# 4. Tutorium : Fourier - Trafo

- Bedeutung : Signale im Zeitbereich in den Frequenzbereich umwandeln

- Fourier - Trafo berechnen
  1. graphisch – Deriviertenmethode (Ableitung)
  2. analytisch – Analysegleichung
  3. mathematisch – Transformationstabelle

- Analysegleichung

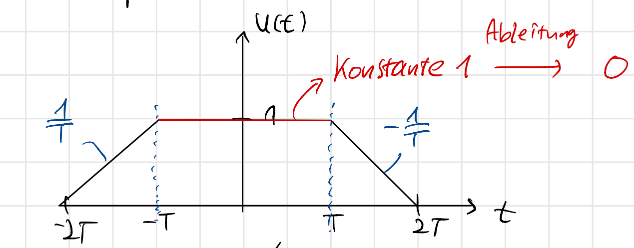
$$U(j\omega) = \mathcal{F}\{u(t)\} = \int_{-\infty}^{\infty} u(t) \cdot e^{-j\omega t} \, dt$$

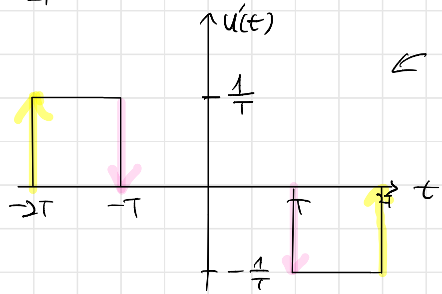
- Transformationstabelle

| Zeitbereich | Frequenzbereich | |
|---|---|---|
| $\frac{d^n(u(t))}{dt^n}$ | $(j\omega)^n \cdot U(j\omega)$ | Abbildungssatz |
| $\delta(t-t_0)$ | $e^{-j\omega t_0}$ | Verschiebungssatz |
| $\delta(t)$ | $1$ | |
| $\sqcap_T(t)$ | $T \cdot si\left(\frac{\omega T}{2}\right)$ | |

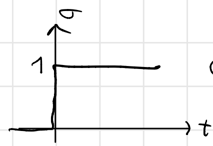
$$\begin{cases} \sin(t) = \frac{1}{2j}\left(e^{jt} - e^{-jt}\right) \\ \cos(t) = \frac{1}{2}\left(e^{jt} + e^{-jt}\right) \end{cases}$$

# Beispiel

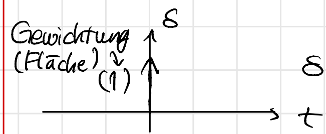



hier werden Rechteckfkt. als Sprungfkt. betrachtet!!

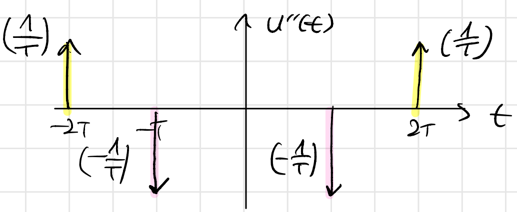



$$\sigma(t) = \begin{cases} 1 & \text{für } t \geq 0 \\ 0, & \text{sonst} \end{cases}$$




$$\delta(t) = \begin{cases} \infty & \text{für } t = 0 \\ 0, & \text{sonst} \end{cases}$$

$$\frac{d}{dt}\sigma(t) = \delta(t)$$




$$u''(t) = \frac{1}{T}\delta(t+2T) - \frac{1}{T}\delta(t+T) - \frac{1}{T}\delta(t-T) + \frac{1}{T}\delta(t-2T)$$

Abb. satz
$$\frac{d^n(u(t))}{dt^n} \circ\!\!-\!\!\bullet\ (j\omega)^n U(j\omega)$$

$\mathcal{F}$ $\circ\!\!-\!\!\bullet$

$$(j\omega)^2 \cdot U(j\omega) = \frac{1}{T}e^{j\omega 2T} - \frac{1}{T}e^{j\omega T} - \frac{1}{T}e^{-j\omega T} + \frac{1}{T}e^{-j\omega 2T}$$

$$j^2\omega^2 = -\omega^2 \qquad j^2 = -1$$

Verschiebungssatz
$$\delta(t-t_0) \circ\!\!-\!\!\bullet\ e^{-j\omega t_0}$$

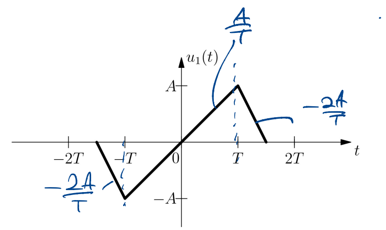
$$\frac{1}{T}\left(e^{j\omega 2T} + e^{-j\omega 2T}\right) = \frac{1}{T}\cdot 2\cos(\omega 2T) \qquad -\frac{1}{T}\left(e^{j\omega T} + e^{-j\omega T}\right) = -\frac{1}{T}\cdot 2\cos(\omega T)$$

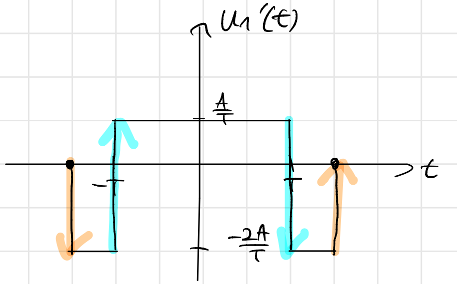
$$= \frac{2}{T}\cos(\omega 2T) - \frac{2}{T}\cos(\omega T)$$

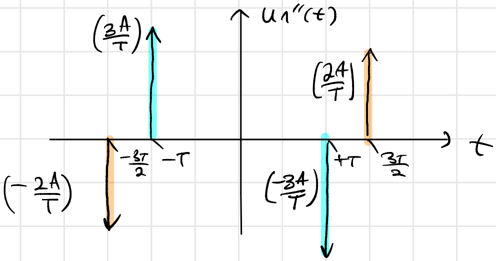
$$\therefore\ U(j\omega) = -\frac{2}{T\omega^2}\cos(2T\omega) + \frac{2}{T\omega^2}\cos(T\omega)$$

# 1.1.a)

a) **[AK]:**




$$u_1''(t) = -\frac{2A}{T}\delta(t+\tfrac{3T}{2}) + \frac{3A}{T}\delta(t+T) - \frac{2A}{T}\delta(t-T) + \frac{2A}{T}\delta(t-\tfrac{3T}{2})$$

$\mathcal{F} \downarrow$

$$\underbrace{(j\omega)^2}_{-\omega^2} U(j\omega) = -\frac{2A}{T}e^{j\omega\frac{3T}{2}} + \frac{3A}{T}e^{j\omega T} - \frac{3A}{T}e^{-j\omega T} + \frac{2A}{T}e^{-j\omega\frac{3T}{2}}$$

$$-\frac{2A}{T}\left(e^{j\omega\frac{3T}{2}} - e^{-j\omega\frac{3T}{2}}\right) \qquad \frac{3A}{T}\left(e^{j\omega T} - e^{-j\omega T}\right)$$

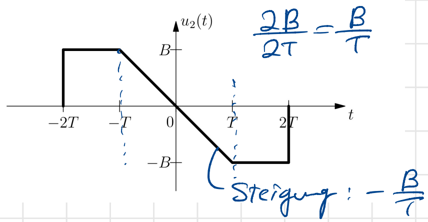
$$= -\frac{2A}{T}\cdot 2j\cdot \sin(\omega\tfrac{3T}{2}) \qquad = \frac{3A}{T}\cdot 2j\cdot \sin(\omega T)$$

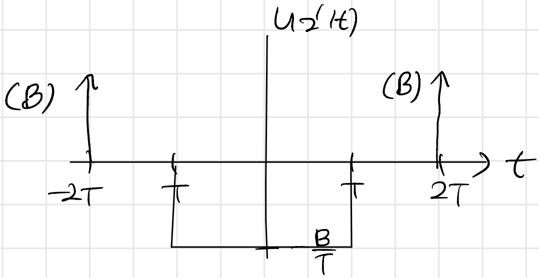
$$\therefore U_1(j\omega) = +\frac{4Aj}{T\omega^2}\sin(\omega\tfrac{3T}{2}) - \frac{6Aj}{T\omega^2}\sin(\omega T)$$

$$\begin{cases} \sin(t) = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) \\ \cos(t) = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \end{cases}$$

# 1. 1. b)

b) **[AK]:**




$$\sqcap_T(t) \circ\!\!-\!\!\bullet\, T \cdot si\left(\frac{\omega T}{2}\right)$$

$$u_2'(t) = B\,\delta(t+2T) + B\,\delta(t-2T) - \frac{B}{T} \sqcap_{2T}(t)$$

$$\mathcal{F} \ \circ\!\!-\!\!\bullet$$

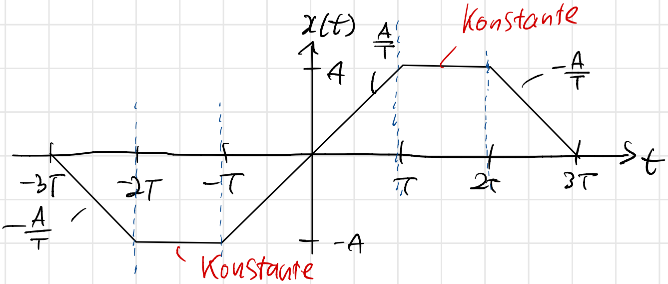
$$\underbrace{(j\omega)}_{\text{einmal abgeleitet}} U(j\omega) = B e^{j\omega 2T} + B e^{-j\omega 2T} - \frac{B}{\cancel{T}} \cdot 2\cancel{T} \cdot si\left(\frac{\omega \cdot \cancel{2}T}{\cancel{2}}\right)$$

$$= B\left(e^{j\omega 2T} + e^{-j\omega 2T}\right) - 2B \cdot si(\omega T)$$

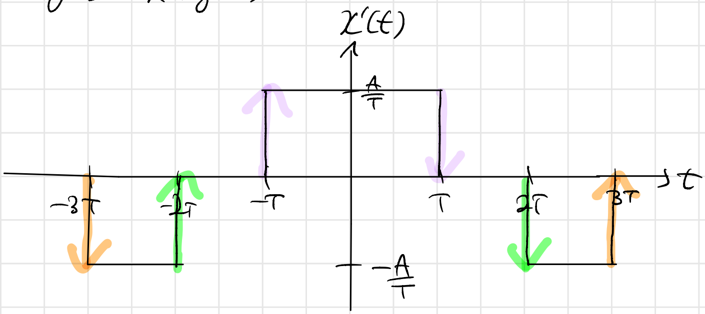
$$= B \cdot 2\cos(\omega 2T) - 2B \cdot si(\omega T)$$

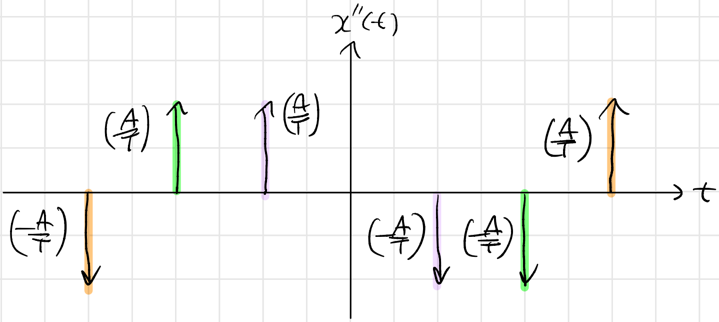
$$\therefore\ U_2(j\omega) = \frac{2B}{j\omega}\cos(2\omega T) - \frac{2B}{j\omega} si(\omega T)$$

# Aufgabe zum Üben




ges.: $X(j\omega)$




$$\frac{d^n(x(t))}{dt^n} \circ\!\!-\!\!\bullet\ (j\omega)^n X(j\omega)$$

$$\delta(t-t_0) \circ\!\!-\!\!\bullet\ e^{-j\omega t_0}$$

$$\sin(t) = \frac{1}{2j}(e^{jt} - e^{-jt})$$

$$\cos(t) = \frac{1}{2}(e^{jt} + e^{-jt})$$

$$x''(t) = -\frac{A}{T}\delta(t+3T) + \frac{A}{T}\delta(t+2T) + \frac{A}{T}\delta(t+T) - \frac{A}{T}\delta(t-T) - \frac{A}{T}\delta(t-2T) + \frac{A}{T}\delta(t-3T)$$

$\mathcal{F}$ ↓

$$\underbrace{(j\omega)^2}_{-\omega^2} X(j\omega) = \underbrace{-\frac{A}{T}e^{j\omega 3T} + \frac{A}{T}e^{-j\omega 3T}}_{-\frac{A}{T}(e^{j\omega 3T} - e^{-j\omega 3T})} + \underbrace{\frac{A}{T}e^{j\omega 2T} - \frac{A}{T}e^{-j\omega 2T}}_{\frac{A}{T}(e^{j\omega 2T} - e^{-j\omega 2T})} + \underbrace{\frac{A}{T}e^{j\omega T} - \frac{A}{T}e^{-j\omega T}}_{\frac{A}{T}(e^{j\omega T} - e^{-j\omega T})}$$

$$= -\frac{A}{T}\cdot 2j \cdot \sin(\omega 3T) \qquad = \frac{A}{T}\cdot 2j\cdot \sin(\omega 2T) \qquad = \frac{A}{T}\cdot 2j \cdot \sin(\omega T)$$

$$\therefore X(j\omega) = \frac{2Aj}{T\omega^2}\sin(3\omega T) - \frac{2Aj}{T\omega^2}\sin(2\omega T) - \frac{2Aj}{T\omega^2}\sin(\omega T)$$

Quiz

**1. Eine Ableitung im Zeitbereich entspricht ...**

- [x] einer Multiplikation des Spektrums U(jω) mit jω im Frequenzbereich. → Abbildungssatz
- [ ] einer Addition des Spektrums U(jω) mit jω im Frequenzbereich.
- [ ] einer Division des Spektrums U(jω) mit jω im Frequenzbereich.

**2. Die Fouriertransformierte eines zur y-Achse symmetrischen Deltaimpulspaares ist ...**

- [ ] eine Sinusfunktion.
- [x] eine Cosinusfunktion.
- [ ] ein Deltaimpulspaar.

**3. Die Fouriertransformierte einer Rechteckfunktion ist eine ...**

- [ ] Sinus-Funktion
- [ ] Konstantwert-Funktion
- [x] Si-funktion

## 2.1. a) Amplituden- und Phasenspektren

**[HA]:** $u_2(t)$

$$U_2(j\omega) = \frac{j}{j} \cdot \left\{ \frac{2B}{j\omega} \cos(2\omega T) - \frac{2B}{j\omega} \operatorname{si}(\omega T) \right\}$$

$$= \frac{2Bj}{\omega \underbrace{j^2}_{-1}} \cos(2\omega T) - \frac{2Bj}{\omega j 2} \operatorname{si}(\omega T)$$

$$= -\frac{2B}{\omega} j \cos(2\omega T) + \frac{2B}{\omega} j \operatorname{si}(\omega T)$$

$$= \frac{2B}{\omega} \left\{ \operatorname{si}(\omega T) - \cos(2\omega T) \right\} j$$

$Im\{U_2(j\omega)\}$

$Re\{U_2(j\omega)\} = 0$

Re, Im: $a + bj$

Amplitudenspektren: $|U_2(j\omega)| = \left|\frac{2B}{\omega}\right| \left| \operatorname{si}(\omega T) - \cos(2\omega T) \right|$

Phasenspektren:
$$\begin{cases} \frac{\pi}{2}, & \frac{2B}{\omega}\left(\operatorname{si}(\omega T) - \cos(2\omega T)\right) > 0 \\ -\frac{\pi}{2}, & \frac{2B}{\omega}\left(\operatorname{si}(\omega T) - \cos(2\omega T)\right) < 0 \end{cases}$$