SuS - 11. Tutorium

DFT und iDFT

$U_{DFT}(n) = U(jn\Delta\Omega) = \sum_{k=0}^{N-1} u(k) \cdot e^{-jkn\Delta\Omega}$

mit $\Delta\Omega = \frac{2\pi}{N}$, $0 \leq n \leq N-1$

es gilt $U_{DFT}(N-n) = U^*_{DFT}(n)$

Inverse: $u(k) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} U_{DFT}(n) \cdot e^{jkn\cdot\Delta\Omega}$

Aufgabe 2

2.1 b)

$u(k) = \{1, -2, 2, 1\}$

aus Rechenübung: $U_{DFT}(n) = \{2, -1+3j, 4, -1-3j\}$

→ Rücktransformation:

$N=4 \Rightarrow \Delta\Omega = \frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$u(k) = \frac{1}{4} \cdot \sum_{n=0}^{3} U_{DFT}(n) \cdot e^{jkn\frac{\pi}{2}}$$

$$= \frac{1}{4} \cdot \left(U_{DFT}(0) \cdot e^{j0} + U_{DFT}(1) \cdot e^{jk\frac{\pi}{2}} + U_{DFT}(2) \cdot e^{jk\pi} + U_{DFT}(3) \cdot e^{jk\frac{3\pi}{2}}\right)$$

$$= \frac{1}{4} \cdot \left(2 + (-1+3j) \cdot e^{jk\frac{\pi}{2}} + 4 \cdot e^{jk\pi} + (-1-3j) \cdot e^{jk\frac{3\pi}{2}}\right)$$

$$= \frac{1}{2} - \frac{1}{4} \cdot e^{jk\frac{\pi}{2}} + \frac{3}{4}j \cdot e^{jk\frac{\pi}{2}} + e^{jk\pi} - \frac{1}{4} e^{jk\frac{3\pi}{2}} - \frac{3}{4}j \cdot e^{jk\frac{3\pi}{2}}$$

$$u(0) = \frac{1}{2} - \frac{1}{4} + \frac{3}{4}j + 1 - \frac{1}{4} - \frac{3}{4}j = 1$$

$$u(1) = \frac{1}{2} - \frac{1}{4} \underbrace{e^{j\frac{\pi}{2}}}_{=j} + \frac{3}{4}j \underbrace{e^{j\frac{\pi}{2}}}_{=j} + \underbrace{e^{j\pi}}_{=-1} - \frac{1}{4} \underbrace{e^{j\frac{3\pi}{2}}}_{=-j} - \frac{3}{4}j \underbrace{e^{j\frac{3\pi}{2}}}_{=-j}$$

$$= \frac{1}{2} - \frac{1}{4}j - \frac{3}{4} - 1 + \frac{1}{4}j - \frac{3}{4} = -2$$

$$u(2) = \frac{1}{2} - \frac{1}{4} \cdot \underbrace{e^{j\pi}}_{=-1} + \frac{3}{4}j \underbrace{e^{j\pi}}_{=-1} + \underbrace{e^{j2\pi}}_{=1} - \frac{1}{4}\underbrace{e^{j3\pi}}_{=-1} - \frac{3}{4}j \cdot \underbrace{e^{j3\pi}}_{=-1}$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{3}{4}j + 1 + \frac{1}{4} + \frac{3}{4}j = 2$$

$$u(3) = \frac{1}{2} - \frac{1}{4} \cdot \underbrace{e^{j\frac{3\pi}{2}}}_{=-j} + \frac{3}{4}j \cdot \underbrace{e^{j\frac{3\pi}{2}}}_{=-j} + \underbrace{e^{j3\pi}}_{=-1} - \frac{1}{4}\underbrace{e^{j\frac{9\pi}{2}}}_{=j} - \frac{3}{4}j \cdot \underbrace{e^{j\frac{9\pi}{2}}}_{=j}$$

$$= \frac{1}{2} + \frac{1}{4}j + \frac{3}{4} - 1 - \frac{1}{4}j + \frac{3}{4} = 1$$

$$u(k) = \{1, -2, 2, 1\}$$

2.2.

Suchen $y(n) = u(n) * v(n)$

$u(n) = \{1, 0, 1\}$, $v(n) = \{0, 2, 1\}$

$\rightarrow$ DFT von u und v

$N = 3 \Rightarrow \Delta\Omega = \frac{2\pi}{3}$

$$U_{DFT}(n) = \sum_{k=0}^{2} u(k) \cdot e^{-jkn \cdot \frac{2\pi}{3}}$$

$$= u(0) \cdot e^{j0} + u(1) e^{-jn\frac{2\pi}{3}} + u(2) \cdot e^{-jn\frac{4\pi}{3}}$$

$$= 1 + e^{-jn\frac{4\pi}{3}}$$

$$U_{DFT}(0) = 1 + e^{j0} = 2$$

$$U_{DFT}(1) = 1 + e^{-j\frac{4\pi}{3}}$$

$$= 1 + \left(-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) = \frac{1}{2} + j\,\frac{\sqrt{3}}{2}$$

$$U_{DFT}(2) = U^*_{DFT}(1) = \frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}$$

$$V_{DFT}(n) = \sum_{k=0}^{2} v(k) \cdot e^{-jkn \cdot \frac{2\pi}{3}}$$

$$= v(0) + v(1) \cdot e^{-jn \cdot \frac{2\pi}{3}} + v(2) \cdot e^{-jn\frac{4\pi}{3}}$$

$$= 2 \cdot e^{-jn\frac{2\pi}{3}} + e^{-jn\frac{4\pi}{3}}$$

$V_{DFT}(0) = 2 + 1 = 3$

$V_{DFT}(1) = 2 \cdot e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}} = -\frac{3}{2} - j \cdot \frac{\sqrt{3}}{2}$

$V_{DFT}(2) = V^*_{DFT}(1) = -\frac{3}{2} + j\frac{\sqrt{3}}{2}$

→ $Y_{DFT}(n)$ ausrechnen

$Y_{DFT}(n) = U_{DFT}(n) \cdot V_{DFT}(n)$

$Y_{DFT}(0) = 2 \cdot 3 = 6$

$Y_{DFT}(1) = \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \cdot \left(-\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}$

$Y_{DFT}(2) = \left(\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right)\left(-\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = j \cdot \sqrt{3}$

$Y_{DFT}(n) = \{6, -j \cdot \sqrt{3}, j\sqrt{3}\}$

→ suchen $y(k)$ → iDFT

$y(k) = \frac{1}{3} \cdot \sum_{n=0}^{2} Y_{DFT}(n) \cdot e^{jkn \cdot \frac{2\pi}{3}}$

$= \frac{1}{3} \cdot \left(6 + (-j \cdot \sqrt{3}) \cdot e^{jk \cdot \frac{2\pi}{3}} + j \cdot \sqrt{3} \cdot e^{jk\frac{4\pi}{3}}\right)$

$y(0) = \frac{1}{3} \cdot (6 - j\sqrt{3} + j\sqrt{3}) = 2$

$y(1) = \frac{1}{3} \cdot (6 - j \cdot \sqrt{3} \cdot e^{j\frac{2\pi}{3}} + j\sqrt{3} \cdot e^{j\frac{4\pi}{3}}) = 3$

$y(2) = \frac{1}{3} \cdot (6 - j\sqrt{3} \cdot e^{j\frac{4\pi}{3}} + j \cdot \sqrt{3} \cdot e^{j\frac{8\pi}{3}}) = 1$

$y(k) = \{2, 3, 1\}$

→ Hinweis: für die Faltung im Zeitbereich muss man die zyklische Faltung ausrechnen