Boosting and Ensemble Learning

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Recap: Statistical Learning setup

Three scenarios: regression, classification & density estimation.
Learn f from examples

$$(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_N, y_N) \in \mathbf{R}^N \times \mathbf{R}^M \text{ or } \{\pm 1\}, \quad \text{generated from } P(\mathbf{x}, y),$$

such that expected number of errors on test set (drawn from $P(\mathbf{x}, y)$),

$$R[f] = \int \frac{1}{2} |f(\mathbf{x}) - y|^2 \, dP(\mathbf{x}, y),$$

is minimal *(Risk Minimization (RM))*.

Problem: P is unknown. $\longrightarrow$ need an *induction principle*.

Empirical risk minimization (ERM): replace the average over $P(\mathbf{x}, y)$ by an average over the training sample, i.e. minimize the training error

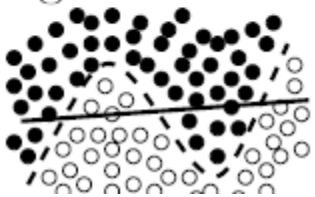
$$R_{emp}[f] = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} |f(\mathbf{x}_i) - y_i|^2$$

Recap: Statistical Learning setup II

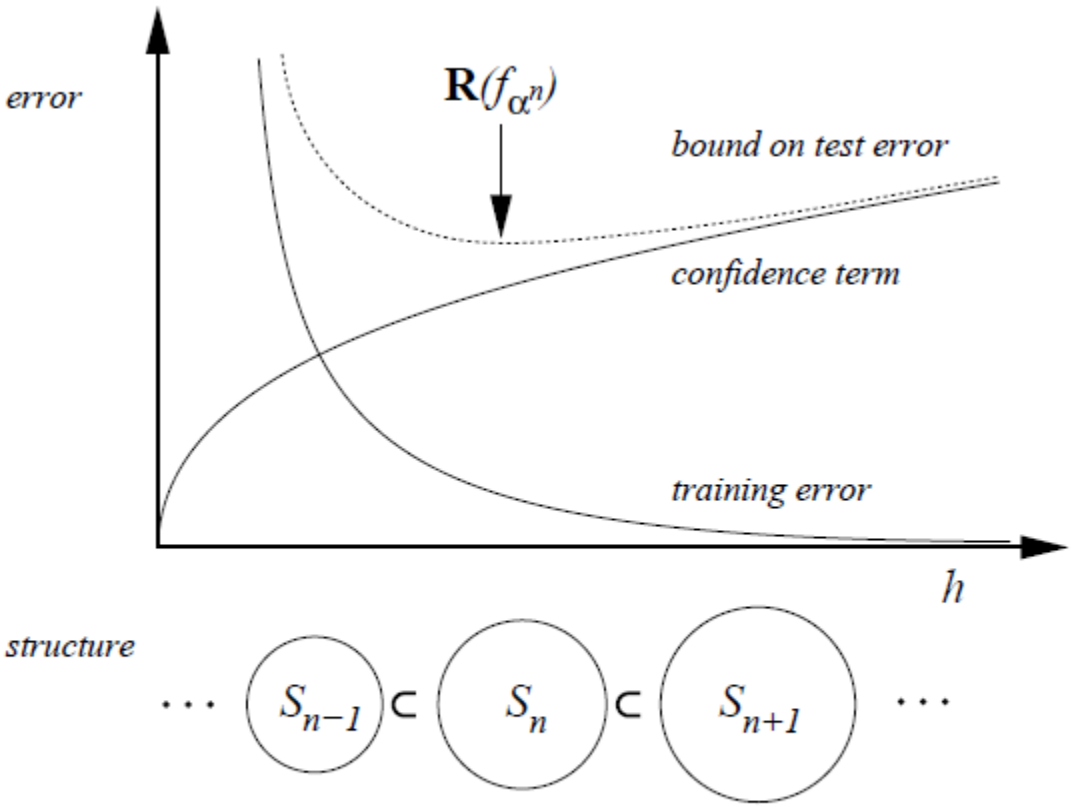
- Law of large numbers: $R_{emp}[f] \to R[f]$ as $N \to \infty$.
  "consistency" of ERM: for $N \to \infty$, ERM should lead to the same result as RM?
- No: *uniform* convergence needed (Vapnik) → VC theory.
  Thm. [classification] (Vapnik 95): with a probability of at least $1 - \eta$,

$$R[f] \leq R_{emp}[f] + \sqrt{\frac{d\left(\log \frac{2N}{d} + 1\right) - \log(\eta/4)}{N}}.$$

- Structural risk minimization (SRM): introduce structure on set of functions $\{f_\alpha\}$ & minimize RHS to get low risk! (Vapnik 95)
- d is VC dimension, measuring complexity of function class



SRM- the picture



Learning f requires small training error *and* small complexity of the set $\{f_\alpha\}$.

SVM vs. Boosting

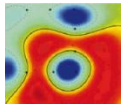
- SVMs

$$R[f] \leq R_{emp}[f] + \mathcal{O}\left(\sqrt{\frac{\log\left(N\theta^2\right)}{\theta^2 N} + \frac{\log(1/\eta)}{N}}\right).$$

- Boosting

$$R[f] \leq R^{\theta}_{emp}[f] + \mathcal{O}\left(\sqrt{\frac{d\log^2\left(\frac{N}{d}\right)}{\theta^2 N} + \frac{\log(1/\delta)}{N}}\right)$$

- **independent of the dimensionality of the space!**



VC dimension: example

VC 维度：例子

在二维实数空间 $\mathbb{R}^2$ 中的半空间：

$$f(x, y) = \text{sgn}(a + bx + cy),$$

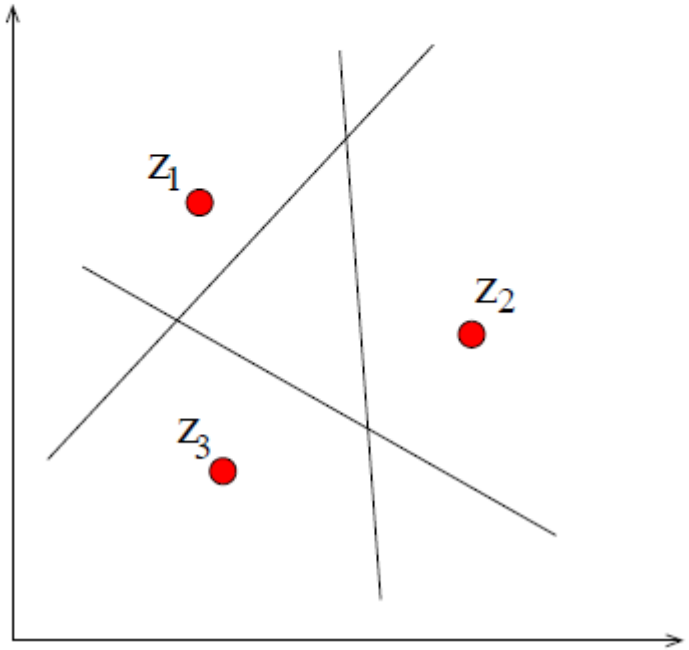
其中参数 $a, b, c \in \mathbb{R}$。

- 显然，我们可以对三个**不共线的点**进行 shatter（完全分类）。
- 但是我们无法对四个点进行 shatter。
- 因此，VC 维度为 $d = 3$。
- 在 n 维空间中：VC 维度为 $d = n + 1$。

Half-spaces in $\mathbf{R}^2$:

$$f(x, y) = \text{sgn}(a + bx + cy), \quad \text{with parameters } a, b, c \in \mathbf{R}$$

- Clearly, we can shatter three non-collinear points.
- But we can never shatter four points.
- Hence the VC dimension is $d = 3$
- in n dimensions: VC dimension is $d = n + 1$



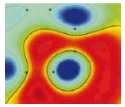
The Basic idea behind boosting

Ensemble Learning and Classification

- Ensemble for binary classification consists of
  - Hypotheses (basis functions) $\{h_t(\mathbf{x}) : t = 1, \ldots, T\}$
    - of some hypothesis ("concept") set
      $\mathcal{H} = \{h \mid h(\mathbf{x}) \mapsto \{\pm 1\}\}$
  - Weights $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_T]$
    - satisfying $\alpha_t \geq 0$
- Classification Output: weighted majority of the votes
  - $f_{\mathrm{Ens}}(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})$
- How to find the hypotheses and their weights?
  - Bagging (Breiman, 1996): $\alpha_t = 1/T$
  - AdaBoost (Freund & Schapire, 1994)

Bagging（Breiman，1996）：所有权重相等，$\alpha_t = 1/T$。

AdaBoost（Freund & Schapire，1994）：根据假设的性能动态调整权重。

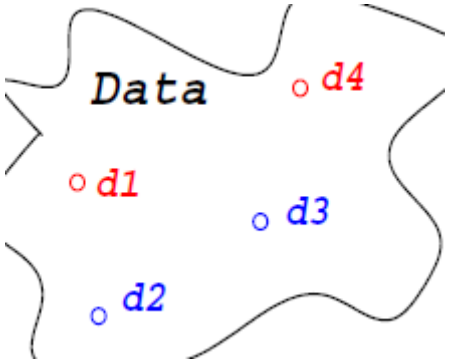


The Adaboost Algorithm

Input: N examples $\{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_N, y_N)\}$

Initialize: $d_i^{(1)} = 1/N$ for all $i = 1 \ldots N$

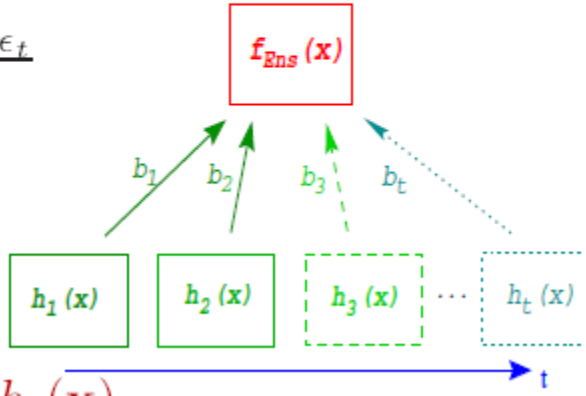
Do for $t = 1, \ldots, T$,



1. Train base learner according to example distribution $\mathbf{d}^{(t)}$ and obtain hypothesis $h_t : \mathbf{x} \mapsto \{\pm 1\}$.
2. compute weighted error $\epsilon_t = \sum_{i=1}^{N} d_i^{(t)} \mathrm{I}(y_i \neq h_t(\mathbf{x}_i))$
3. compute hypothesis weight $\alpha_t = \frac{1}{2} \log \frac{1-\epsilon_t}{\epsilon_t}$
4. update example distribution

$$d_i^{(t+1)} = d_i^{(t)} \exp\left(-\alpha_t y_i h_t(\mathbf{x}_i)\right) / Z_t$$

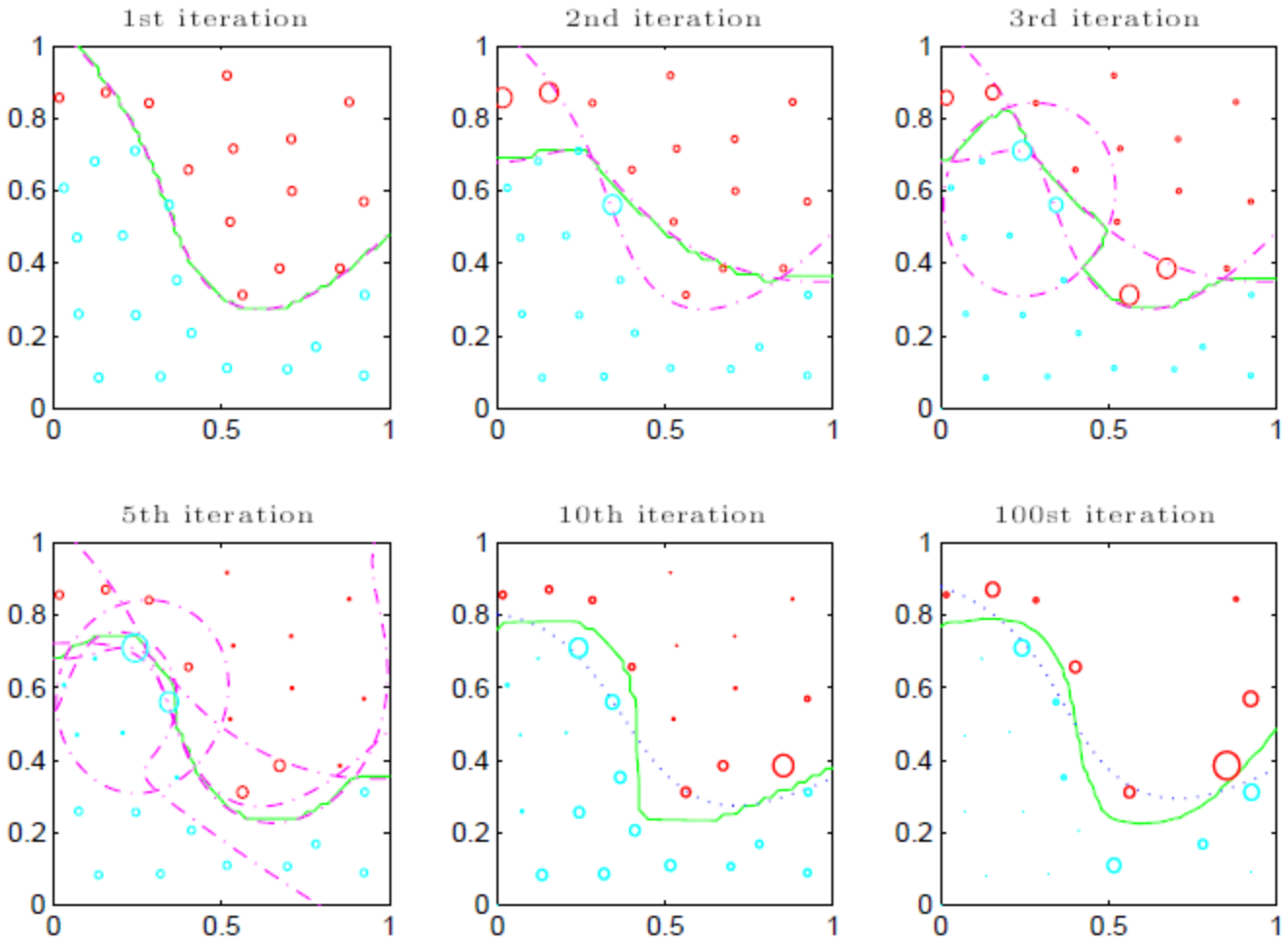
其中 Z_t 是归一化因子，保证权重总和为 1。



Output: final hypothesis $f_{\mathrm{Ens}}(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})$

- AdaBoost 的核心思想是：每次迭代后，更加关注那些被当前弱分类器错误分类的样本。
- 权重的更新公式使得分类正确的样本逐渐"被忽略"，而分类错误的样本在后续迭代中"受到更多关注"，从而提升整体模型的性能。

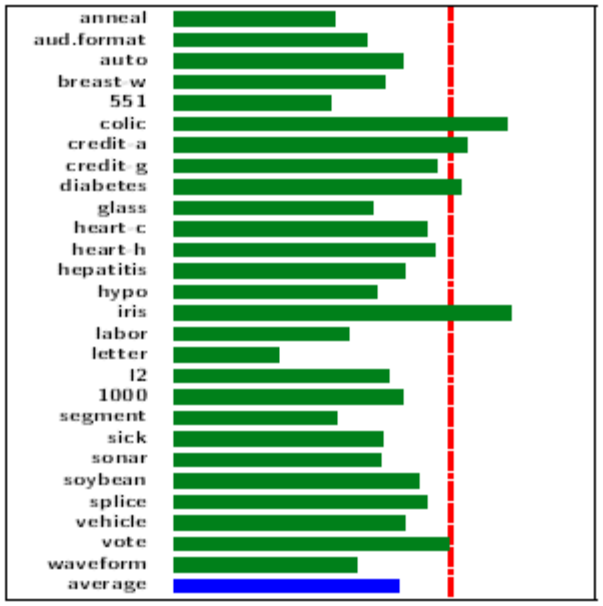
Adaboost Algorithm: illustration



Experimental Motivation

Architecture	Test Error
LeNet 1	1.7%
LeNet 4	1.1%
LeNet 5	0.9%
SVM polynom.	1.4%
SVM virt. SV	0.8%
boosted LeNet 4	**0.7%**

Comparison on NIST handwritten character recognition data set (LeCun et al. (1995))



Comparison on UCI repository data (Quinlan (1998))

Error Function of Adaboost

- AdaBoost stepwise minimizes a function of $y_i f_{\boldsymbol{\alpha}}(x_i) = y_i \sum_t \alpha_t h_t(\mathbf{x}_i)$

$$\mathcal{G}(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \exp\{-y_i f_{\boldsymbol{\alpha}}(\mathbf{x}_i)\}$$

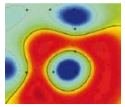
- The gradient of $\mathcal{G}(\boldsymbol{\alpha}^{(t)})$ gives exactly the example weights used for AdaBoost:

$$\frac{\partial \mathcal{G}(\boldsymbol{\alpha}^{(t)})}{\partial f(\mathbf{x}_i)} \sim \exp\{-y_i f_{\boldsymbol{\alpha}}(\mathbf{x}_i)\} \sim d_i^{(t+1)}$$

- The hypothesis coefficient α_t is chosen, such that $\mathcal{G}(\boldsymbol{\alpha}^{(t)})$ is minimized:

$$\alpha_t = \underset{\alpha_t \geq 0}{\operatorname{argmin}}\, \mathcal{G}(\boldsymbol{\alpha}^{(t)}) = \frac{1}{2} \log \frac{1-\epsilon_t}{\epsilon_t}$$

- **AdaBoost is a gradient descent method to minimizes $\mathcal{G}(\boldsymbol{\alpha})$.**
  $\Longrightarrow$ Bregman Divergences (Entropy Projections, ...)
  $\Longrightarrow$ Coordinate Descent Methods & Column Generation



Theoretical Motivation PAC boosting

定理 1 (Schapire et al. 1997): 这一定理说明了使用 AdaBoost 生成加权训练误差 $\epsilon_1, \ldots, \epsilon_T$ 的假设下的训练误差界。

PAC Boosting – exponential convergence

- 左侧：训练错误样本的加权总数。
- 右侧：误差的指数界限，随着迭代次数 T 增大，训练误差会下降。

Theorem 1 (Schapire et al. 1997) *Suppose AdaBoost generates hypotheses with weighted training errors* $\epsilon_1, \ldots, \epsilon_T$. *Then we have*

$$\sum_{i=1}^{N} \mathrm{I}(y_i \neq \mathrm{sign}(f_{Ens}(\mathbf{x}_i))) \leq 2^T \prod_{t=1}^{T} \sqrt{\epsilon_t(1-\epsilon_t)}$$

其中 γ 表示每个基分类器的“余量” (margin)。

If $\epsilon_t < \frac{1}{2} - \frac{1}{2}\gamma$ (for all $t = 1, \ldots, T$), then the training error will decrease exponentially fast, i.e. will be **zero** after only

$$\frac{2\log(N)}{\gamma^2} = \mathcal{O}(\log(N))$$

iterations.

PAC Boosting – VC dimension of combined Hypothesis

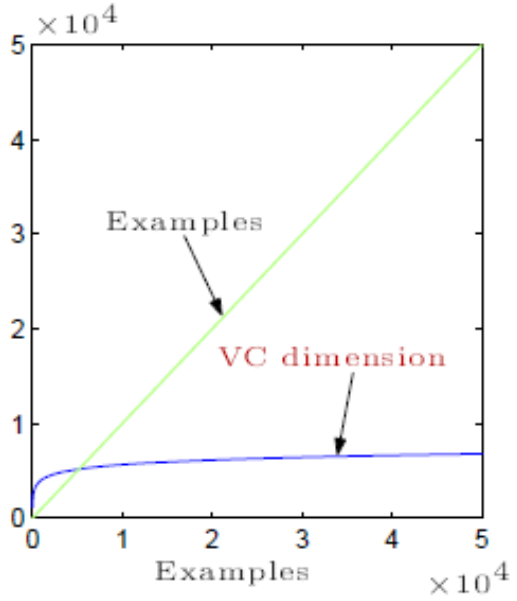
若单个基分类器的 VC 维为 d，则组合分类器的 VC 维为：

Let d be the VC dimension of the base hypothesis class $\mathcal{H}$.
Then the VC dimension of the class of combined functions is

$$d_{Ens}(N,\gamma) = \mathcal{O}\Big(\underbrace{d\,\frac{\log(N)}{\gamma^2}}_{\sim T}\log\Big(\frac{\log(N)}{\gamma^2}\Big)\Big) = \mathcal{O}\left(d\log(N)\log^2(N)\right).$$

An Example

- VC dimension $d = 2$
  (e.g. decision stumps)
- $\epsilon_t \leq 0.4 = \frac{1}{2} - \frac{1}{2}\gamma$
  $\Rightarrow \gamma \geq 0.2$

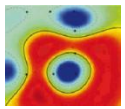


PAC Boosting – Digestion

- properties of weak learner imply exponential convergence to a consistent hypothesis
- Fast convergence ensures small VC dimension of the combined hypothesis
- small VC implies small deviation from the empirical risk
- for any $\varepsilon > 0$ and $\delta > 0$ exists a sample size N, such that with probability $1 - \delta$ the expected risk is smaller than ε
- Any weak learner can be boosted to achieve an arbitrary high accuracy! ($\rightsquigarrow$ strong learner)

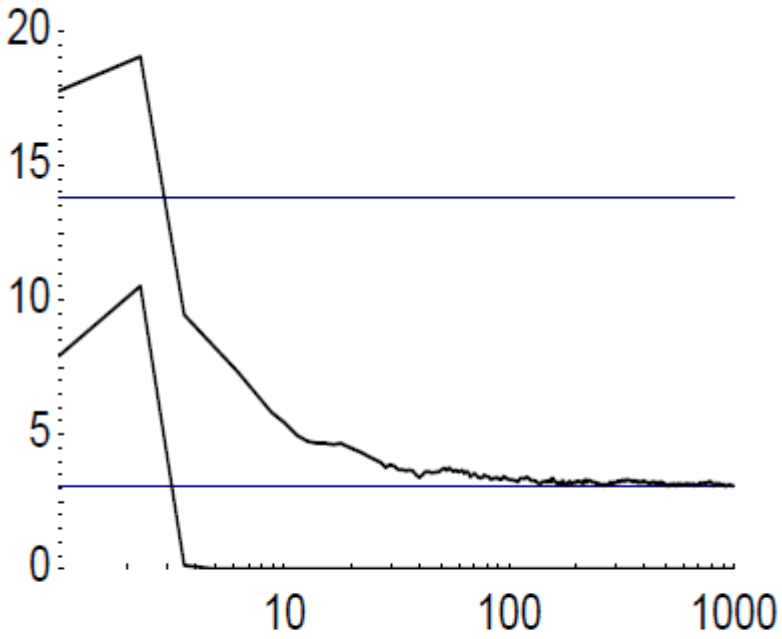
PAC 提升 - 摘要

- 弱学习器的特性意味着 **指数收敛** 到一个一致的假设。
- 快速收敛确保组合假设的 **VC 维度小**。
- VC 维度小意味着与经验风险的 **偏差小**。
- 对于任何 $\epsilon > 0$ 和 $\delta > 0$，都存在一个样本大小 N，使得以概率 $1 - \delta$ 达到的期望风险小于 ϵ。
- **任何弱学习器都可以通过提升达到任意高的准确性！**（即转换为强学习器）



A strange Phenomenon

boosting C4.5 on "letter" data



- test error does not increase
  ⇝ even after 1000 iterations!
- it continues to drop
  ⇝ even after training error is 0!
- Occam's razor predicts simpler rule is better
  ⇝ wrong in this case!?

Needs a better explanation!

Theoretical Motivation margin distributions

Margin Distributions - definitions

这个定义表示 **函数** f 是通过权重 α_h 的线性组合，从假设集合 $\mathcal{H}$ 中的弱学习器 h 构造出来的。

满足条件：所有 $\alpha_h \geq 0$ 且权重之和为 1。

这个函数集合是 **假设集合** $\mathcal{H}$ **的凸包（Convex Hull）**。

- Function set used in boosting: Convex Hull of $\mathcal{H}$

$$S := \left\{ f : \mathbf{x} \mapsto \sum_{h \in \mathcal{H}} \alpha_h h(\mathbf{x}) \;\middle|\; \alpha_h \geq 0, \sum_{h \in \mathcal{H}} \alpha_h = 1 \right\}$$

- the α's are the parameters
- Find a hyperplane in the Feature Space spanned by the hypotheses set $\mathcal{H} = \{h_1, h_2, \ldots\}$
- Margin ρ for an example $(\mathbf{x}_i, y_i)$ by

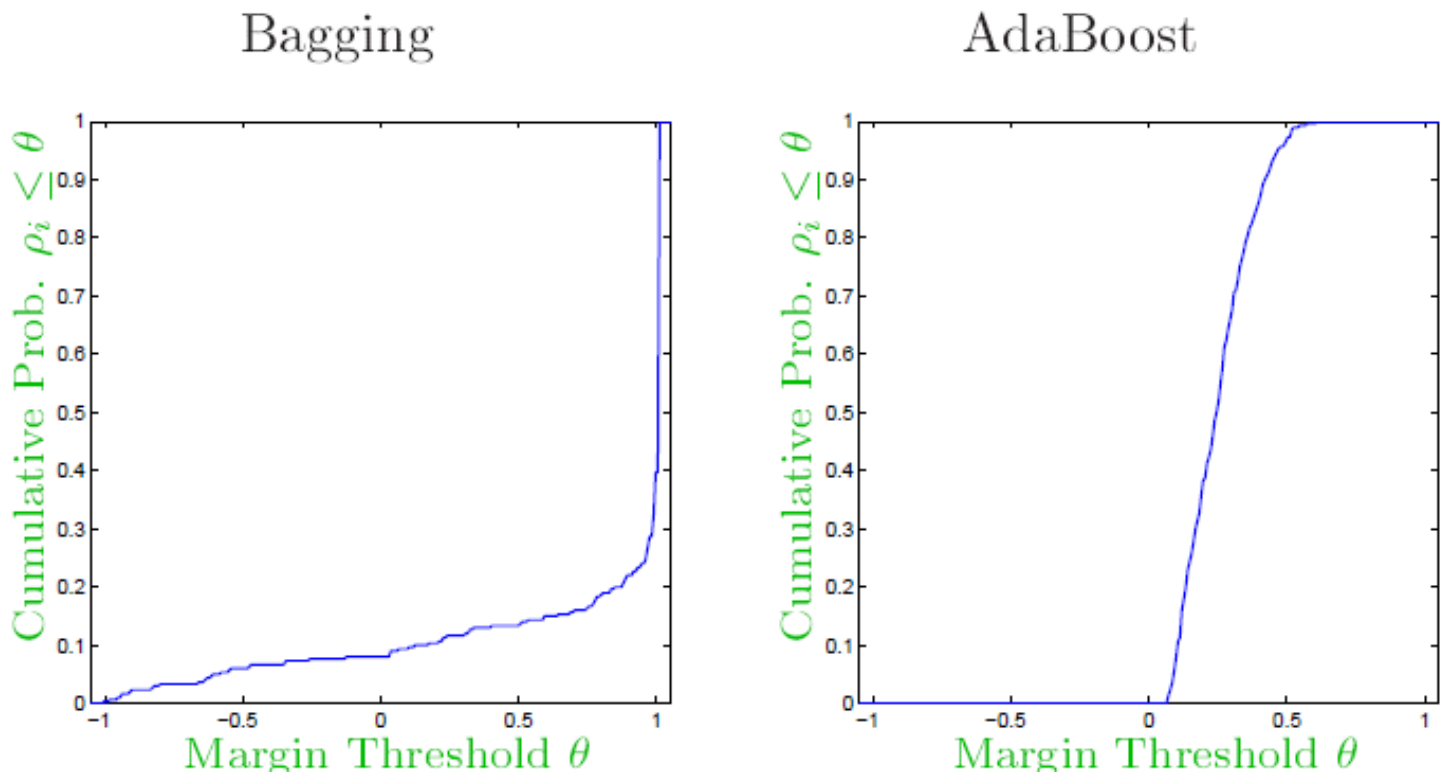
$$\rho_i(\boldsymbol{\alpha}) := y_i f_{\text{Ens}}(\mathbf{x}_i) = y_i \sum_{t=1}^{T} \frac{\alpha_t}{\sum_t \alpha_t} h_t(\mathbf{x}_i)$$

- Margin ϱ for a function f_{Ens} by $\varrho(\boldsymbol{\alpha}) := \min_{i=1,\ldots,N} \rho_i(\boldsymbol{\alpha})$

- $f_{\text{Ens}}(\mathbf{x}_i)$：组合模型在样本 $\mathbf{x}_i$ 上的预测值。
- y_i：样本的真实标签（通常为 -1 或 $+1$）。
- α_t：组合模型中第 t 个假设的权重。
- 这个公式衡量样本预测的 **信心程度**，即预测是否与真实标签一致。

Margin Distributions - illustration

AdaBoost tends to increase small margins, while decreasing large margins

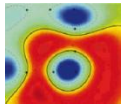


Margin Distributions – lower bounding the margin

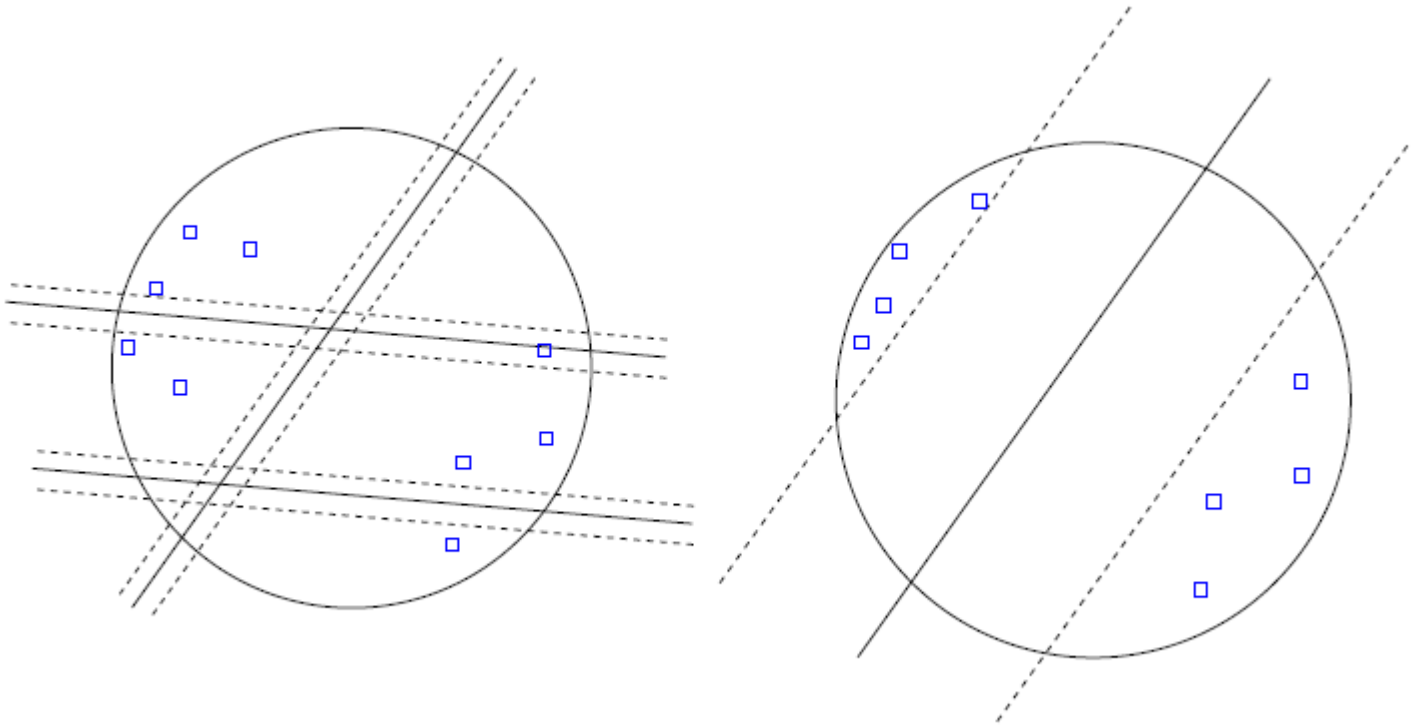
Theorem 2 *Suppose the base learning algorithm generates hypothesis with weighted training errors* $\epsilon_1, \ldots, \epsilon_T$. *Then we have for any* θ

$$P_{\mathbf{Z}}(y f_{Ens}(\mathbf{x}) \leq \theta) \leq 2^T \prod_{t=1}^{T} \sqrt{\epsilon_t{}^{1-\theta}(1-\epsilon_t)^{1+\theta}}$$

Corollary 3 *If the base learning algorithm always achives* $\epsilon_t \leq \frac{1}{2} - \frac{1}{2}\gamma$ *then AdaBoost will generate a combined hyperplane with margin at least* $\frac{1}{2}\gamma$.



Margin Distributions - Large Margin Hyperplanes



Margin Distributions – a bound

Theorem 4 *Let D be a distribution over $X \times \{\pm 1\}$ and let $\mathbf{Z}$ be a sample of N examples chosen independently at random according to D. Suppose the base-hypothesis space $\mathcal{H}$ has VC-dimension d, and let $\delta > 0$. Then with probability at least $1 - \delta$, the expected risk is bounded for $\theta > 0$ by*

$$R[f_{Ens}] \leq P_{\mathbf{Z}}(y f_{Ens}(\mathbf{x}) \leq \theta) + \mathcal{O}\left(\sqrt{\frac{d \log^2(N/d)}{N\theta^2} + \frac{\log(1/\delta)}{N}}\right)$$

SVM vs. Boosting

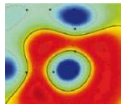
- SVMs

$$R[f] \leq R_{emp}[f] + \mathcal{O}\left(\sqrt{\frac{\log\left(N\theta^2\right)}{\theta^2 N} + \frac{\log(1/\eta)}{N}}\right).$$

- Boosting

$$R[f] \leq R^{\theta}_{emp}[f] + \mathcal{O}\left(\sqrt{\frac{d\log^2\left(\frac{N}{d}\right)}{\theta^2 N} + \frac{\log(1/\delta)}{N}}\right)$$

- **independent of the dimensionality of the space!**



Boosting in the limit

An error function for Adaboost

- AdaBoost stepwise minimizes a function of $y_i f_{\boldsymbol{\alpha}}(x_i) = y_i \sum_t \alpha_t h_t(\mathbf{x}_i)$

$$\mathcal{G}(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \exp\{-y_i f_{\boldsymbol{\alpha}}(\mathbf{x}_i)\}$$

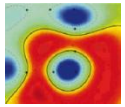
- The gradient of $\mathcal{G}(\boldsymbol{\alpha}^{(t)})$ gives exactly the example weights used for AdaBoost:

$$\frac{\partial \mathcal{G}(\boldsymbol{\alpha}^{(t)})}{\partial f(\mathbf{x}_i)} \sim \exp\{-y_i f_{\boldsymbol{\alpha}}(\mathbf{x}_i)\} \sim d_i^{(t+1)}$$

- The hypothesis coefficient α_t is chosen, such that $\mathcal{G}(\boldsymbol{\alpha}^{(t)})$ is minimized:

$$\alpha_t = \underset{\alpha_t \geq 0}{\operatorname{argmin}}\, \mathcal{G}(\boldsymbol{\alpha}^{(t)}) = \frac{1}{2} \log \frac{1-\epsilon_t}{\epsilon_t}$$

- **AdaBoost is a coordinate gradient descent method which minimizes $\mathcal{G}(\alpha)$ stepwise.**



- AdaBoost 是一种 **坐标梯度下降法**，通过逐步最小化误差函数 $G(\alpha)$ 来提升模型性能。

What happens in the long run?

- Explicit expression for $d_i^{(t+1)}$:

$$d_i^{(t+1)} = \frac{\exp\left\{-\rho_i\left(\boldsymbol{\alpha}^{(t)}\right)\right\}^{\|\boldsymbol{\alpha}^{(t)}\|}}{\sum_{j=1}^{N} \exp\left\{-\rho_j\left(\boldsymbol{\alpha}^{(t)}\right)\right\}^{\|\boldsymbol{\alpha}^{(t)}\|}}$$

$\rightsquigarrow$ **Soft-Max Function** with parameter $\|\boldsymbol{\alpha}^{(t)}\|_1$

- $\|\boldsymbol{\alpha}\|_1$ will increase monotonicaly ($\sim$ linear)

$\rightsquigarrow$ the d's concentrate on a **few** difficult patterns

$\rightarrow$ **Support Patters**

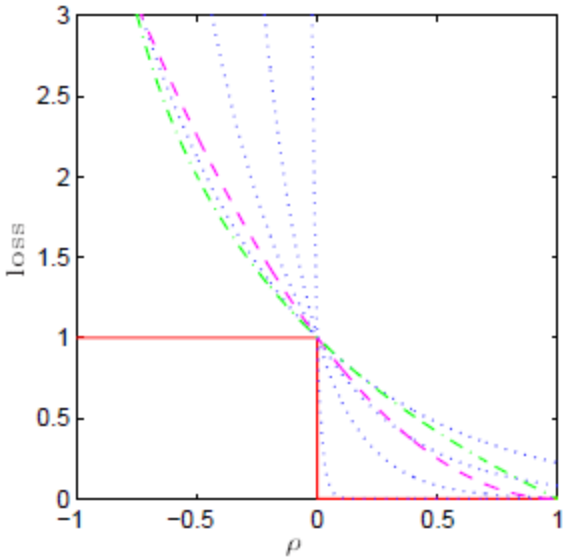
$\rightsquigarrow$ Annealing Process:

退火过程（**Annealing Process**）

$$\mathcal{G}(\boldsymbol{\alpha}) = \sum \exp\left\{-\rho_i(\boldsymbol{\alpha})\right\}^{\|\boldsymbol{\alpha}\|_1}$$

$\rightarrow$ $0/\infty$-Loss approximated asymptoticaly

$\rightarrow$ Barrier Optimization



- ρ_i：样本的边缘值。
- 权重更新类似于 **Soft-Max 函数**。

长期动态

1. **权重变化**：
   - $\|\alpha\|_1$（权重之和）会单调增加（大约线性增长）。
2. **聚焦难点样本**：
   - 权重 d 会集中在少数难分类的样本上（支持样本，Support Patterns）

- 当 $\|\alpha\|_1 \to \infty$，模型逐渐逼近一个极限解。

优化特性

- AdaBoost 的优化目标接近 **Barrier Optimization（屏障优化）**，逼近 0/∞ 损失函数。

1. 退火过程在 AdaBoost 中的含义

在 AdaBoost 中，**退火过程**主要指：

- **样本权重的动态变化**：随着训练轮次 t 的增加，样本的权重会逐渐集中到那些 **难以分类的样本** 上（称为“支持样本”）。
- **优化目标逼近极限**：模型的误差函数 $G(\alpha)$ 在训练过程中逐步收敛，最终达到一个极限解。

2. 样本权重的表达式

样本权重 $d_i^{(t+1)}$ 的公式：

$$d_i^{(t+1)} = \frac{\exp\{-\rho_i(\alpha^{(t)})\|\alpha^{(t)}\|\}}{\sum_{j=1}^{N}\exp\{-\rho_j(\alpha^{(t)})\|\alpha^{(t)}\|\}}$$

- $\rho_i(\alpha^{(t)})$：样本 i 的边缘（Margin），衡量当前分类的信心程度。
  - $\rho_i > 0$：分类正确且信心高。
  - $\rho_i < 0$：分类错误或信心不足。
- $\|\alpha^{(t)}\|$：模型的权重规模，随着训练轮次 t 增加，模型的能力增强，$\|\alpha^{(t)}\|$ 会不断增大。

理解权重变化：

1. **正确分类样本**（边缘较大）：权重 $d_i^{(t+1)}$ 会逐渐减少，因为模型已经很好地掌握了这些样本。
2. **难分类样本**（边缘较小甚至为负）：权重 $d_i^{(t+1)}$ 会逐渐增加，使模型更多关注这些样本。

这种机制类似于 **退火过程中降低温度** 的动态——只保留难点，逐步优化模型。

3. 优化目标和退火特性

AdaBoost 的目标函数是：

$$G(\alpha) = \sum_{i=1}^{N}\exp\{-\rho_i(\alpha)\|\alpha\|_1\}$$

1. **随着 $\|\alpha\|_1$ 增大：**
   - 如果边缘 $\rho_i(\alpha)$ 很大（分类正确且信心高），对应的 $\exp\{-\rho_i(\alpha)\|\alpha\|_1\}$ 会趋近于 0。
   - 只有 **分类难的样本** 会对目标函数 $G(\alpha)$ 产生重要影响。
2. **退火过程**：
   - 初始阶段，模型权重较小（温度高），大多数样本权重 d_i 的差异不明显。
   - 随着训练的进行（温度降低），模型聚焦于 **少数难分类的样本**，从而提升整体分类能力。

4. 支持模式（Support Patterns）

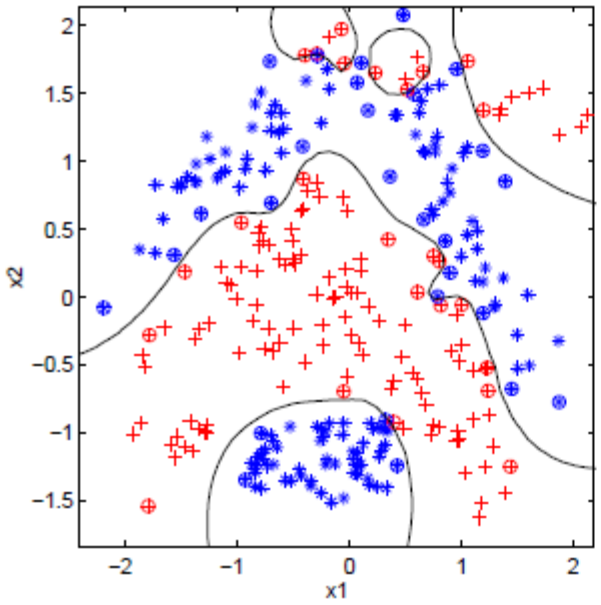
退火的结果是：

- 样本权重 d_i 最终集中在 **支持样本（Support Patterns）** 上。
- 这些支持样本是那些对模型性能至关重要的难点。

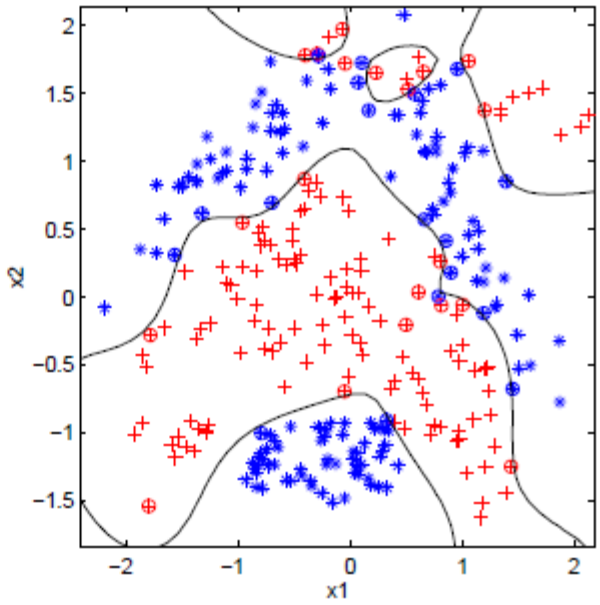
这类似于 SVM 中的 **支持向量**，但 AdaBoost 的支持样本更多是动态调整的，而不是固定的点

Support Vector vs Support Patterns

- AdaBoost 更倾向于聚焦难分类样本（支持模式），生成更复杂的决策边界。
- SVM 则受支持向量的主导，边界更平滑。



AdaBoost's decision line



SVM's decision line

These decision lines are for a low noise case with similar generalisation errors. In AdaBoost, RBF networks with 13 centers were used.

Mathematical Programs: SVMs vs. Boosting

Mathematical Program Formulation- SVMs

The SVM minimization of

$$\begin{aligned} \min_{\mathbf{w}\in\mathcal{F}_\Phi} \quad & \tfrac{1}{2}\|\mathbf{w}\|^2 \\ \text{subject to} \quad & y_i\langle \mathbf{w}, \Phi(\mathbf{x}_i)\rangle \geq 1, \quad i = 1,\ldots,N. \end{aligned} \tag{1}$$

reformulate as maximization of the margin ρ

$$\begin{aligned} \max_{\mathbf{w}\in\mathcal{F}_\Phi,\rho\in\mathbf{R}_+} \quad & \rho \\ \text{subject to} \quad & y_i \sum_{j=1}^{D} w_j \Phi_j(\mathbf{x}_i) \geq \rho \quad \text{for} \quad i = 1,\ldots,N \\ & \|\mathbf{w}\|_2 = 1, \end{aligned} \tag{2}$$

where $D = \dim(\mathcal{F})$ and Φ_j is the j-th component of Φ in feature space:

$$\Phi_j = P_j[\Phi]$$

Boosting as a Mathematical Program

主分类器是多个基分类器的加权线性组合。

- master hypothesis

$$f(\mathbf{x}) = \sum_{t=1}^{T} \frac{w_t}{\|\mathbf{w}\|_1} h_t(\mathbf{x})$$

- base hypotheses h_t produced by the base learning algorithm.

Arc-GV solution is asymptotically the same as linear program solution, maximizing smallest margin ρ:

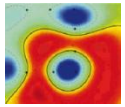
它的解法在渐近意义上与线性规划解相同，目标是**最大化最小间隔** ρ:

$$\max_{\mathbf{w}\in\mathbf{R}^J, \rho\in\mathbf{R}_+} \quad \rho$$

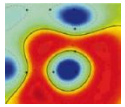
$$\text{subject to} \quad y_i \sum_{j=1}^{J} w_j h_j(\mathbf{x}_i) \geq \rho \quad \text{for} \quad i = 1, \ldots, N \qquad (3)$$

$$\|\mathbf{w}\|_1 = 1,$$

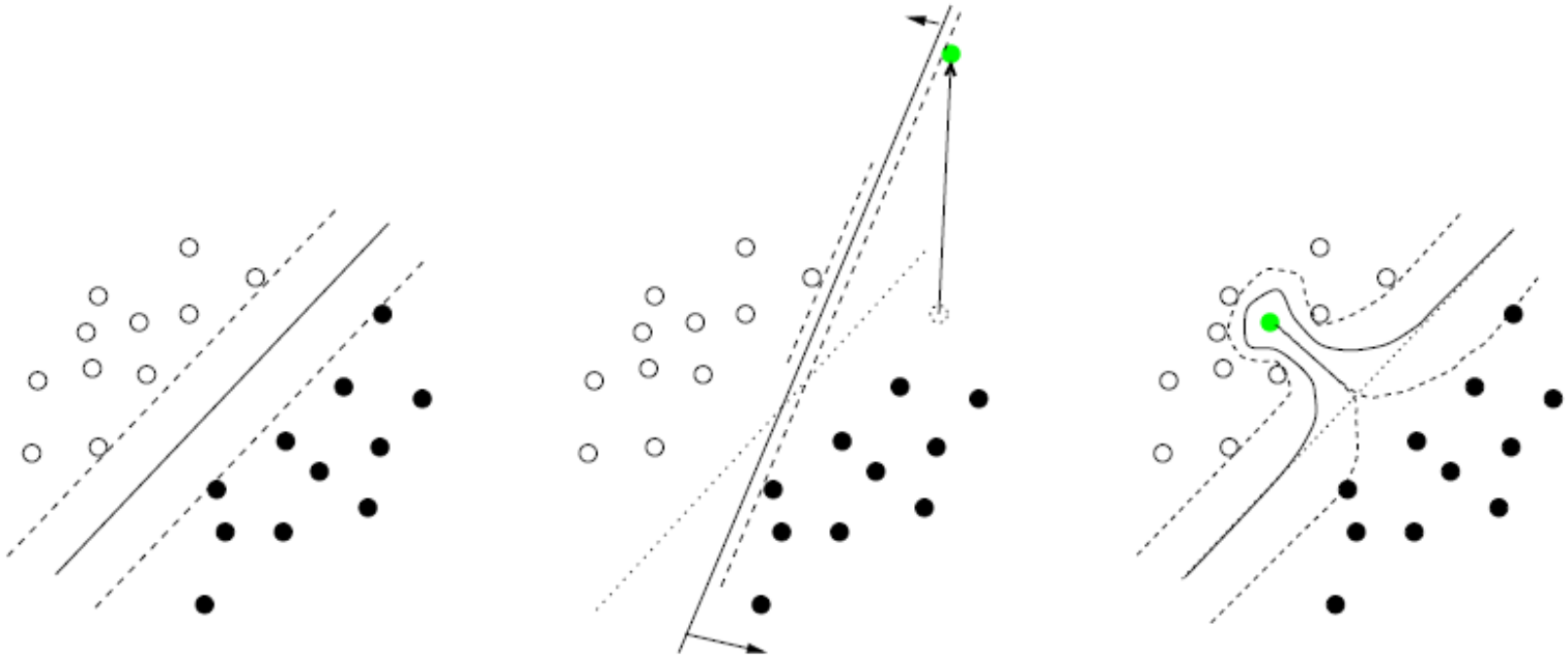
where J is the number of hypotheses in $\mathcal{H}$.



Soft Margins



Hard Margin Classification



- The problem of finding a maximum margin "hyper-plane" on reliable data (left), data with outlier (middle) and a mislabeled pattern (right). The hard margin implies **noise sensitivity**.

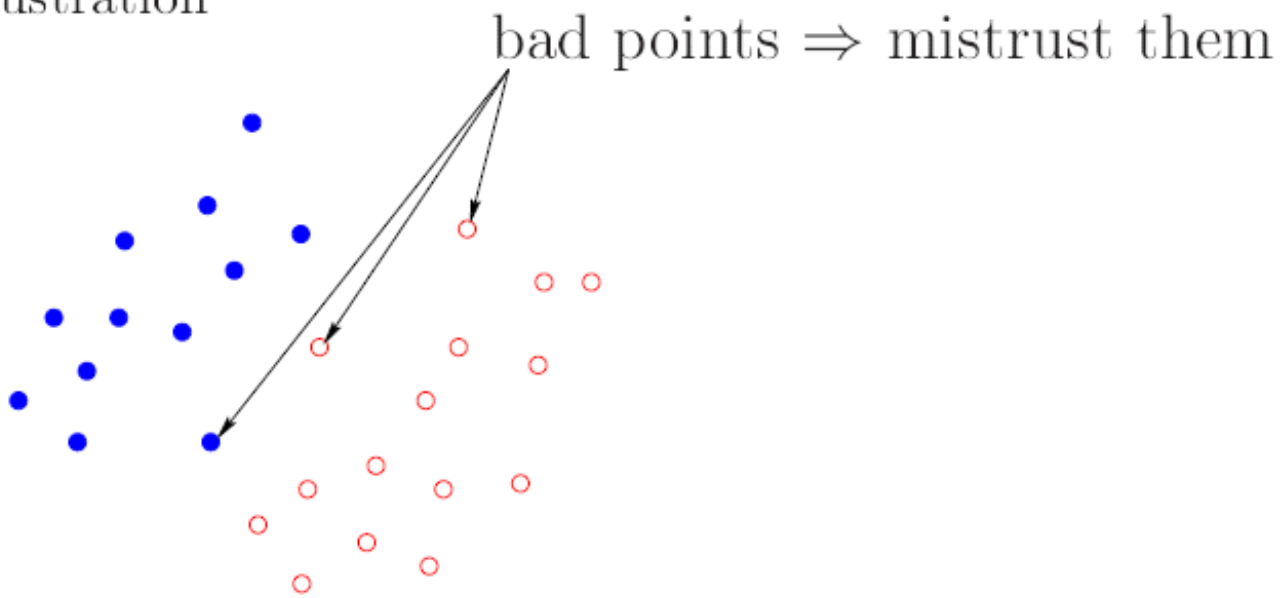
Adaboost with Soft Margins

- Define a Soft Margin

$$\tilde{\rho}_n(\boldsymbol{\alpha}) = \rho_n(\boldsymbol{\alpha}) + \zeta_n,$$

  - where ζ_n is the amount of uncertainty in example $(\mathbf{x}_n, y_n)$

- Illustration



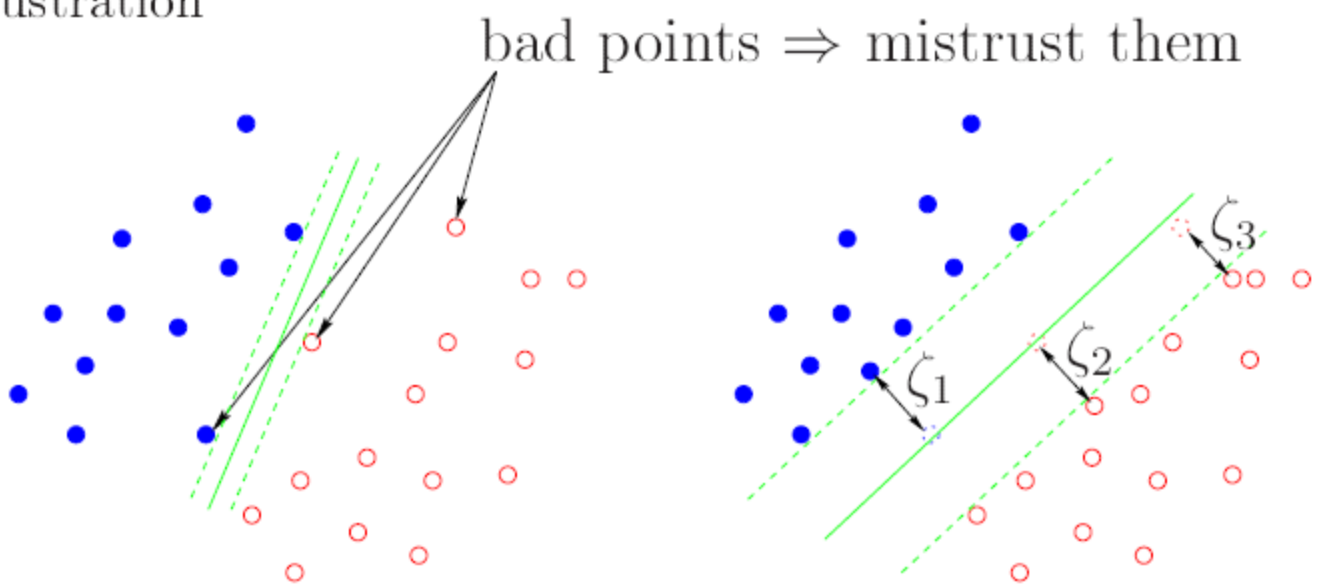
Adaboost with Soft Margins

- Define a Soft Margin

$$\tilde{\rho}_n(\boldsymbol{\alpha}) = \rho_n(\boldsymbol{\alpha}) + \zeta_n$$

  - where ζ_n is the amount of uncertainty in pattern $\mathbf{z}_n$

- Illustration



Adaboost with Soft Margins

- 一旦我们定义了**不确定性测度** ζ_n，我们就可以轻松得到一个新的**正则化的Boosting算法**。
- 通过引入**软边界**来改进误差函数：

- Once we have defined the uncertainity measure ζ_n, we can easily get a new regularized Boosting algorithm.

  $\Rightarrow$ Improve the Error Function by plugging-in the Soft Margin

$$\tilde{G}(\boldsymbol{\alpha}) = \sum_{n=1}^{N} \exp\left\{-\|\boldsymbol{\alpha}\|_1 \tilde{\rho}_n(\boldsymbol{\alpha})\right\}$$

$$d_n^{t+1} = \frac{\partial \tilde{G}(\boldsymbol{\alpha})}{\partial f_{\boldsymbol{\alpha}}(\mathbf{x}_n))}$$

$$\alpha_t = \underset{\alpha_t \geq 0}{\operatorname{argmin}}\, \tilde{G}(\boldsymbol{\alpha})$$

Regularizing Adaboost – Reducing the *Influence*

- How can we know which patterns are unreliable?

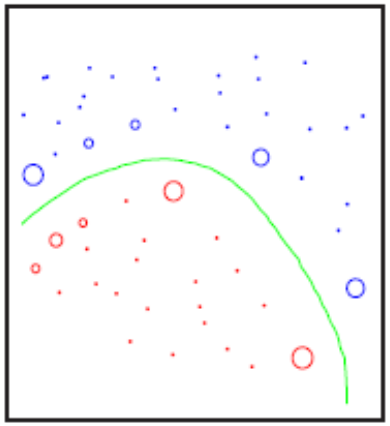
  AdaBoost focuses on difficult-to-learn patterns by assigning high pattern weights d_n that we can exploit. Hence, we define the Influence of a pattern

我们如何知道哪些模式（样本）是不可靠的?

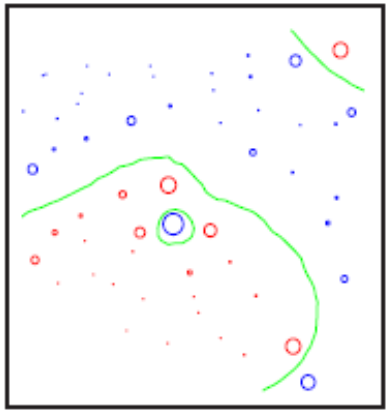
AdaBoost 通过为难以学习的模式分配较高权重 d_n 来解决此问题。

因此，我们定义了模式的“影响”（Influence）。

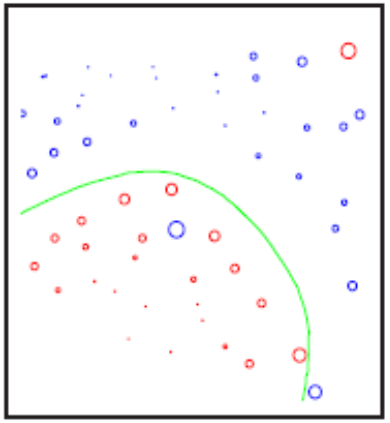
$$\mu_n^t = \sum_{r=1}^{t} \frac{\alpha_r}{\|\boldsymbol{\alpha}\|_1} d_n^r \qquad \zeta_n = C(\mu_n^t)^2$$



AdaBoost without “Noise”



AdaBoost with “Noise”



AdaBoost$_{Reg}$ with “Noise”

Regularizing Adaboost

Positive:

- first algorithm that addresses overfitting in Boostin
- much improved results

Negative:

- Modification on algorithmic level
- hard to analyze
  - which optimization problem is solved
  - no generalization results

Idea:

- go back to beginning and redesign optimization problem
- use convergence results for leveraging
- apply margin bounds

优点（Positive）：

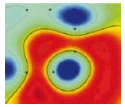
- 第一个解决Boosting中过拟合问题的算法。
- 结果显著改进。

缺点（Negative）：

- 算法级别的修改，难以分析。
  - 解决了哪个优化问题?
  - 缺乏泛化性结果。

思路（Idea）：

- 回到算法起点，重新设计优化问题。
- 使用收敛性结果来改进模型性能。
- 应用间隔边界理论。



Benchmark Comparison

- 10 datasets (from UCI, DELVE and STATLOG repositories)
- Non binary problems partitioned into two-class problems.
- 100 partitions into test and training set (about 60%:40%).
- On each data sets we trained and tested all classifiers. Results are average test errors over 100 runs and standard deviations.
- Parameters estimated by 5-fold cross validation on first 5 realizations of dataset.
- For SVM we used Gaussian kernel
- For Boosting we used RBF networks as base learner

实验设置：

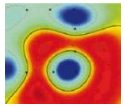
- 使用10个数据集（来自UCI、DELVE和STATLOG数据库）。
- 非二分类问题被划分为二分类问题。
- 数据集被随机分为训练集和测试集，比例约为60%：40%。
- 对每个数据集的所有分类器进行训练和测试。
- 实验结果是100次运行中测试误差的平均值及标准差。

参数估计：

- 使用5折交叉验证，基于数据集的前5个分割。

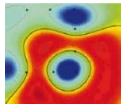
方法：

- 对于SVM，使用高斯核。
- 对于Boosting，使用RBF网络作为基学习器。



Experimental Results

	KNN	C4.5	RBF	AB	AB_R	SVM
Banana	15.0±1.0	16.1±2.8	10.8±0.6	12.3±0.7	10.9±0.4	11.5±0.7
B.Cancer	28.4±4.4	24.6±4.5	27.6±4.7	30.4±4.7	26.5±4.5	26.0±4.7
Diabetes	28.9±2.4	26.0±2.4	24.3±1.9	26.5±2.3	23.8±1.8	23.5±1.7
German	28.9±1.9	28.1±2.4	24.7±2.4	27.5±2.5	24.3±2.1	23.6±2.1
Heart	15.8±3.3	20.4±4.6	17.6±3.3	20.3±3.4	16.5±3.5	16.0±3.3
Ringnorm	35.9±1.3	15.3±1.5	1.7±0.2	1.9±0.3	1.6±0.1	1.7±0.1
F.Solar	37.8±2.8	33.2±1.9	34.4±2.0	35.7±1.8	34.2±2.2	32.4±1.8
Thyroid	5.8±2.8	8.7±3.3	4.5±2.1	4.4±2.2	4.6±2.2	4.8±2.2
Titanic	25.5±3.8	22.9±1.5	23.3±1.3	22.6±1.2	22.6±1.2	22.4±1.0
Waveform	11.4±0.8	17.8±1.0	10.7±1.1	10.8±0.6	9.8±0.8	9.9±0.4
Mean%	2400±6800	1200±2700	5.8±3.7	13.4±9.2	2.7±2.5	2.9±3.5



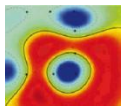
Other Applications

Some examples:

Text classification	Schapire and Singer - Used stumps with normalized term frequency and multi-class encoding
OCR	Schwenk and Bengio (neural networks)
Natural language Processing	Collins; Haruno, Shirai and Ooyama
Image retrieval	Thieu and Viola
Medical diagnosis	Merle *et al.*
Fraud Detection	Rätsch & Müller 2001
Drug Discovery	Rätsch, Demiriz, Bennett 2002
Elect. Power Monitoring	Onoda, Rätsch & Müller 2000

Fuller list: Schapire's 2002, Meir & Rätsch 2003 review

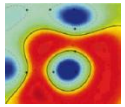
Recently more…



Conclusions

- Boosting algorithms
  - AdaBoost
  - PAC Motivation
  - Boosting with Large Margins
  - Strategies for Dealing with High Dimensional Spaces
  - Relations to Mathematical Programming & SVMs

`Boosting Homepage:` **http://www.boosting.org**



Sources of Information

Internet http://www.boosting.org
http://www.cs.princeton.edu/~schapire/boost.html

Conferences Computational Learning Theory (COLT), Neural Information Processing Systems (NIPS), Int. Conference on Machine Learning (ICML), ...

Journals Machine Learning, Journal of Machine Learning Research, Information and Computation, Annals of Statistics

People List available at http://www.boosting.org

Software Only few implementations (algorithms 'too simple')
(cf. http://www.boosting.org)

Acknowledgements to Gunnar Rätsch