SuS 3. Tutorium

Faltung

$$u(t) * v(t) = \int_{-\infty}^{\infty} u(\tau) \cdot v(t-\tau)\, d\tau$$

Kreuz- und Autokorrelationsfunktion (KKF und AKF)

KKF: $r_{uv}(\tau) = \int_{-\infty}^{\infty} u(t) \cdot v(t+\tau)\, dt$

→ Überlagerung von $u(t)$ und $v(t)$

AKF: $r_{uu}(\tau) = \int_{-\infty}^{\infty} u(t) \cdot u(t+\tau)\, dt$

→ wie ähnlich ist $u(t)$ mit einer zeitverschobenen Version von sich selbst?

Aufgabe 1

1.1 b)

zu zeigen: $u(t) * v(t) = v(t) * u(t)$

$$u(t) * v(t) = \int_{-\infty}^{\infty} u(\tau) \cdot v(t-\tau)\, d\tau$$

→ Substitution: $x = t - \tau \Rightarrow \tau = t - x$

$d\tau = -dx$

obere Grenze: $\tau = \infty \rightarrow x = t - \tau = t - \infty = -\infty$

untere Grenze: $\tau = -\infty \rightarrow x = t - (-\infty) = \infty$

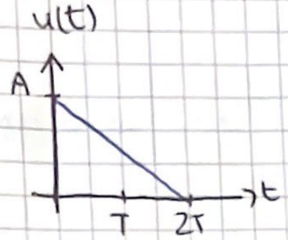
$$u(t) * v(t) = -\int_{\infty}^{-\infty} u(t-x) \cdot v(t-(t-x))\, dx$$

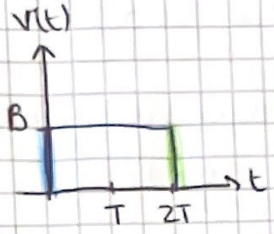
$$= \int_{-\infty}^{\infty} u(t-x) \cdot v(x)\, dx$$

$$\overset{x=\tau}{=} \int_{-\infty}^{\infty} u(t-\tau) \cdot v(\tau)\, d\tau = v(t) * u(t)$$

Aufgabe 2

2.1 b)





gesucht: $r_{uv}(\tau) = \int\limits_{-\infty}^{\infty} u(t) \cdot v(t+\tau)\, dt$

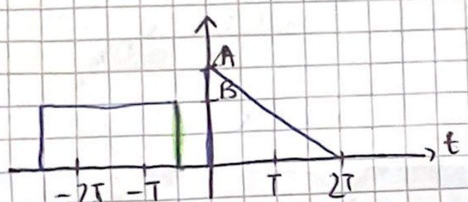
$u(t) = -\frac{A}{2T} \cdot (t-2T) \cdot \sqcap_{2T}(t-T)$

$v(t) = B \cdot \sqcap_{2T}(t-T)$

linke Grenze : $t+\tau = 0 \iff t = -\tau$

rechte Grenze : $t+\tau = 2T \iff t = 2T-\tau$

1. Fall:

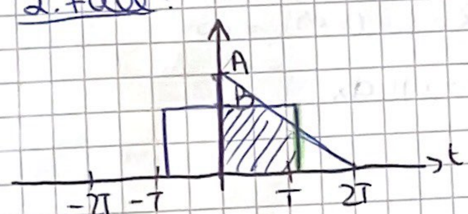


$2T - \tau < 0$

$\iff \tau > 2T$

$r_{uv}(\tau) = 0$

2. Fall:



$0 \leq 2T-\tau < 2T$

$\iff 0 < \tau \leq 2T$

$$r_{uv}(\tau) = \int\limits_{0}^{2T-\tau} u(t) \cdot v(t+\tau)\, dt$$

$$= \int\limits_{0}^{2T-\tau} -\frac{A}{2T}(t-2T) \cdot B \, dt$$

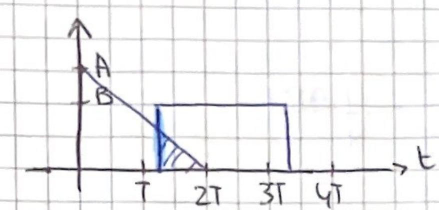
$$= -\frac{AB}{2T} \cdot \left[\frac{1}{2}t^2 - 2T \cdot t\right]_0^{2T-\tau}$$

$$= -\frac{AB}{2T} \cdot \left[\frac{1}{2}(2T-\tau)^2 - 2T \cdot (2T-\tau)\right]$$

$$= -\frac{AB}{2T} \cdot \left[\frac{1}{2} \cdot (4T^2 - 4T \cdot \tau + \tau^2) - 4T^2 + 2T \cdot \tau \right]$$

$$= -\frac{AB}{2T} \cdot \left(-2T^2 + \frac{1}{2} \tau^2 \right)$$

<u>3. Fall:</u>



$$0 \leq -\tau < 2T$$

$$\Leftrightarrow -2T < \tau \leq 0$$

$$r_{uv}(\tau) = \int_{-\tau}^{2T} -\frac{A}{2T} (t - 2T) \cdot B \, dt$$

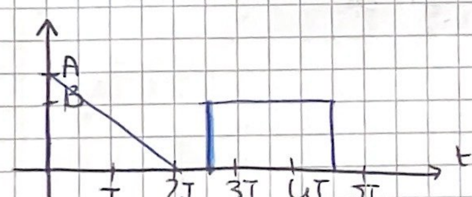
$$= -\frac{AB}{2T} \cdot \left[\frac{1}{2} t^2 - 2T \cdot t \right]_{-\tau}^{2T}$$

$$= -\frac{AB}{2T} \cdot \left[2T^2 - 4T^2 - \frac{1}{2} \tau^2 - 2T \cdot \tau \right]$$

$$= -\frac{AB}{2T} \cdot \left(-2T^2 - \frac{1}{2} \tau^2 - 2T \cdot \tau \right)$$

$$= AB \cdot \left(T + \frac{1}{4T} \cdot \tau^2 + \cancel{2} \tau \right)$$

<u>4. Fall:</u>



$$-\tau \geq 2T$$

$$\Leftrightarrow \tau \leq -2T$$

$$r_{uv}(\tau) = 0$$

$$r_{uv}(\tau) = \begin{cases} 0, & \tau \leq -2T \\ AB\left(\frac{1}{4T}\cdot\tau^2 + \tau + T\right), & -2T < \tau \leq 0 \\ -\frac{AB}{2T}\cdot\left(\frac{1}{2}\tau^2 - 2T^2\right), & 0 < \tau \leq 2T \\ 0, & \tau > 2T \end{cases}$$

$$r_{uv}(-T) = AB\cdot\left(\frac{1}{4T}\cdot T^2 - T + T\right) = \frac{1}{4}ABT$$

$$r_{uv}(0) = ABT$$

$$r_{uv}(T) = -\frac{AB}{2T}\cdot\left(\frac{1}{2}T^2 - 2T^2\right) = \frac{3}{4}ABT$$