3. Tutorium : Korrelation

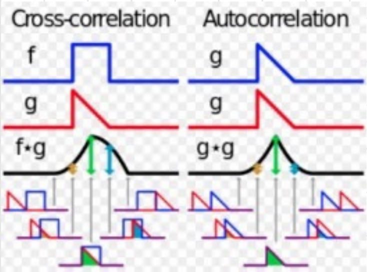
- Bedeutung : Ähnlichkeit zweier Signale berechnen

- $r_{uv}(\tau) = \int_{-\infty}^{\infty} u(t) \cdot v(t+\tau)\, dt$

  Zeitverschiebung Tau

  $\rightarrow$ $v(t)$ wird über das Signal $u(t)$ geschoben.

- Kreuzkorrelation, wenn $u(t) \neq v(t)$
  Autokorrelation, wenn $u(t) = v(t)$

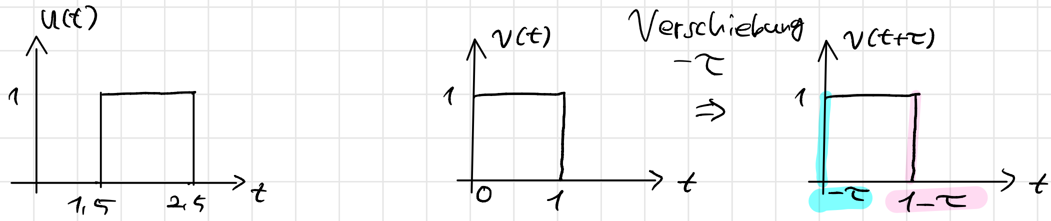


Schrittfolge der Korrelation

1. Verschiebung von $v(t)$ um $-\tau$
2. Bildung des Produktes $u(t) \cdot v(t+\tau)$
3. Integration des Produktes im Bereich der Überdeckungen
4. Wiederholung der Schritte 1 bis 3 bis zur Erfassung aller Überdeckungen

Aufgabe 1.1.6 geht um Faltung $\rightarrow$ machen wir im 5. Tutorium!

Beispiel: ges.: $r_{uv} = \int_{-\infty}^{\infty} u(t) \cdot v(t+\tau)\, dt$



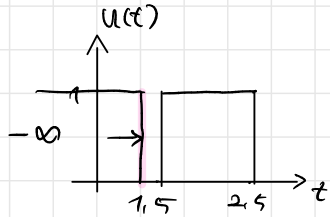
$u(t) = 1 \cdot \Pi_1(t-2)$ $\quad v(t) = 1 \cdot \Pi_1(t-0{,}5)$

1. Fall

$1-\tau < 1{,}5$

$\Leftrightarrow -0{,}5 < \tau$

$r_{uv} = 0$



$v(t+\tau)$ wird von links nach rechts geschoben. Noch keine Überdeckung $\rightarrow r_{uv} = 0$

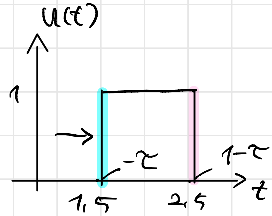
2. Fall

$1{,}5 \leq 1-\tau < 2{,}5$

$\Leftrightarrow -1{,}5 < \tau < 0{,}5$

$\int_{1,5}^{1-\tau} 1\, dt = 1-\tau-1{,}5$

$= -\tau - 0{,}5$



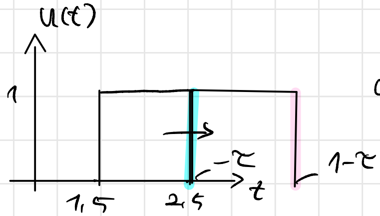
geschoben bis zur völligen Überdeckung

3. Fall

$1{,}5 < -\tau < 2{,}5$

$\Leftrightarrow -2{,}5 < \tau < -1{,}5$

$\int_{-\tau}^{2,5} 1\, dt = 2{,}5 + \tau$



weiter geschoben nach rechts

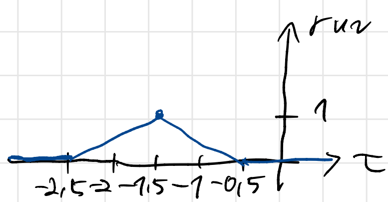
4. Fall

$-\tau > 2{,}5$

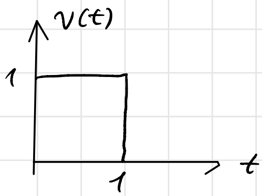
$\Leftrightarrow \tau < -2{,}5$

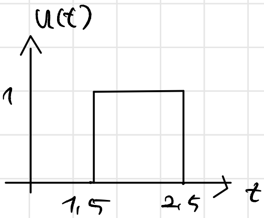
$r_{uv} = 0$

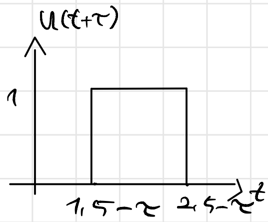
keine Überdeckung $\rightarrow r_{uv} = 0$



Frage: Wenn $u(t)$ über $v(t)$ geschoben wird?







$v(t) = 1 \cdot \Pi_1(t-0{,}5)$ $\qquad$ $u(t) = 1 \cdot \Pi_1(t-2)$

$$\int_{-\infty}^{\infty} v(t) \cdot u(t+\tau)\, dt$$

1. Fall

$2{,}5-\tau < 0$

$r_{vu} = 0$

2. Fall

$0 \leq 2{,}5-\tau < 1$

$$\int_0^{2{,}5-\tau} 1\, dt$$

$= 2{,}5-\tau$

3. Fall

$0 < 1{,}5-\tau < 1$

$$\int_{1{,}5-\tau}^{1} 1\, dt$$

$= 1-(1{,}5-\tau)$

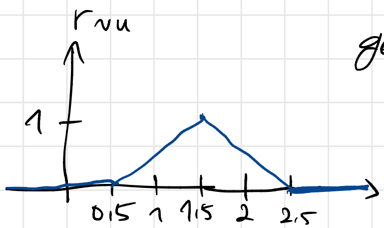
$= \tau - 0{,}5$

4. Fall

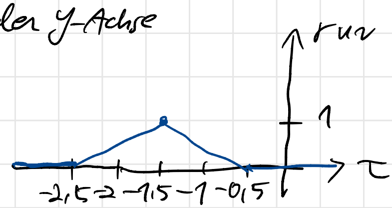
$1{,}5-\tau > 1$

$\Leftrightarrow 0{,}5 > \tau$

$r_{vu} = 0$

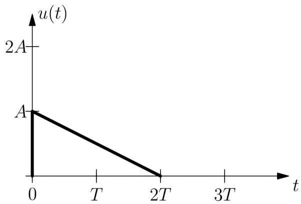


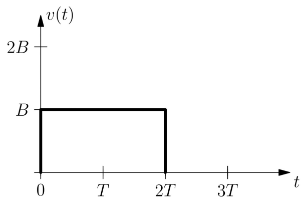
gespiegelt an der y-Achse



$\therefore$ Eigenschaft der Korrelation: $r_{uv}(\tau) = r_{vu}(-\tau)$

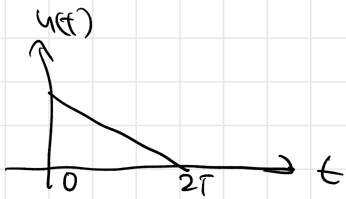
2.1. b) KKF

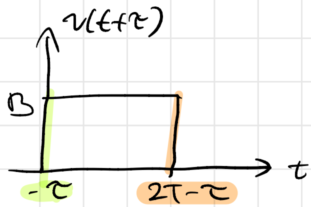




$$u(t) = \left(-\frac{A}{2T}t + A\right)\Pi_{2T}(t-T) \qquad v(t) = B \cdot \Pi_{2T}(t-T)$$

$$\int_{-\infty}^{\infty} u(t) \cdot v(t+\tau)\, dt$$





1. Fall

$2T-\tau < 0$

$r_{uv} = 0$

2. Fall

$0 \leq 2T-\tau < 2T$

$\Leftrightarrow \quad 0 < \tau \leq 2T$

$$\int_0^{2T-\tau} \left(-\frac{A}{2T}t + A\right) \cdot B \, dt$$

$$= -\frac{AB}{4T}\tau^2 + ABT$$

3. Fall

$0 < -\tau < 2T$

$\Leftrightarrow \quad -2T < \tau < 0$

$$\int_{-\tau}^{2T} \left(-\frac{A}{2T}t + A\right) B \, dt$$

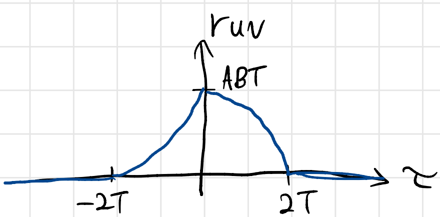
$$= \frac{AB}{4T}\tau^2 + AB\tau + ABT$$

4. Fall

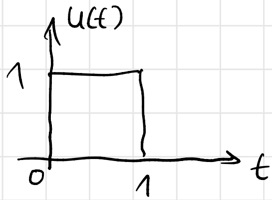
$-\tau > 2T$

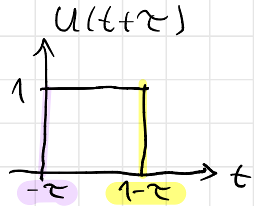
$\Leftrightarrow \tau < -2T$

$r_{uv} = 0$



AKF Autokorrelation





$u(t) = 1 \cdot \Pi_1(t-0{,}5)$

1. Fall

$1-\tau < 0$

$\Leftrightarrow 1 < \tau$

$r_{uu} = 0$

2. Fall

$0 \leq 1-\tau < 1$

$\Leftrightarrow 0 < \tau \leq 1$

$\int_0^{1-\tau} 1 \, dt$

$= 1-\tau$

3. Fall

$0 < -\tau < 1$

$\Leftrightarrow -1 < \tau < 0$

$\int_{-\tau}^{1} 1 \, dt$

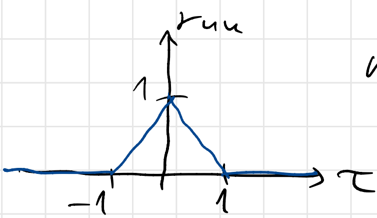
$= 1+\tau$

4. Fall

$-\tau > 1$

$\Leftrightarrow \tau < -1$

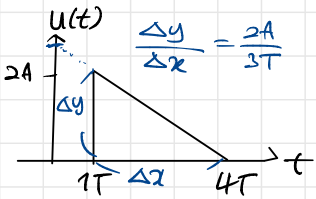
$r_{uu} = 0$



Wenn $\tau = 0 \rightarrow$ maximale Ähnlichkeit

Extra Aufgabe zum Üben

Berechne die $r_{uu}(\tau)$!



$$\frac{\Delta y}{\Delta x} = \frac{2A}{3T}$$

Geradegleichung

$y = ax + b$

$$a = -\frac{2A}{3T}$$

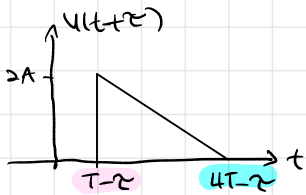
$$y = -\frac{2A}{3T}x + b$$

$$0 = -\frac{2A}{3T} \cdot 4T + b$$

$$\therefore b = \frac{2A}{3T} \cdot 4T = \frac{8A}{3}$$

$$u(t) = \left(-\frac{2A}{3T}t + \frac{8A}{3}\right) \cdot \sqcap_{3T}(t - 2.5T)$$

$$r_{uu} = \int_{-\infty}^{\infty} u(t) \cdot u(t+\tau)\, dt$$



1. Fall

$4T - \tau < T$

$\Leftrightarrow 3T < \tau$

$r_{uu} = 0$

2. Fall

$1T \leq 4T - \tau < 4T$

$\Leftrightarrow -3T \leq -\tau < 0$

$\Leftrightarrow 0 < \tau \leq 3T$

$$\int_{T}^{4T-\tau} \left(-\frac{2A}{3T}t + \frac{8A}{3}\right)^2 dt$$

3. Fall

$T < T - \tau < 4T$

$\Leftrightarrow 0 < -\tau < 3T$

$\Leftrightarrow -3T < \tau < 0$

$$\int_{T-\tau}^{4T} \left(-\frac{2A}{3T}t + \frac{8A}{3}\right)^2 dt$$

4. Fall

$T - \tau > 4T$

$\Leftrightarrow \tau < -3T$

$r_{uu} = 0$