13. Tutorium: Zeitdiskrete Filter,
PN-Diagramme zeitdiskreter Systeme

Eigenschaften zeitdiskreter Filter

reellwertig: rein reell oder konjugiert komplex

stabil: PS im Einheitskreis
(bedingt: auf EHK)

kausal: Anzahl der PS $\geq$ Anzahl der NS

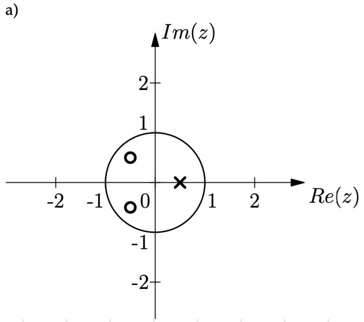
gleichzeitig nicht möglich

linearphasig: • NS symm. am EHK
• keine NS im Ursprung

Allpass: • PS und NS symm. am EHK
Abstand vom Ursprung reziprok; $r_{PS} = \frac{1}{r_{NS}}$
• PS, NS dürfen im Ursprung liegen

minimalphasig: PS, NS im EHK

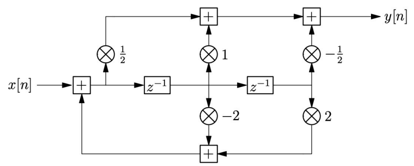
a)



ja	nein	
X		reell
X		stabil
	X	kausal
	X	linearphasig
	X	Allpass
X		minimalphasig

AR 10, 2015

3.2 Gegeben sei das folgende zeitdiskrete Filter. 6 P



a) Bestimmen Sie die ersten sechs Elemente der Impulsantwort. 1,5 P

Differenzengl. : $y(n) = \frac{1}{2}x(n) + x(n-1) - \frac{1}{2}x(n-2) - 2y(n-1) + 2y(n-2)$

$x(0) = 1$

n	$x(n)$	$x(n-1)$	$x(n-2)$	$y(n-1)$	$y(n-2)$	$h(n) = y(n)$
0	1					$\frac{1}{2} = y(0)$
1		1		$\frac{1}{2}$		$0 = y(1)$
2			1	0	$\frac{1}{2}$	$\frac{1}{2} = y(2)$
3				$\frac{1}{2}$	0	$-1 = y(3)$
4				-1	$\frac{1}{2}$	3
$\vdots$						

$\therefore h(n) = \{\frac{1}{2}, 0, \frac{1}{2}, -1, 3, -8\}$

AK 07.2015

3.2 Gegeben sei die Systemfunktion $H(z) = \frac{z^2+2z+1}{z^3+z}$. 3 P

a) Bestimmen Sie die Differenzengleichung $y(n)$. 2 P

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2+2z+1}{z^3+z} \cdot \frac{z^{-3}}{z^{-3}} = \frac{z^{-1}+2z^{-2}+z^{-3}}{1+z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{z^{-1}+2z^{-2}+z^{-3}}{1+z^{-2}}$$

$$Y(z)(1+z^{-2}) = X(z)(z^{-1}+2z^{-2}+z^{-3})$$

z^{-1} ↓

$$y(n) + y(n-2) = x(n-1) + 2x(n-2) + x(n-3)$$

$$\therefore y(n) = x(n-1)+2x(n-2)+x(n-3)-y(n-2)$$

b) Bestimmen Sie die Lage der Pol- und Nullstellen von $H(z)$ und zeichnen Sie das zugehörige PN-Diagramm. 1 P

$$H(z) = \frac{z^2+2z+1}{z^3+z}$$

$z^2+2z+1=0 \leftarrow$ NS

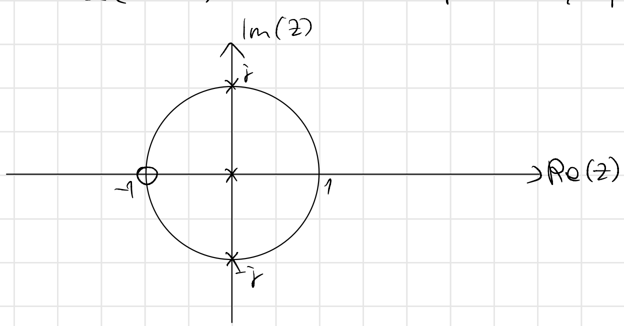
$z^3+z=0 \leftarrow$ PS

$(z+1)^2 = z^2+2z+1$

$z(z^2+1)$

$z_{0\,1,2} = -1$

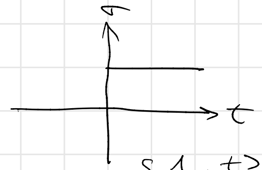
$z_{p1} = 0, \; z_{p2,3} = \pm j$



1.1 Gegeben sei das zeitkontinuierliche Signal $u_1(t)$ mit: 3,5 P

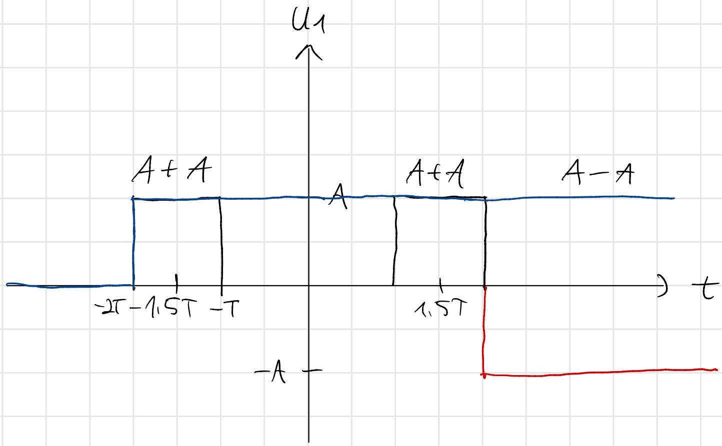
$$u_1(t) = A \cdot \sqcap_T(t + 1,5T) + A \cdot \sqcap_T(t - 1,5T) + A \cdot \sigma(t + 2T) - A \cdot \sigma(t - 2T)$$

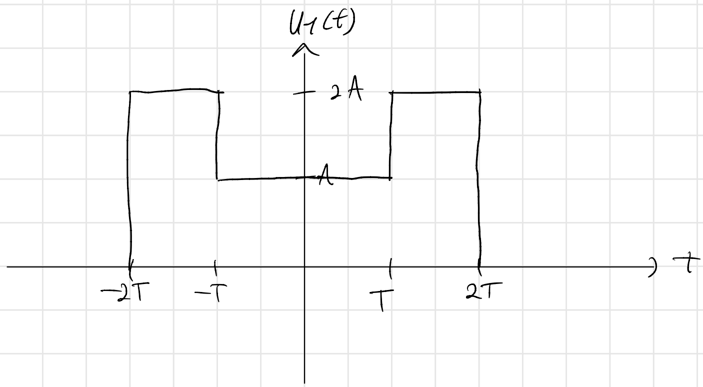
a) Zeichnen Sie das Signal $u_1(t)$. 1 P



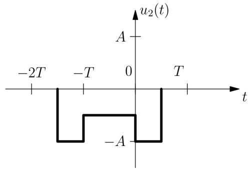
$$\sigma(t) = \begin{cases} 1, & t > 0 \\ 0, & \text{sonst} \end{cases}$$

$-A \cdot \sigma(t - 2T)$



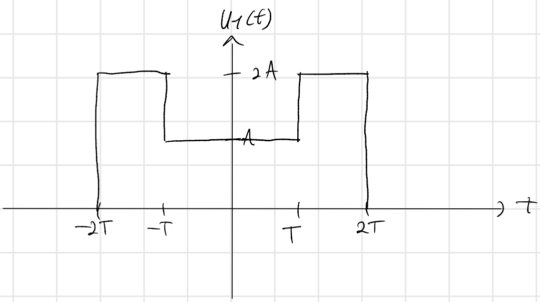


b) Bestimme die Funktion des zeittransformierten Signals $u_2(t)$ in Abhängigkeit von $u_1(t)$. 1,5 P



$u(\alpha(t-\beta T))$

1. Skalierung um $\frac{1}{\alpha}$
2. Verschiebung βT
3. Inversion

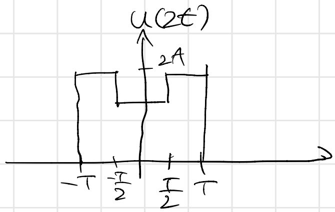


1.

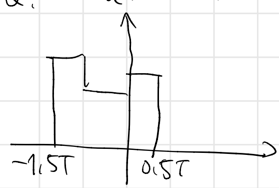
$4T \to 2T$

$\frac{1}{2} = \frac{1}{\alpha} \quad \alpha = 2$

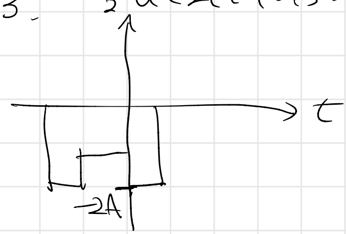
$u(2t)$



2. $u(2(t+0{,}5T))$



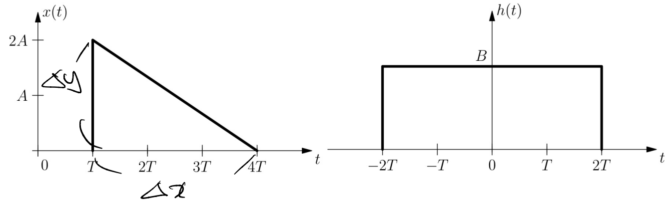
3. $-\frac{1}{2}u(2(t+0{,}5T))$



$-2A \xrightarrow[\frac{1}{2}]{} -A$

$$\therefore\ u_2(t) = -\frac{1}{2}u(2(t+0{,}5T))$$
$$= -\frac{1}{2}u(2t+T))$$

1.2 Gegeben seien das Eingangssignal $x(t)$ und ein Filter mit der Impulsantwort $h(t)$. 6 P



a) Bestimmen Sie das Ausgangssignal $y(t)$. 4,5 P

$$x(t) * h(t) = y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

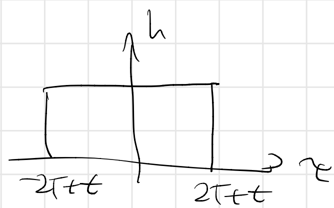
$$x(t) = \left(-\frac{2A}{3T}t + \frac{8A}{3T}\right)\cdot \Pi_{3T}(t-2{,}5T)$$

$$h(t) = B\,\Pi_{4T}(t)$$

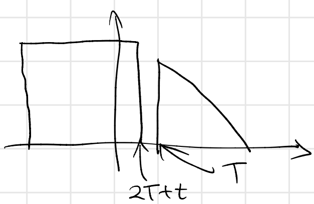
1. Variable ersetzen

$$x(\tau) = \left(-\frac{2A}{3T}\tau + \frac{8A}{3T}\right)\cdot \Pi_{3T}(\tau-2{,}5T)$$

$$h(\tau) = B\,\Pi_{4T}(\tau)$$



1. Fall



$y(t) = 0$

$2T+t < T$

$\Leftrightarrow t < -T$

2. Fall

$T \leq 2T+t < 4T$

$\Leftrightarrow -T \leq t < 2T$

$$\int_{T}^{2T+t} \left(-\frac{2A}{3T}\tau + \frac{8A}{3T}\right) B\, d\tau$$

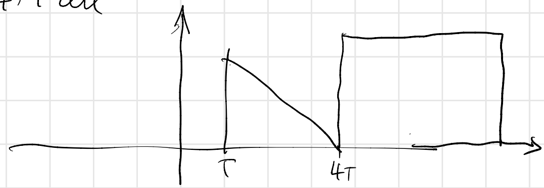
3. Fall

$0 \leq -2T+t < T$

$\Leftrightarrow 2T \leq t < 3T$

$$\int_{T}^{4T} \left(-\frac{2A}{3T}\tau + \frac{8A}{3T}\right) B\, d\tau$$

4. Fall



$T \leq -2T+t < 4T \Leftrightarrow 3T \leq t < 6T$

$$\int_{-2T+t}^{4T} \left(-\frac{2A}{3T}\tau + \frac{8A}{3T}\right) B \, d\tau$$

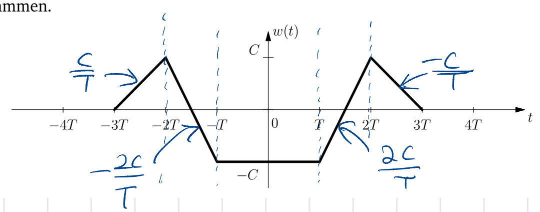
5. Fall

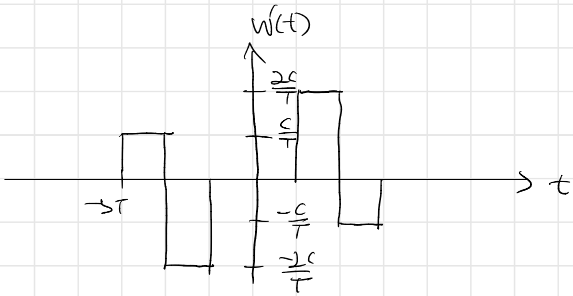
$-2T+t \geq 4T$

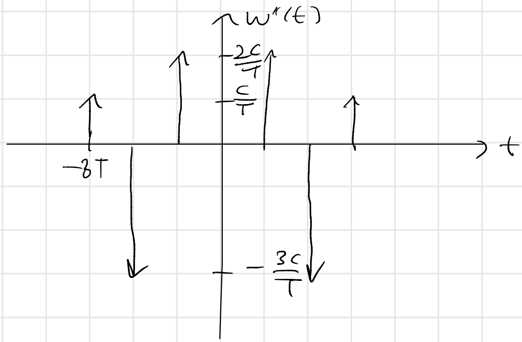
$\Leftrightarrow t \geq 6T$

$y(t) = 0$

1.3 Berechnen Sie die Fouriertransformierte des folgenden Signals $w(t)$. Fassen Sie das Ergebnis so weit wie möglich zu trigonometrischen Funktionen zusammen. 2 P







$$w''(t) = \frac{C}{T}\delta(t+3T) - \frac{3C}{T}\delta(t+2T) + \frac{2C}{T}\delta(t+T) + \frac{C}{T}\delta(t-3T) - \frac{3C}{T}\delta(t-2T) + \frac{2C}{T}\delta(t-T)$$

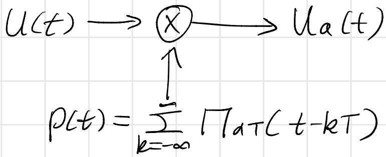
$\mathcal{F}\{$

$$(j\omega)^2 W(j\omega) = \frac{C}{T}(e^{3j\omega T} + e^{-3j\omega T}) - \frac{3C}{T}(e^{2j\omega T} + e^{-2j\omega T}) + \frac{2C}{T}(e^{j\omega T} + e^{-j\omega T})$$

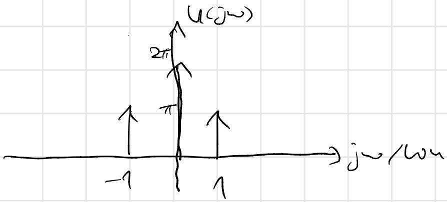
$$\therefore W(j\omega) = -\frac{2C}{\omega^2 T}(\cos(3\omega T) - 3C\cos(2\omega T) + 2\cos(\omega T))$$

2.1

c) Blockschaltbild des ST-Samplings



d) Nun werde das Signal u(t) mittels Shape-Top-Sampling ($\omega_T = 3\omega_u$, $\alpha = 0,7$) abgetastet. Skizzieren Sie $U_A(j\omega)$ im Bereich $-6\omega_u \leq \omega \leq 6\omega_u$.



$$u_{a,ST}(t) = u(t)\,(S_{T}(t) * \Pi_{\alpha T_T}(t))$$

$$U_A(j\omega) = \alpha \cdot U(j\omega) * \left(si\left(\frac{\omega \alpha T}{2}\right) \cdot S_{\omega_T}(\omega)\right)$$

$$= \alpha \sum_{k=-\infty}^{\infty} \left(si(k\pi\alpha) \cdot U(j(\omega - k\omega_T)\right)$$

* Nullstellen der Si-Fkt bestimmen

$$\omega = k \cdot \frac{\omega_T}{\alpha} = k \cdot \frac{3\omega_u}{0{,}7}$$

bei $k = 1 \rightarrow 4{,}3\,\omega_u$

$2\pi \cdot 0{,}7 = 1{,}4\pi$

Max. Amplitude vom Spektrum

Amplituden zur si-Fkt anpassen