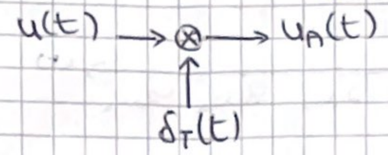
SuS - 9. Tutorium

ideale Abtastung



$u_A(t) = u(t) \cdot \delta_T(t)$

$U_A(j\omega) = \frac{1}{2\pi} \cdot \left(U(j\omega) * \delta_{\omega_T}(\omega) \cdot \omega_T \right)$

setzen $\omega_T = \frac{2\pi}{T}$ ein

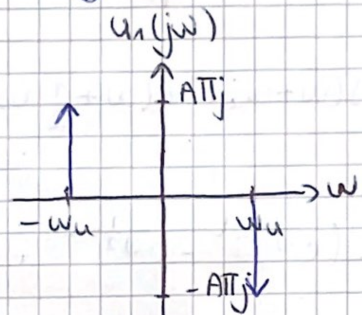
$U_A(j\omega) = \frac{1}{T} \cdot U(j\omega) * \delta_{\omega_T}(\omega)$

$= \frac{1}{T} \cdot \sum_{k=-\infty}^{\infty} U(j(\omega - k \cdot \omega_T))$

Aufgabe 1

gegeben $u_1(t) = A \cdot \sin(\omega_u \cdot t)$

$\Rightarrow U_1(j\omega) = A \cdot \pi j \cdot (\delta(\omega + \omega_u) - \delta(\omega - \omega_u))$ → siehe Rechenübung



1.1.c)

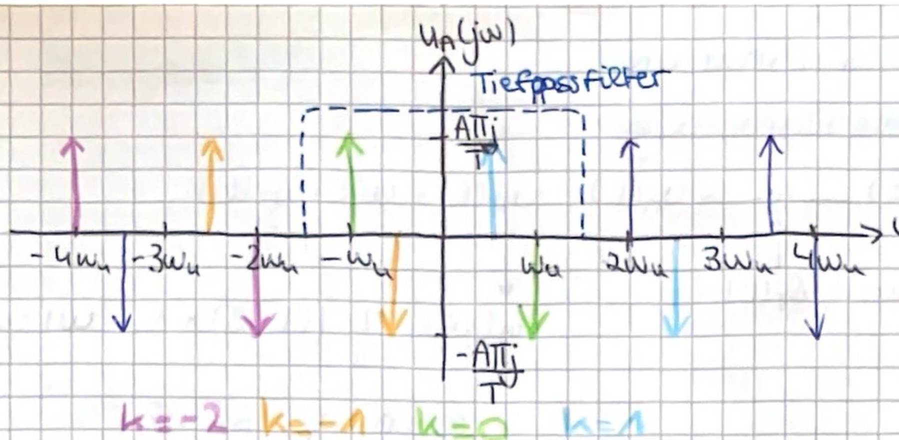
→ mit $\omega_T = 1{,}5\omega_u$ abtasten

$U_A(j\omega) = \frac{1}{T} \cdot U_1(j\omega) * \delta_{\omega_T}(\omega)$

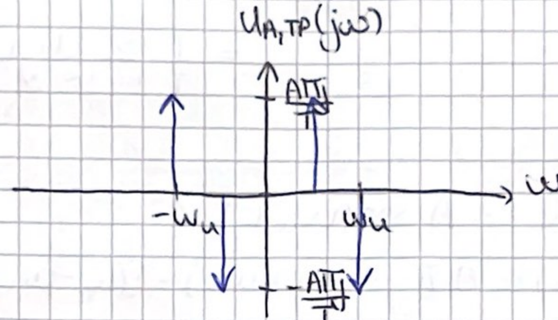
$= \frac{1}{T} \cdot \sum_{k=-\infty}^{\infty} U_1(j(\omega - k \cdot 1{,}5\omega_u))$

$= \frac{1}{T} \cdot \left(\ldots + U_1(j(\omega + 3\omega_u)) + U_1(j(\omega + 1{,}5\omega_u)) \right.$

$\left. + U_1(j\omega) + U_1(j(\omega - 1{,}5\omega_u)) + \ldots \right)$



Tiefpass zur Rekonstruktion: $\omega_g = \frac{3}{2}\omega_u$



$$U_{A,TP}(j\omega) = \frac{1}{T}\cdot A\pi j \cdot \left(\delta(\omega+\omega_u) - \delta(\omega + \tfrac{1}{2}\omega_u) + \delta(\omega - \tfrac{1}{2}\omega_u) - \delta(\omega-\omega_u)\right)$$

Rücktransformation: $e^{j\omega_0 \cdot t} \circ\!\!-\!\!\bullet\ 2\pi\cdot\delta(\omega-\omega_0)$

$$u_{A,TP}(t) = \frac{1}{T}\cdot A\pi j\cdot\frac{1}{2\pi}\cdot\left(e^{-j\omega_u t} - e^{-j\frac{1}{2}\omega_u t} + e^{j\frac{1}{2}\omega_u t} - e^{j\omega_u t}\right)$$

$$= \frac{1}{2\pi}\cdot Aj\cdot\left(-2j\cdot\sin(\omega_u\cdot t) + 2j\cdot\sin(\tfrac{1}{2}\omega_u\cdot t)\right)$$

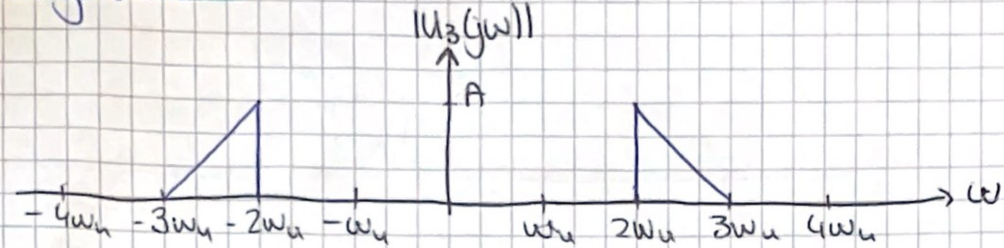
$$= \frac{A}{T}\left(\sin(\omega_u\cdot t) \cancel{+} - \sin(\tfrac{1}{2}\omega_u t)\right)$$

→ Abtasttheorem:

$\omega_T \geq 2\cdot\omega_{max}$
bei Deltaimpulse: $\omega_T > 2\omega_{max}$

→ wurde verletzt

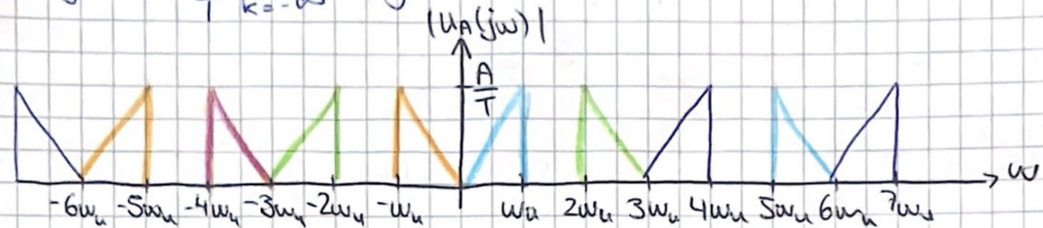
Aufgabe 2



2.1.b)

$\omega_T = 3\omega_u$

$U_A(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} U_3(j(\omega - k\cdot 3\omega_u))$

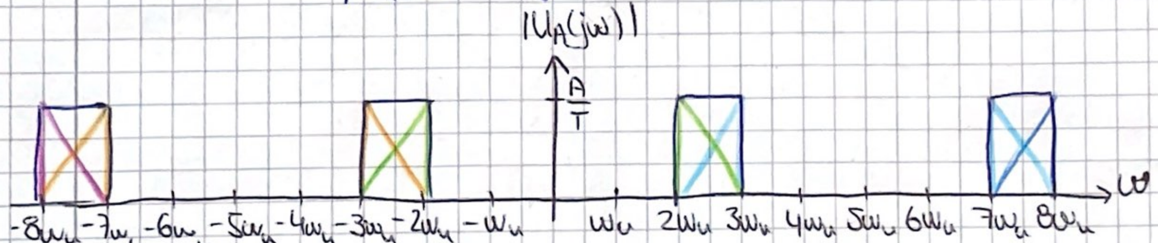


$k=-2$ $k=-1$ $k=0$ $k=1$

2.1.c)

$\omega_T = 5\omega_u$

$U_A(j\omega) = \frac{1}{T}\cdot\sum_{k=-\infty}^{\infty} U_3(j(\omega - k5\omega_u))$



$k=-2$ $k=-1$ $k=0$ $k=1$ $k=2$