SuS_4. Tutorium

Fouriertransformation

Analysegleichung: $U(j\omega) = \mathcal{F}\{u(t)\} = \int_{-\infty}^{\infty} u(t) \cdot e^{-j\omega t} \, dt$

Synthesegleichung: $u(t) = \mathcal{F}^{-1}\{U(j\omega)\} = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} U(j\omega) \cdot e^{j\omega t} \, d\omega$

Sätze: Ähnlichkeitssatz: $u(a \cdot t) \circ\!\!-\!\!\bullet \frac{1}{|a|} \cdot U\left(j \frac{\omega}{a}\right)$

Zeitverschiebung: $u(t-t_0) \circ\!\!-\!\!\bullet U(j\omega) \cdot e^{-j\omega t_0}$

Ableitung: $\frac{d^n u(t)}{dt^n} \circ\!\!-\!\!\bullet (j\omega)^n \cdot U(j\omega)$

wichtige Fouriertransformationen

Rechteckfunktion: $\sqcap_T(t) \circ\!\!-\!\!\bullet T \cdot si\left(\omega \cdot \frac{T}{2}\right)$

Deltaimpuls: $\delta(t) \circ\!\!-\!\!\bullet 1$

$\delta(t-t_0) \circ\!\!-\!\!\bullet e^{-j\omega t_0}$

Deltakamm: $\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - k \cdot T) \circ\!\!-\!\!\bullet \delta_{\omega_T}(\omega)$ mit $\omega_T = \frac{2\pi}{T}$

Amplituden- und Phasenspektrum

→ komplexe Funktion: $U(j\omega) = Re(\omega) + j \cdot Im(\omega)$

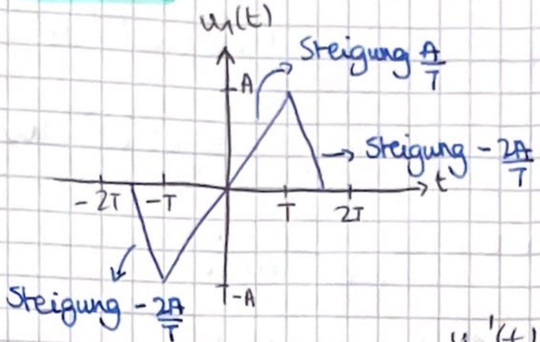
$= A(\omega) \cdot e^{j\varphi(\omega)}$

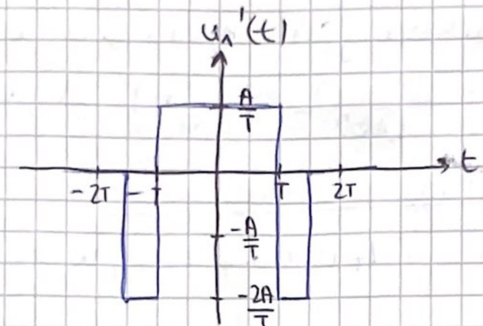
$A(\omega) = |U(j\omega)| = \sqrt{Re^2(\omega) + Im^2(\omega)}$

$$\varphi(\omega) = \begin{cases} \arctan\left(\frac{Im}{Re}\right), & Re > 0 \\ \arctan\left(\frac{Im}{Re}\right) + \pi, & Re < 0, Im \geq 0 \\ \arctan\left(\frac{Im}{Re}\right) - \pi, & Re < 0, Im < 0 \\ \frac{\pi}{2}, & Re = 0, Im > 0 \\ -\frac{\pi}{2}, & Re = 0, Im < 0 \\ \text{unbestimmt}, & Re = Im = 0 \end{cases}$$

Aufgabe 1

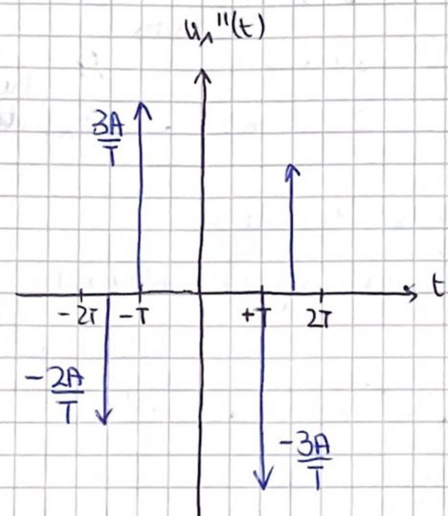
1.1.a)





Erinnerung:

$$\delta(t) = \frac{d\sigma(t)}{dt}$$



$$u_1''(t) = -\frac{2A}{T} \cdot \delta\left(t + \frac{3}{2}T\right) + \frac{3A}{T} \cdot \delta(t + T) - \frac{3A}{T} \cdot \delta(t - T) + \frac{2A}{T} \cdot \delta\left(t - \frac{3}{2}T\right)$$

$$\circ\!\!-\!\!\bullet$$

$$(j\omega)^2 \cdot U_1(j\omega) = -\frac{2A}{T} \cdot e^{j\omega\frac{3}{2}T} + \frac{3A}{T} \cdot e^{j\omega T} - \frac{3A}{T} \cdot e^{-j\omega T} + \frac{2A}{T} \cdot e^{-j\omega\frac{3}{2}T}$$

$$\sin(x) = \frac{1}{2j} \cdot \left(e^{jx} - e^{-jx}\right)$$

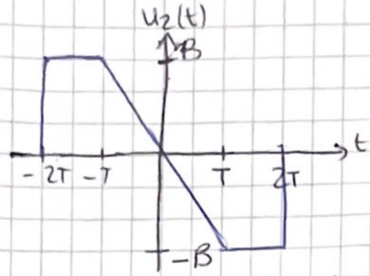
$$\cos(x) = \frac{1}{2} \cdot \left(e^{jx} + e^{-jx}\right)$$

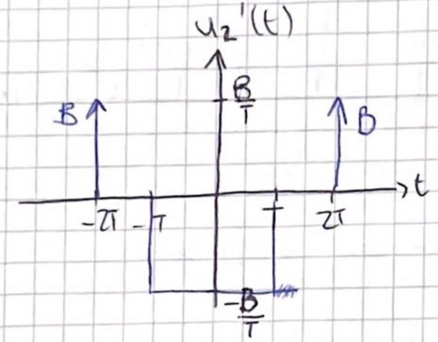
$$(j\omega)^2 \cdot U_1(j\omega) = -\frac{2A}{T}\cdot\left(e^{j\omega\frac{3}{2}T} - e^{-j\omega\frac{3}{2}T}\right) + \frac{3A}{T}\left(e^{j\omega T} - e^{-j\omega T}\right)$$

$$= -\frac{2A}{T}\cdot 2j\cdot\sin\left(\omega\tfrac{3}{2}T\right) + \frac{3A}{T}\cdot 2j\cdot\sin(\omega T)$$

$$U_1(j\omega) = \frac{4Aj}{T\omega^2}\cdot\sin\left(\omega\tfrac{3}{2}T\right) - \frac{6Aj}{T\omega^2}\cdot\sin(\omega T)$$

1.1 b)





$$u_2'(t) = B\cdot\delta(t+2T) - \frac{B}{T}\cdot\sqcap_{2T}(t) + B\cdot\delta(t-2T)$$

$$j\omega\cdot U_2(j\omega) = B\cdot e^{+j\omega 2T} - \frac{B}{T}\cdot 2T\cdot si\left(\omega\frac{2T}{2}\right) + B\cdot e^{-j\omega T}$$

$$= B\left(e^{j\omega 2T} + e^{-j\omega 2T}\right) - 2B\cdot si(\omega T)$$

$$U_2(j\omega) = -\frac{Bj}{\omega}\cdot 2\cos(\omega 2T) + \frac{2Bj}{\omega}\cdot si(\omega T)$$

$$\frac{1}{j} = -j$$

Aufgabe 2

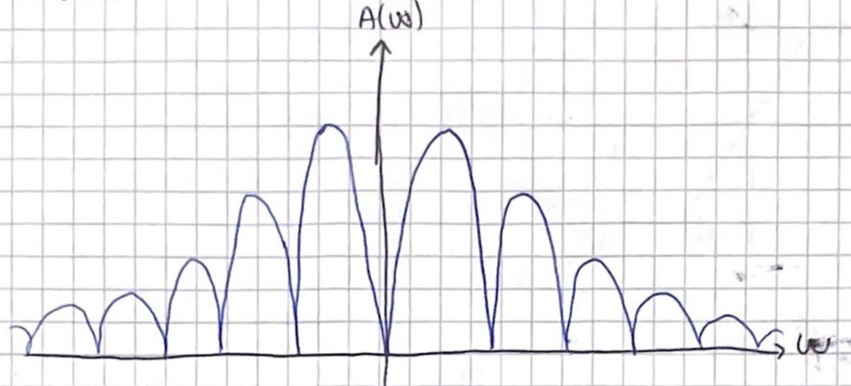
2.1.a)

$$U_2(j\omega) = \frac{2Bj}{\omega} \cdot (si(\omega T) - \cos(\omega 2T))$$

$$A(\omega) = |U_2(j\omega)| = \left|\frac{2Bj}{\omega}\right| \cdot |si(\omega T) - \cos(\omega 2T)|$$

$\hookrightarrow |j| = 1$

$$A(\omega) = \left|\frac{2B}{\omega}\right| \cdot |si(\omega T) - \cos(\omega 2T)|$$



→ hier $Re = 0$

$$\varphi(\omega) = \begin{cases} \frac{\pi}{2}, & \frac{1}{\omega}(si(\omega T) - \cos(\omega 2T)) > 0 \\ -\frac{\pi}{2}, & \frac{1}{\omega}(si(\omega T) - \cos(\omega 2T)) < 0 \\ \text{unbestimmt}, & si(\omega T) - \cos(\omega 2T) = 0 \end{cases}$$