

Cognitive Algorithms

Lecture 1 - Introduction to ML & Linear Classification

Klaus-Robert Müller, Johannes Niediek,
Augustin Krause, Joanina Oltersdorff, Ken Schreiber

Technische Universität Berlin
Machine Learning Group

What will you learn in this lecture?

Introductory course into Machine Learning → “Machine Learning Light”

- Learning the basics of ML
 - Formalizing learning problems
 - Evaluation of models
 - Standard concepts
 - Explain the underlying maths

With this course you will be able to

- Understand what the state of the art does especially via the elective courses
- See if you like this field

But you will not learn about

- Contemporary machine learning research
- Tools like pytorch or tensorflow

For a mathematically more advanced, in-depth treatment, we recommend:

- M.Sc. module Machine Learning 1/2 (9 ECTS)
- M.Sc. module Deep Learning 1/2 (9 ECTS)

For an advanced practical treatment, we recommend:

- M.Sc. module "Project Machine Learning" (9 ECTS)

More information can be found on

<https://web.ml.tu-berlin.de/teaching/courses/>

Integrated lecture "Cognitive Algorithms"

Overall responsibility (Modulverantwortlicher)

Prof. Dr. Klaus-Robert Müller

Lectures

Dr. Johannes Niediek

Tutorials, grading of assignments

Augustin Krause

Joanina Oltersdorff

Ken Schreiber

Electives

The concept of this module

The B.Sc. Module Cognitive Algorithms (6 ECTS) consists of

- Lectures with tutorials (3 ECTS)
- Elective course or seminar (3 ECTS)

The module exam is a written exam at the end of the semester

Important

Each student has to complete an elective before taking the module exam

Electives I

Important

- Each student has to complete an elective before taking the module exam
- We offer elective *courses* and *seminars*

Kickoff meeting for elective seminars

- Elective *seminars* will be presented in one meeting
- Monday, October 23rd, 12–14, room H3006
- <https://isis.tu-berlin.de/course/view.php?id=34907>
- The selection of seminars will happen via this ISIS page, too

List of electives

Courses

- Python Programming for Machine Learning (PyML)
- Julia programming for Machine Learning (JuML)
- Mathematical Foundations for Machine Learning (MathML)
- Bayesian Inference

Seminars

- Classical Topics in Machine Learning
- Cognitive Algorithms
- Machine Learning for Quantum Chemistry
- Machine Learning for Data Management Systems
- Machine Learning in Science and Industry
- Machine Learning in Neuroscience

About
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Organizational
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Cognition, AI, and ML
oooooooo

ML Basics
oooooooo

Prototypes
ooooo

Biological Neurons
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Perceptron
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oo

Summary
ooo

Questions regarding electives?

ISIS Quizzes and voluntary homework

ISIS Quizzes

- Multiple choice and programming exercises
- Automatically graded
- **Counted as some bonus points for the exam**
details on bonus points will follow

Assignment sheets

- Great way to practice the relevant mathematics
- Graded by the tutors
- Submission of homework *in groups of up to two*

Schedule

All announcements and updates will be via ISIS

Lectures

- Week 1, Tuesday 12–14:
Lecture in H0107
- Week 1, Tuesday afternoon:
Assignment sheet opens
- Week 2, Tuesday 09:59:
Assignment sheet closes

Tutorials

- Tuesday 10–12 FH 313
24th Oct: PC 203
- Wednesday 16–18 FH 302
- Thursday 14–16 FH 311
26th Oct, 18th Jan: MAR 4033

Exams

- Written exams on **February 19th, 2024** and **TBD**
- **Remember that you need to pass an elective before the exam**
- Registration to the exam via **Moses**

Literature

Two standard textbooks of Machine Learning are:

Hastie T. et al. - The Elements of Statistical Learning

Bishop C. - Pattern Recognition and Machine Learning

A less mathematical treatment with practical examples in Python:

Marsland S. - Machine Learning, An algorithmic Perspective

A book for practitioners:

James G. et al. - An Introduction to Statistical Learning

For most lectures, optional reading material can be found in ISIS

Helpful prerequisites

- Knowledge in linear algebra:
vector norm, scalar product, matrix operations, eigenvectors
- Knowledge in calculus:
derivatives of functions that depend on more than one variable
(gradient, partial derivatives)
- Basic programming knowledge, programming in Python
(not necessary for the exam)

We also upload a math refresher,
where you can go through the mathematical basics of this lecture

要执行的自包含操作序列
执行计算、数据处理、自动推理任务等。
认识

源自认知（拉丁语）：学习，学习，知识，探究，...指在人脑中处理信息
注意力、记忆力、解决问题的能力、决策能力、语言能力。认知过程使用
现有知识并产生新知识

Why is it called “Cognitive Algorithms”?

Algorithms

- Self-contained sequences of actions to be performed
- Perform calculation, data processing, automated reasoning tasks, ...

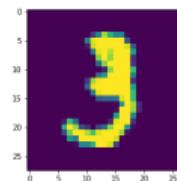
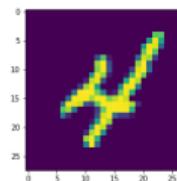
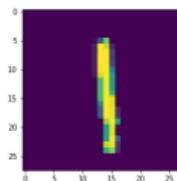
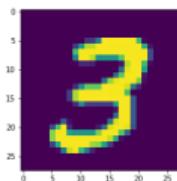
Cognition

- Derived from *cognitio* (latin): *learning, study, knowledge, inquiry, ...*
- Refers to processing of information in the human brain
 - attention, memory, problem solving, decision making, language, ...
- Cognitive processes use existing knowledge and generate new knowledge

Practical examples of cognition

Let's do some cognition together.

Name the numbers



For us this is very easy, but can you write a program that does it?

Name what you see



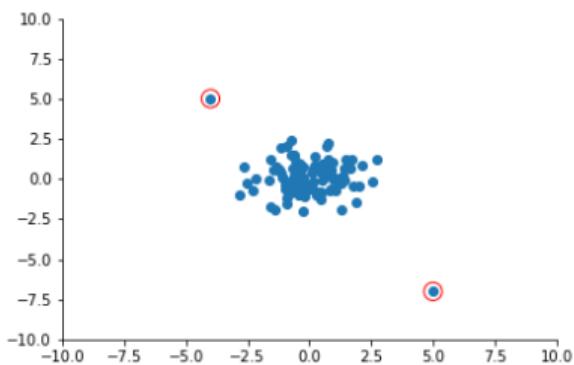
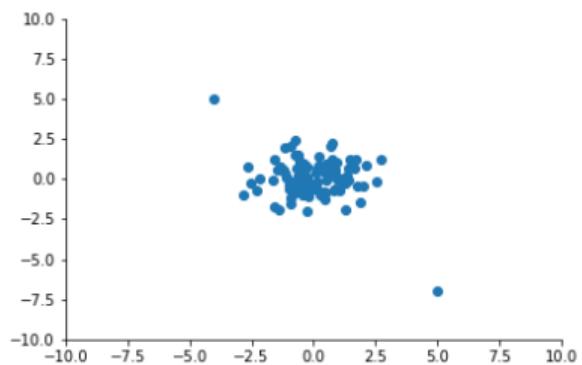
For us this is very easy, but can you write a program that does it?

Select the logically correct sentence

- 1** Leyla is 25 years old. She is 30 years old.
- 2** Leyla is 25 years old. She is not.
- 3** Leyla is 25 years old. She is not 30 years old.

For us this is very easy, but can you write a program that does it?

Mark the outliers



For us this is very easy, but can you write a program that does it?

Artificial General Intelligence vs. Machine Learning

- Artificial General Intelligence (AGI)/“Strong AI”
“Machine intelligence with the full range of human intelligence” [Kurzweil 2005]
- Machine Learning - focus on specific problems
E.g. image classification, sales prediction, text translation, go playing, ...
- In this course: focus on key ideas and methods that are applied in different subproblems. This course is not about the definition or history of AI.

Turing Test [Turing, 1950]:
Can machines think?

Perceptron [Rosenblatt, 1958]:
Artificial “brains”

Eliza [Weizenbaum, 1966]
One of the first chatbots!

Large Language Models such as GPT-4 [OpenAI, 2023]:
Some say this is the start of AGI, but who knows... .

Types of Machine Learning problems

Supervised learning

Given labeled training examples, learn a mapping from inputs (features) to output (label)
e.g. classification, regression

监督学习

给定标记的训练示例，学习从输入（特征）到输出（标签）的映射，例

Unsupervised learning

Find structure in data
e.g. clustering, dimensionality reduction

如分类、回归
无监督学习

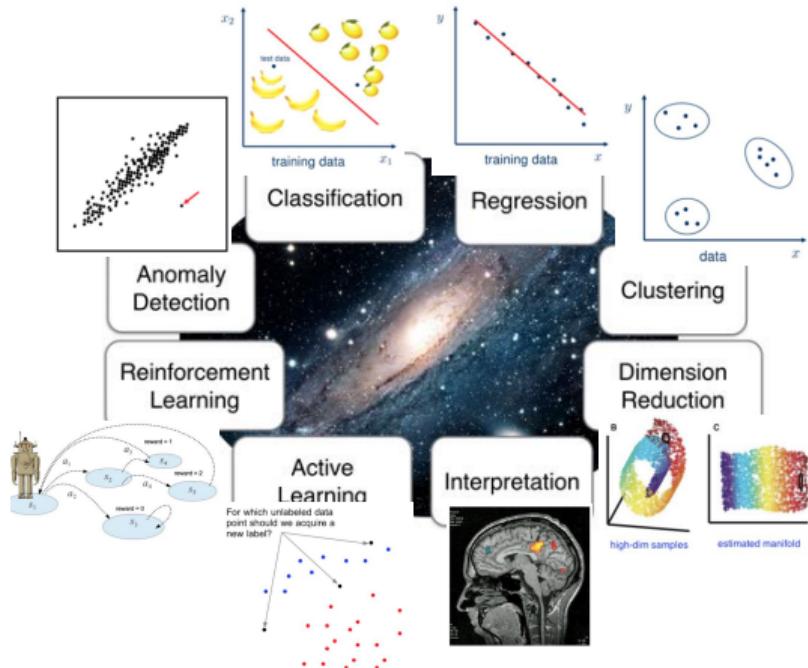
在数据中查找结构
例如聚类、降维
强化学习

在没有确切环境模型的情况下学习最大化某些奖励概念的环境中的行为
例如玩游戏（围棋，星际争霸），优化流程（自动驾驶汽车）

Reinforcement Learning

Learn actions in an environment which maximize some notion of reward, without exact model of the environment
e.g. game playing (Go, Starcraft), optimizing processes (self-driving cars)

The Machine Learning Universe



What does a ML problem look like?

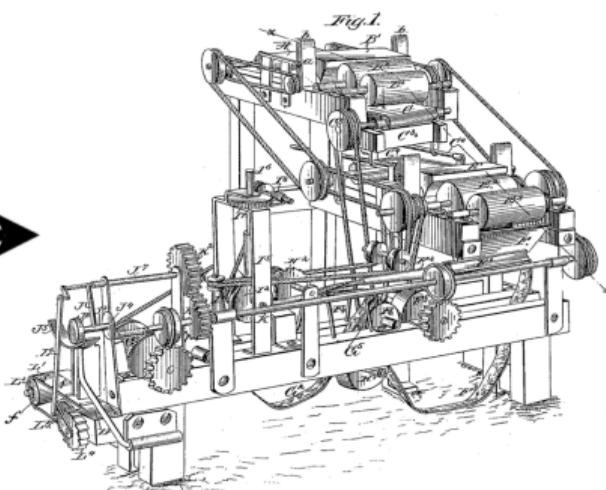
Daniel
likes



Daniel
dislikes



The learning machine



Training Set

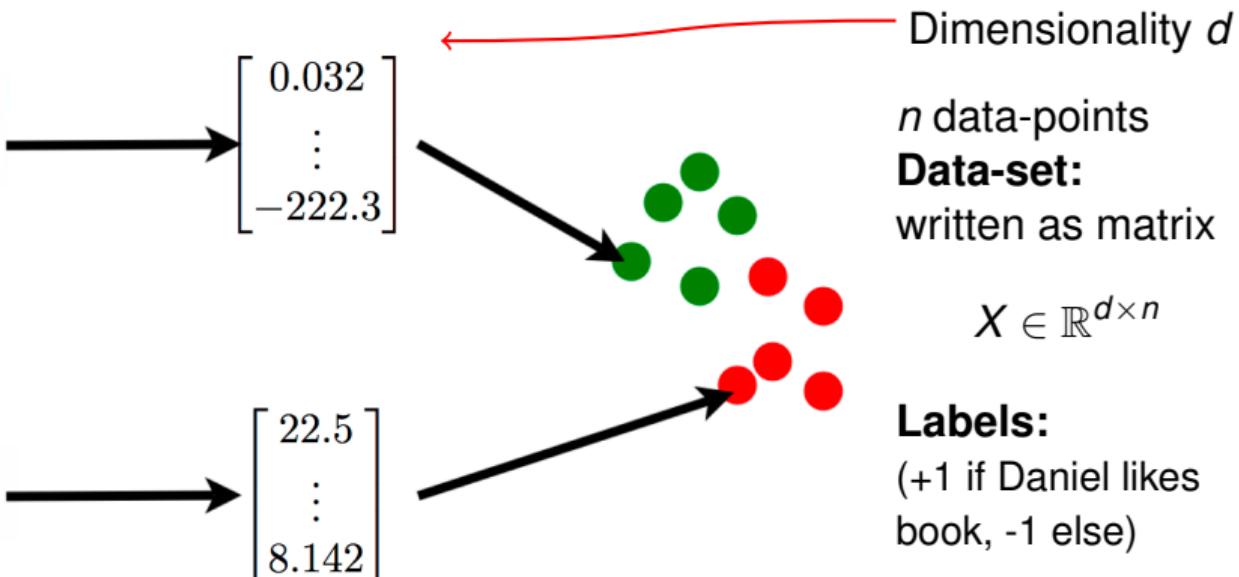
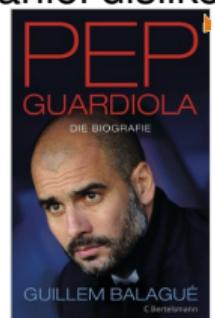
Model

Feeding data to the model: Feature extraction

Daniel likes:

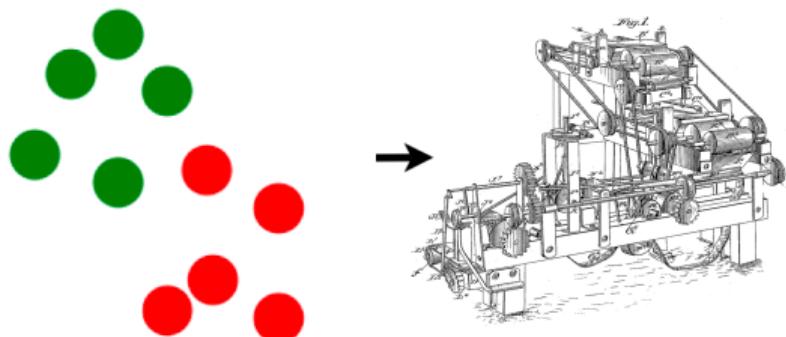


Daniel dislikes:



Every book is a vector in feature space

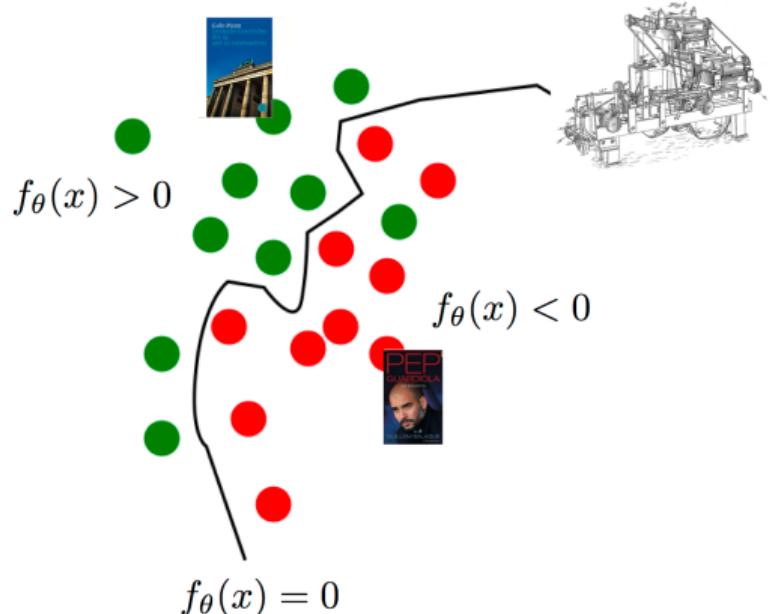
Training



We train the parameters θ of the model with the data X, y

More on that later

Discriminant function



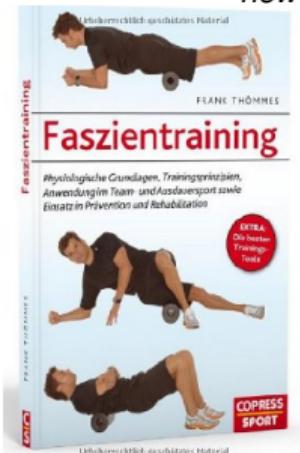
学习机器学习类之间最好的判别函数 $f_{\theta}(x)$ 。
决策边界为 其中 $f_{\theta}(x) = 0$

The learning machine learns the best possible **discriminant function** $f_{\theta}(x)$ between the classes.

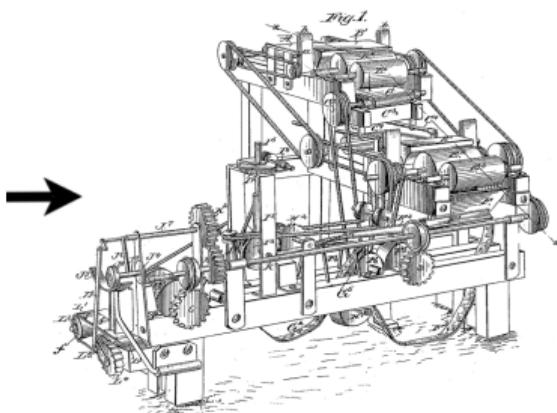
The **decision boundary** is where $f_{\theta}(x) = 0$

Prediction for a new object

New book \mathbf{x}_{new}



Does Daniel like it?
 $f_{\theta}(\mathbf{x}_{new})$



Prediction

→ Yes / No

Handcrafted classifier

Let's look at digit recognition

MNIST, Modified National Institute of Standards and Technology database

- Widely used data-set with images of handwritten digits
- The “Hello World” of machine learning
- You classified MNIST numbers today!

Let's try to classify only sixes and nines:

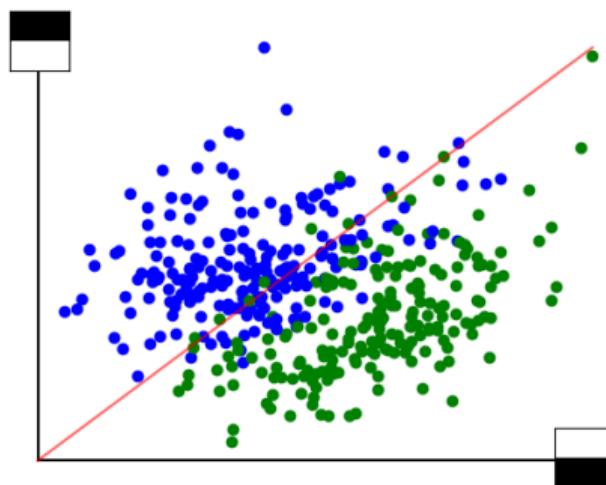


Handcrafted classifier

Classifying 6 and 9

We could try that with a simple program:

- We know that nines have more mass in the upper half and sixes in the lower half
- We can build a simple classifier



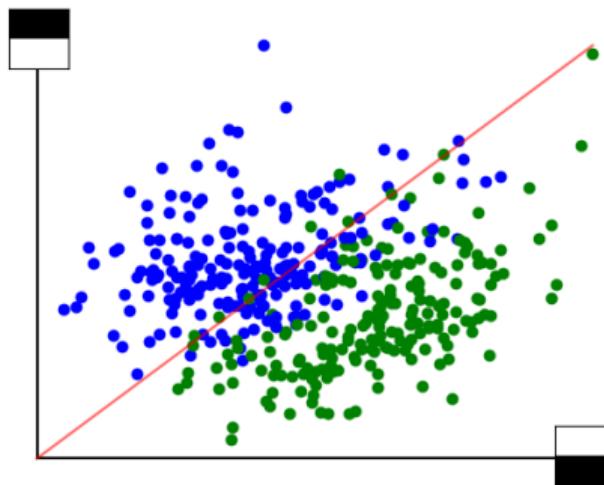
`if(sum(lower pixels) > sum(upper pixels)) ⇒ classify as 6`

→ 82% correct classifications (“accuracy”)

Handcrafted classifier

In this approach we:

- Had a simple problem
- Made many decisions by hand:
 - 1 The model (linear model)
 - 2 Feature extraction
 - 3 Parameters → position of decision boundary



$$1 \cdot \text{sum(lower pixels)} - 1 \cdot \text{sum(upper pixels)} > 0$$

In this course you learn how to automate steps 2. and 3.

Handcrafted classifier

Quick summary

- **Feature extraction:** Transforming input into numeric features
- Classification is performed using **discriminant function** $f_{\theta}(x)$ parametrized by parameters θ
- If $f_{\theta}(x) > 0$, class 0, if $f_{\theta}(x) < 0$, class 1. $f_{\theta}(x) = 0$ is the **decision boundary**.
- We can evaluate a classification model using **accuracy**

$$\text{accuracy} = \frac{\#\text{correct classifications}}{\#\text{samples}}$$

Linear classification

- In MNIST example: decision boundary was a straight line
- In general, linear classification has a **linear decision boundary**, a hyperplane
- The discriminant function has the form

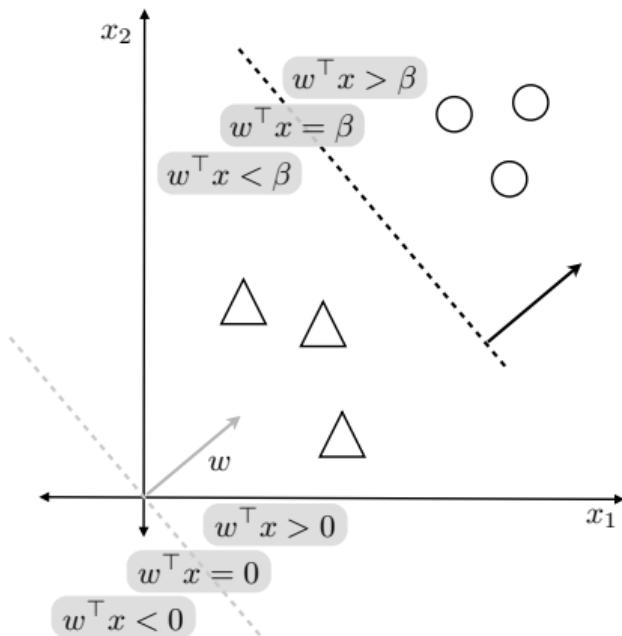
$$f_{\mathbf{w}, \beta}(\mathbf{x}) = \mathbf{w}^T \mathbf{x} - \beta$$

where $\mathbf{x}, \mathbf{w} \in \mathbb{R}^d$, $\beta \in \mathbb{R}$

- The position of the decision boundary is determined by **weights** w_j for every dimension $j \in \{1, \dots, d\}$ and an **offset** β

Handcrafted classifier

Linear classification



$$\mathbf{w}^\top \mathbf{x} - \beta = \begin{cases} > 0 & \text{if } \mathbf{x} \text{ belongs to } \textcolor{blue}{o} \\ < 0 & \text{if } \mathbf{x} \text{ belongs to } \textcolor{red}{\Delta} \end{cases}$$

The decision boundary is orthogonal to \mathbf{w}

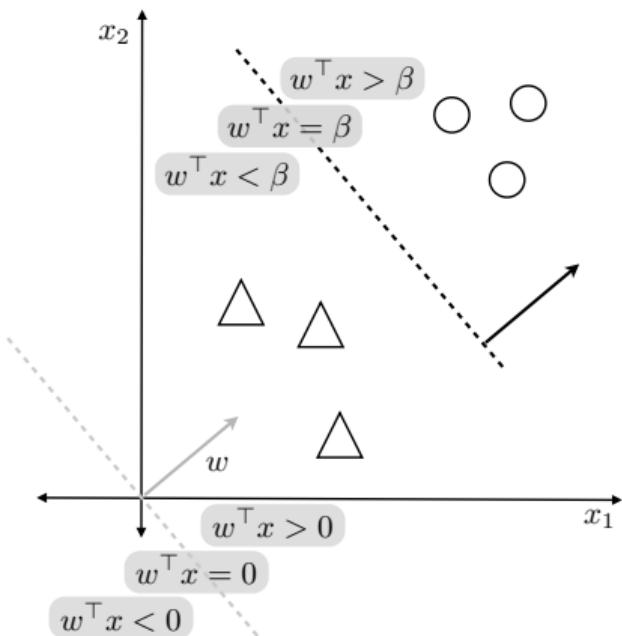
$$\begin{aligned} \mathbf{w}^\top \mathbf{x} &= \cos(\angle(\mathbf{w}, \mathbf{x})) \cdot \|\mathbf{w}\| \cdot \|\mathbf{x}\| \\ \Rightarrow \mathbf{w}^\top \mathbf{x} = 0 &\Leftrightarrow \angle(\mathbf{w}, \mathbf{x}) = \frac{\pi}{2} \quad (\text{if } \mathbf{w}, \mathbf{x} \neq \mathbf{0}) \end{aligned}$$

here, $\|\mathbf{w}\|$ is the Euclidean norm $\|\mathbf{w}\| = \sqrt{\sum_i^d w_i^2}$.

The decision boundary is shifted by β .

Handcrafted classifier

Linear classification: simplifying notation



$$\mathbf{w}^\top \mathbf{x} - \beta = \begin{cases} > 0 & \text{if } \mathbf{x} \text{ belongs to } o \\ < 0 & \text{if } \mathbf{x} \text{ belongs to } \Delta \end{cases}$$

The *offset* β can be included in \mathbf{w}

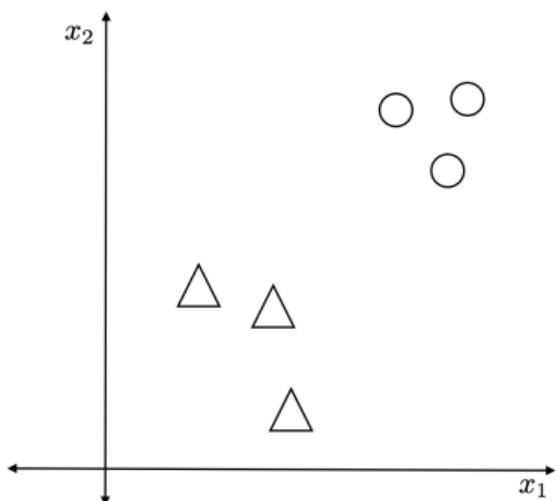
$$\tilde{\mathbf{x}} := \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \quad \tilde{\mathbf{w}} := \begin{bmatrix} -\beta \\ \mathbf{w} \end{bmatrix}$$

such that

$$\tilde{\mathbf{w}}^\top \tilde{\mathbf{x}} = \mathbf{w}^\top \mathbf{x} - \beta.$$

Prototypes: psychological models of abstract ideas

How can we obtain \mathbf{w} and β from the data?



Psychologists postulated that we learn **prototypes** [Jäkel, 2007; Posner and Keele, 1968]

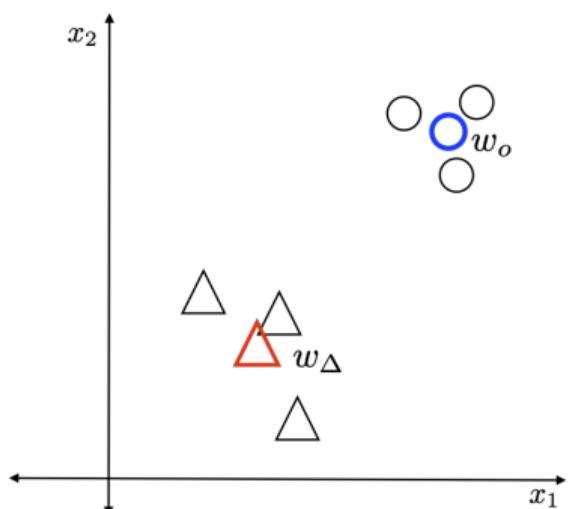
Toy data example:

Two dimensional input $\mathbf{x} \in \mathbb{R}^2$

Two classes of data, Δ and \circ

Prototypes: psychological models of abstract ideas

原型：抽象概念的心理模型



Prototypes \mathbf{w}_Δ and \mathbf{w}_o can be class means

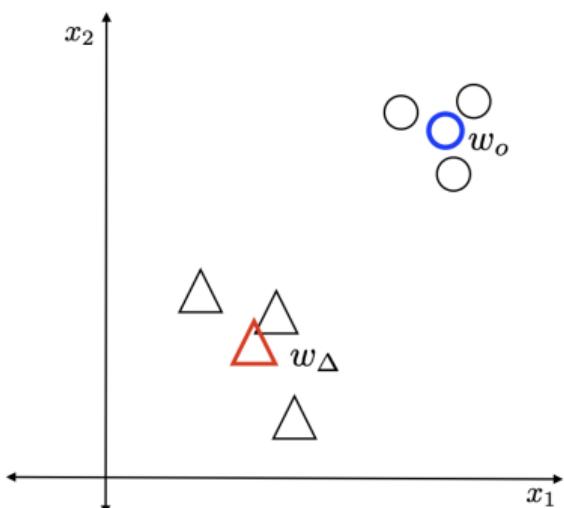
$$\mathbf{w}_\Delta = \frac{1}{N_\Delta} \sum_{n=1}^{N_\Delta} \mathbf{x}_{\Delta,n}$$

$$\mathbf{w}_o = \frac{1}{N_o} \sum_n \mathbf{x}_{o,n}$$

Distance from \mathbf{w}_Δ to new data \mathbf{x}

$$\|\mathbf{w}_\Delta - \mathbf{x}\| = \sqrt{\sum_{j=1}^2 (w_{\Delta j} - x_j)^2}$$

Prototypes: psychological models of abstract ideas



For new data x check:
Is x more similar to w_o ?

$$\|w_\Delta - x\| > \|w_o - x\|$$

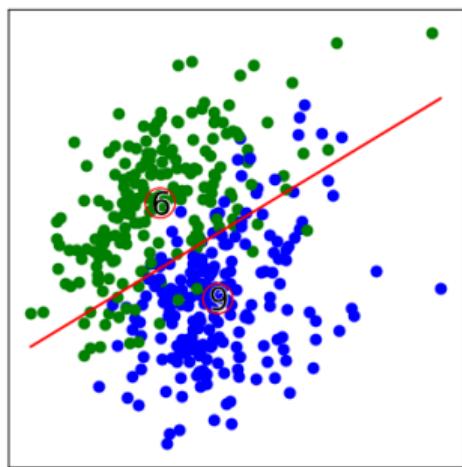
yes? $\rightarrow x$ belongs to w_o
no? $\rightarrow x$ belongs to w_Δ

What does the decision boundary look like?

The nearest centroid classifier (NCC)

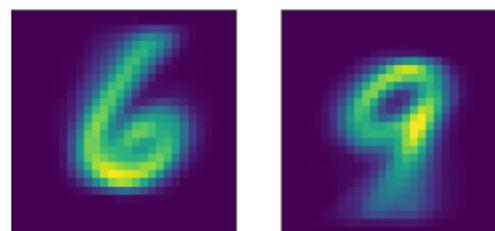
$$\begin{aligned} \text{distance}(\mathbf{x}, \mathbf{w}_\Delta) &> \text{distance}(\mathbf{x}, \mathbf{w}_o) \\ \|\mathbf{x} - \mathbf{w}_\Delta\| &> \|\mathbf{x} - \mathbf{w}_o\| \\ \Leftrightarrow \|\mathbf{x} - \mathbf{w}_\Delta\|^2 &> \|\mathbf{x} - \mathbf{w}_o\|^2 \\ \Leftrightarrow (\mathbf{x} - \mathbf{w}_\Delta)^\top (\mathbf{x} - \mathbf{w}_\Delta) &> (\mathbf{x} - \mathbf{w}_o)^\top (\mathbf{x} - \mathbf{w}_o) \\ \Leftrightarrow \mathbf{x}^\top \mathbf{x} - \mathbf{x}^\top \mathbf{w}_\Delta - \mathbf{w}_\Delta^\top \mathbf{x} + \mathbf{w}_\Delta^\top \mathbf{w}_\Delta &> \mathbf{x}^\top \mathbf{x} - \mathbf{x}^\top \mathbf{w}_o - \mathbf{w}_o^\top \mathbf{x} + \mathbf{w}_o^\top \mathbf{w}_o \\ \Leftrightarrow -2\mathbf{w}_\Delta^\top \mathbf{x} + \mathbf{w}_\Delta^\top \mathbf{w}_\Delta &> -2\mathbf{w}_o^\top \mathbf{x} + \mathbf{w}_o^\top \mathbf{w}_o \\ \Leftrightarrow 0 &< \underbrace{(\mathbf{w}_o - \mathbf{w}_\Delta)^\top \mathbf{x}}_w - \underbrace{\frac{1}{2}(\mathbf{w}_o^\top \mathbf{w}_o - \mathbf{w}_\Delta^\top \mathbf{w}_\Delta)}_\beta \end{aligned}$$

Applying NCC to 6 vs 9



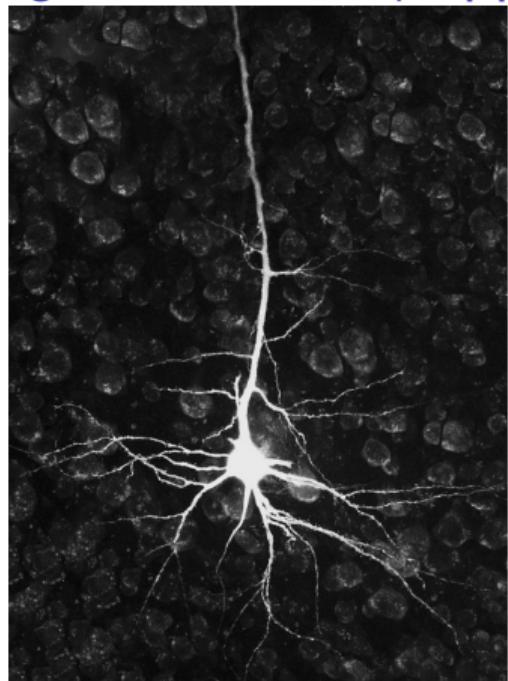
Accuracy of 88%
on training set
before: 82%

Even better: use all 28×28 pixels as input
→ vectorize and get $d = 784$
Prototypes:



Accuracy of 98.8%
on training set

Biological neurons (supplementary)



Cortical Neuron

生物神经元 (补充)

皮质神经元

神经元是带电的电池 (-50 mV至-70 mV)

它们通过膜电位的变化来处理信息

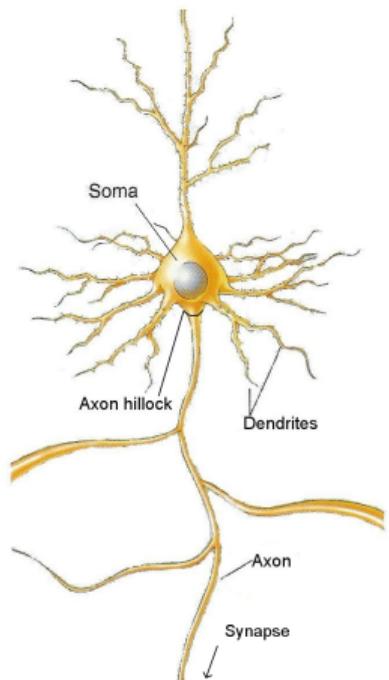
人脑有 10^{11} 个神经元和 10^{14} 个突触

Neurons are electrically charged cells (-50 mV to -70 mV)

They process information by changes in membrane potential

Human brain has 10^{11} neurons and 10^{14} synapses

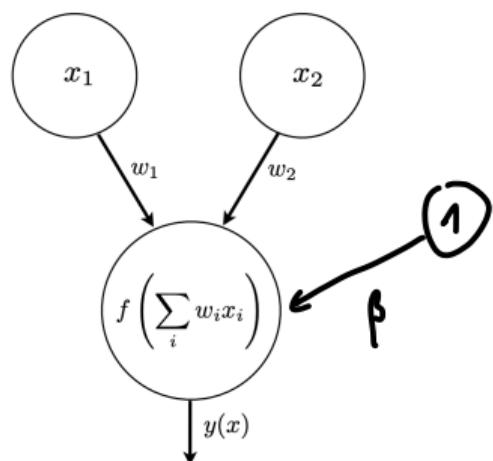
The prototypical neuron (supplementary)



Parts of a neuron

| Part | Function |
|---------------------|--|
| Dendrites | Receive incoming 'messages' from other neurons. |
| Soma | Combines all incoming 'messages' |
| Axon hillock | If the membrane potential at the axon hillock reaches a threshold value, the axon 'fires' an action potential. |
| Axon | Carries the action potential over short (<1 mm) or long (>1 m) distances. |

Perceptron 感知器



Input nodes x_i receive information

Inputs are multiplied with a weighting factor w_i and summed up

Averaged input is mapped through some (non-linear) function $f(\cdot)$, e.g.

$$f(x) = \text{sign}(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Loss Function:

$$\mathcal{L}_{\mathcal{P}}(\mathbf{w}) = - \sum_{m \in \mathcal{M}} \mathbf{w}^\top \mathbf{x}_m y_m$$

Rosenblatt's Perceptron



Frank Rosenblatt
(1928–1969)

Rosenblatt proposed the **perceptron**
[Rosenblatt, 1958]

Perceptrons gave rise to the field of artificial
neural networks

The perceptron learning algorithm

Goal Binary classification of data $\mathbf{x} \in \mathbb{R}^d$

Input Learning rate η and n tuples (\mathbf{x}_i, y_i) where

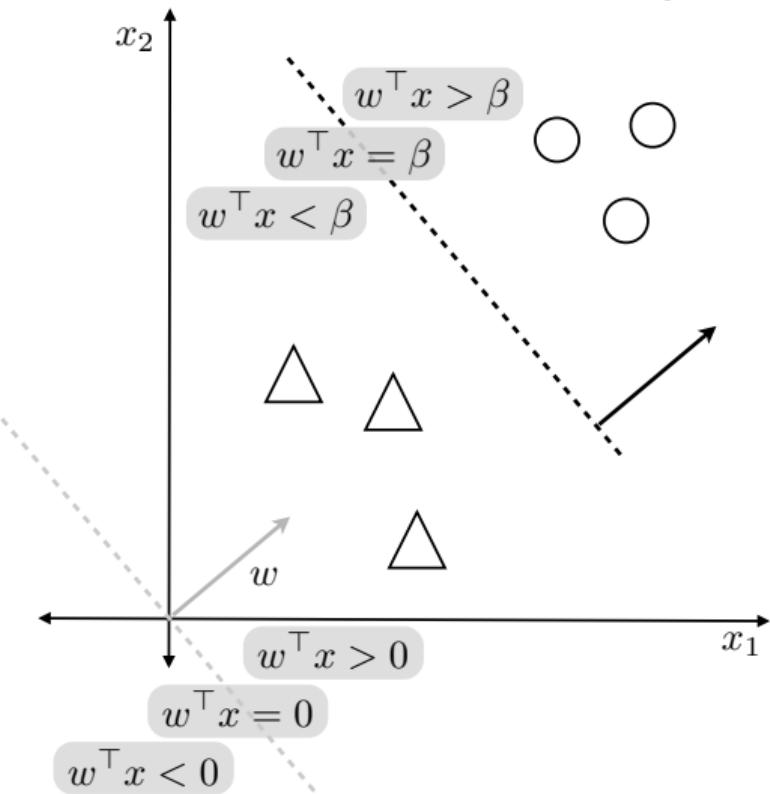
$\mathbf{x}_i \in \mathbb{R}^d$ is a d -dimensional data point

$y_i \in \{-1, +1\}$ is the label corresponding to \mathbf{x}_i

Output Weight vector $\mathbf{w} \in \mathbb{R}^d$ such that

$$\mathbf{w}^\top \mathbf{x}_i \stackrel{!}{=} \begin{cases} \geq 0 & \text{if } y_i = +1 \\ < 0 & \text{if } y_i = -1 \end{cases} \quad \text{i.e.} \quad f(\mathbf{w}^\top \mathbf{x}_i) = y_i$$

Linear classification and the perceptron



What is a good \mathbf{w} ?

- We need a **loss function** that tells us how good \mathbf{w} is.
- Then we choose \mathbf{w} such that the loss function is minimized.

The perceptron loss function

Perceptron error

The perceptron error or **loss function** $\mathcal{L}_{\mathcal{P}}$ is a function of the weights \mathbf{w}

$$\mathcal{L}_{\mathcal{P}}(\mathbf{w}) = - \sum_{m \in \mathcal{M}} \mathbf{w}^\top \mathbf{x}_m y_m$$

where \mathcal{M} denotes the index set of all *misclassified* data \mathbf{x}_m

The optimal weights are $\mathbf{w}^* = \operatorname{argmin}_{\mathbf{w}} (\mathcal{L}_{\mathcal{P}}(\mathbf{w}))$.

Gradient vector

For the loss function

$$\mathcal{L}_P(\mathbf{w}) : \mathbb{R}^p \rightarrow \mathbb{R},$$

the gradient vector is

$$\frac{\partial \mathcal{L}_P(\mathbf{w})}{\partial \mathbf{w}} : \mathbb{R}^p \rightarrow \mathbb{R}^p, \quad \left[\frac{\partial \mathcal{L}_P(\mathbf{w})}{\partial \mathbf{w}} \right]_j = \frac{\partial \mathcal{L}_P(\mathbf{w})}{\partial w_j}$$

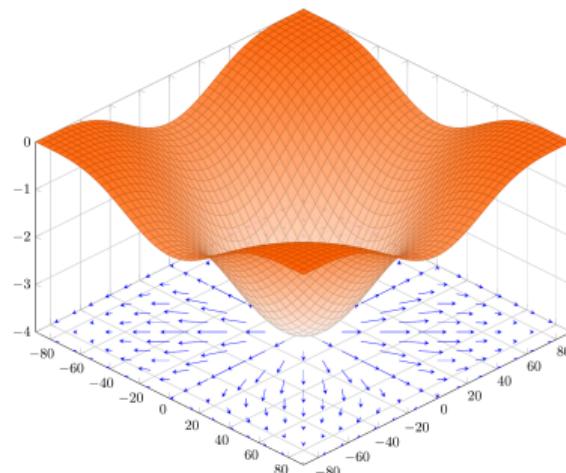
The gradient vector gives the “direction of steepest increase”

梯度矢量给出“最陡峭增加的方向”

$$L_P(\mathbf{w}) = - \sum_{m \in M} \mathbf{w}^T \mathbf{x}_m y_m$$

$$\frac{\partial L_P(\mathbf{w})}{\partial \mathbf{w}} = - \sum_{m \in M} \mathbf{x}_m y_m = 0 \Leftrightarrow \mathbf{w} = ?$$

$$\mathbf{w}^{new} = \mathbf{w}^{old} + \eta \sum_{m \in M} \mathbf{x}_m y_m$$



from Wikipedia

Often the notation $\nabla_{\theta} \mathcal{L} \equiv \frac{\partial \mathcal{L}}{\partial \theta}$ is used.

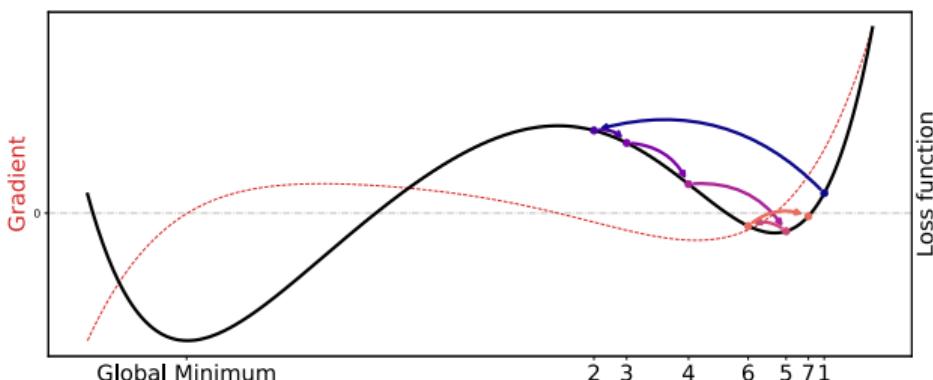
Gradient descent

梯度下降

通过迭代找到最优损耗 \mathcal{L} 的算法

Algorithm which finds **an** optimum of loss \mathcal{L} by iterating

$$\theta \leftarrow \theta - \eta \frac{\partial \mathcal{L}}{\partial \theta}, \quad \text{where} \quad \eta > 0$$



- Not guaranteed to find global optimum
- Adaptive η can yield improvements
- Gradient descent is basis for state-of-the art ML

不保证找到全局最佳
自适应 η 可以带来改进
梯度下降是最先进的ML的
基础

The perceptron learning algorithm

$$\text{Perceptron loss } \mathcal{L}_P(\mathbf{w}) = -\sum_{m \in \mathcal{M}} \mathbf{w}^\top \mathbf{x}_m y_m$$

can be minimized *iteratively* using **stochastic** gradient descent

[Bottou, 2010; Robbins and Monro, 1951]

1 Initialize \mathbf{w}^{old} (randomly, $1/n, \dots$)

虽然存在错误分类的数据点

2 While there are misclassified data points

随机选择一个错误分类的数据点 \mathbf{x}_m
单个数据点 \mathbf{x}_m 处梯度方向的下降

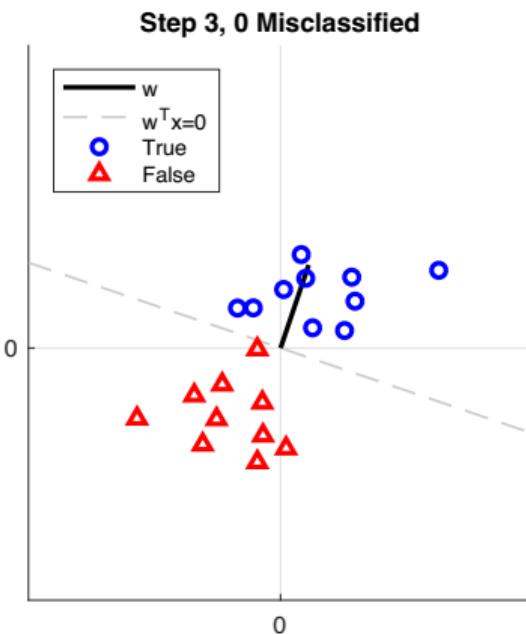
Pick a random misclassified data point \mathbf{x}_m

Descent in direction of the gradient at single data point \mathbf{x}_m

$$\frac{\partial \mathcal{L}_m}{\partial \mathbf{w}} \approx \nabla_{\mathbf{w}} \mathcal{L}_m(\mathbf{w}) = -\mathbf{x}_m y_m$$

$$\mathbf{w}^{\text{new}} \leftarrow \mathbf{w}^{\text{old}} - \eta \nabla_{\mathbf{w}} \mathcal{L}_m(\mathbf{w}^{\text{old}}) = \mathbf{w}^{\text{old}} + \eta \mathbf{x}_m y_m$$

The perceptron learning algorithm in action



The perceptron learning algorithm

单次更新后，该数据点的损失没有增加：

$$\mathbf{w}^{\text{new}} \leftarrow \mathbf{w}^{\text{old}} + \eta \mathbf{x}_m y_m$$

After a single update the loss at that data point has not increased:

$$\begin{aligned}\mathcal{L}_m(\mathbf{w}^{\text{new}}) &= -\mathbf{w}^{(\text{new})\top} \mathbf{x}_m y_m = -\mathbf{w}^{(\text{old})\top} \mathbf{x}_m y_m - \eta (\mathbf{x}_m y_m)^\top \mathbf{x}_m y_m \\ &\leq -\mathbf{w}^{(\text{old})\top} \mathbf{x}_m y_m\end{aligned}$$

because $(\mathbf{x}_m y_m)^\top \mathbf{x}_m y_m = \|\mathbf{x}_m y_m\|^2 \geq 0$

Note: This does not imply that the total loss decreased!

[Novikoff, 1962; Rosenblatt, 1962]

If the data is linearly separable, the perceptron algorithm
will find a solution in a finite number of steps

如果数据是线性可分的，感知器算法将在有限的步长中找到解决方案。

The learning rate η

[Novikoff, 1962; Rosenblatt, 1962]:

If there is a solution, the perceptron algorithm will find it in a finite number of steps

如果有解决方案，感知器算法将在有限步数内找到它
线性可分集合的收敛：

Convergence on linearly separable sets:

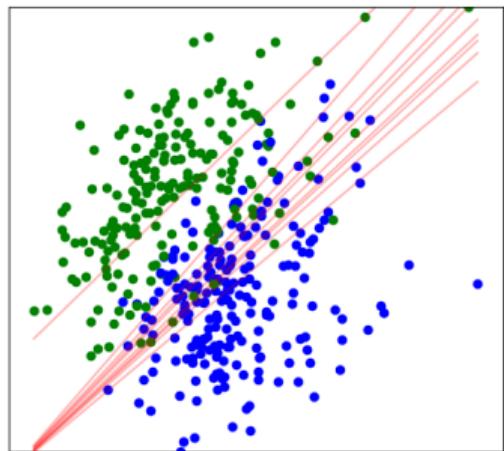
$$\mathbf{w}^{\text{new}} \leftarrow \mathbf{w}^{\text{old}} + \eta \mathbf{x}_m y_m$$

- Proven for variable learning rate $\eta(t)$, with $\sum_t \eta(t) = \infty$ and $\sum_t (\eta(t))^2 < \infty$
- Good convergence speed can be achieved for $\eta(t) \sim \frac{1}{t}$

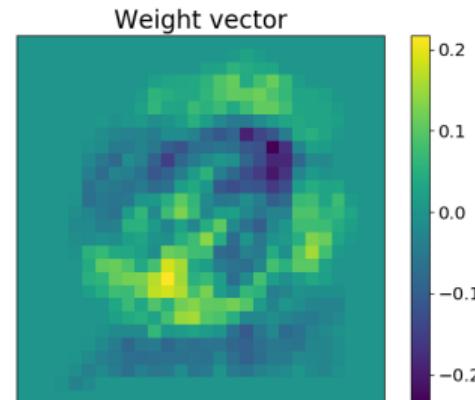
No convergence for non-linearly separable data

reviewed in [Bottou, 2010]

Applied to MNIST



- After 10 tries average 82%
- Using all 784 pixels accuracy of 100%



Problems with Perceptrons

- Each update might lead to new misclassifications
- Many solutions might exist; which one is the best?
- Convergence might be slow
- Only solves linearly separable problems

训练

NCC 可以封闭形式进行优化

感知器使用随机梯度下降 (SGD) 进行迭代训练

梯度下降

梯度下降计算整个数据集的梯度

随机梯度下降计算数据集随机采样子集的梯度

■ Training

- NCC can be optimized in **closed form**
- Perceptron is trained **iteratively** with Stochastic Gradient Descent (SGD)

■ Gradient Descent

- Gradient Descent computes the gradient on the **whole dataset**
- **Stochastic** Gradient Descent computes the gradient on randomly sampled subsets of the dataset
- Learning rate η is not a learnable parameter, but a **hyperparameter**, i.e. it is given to the model, rather than trained

Summary

Linear Classification

Linear decision boundary

Linearly project data on 1d-plane

Prototypes inspired by human learning

Prototypes can be the class means

Yields linear classification model (NCC)

Perceptron

Inspired by biological neural networks

Perceptron algorithm still competitive, good for large problems

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