5. Tutorium : Faltung, Lineare Systeme im Zeitbereich

- Lineare Systeme im Zeitbereich

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Eingangssignal

System / Impulsantwort, wenn $h(t) = y(t) \mid x(t) = \delta(t)$.

Ausgangssignal / Systemantwort

- Faltung : $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$

- Faltung mit Deltaimpuls

$$\delta(t) * u(t) = u(t)$$
$$\delta(t-T) * u(t) = u(t-T)$$

- $y(t) = x(t) * h(t) = h(t) * x(t) \rightarrow$ kommutativ

1.1.b aus 3. Tutorium

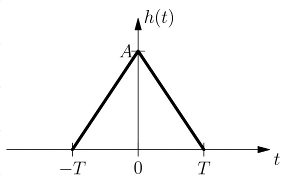
$$u(t) * v(t) = \int_{-\infty}^{\infty} u(\tau) \cdot v(t-\tau)\, d\tau$$
$$= \int_{\infty}^{-\infty} u(t-x) \cdot v(x)\; -dx$$
$$= \int_{-\infty}^{\infty} v(x) \cdot u(t-x)\, dx$$
$$= v(t) * u(t)$$

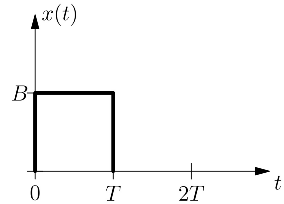
Substitution
$x = t - \tau$ $\begin{cases} \frac{dx}{d\tau} = -1 \\ \tau = t - x \end{cases}$

Schrittfolge der Faltung

1. Zeitvariable t durch τ ersetzen $\rightarrow x(\tau), h(\tau)$
2. Spiegelung von $h(\tau)$ an der y-Achse $\rightarrow h(-\tau)$
3. Verschiebung von $h(-\tau)$ um $t \rightarrow = h(-(\tau - t)) = h(-\tau + t)$
4. Integration über $x(\tau) \cdot h(t-\tau)$ im Bereich der Überdeckungen
5. Wiederholung der Schritte 3 und 4 bis zur Erfassung aller Überdeckungen

1.1. b)

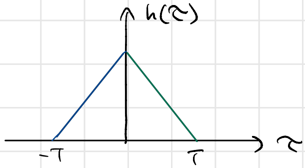


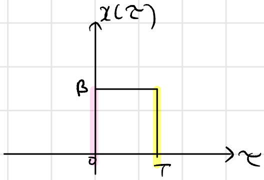


$$h(t) = \left(\frac{A}{T}t + A\right)\cdot \Pi_T\left(t+\frac{T}{2}\right) + \left(-\frac{A}{T}t + A\right)\cdot \Pi_T\left(t-\frac{T}{2}\right)$$

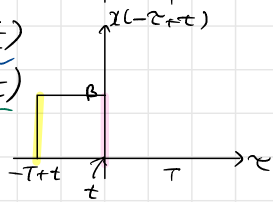
$$x(t) = B\cdot \Pi_T\left(t-\frac{T}{2}\right)$$

$$y = h(t) * x(t) = x(t) * h(t)$$

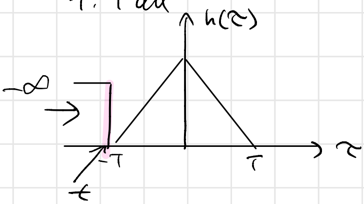




$$h(\tau) = \left(\frac{A}{T}\tau + A\right)\cdot \Pi_T\left(\tau+\frac{T}{2}\right) + \left(-\frac{A}{T}\tau + A\right)\cdot \Pi_T\left(\tau-\frac{T}{2}\right)$$



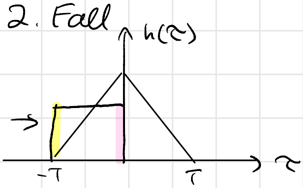
1. Fall



$t < -T$

$y(t) = 0$

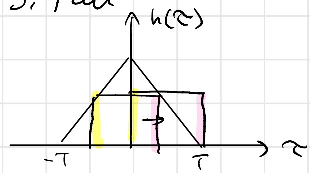
2. Fall



$-T \le t < 0$

$$\int_{-T}^{t} \left(\frac{A}{T}\tau + A\right)\cdot B\, d\tau = \frac{AB}{T}\left(\frac{t^2}{2} + Tt + \frac{T^2}{2}\right)$$

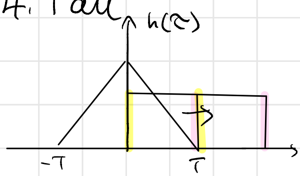
3. Fall



$0 \le t < T$

$$\int_{-T+t}^{0} \left(\frac{A}{T}\tau + A\right)\cdot B\, d\tau + \int_{0}^{t} \left(-\frac{A}{T}\tau + A\right)\cdot B\, d\tau = \frac{AB}{T}\left(-t^2 + Tt + \frac{T^2}{2}\right)$$

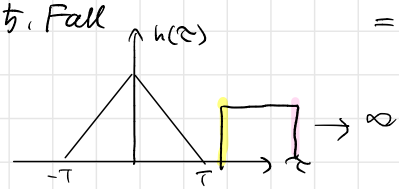
4. Fall



$0 \le -T+t < T \Leftrightarrow T \le t < 2T$

$$\int_{0}^{-T+t} \left(-\frac{A}{T}\tau + A\right)\cdot B\, d\tau = \frac{AB}{T}\left(\frac{t^2}{2} - 2Tt + 2T^2\right)$$

5. Fall

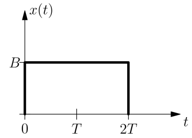


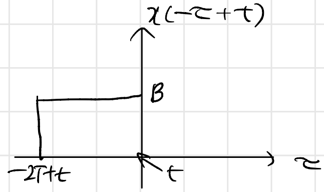
$-T+t \ge T \Leftrightarrow t \ge 2T$

$y(t) = 0$

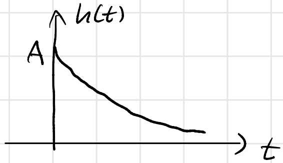
1.1. c)

$$h(t) = \begin{cases} A \cdot e^{-\frac{t}{2T}}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$





$h(t) =$



1. Fall

$t < 0, \; y(t) = 0$

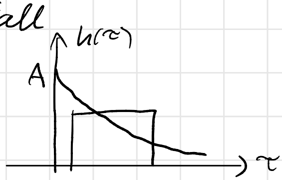
2. Fall

$0 \leq t \wedge -2T+t < 0 \iff 0 \leq t < 2T$

$$\int_0^t A \cdot e^{-\frac{\tau}{2T}} \cdot B \, d\tau$$

$$= AB \cdot 2T \cdot (1 - e^{-\frac{t}{2T}})$$

3. Fall

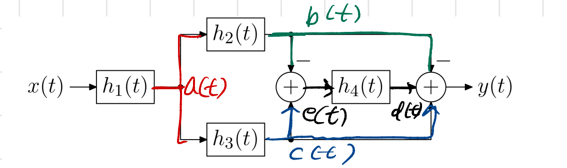


$-2T+t > 0 \iff t \geq 2T$

$$\int_{-2T+t}^{t} A \cdot e^{-\frac{\tau}{2T}} \cdot B \, d\tau$$

$$= AB \cdot 2T \cdot e^{-\frac{t}{2T}} (e^1 - 1)$$

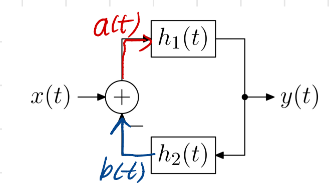
2.1. d)



$$y(t) = -b(t) + c(t) + d(t)$$
$$b(t) = a(t) * h_2(t) = x(t) * h_1(t) * h_2(t)$$
$$a(t) = x(t) * h_1(t)$$
$$c(t) = a(t) * h_3(t) = x(t) * h_1(t) * h_3(t)$$
$$d(t) = e(t) * h_4(t) = (-b(t) + c(t)) * h_4(t)$$
$$e(t) = -b(t) + c(t)$$

$$\therefore y(t) = -x(t) * h_1(t) * h_2(t) + x(t) * h_1(t) * h_3(t)$$
$$- b(t) * h_4(t) + c(t) * h_4(t)$$

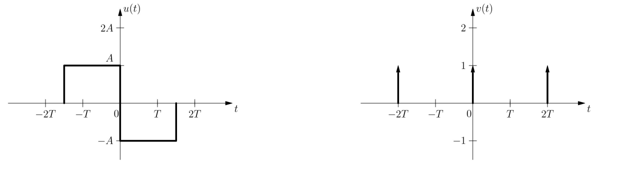
2.1. e)



$$y(t) = a(t) * h_1(t)$$
$$a(t) = x(t) - b(t) = x(t) - y(t) * h_2(t)$$
$$b(t) = y(t) * h_2(t)$$

$$\therefore y(t) = (x(t) - y(t) * h_2(t)) * h_1(t)$$
$$= x(t) * h_1(t) - y(t) * h_2(t) * h_1(t)$$

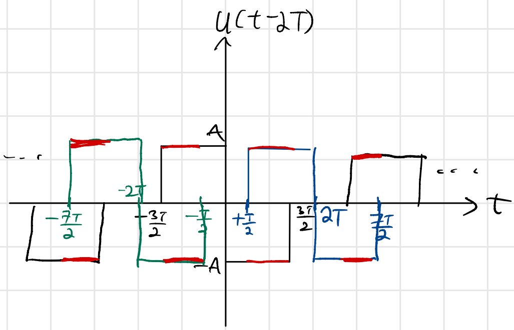
3. a)



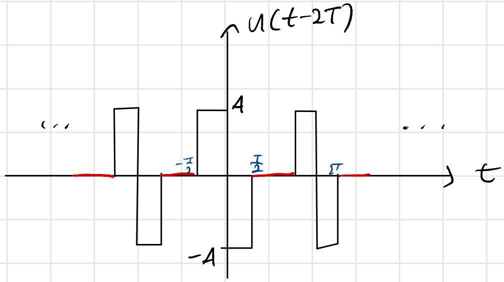
Deltakamm $v(t) = \delta(t-2T)$

$$\ast\begin{cases} \delta(t) \ast u(t) = u(t) \\ \delta(t-\underbrace{T}_{\text{Periodendauer}}) \ast u(t) = u(t-T) \end{cases}$$

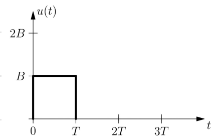
$$v(t) \ast u(t) = \delta(t-2T) \ast u(t) = \underbrace{u(t-2T)}_{\text{periodisches Signal mit } T_p = 2T}$$

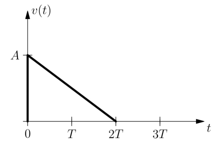


wenn die Signale sich überlappen
→ Werte addieren!
$A + (-A) = 0$

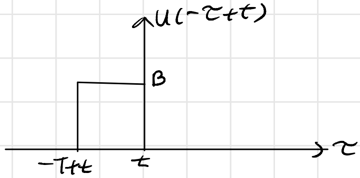


3. b)





$u(t) = B \cdot \Pi_T(t - \frac{T}{2})$ $\quad v(t) = (-\frac{A}{2T}t + A) \cdot \Pi_{2T}(t - T)$

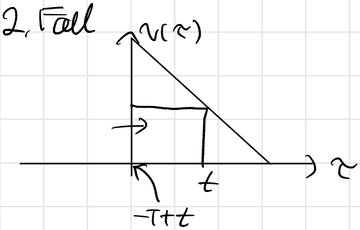


1. Fall

$t < 0$

$y(t) = 0$

2. Fall

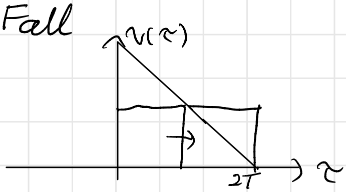


$t \geq 0 \wedge -T+t < 0 \Leftrightarrow 0 \leq t < T$

$$\int_0^t (-\frac{A}{2T}\tau + A) \cdot B \, d\tau$$

$$= -\frac{AB}{2T}\left(\frac{t^2}{2} - 2Tt\right)$$

3. Fall

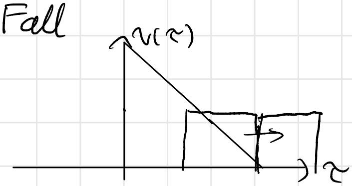


$T \leq t < 2T$

$$\int_{-T+t}^{t} (-\frac{A}{2T}\tau + A) \cdot B \, d\tau$$

$$= -\frac{AB}{2T}\left(Tt - \frac{5T^2}{2}\right)$$

4. Fall



$T \leq -T+t < 2T \Leftrightarrow 2T \leq t < 3T$

$$\int_{-T+t}^{2T} (-\frac{A}{2T}\tau + A) \cdot B \, d\tau$$

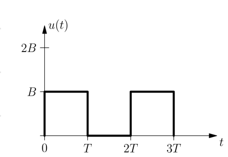
$$= -\frac{AB}{2T}\left(-\frac{9T^2}{2} - \frac{1}{2}t^2 + 3Tt\right)$$

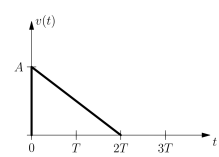
5. Fall

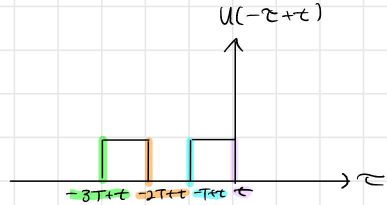
$-T+t \geq 2T \Leftrightarrow t \geq 3T$

$y(t) = 0$

3. c)



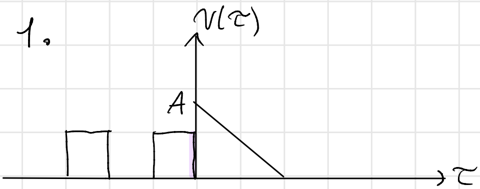




$$u(t) = B\left\{\left(\Pi_T\left(t-\tfrac{T}{2}\right) + \Pi_T\left(t-\tfrac{5T}{2}\right)\right)\right\}$$

$$v(t) = \left(-\frac{A}{2T}t + A\right)\cdot \Pi_{2T}(t-T)$$

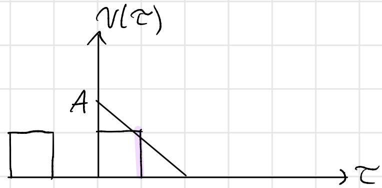
1.



$t<0$

$y=0$

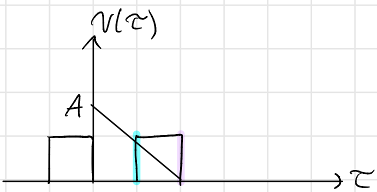
2.



$0 \le t < T$

$$\int_0^t B\cdot\left(-\frac{A}{2T}\tau + A\right)d\tau$$

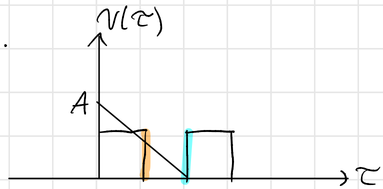
3.



$T \le t < 2T$

$$\int_{t-T}^{t} B\cdot\left(-\frac{A}{2T}\tau + A\right)d\tau$$

4.

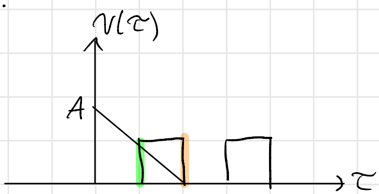


$T \le t-T < 2T$

$\Leftrightarrow 2T \le t < 3T$

$$\int_0^{t-2T} B\cdot\left(-\frac{A}{2T}\tau + A\right)d\tau + \int_{t-T}^{2T} B\cdot\left(-\frac{A}{2T}\tau + A\right)d\tau$$

5.



$0 \le t-3T < T \Leftrightarrow 3T \le t < 4T$

$$\int_{t-3T}^{t-2T} B\cdot\left(-\frac{A}{2T}\tau + A\right)d\tau$$

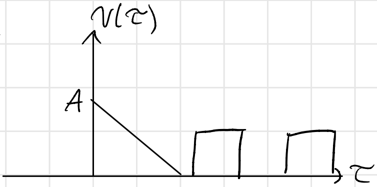
6.

$2T \le t-2T \wedge t-3T < 2T$

$\Leftrightarrow 4T \le t < 5T$

$$\int_{t-3T}^{2T} B\cdot\left(-\frac{A}{2T}\tau + A\right)d\tau$$

7.



$t - 3T \geq 2T$

$\Leftrightarrow t \geq 5T$

$y(t) = 0$

Quiz

1. Für die Faltungsoperation gilt das ..

- ○ Assoziativgesetz.
- ○ Kommutativgesetz.
- ○ Distributivgesetz.

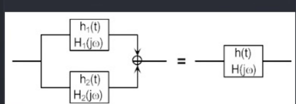
2. Gegeben sei ein lineares System mit der Impulsantwort h(t) . Welche Systemantwort ergibt sich für das Eingangssignal u(t) = δ(t) ?

- ○ $y(t)=\delta(t)$
- ○ $y(t)=\delta(t)\cdot h(t)$
- ○ $y(t)=h(t)$

3. Die Impulsantwort ist die Antwort des Systems auf einen ...

- ☐ Deltaimpuls.
- ☐ Rechteckimpuls.
- ☐ Dreiecksimpuls.

4. Für die Parallelschaltung zweier LTI-Systeme gilt:



- ☐ Für das Ausgangssignal des Gesamtsystems gilt: y(t) = u (t) * (h_1(t) + h_2(t))
- ☐ Die Gesamtimpulsantwort ist das Produkt der Einzelimpulsantworten.
- ☐ Die Einzelimpulsantworten addieren sich.

Lösung : 1 – Kommutativgesetz
2 – $y(t) = h(t)$
3 – Deltaimpuls
4 – $y(t) = u(t) * (h_1 + h_2)$,
Einzelimpulsantworten addieren sich