SuS - 5. Tutorium

LTI-Systeme

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$x(t)$: Eingang
$h(t)$: Impulsantwort
$y(t)$: Ausgang

$y(t) = x(t) * h(t)$

$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \underbrace{h(t-\tau)}\, d\tau$

↳ Verschiebung und Spiegelung

→ Faltung: wie verzerrt die Funktion $x(t)$ eine Funktion $h(t)$?

→ Beispielsanwendungen Faltung vs. Korrelation

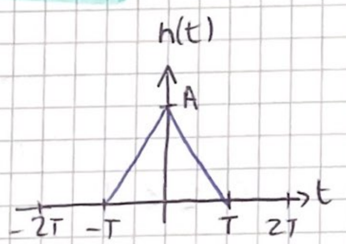
↳ in der Bildbearbeitung:

- Faltung → verschiedene Filter auf Bild anwenden (zB. Filter um Bild unscharf zu machen)
- Korrelation → bestimmtes Muster in einem Bild erkennen

es gilt: $x(t) * h(t) = h(t) * x(t)$

Aufgabe 1

1.1.b)



gesucht: $y(t) = x(t) * h(t) = h(t) * x(t)$

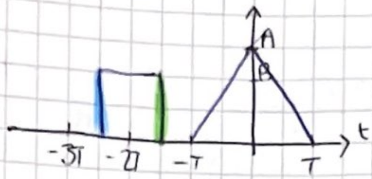
$h(t) = \frac{A}{T} \cdot (t+T) \cdot \sqcap_T(t+\frac{1}{2}T) - \frac{A}{T} \cdot (t-T) \cdot \sqcap_T(t-\frac{1}{2}T)$

$x(t) = B \cdot \sqcap_T(t-\frac{1}{2}T)$

Grenze 1: $t-\tau = 0 \Leftrightarrow \tau = t$

Grenze 2: $t-\tau = T \Leftrightarrow \tau = t-T$

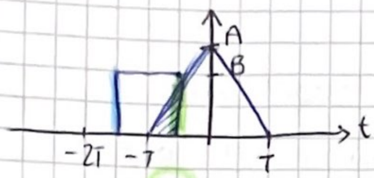
1. Fall:



$t < -T$

$\rightarrow$ keine Überlagerung: $y(t) = 0$

2. Fall:



$t \geq -T$ und $t - T < -T$

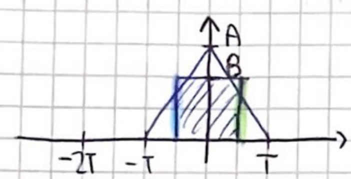
$\Leftrightarrow -T \leq t < 0$

$$y(t) = \int_{-T}^{t} \frac{A}{T} \cdot (\tau + T) \cdot B \, d\tau$$

$$= \frac{AB}{T} \cdot \left[\frac{1}{2}\tau^2 + T \cdot \tau \right]_{-T}^{t}$$

$$= \frac{AB}{T} \cdot \left[\frac{1}{2}t^2 + T \cdot t - \frac{1}{2}T^2 + T^2 \right] = \frac{AB}{T} \cdot \left[\frac{1}{2}t^2 + T \cdot t + \frac{1}{2}T^2 \right]$$

3. Fall:



$t < T$ und $t - T \geq -T$

$\Leftrightarrow 0 \leq t < T$

$$y(t) = \int_{t-T}^{0} \frac{A}{T}(\tau + T) \cdot B \, d\tau + \int_{0}^{t} \left(-\frac{A}{T}\right) \cdot (\tau - T) \cdot B \, d\tau$$

$$= \frac{AB}{T} \cdot \left[-t^2 + \frac{1}{2}T^2 + T \cdot t \right]$$

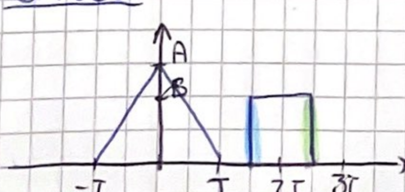
4. Fall:

$t \geq T$ und $t - T < T$

$\Leftrightarrow T \leq t < 2T$

$$y(t) = \int_{t-T}^{T} \left(-\frac{A}{T}\right) \cdot (\tau - T) \cdot B \, d\tau = \frac{AB}{T} \cdot \left[\frac{1}{2}t^2 - 2T t + 2T^2 \right]$$

5. Fall:

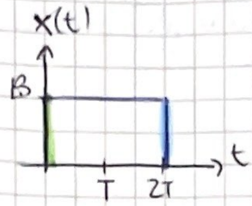


$t - T \geq T \Rightarrow t \geq 2T$

$y(t) = 0$

1.1.c)

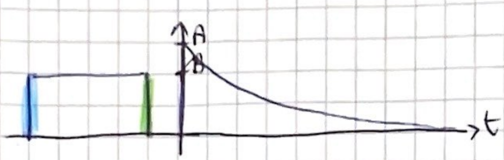
$$h(t) = \begin{cases} A \cdot e^{-\frac{t}{2T}} & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



Grenze 1: $t - \tau = 0 \Leftrightarrow \tau = t$

Grenze 2: $t - \tau = 2T \Leftrightarrow \tau = t - 2T$

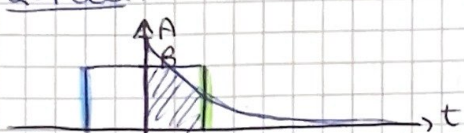
1. Fall:



$t < 0$

$y(t) = 0$

2. Fall:



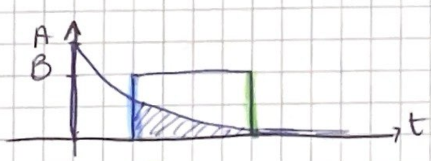
$t \geq 0$ und $t - 2T < 0$

$\Leftrightarrow \quad 0 \leq t < 2T$

$$y(t) = \int_0^t A \cdot e^{-\frac{\tau}{2T}} \cdot B \, d\tau$$

$$= AB \cdot 2T \cdot \left[1 - e^{-\frac{t}{2T}}\right]$$

3. Fall:



$t - 2T \geq 0$

$\Leftrightarrow \quad t \geq 2T$

$$y(t) = \int_{t-2T}^{t} A \cdot e^{-\frac{\tau}{2T}} \cdot B \, d\tau$$

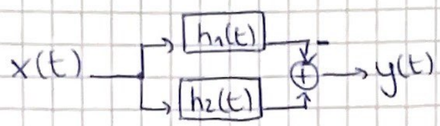
$$= AB \cdot 2T \cdot e^{-\frac{t}{2T}} \cdot \left[e^{1} - 1\right]$$

Aufgabe 2

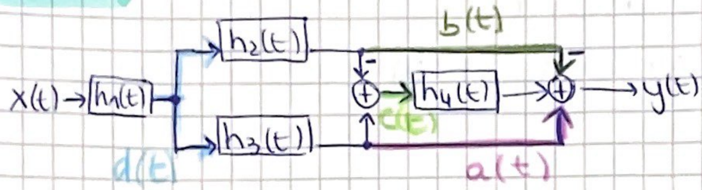
→ 2 Impulsantworten in Reihe werden gefaltet

$$x(t) \to \boxed{h_1(t)} \to \boxed{h_2(t)} \to y(t)$$

→ 2 Impulsantworten parallel werden addiert / substrahiert



2.1 d)



$a(t) = d(t) * h_3(t)$

$b(t) = d(t) * h_2(t)$

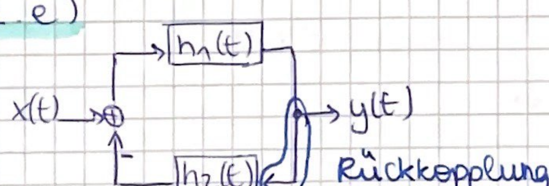
$c(t) = a(t) - b(t)$

$d(t) = x(t) * h_1(t)$

$$\begin{aligned} y(t) &= a(t) - b(t) + c(t) * h_4(t) \\ &= d(t) * h_3(t) - d(t) * h_2(t) + h_4(t) * (a(t) - b(t)) \\ &= x(t) * h_1(t) * h_3(t) - x(t) * h_1(t) * h_2(t) \\ &\quad + h_4(t) * [x(t) * h_1(t) * h_3(t) - x(t) * h_1(t) * h_2(t)] \end{aligned}$$

$$y(t) = x(t) * h_1(t) * [h_3(t) - h_2(t) + h_4(t) * h_3(t) - h_4(t) * h_2(t)]$$

2.1 e)



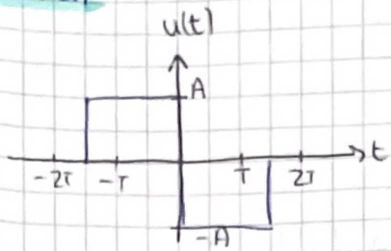
$y(t) = h_1(t) * (x(t) - h_2(t) * y(t))$

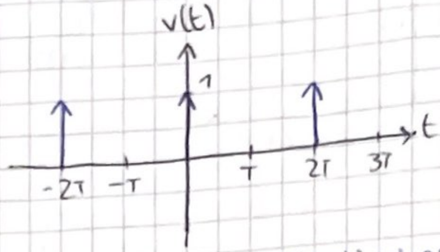
$y(t) + h_1(t) * h_2(t) * y(t) = x(t) * h_1(t)$

$y(t) \cdot [1 + h_1(t) * h_2(t)] = x(t) * h_1(t)$

Aufgabe 3

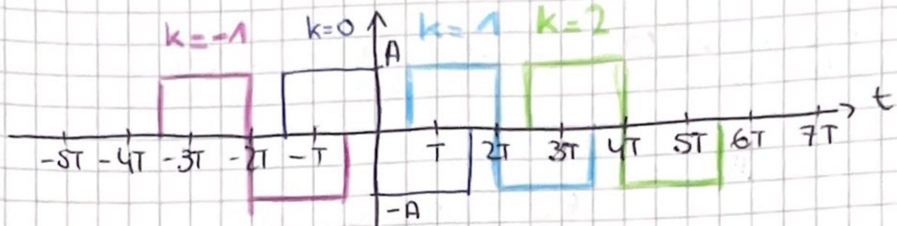
3.1 a)

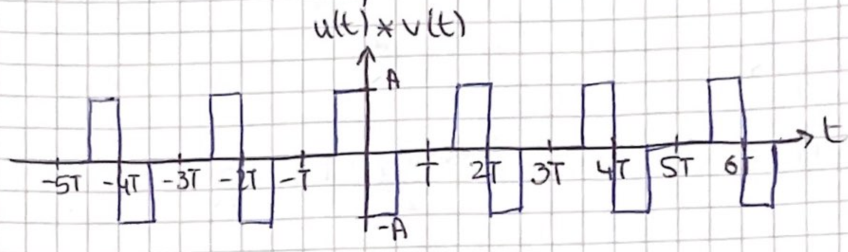




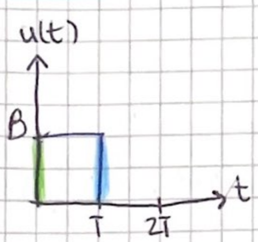
↳ 2T-periodisches Deltakamm

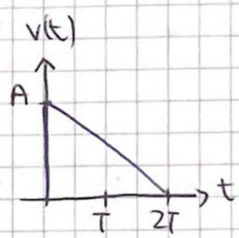
$$u(t) * v(t) = u_p(t) = \sum_{k=-\infty}^{\infty} u(t-k\cdot 2T)$$





3.1 b)





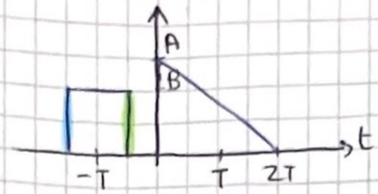
$\rightarrow u(t) * v(t) = v(t) * u(t)$

$$v(t) = -\frac{A}{2T}\cdot(t-2T)\cdot \Pi_{2T}(t-T)$$

Grenze 1: $t - \tau = 0 \iff \tau = t$

Grenze 2: $t - \tau = T \iff \tau = t - T$

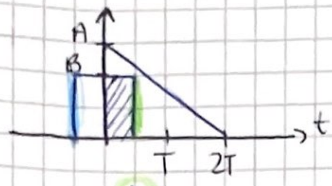
1. Fall:



$t < 0$

$y(t) = 0$

2. Fall:

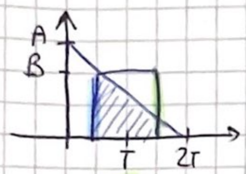


$t \geq 0$ und $t - T < 0$

$\Leftrightarrow \quad 0 \leq t < T$

$$y(t) = \int_0^t -\frac{A}{2T}(\tau - 2T)\cdot B\, d\tau = -\frac{AB}{2T}\cdot\left[\frac{1}{2}t^2 - 2T\cdot t\right]$$

3. Fall:

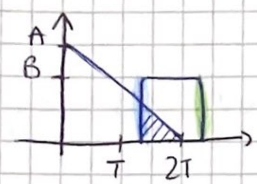


$t < 2T$ und $t - T \geq 0$

$\Leftrightarrow T \leq t < 2T$

$$y(t) = \int_{t-T}^{t} -\frac{A}{2T}(\tau - 2T)\cdot B\, d\tau = -\frac{AB}{2T}\cdot\left(T\cdot t - \frac{5}{2}T^2\right)$$

4. Fall:

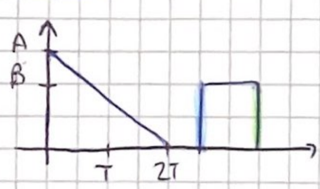


$t \geq 2T$ und $t - T < 2T$

$\Leftrightarrow \quad 2T \leq t < 3T$

$$y(t) = \int_{t-T}^{2T} -\frac{A}{2T}(\tau - 2T)\cdot B\, d\tau = -\frac{AB}{2T}\cdot\left(-\frac{9}{2}T^2 - \frac{1}{2}t^2 + 3T\cdot t\right)$$

5. Fall:



$t - T \geq 2T \Leftrightarrow t \geq 3T$

$y(t) = 0$