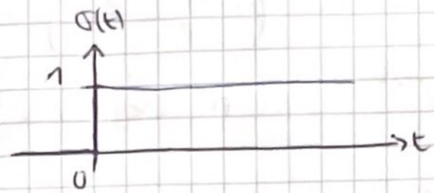
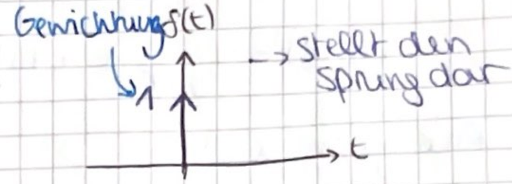
SuS 2 Tutorium

weitere Elementarsignale

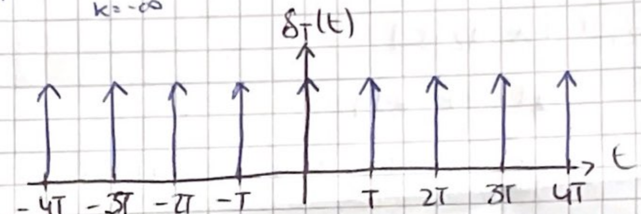
Sprungfunktion: $\sigma(t) = \begin{cases} 0, & \text{für } t<0 \\ 0{,}5, & \text{für } t=0 \\ 1, & \text{für } t>0 \end{cases}$



Deltaimpuls: $\delta(t) = \frac{d\sigma(t)}{dt}$



Deltakamm: $\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - k \cdot T)$



$\rightarrow u_p(t) = u(t) * \delta_T(t) = \sum_{k=-\infty}^{\infty} u(t - k \cdot T)$

Energie, Leistung, Mittelwert, Varianz

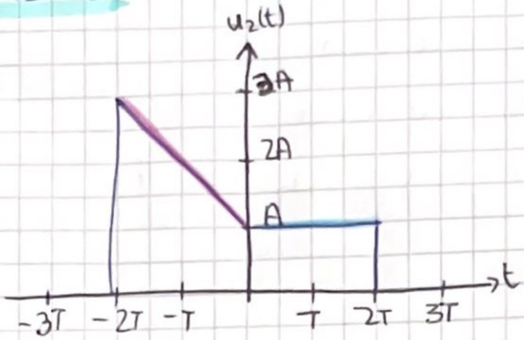
	nicht periodisch	periodisch
Energie	$W_u = \int_{-\infty}^{\infty} u^2(t)\,dt$	$\rightarrow \infty$
Leistung	$P_u = \lim_{x \to \infty} \frac{1}{x} \cdot \int_x u^2(t)\,dt$	$P_u = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} u_p^2(t)\,dt$
Mittelwert	$m_u(t_1, t_2) = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} u(t)\,dt$	$m_u = \frac{1}{T_p} \cdot \int_{-\frac{1}{2}T_p}^{\frac{1}{2}T_p} u_p(t)\,dt$
Varianz	$\sigma_u^2 = P_u - m_u^2$	$\sigma_u^2 = P_u - m_u^2$

Energiesignale:
- endliche Gesamtenergie
- Leistung = 0

Leistungssignale:
- Energie $\rightarrow \infty$
- endliche Leistung

Aufgabe 1

1.1.b)



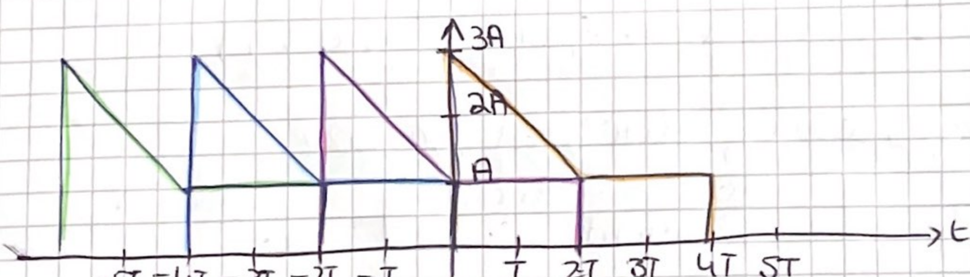
- Gerade: Steigung $-\frac{A}{T}$, Schnittpunkt mit y-Achse bei A
  $\Rightarrow \left(-\frac{A}{T}t + A\right) \cdot \Pi_{2T}(t+T)$
- Rechteck: $A \cdot \Pi_{2T}(t-T)$

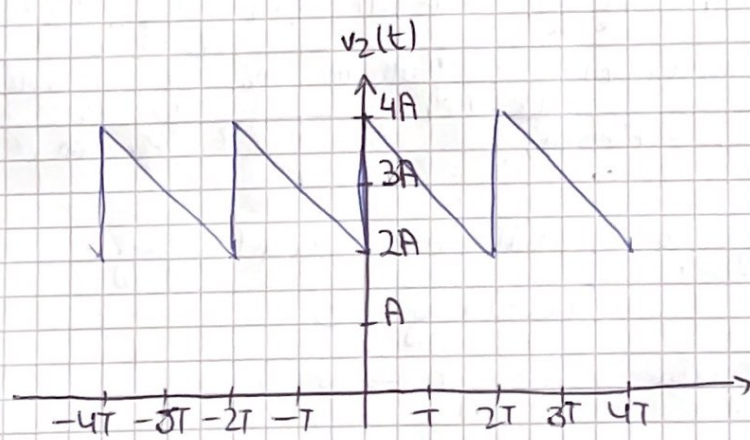
$$\Rightarrow u_2(t) = \left(-\frac{A}{T} \cdot t + A\right) \cdot \Pi_{2T}(t+T) + A \cdot \Pi_{2T}(t-T)$$

Aufgabe 2

2.1.b)

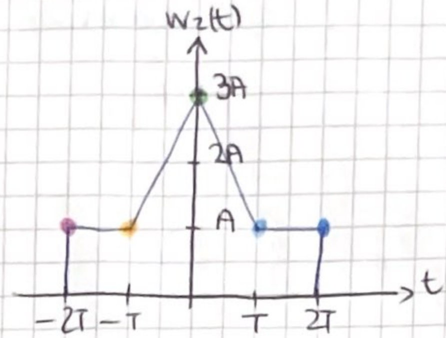
$$v_2(t) = \delta_{2T}(t) * u_2(t)$$
$$= \sum_{k=-\infty}^{\infty} u_2(t - k \cdot 2T)$$
$$= \ldots + \underline{u_2(t+4T)} + \underline{u_2(t+2T)} + \underline{u_2(t)} + \underline{u_2(t-2T)} + \ldots$$



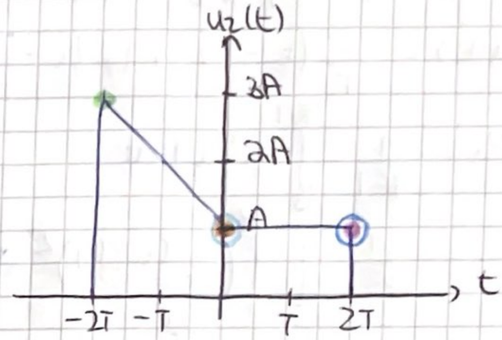


Aufgabe 3

3.1.b)

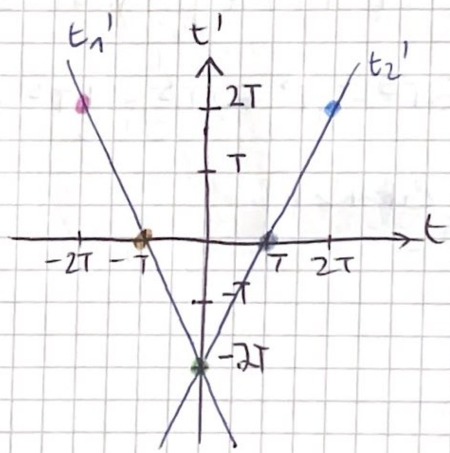


→ aus Aufgabe 1.1



→ 2 Hilfsgeraden: $\cancel{w_2(t_1')}$ +

$$w_2(t) = u_2(t_1') + u_2(t_2')$$



t_1': Steigung -2
Verschiebung -T

$\Rightarrow t_1' = -2(t+T)$

t_2': Steigung 2
Verschiebung T

$\Rightarrow t_2' = 2(t-T)$

$$\Rightarrow w_2(t) = u_2(-2(t+T)) + u_2(2(t-T))$$

Aufgabe 4

gesucht: Leistung von $u_p(t)$

→ Periodendauer $16T$,

$T = 1$

$$P_u = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} u_p^2(t)\, dt$$

mit $t_1 = 0$, $t_2 = 16T$

$$u(t) = \frac{2}{T}(t - T) \cdot \Pi_T\left(t - \frac{3}{2}T\right) + \Pi_{2T}(t - 3T)$$

$$P_u = \frac{1}{16T} \cdot \int_0^{16T} u_p^2(t)\, dt$$

$$= \frac{1}{16T} \cdot \left[\int_T^{2T} \left(\frac{2}{T}(t-T)\right)^2 dt + \int_{2T}^{4T} 1^2\, dt \right]$$

$$= \frac{1}{16T} \cdot \left(\frac{4}{T^2} \cdot \int_T^{2T} (t^2 - 2t \cdot T + T^2)\, dt + [t]_{2T}^{4T} \right]$$

$$= \frac{1}{16T} \left[\frac{4}{T^2} \cdot \left[\frac{1}{3}t^3 - t^2 \cdot T + T^2 \cdot t\right]_T^{2T} + 4T - 2T \right]$$

$$= \frac{1}{16T} \left[\frac{4}{T^2} \left(\frac{1}{3} \cdot 8T^3 - 4T^3 + 2T^3 - \frac{1}{3}T^3 + T^3 - T^3 \right) + 2T \right]$$

$$= \frac{1}{16T} \cdot \left[4 \cdot \frac{7}{3}T - 8T + 2T \right]$$

$$= \frac{5}{24}$$