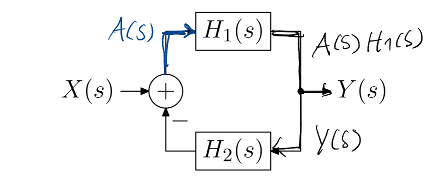
7. Tutorium: Lineare Systeme im Frequenzbereich, Blockschaltbilder

2.1. c)



ges.: $H_{ges}(s)$, PN-Verteilungen

$$H(s) = \frac{\text{Ausgang}}{\text{Eingang}} = \frac{Y(s)}{X(s)}$$

$$Y(s) = A(s) \cdot H_1(s) = X(s)H_1(s) - H_1(s)H_2(s)Y(s)$$

$$A(s) = X(s) - H_2(s) \cdot Y(s)$$

$$Y(s) = X(s)H_1(s) - H_1(s)H_2(s)Y(s)$$

$$Y(s) + H_1(s)H_2(s)Y(s) = X(s)H_1(s)$$

$$Y(s)(1 + H_1H_2) = X(s)H_1(s)$$

$$\therefore Y(s) = \frac{X(s)H_1(s)}{1+H_1H_2}$$

1. Aufgabe

$H_1(s) = \frac{s-1}{s+1}$

$H_2(s) = (s-j)(s+j) = s^2 - j^2 = s^2+1$

$$\therefore H_{ges}(s) = \frac{Y(s)}{X(s)} = \frac{\cancel{X(s)}H_1(s)}{1+H_1H_2} \cdot \frac{1}{\cancel{X(s)}} = \frac{H_1(s)}{1+H_1H_2} = \frac{\frac{s-1}{s+1}}{1+\frac{s-1}{s+1}\cdot s^2+1} = \frac{\frac{s-1}{s+1}}{1+\frac{s^3+s-s^2-1}{s+1}}$$

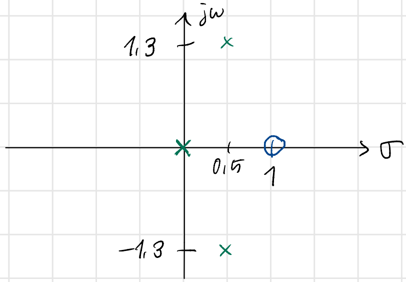
$$\frac{\frac{s-1}{s+1}}{1+\frac{s^3+s-s^2-1}{s+1}} = \frac{\frac{s-1}{\cancel{s+1}}}{\frac{s+\cancel{1}+s^3+s-s^2-\cancel{1}}{\cancel{s+1}}} = \frac{s-1}{s^3-s^2+2s}$$

$s-1=0$ $\quad S_N = 1$ Nullstelle

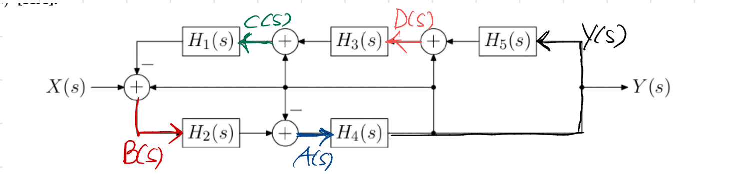
$s^3-s^2+2s=0$ $\quad s(s^2-s+2)=0$ Polstellen

$$\begin{cases} S_{p1} = 0 \\ S_{p2,3} = \frac{1}{2} \pm \sqrt{\frac{1}{4}-2} = \frac{1}{2} \pm \sqrt{-\frac{7}{4}} = \frac{1}{2} \pm \frac{\sqrt{7}}{2} j \end{cases}$$

$\frac{\sqrt{7}}{2} \approx 1,3$



2.1. e)



ges.: $H_{ges}(s) = \frac{Y(s)}{X(s)}$

$$Y(s) = A(s)\cdot H_4(s) = X(s)H_2H_4 - Y(s)\underbrace{(H_4 - H_2H_4 + H_1H_2H_4 + H_1H_2H_3H_4 + H_1H_2H_3H_4H_5)}_{K}$$

$$A(s) = -Y(s) + B(s)\cdot H_2(s) = X(s)H_2(s) - Y(s)(1 - H_2 + H_1H_2 + H_1H_2H_3 + H_1H_2H_3H_5)$$

$$B(s) = X(s) + Y(s) - C(s)H_1(s) = X(s) + Y(s)(1 - H_1 - H_1H_3 - H_1H_3H_5)$$

$$C(s) = Y(s) + D(s)H_3(s) = Y(s)(1 + H_3(s) + H_3H_5)$$

$$D(s) = Y(s) + H_5(s)Y(s) = Y(s)(1 + H_5)$$

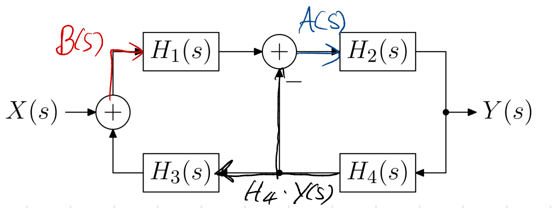
$$Y(s) = X(s)H_2H_4 - Y(s)\cdot K$$

$$Y(s)(1+K) = X(s)H_2H_4$$

$$\Leftrightarrow \quad Y(s) = \frac{X(s)H_2H_4}{1+K}$$

$$\therefore H_{ges}(s) = \frac{Y(s)}{X(s)} = \frac{\cancel{X(s)}H_2H_4}{1+K} \times \frac{1}{\cancel{X(s)}} = \frac{H_2H_4}{1+K}$$

2.1. d)

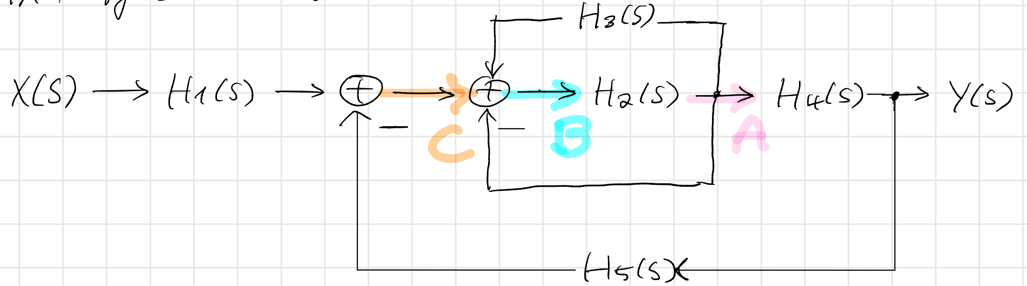


$$Y(S) = A(S)\cdot H_2(S) = X(S)H_1H_2 + H_1H_2H_4Y(S) - H_2H_4Y(S)$$
$$A(S) = B(S)\cdot H_1(S) - H_4(S)\cdot Y(S) = X(S)H_1 + H_1H_4Y(S) - H_4Y(S)$$
$$B(S) = X(S) + H_4(S)\cdot Y(S)$$

$$Y(S) - H_1H_2H_4Y(S) + H_2H_4Y(S) = X(S)H_1H_2$$
$$\Leftrightarrow Y(S)(1 - H_1H_2H_4 + H_2H_4) = X(S)H_1H_2$$
$$\Leftrightarrow Y(S) = \frac{X(S)H_1H_2}{1 - H_1H_2H_4 + H_2H_4}$$

$$\therefore H_{ges}(S) = \frac{Y(S)}{X(S)} = \frac{\cancel{X(S)}H_1H_2}{1 - H_1H_2H_4 + H_2H_4}\cdot\frac{1}{\cancel{X(S)}}$$
$$= \frac{H_1H_2}{1 - H_1H_2H_4 + H_2H_4}$$

AK Aufgabe zum Üben



Bestimme A, B und C
Y(S) und H(S)

$$Y(S) = A(s)\,H_4(s) = B(S)\,H_2H_4$$
$$A(s) = B(S)\,H_2(s)$$
$$B(S) = C(S) + A(S)\,H_3(S) - A(S) = X(S)H_1 - H_5Y(s) + A(S)H_3(S) - A(S)$$
$$C(S) = X(S)H_1(S) - H_5(S)\,Y(S)$$

$$B(S) = X(S)H_1 - H_5Y(s) + A(S)H_3(S) - A(S)$$
$$= X(S)H_1 - H_5Y(S) + B(S)H_2H_3 - B(S)H_2(S)$$

$$B(S)(1 - H_2H_3 + H_2) = X(S)H_1 - H_5Y(S)$$
$$\Leftrightarrow B(S) = \frac{X(S)H_1 - H_5Y(S)}{1 - H_2H_3 + H_2}$$
$$Y(S) = B(S)H_2H_4 = \frac{X(S)H_1H_2H_4 - H_2H_4H_5Y(S)}{1 - H_2H_3 + H_2}$$
$$= \frac{X(S)H_1H_2H_4}{1-H_2H_3+H_2} - \frac{H_2H_4H_5Y(S)}{1-H_2H_3+H_2}$$

$$\Leftrightarrow Y(S)\left(1 + \frac{H_2H_4H_5}{1-H_2H_3+H_2}\right) = \frac{X(S)H_1H_2H_4}{1-H_2H_3+H_2}$$

$$\Leftrightarrow Y(S) = \frac{X(S)H_1H_2H_4}{\cancel{1-H_2H_3+H_2}} \cdot \frac{\cancel{1-H_2H_3+H_2}}{1-H_2H_3+H_2+H_2H_4H_5} = \frac{X(S)H_1H_2H_4}{1-H_2H_3+H_2+H_2H_4H_5}$$

$$\therefore H(S) = \frac{\cancel{X(S)}H_1H_2H_4}{1-H_2H_3+H_2+H_2H_4H_5} \cdot \frac{1}{\cancel{X(S)}} = \frac{H_1H_2H_4}{1-H_2H_3+H_2+H_2H_4H_5}$$