tau} x - \widetilde{\tau} x^\* -\frac{bx(\frac{k}{x})^{\frac{1}{b}}}{b-1}+\frac{bx^\*(\frac{k}{x^\*})^{\frac{1}{b}}}{b-1},& x > x^\*

\end{cases}

$$

\begin{minted}{python}

def cost(self, period, mitigation, ave\_mitigation):

"""Calculates the mitigation cost for the period. For details about the cost function

see DLW-paper.

Parameters

----------

period : int

period in tree for which mitigation cost is calculated

mitigation : ndarray

current mitigation values for period

ave\_mitigation : ndarray

average mitigation up to this period for all nodes in the period

Returns

-------

ndarray

cost

"""

years = self.tree.decision\_times[period]

tech\_term = (1.0 - ((self.tech\_const + self.tech\_scale\*ave\_mitigation) / 100.0))\*\*years

cbs = self.g \* (mitigation\*\*self.a) #cbs is a power function of mitigation

bool\_arr = (mitigation < self.cbs\_level).astype(int) # check if backstop technology is implemented

if np.all(bool\_arr): # cost of traditional mitigation

c = (cbs \* tech\_term) / self.cons\_per\_ton

else: # cost with backstop technology

base\_cbs = self.g \* self.cbs\_level\*\*self.a #cost of normal mitigation

bool\_arr2 = (mitigation > self.cbs\_level).astype(int)

extension = ((mitigation-self.cbs\_level) \* self.max\_price

- self.cbs\_b\*mitigation \* (self.cbs\_k/mitigation)\*\*(1.0/self.cbs\_b) / (self.cbs\_b-1.0)

+ self.cbs\_b\*self.cbs\_level \* (self.cbs\_k/self.cbs\_level)\*\*(1.0/self.cbs\_b) / (self.cbs\_b-1.0))

#cost of implementing backstop technology

c = (cbs \* bool\_arr + (base\_cbs + extension)\*bool\_arr2) \* tech\_term / self.cons\_per\_ton

return c

\end{minted}

\textbf{price} determines the price, or marginal cost, given a period, corresponding fractional-mitigation, and average mitigation level.

$$\text{price for each level of fractional-mitigation}=

\begin{cases}

\tau(x)=m\cdot \alpha x^{\alpha-1}[1-\phi\_0-\phi\_1 X\_t]^t, & x\leq x^\*\\

B(x)=\widetilde{\tau}-\left(\frac{k}{x^\*}\right)^{\frac{1}{b}}[1-\phi\_0-\phi\_1 X\_t]^t, & x>x^\*

\end{cases}

$$

\begin{minted}{python}

def price(self, years, mitigation, ave\_mitigation):

"""Inverse of the cost function. Gives emissions price for any given

degree of mitigation, average\_mitigation, and horizon.

Parameters

----------

years : int y

years of technological change so far

mitigation : float

mitigation value in node

ave\_mitigation : float

average mitigation up to this period

Returns

-------

float

the price.

"""

tech\_term = (1.0 - ((self.tech\_const + self.tech\_scale\*ave\_mitigation) / 100))\*\*years

if mitigation < self.cbs\_level:

return self.g \* self.a \* (mitigation\*\*(self.a-1.0)) \* tech\_term #price under traditional mitigation

else:

return (self.max\_price - (self.cbs\_k/mitigation)\*\*(1.0/self.cbs\_b)) \* tech\_term # marginal price with backstop technology

\end{minted}

\begin{minted}{python}

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\end{document}