SuS - 6. Tutorium

Laplace-Transformation

$e^{at} \circ\!\!-\!\!\bullet \frac{1}{s-a}$, $\boxed{\forall t > 0}$

$f'(t) \circ\!\!-\!\!\bullet s \cdot F(s) - f(0)$

$\int_0^t f(t)\, dt \circ\!\!-\!\!\bullet \frac{1}{s} \cdot F(s)$

Spannung über Widerstand, Spule und Kondensator

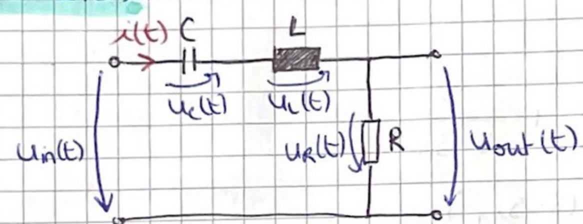
Widerstand: $u_R(t) = R \cdot i_R(t)$

Spule: $u_L(t) = L \cdot \frac{di_L(t)}{dt}$ $\qquad (i_L(0) = 0)$

Kondensator: $i_C(t) = C \cdot \frac{du_C(t)}{dt} \Rightarrow u_C(t) = \frac{1}{C} \cdot \int_0^t i_C(t)\, dt + \underbrace{u_C(0)}_{=0}$

Aufgabe 1

1.1 b)



gesucht: $H(s) = \frac{U_{out}(s)}{U_{in}(s)} \Rightarrow h(t)$

Maschengleichung: $u_C(t) + u_L(t) + u_R(t) - u_{in}(t) = 0$

$$\Leftrightarrow u_{in}(t) = u_C(t) + u_L(t) + u_R(t)$$
$$= \frac{1}{C} \cdot \int_0^t i_C(t)\, dt + L \cdot \frac{di_L(t)}{dt} + R \cdot i_R(t)$$

$\hookrightarrow i_C(t) = i_L(t) = i_R(t) = i(t)$

$u_{out}(t) = u_R(t) = R \cdot i(t)$

Laplace-Trafo:

$$U_{in}(s) = \frac{1}{C \cdot s} \cdot I(s) + L \cdot (s \cdot I(s) - \underbrace{i_L(0)}_{=0}) + R \cdot I(s)$$
$$= I(s) \cdot \left(\frac{1}{s \cdot C} + s \cdot L + R \right)$$

$U_{out}(s) = R \cdot I(s)$

$$\Rightarrow H(s) = \frac{R \cdot I(s)}{I(s) \cdot \left(\frac{1}{s \cdot C} + s \cdot L + R\right)} = \frac{R}{\frac{1}{sC} + sL + R} \cdot \frac{sC}{sC}$$

$$= \frac{s \cdot RC}{s^2 \cdot LC + s \cdot RC + 1} \cdot \frac{\frac{1}{LC}}{\frac{1}{LC}}$$

$$= \frac{s \cdot \frac{R}{L}}{s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}}$$

↳ wollen für die Rücktransformation die Form:

$$A \cdot \frac{1}{s-a} + B \cdot \frac{1}{s-b}$$

Nullstellen von $s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}$:

$$s_{x1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_{x2} = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Partialbruchzerlegung :

$$\frac{s \cdot \frac{R}{L}}{(s - s_{x1})(s - s_{x2})} = \frac{A}{s - s_{x1}} + \frac{B}{s - s_{x2}}$$

$$\Leftrightarrow s \cdot \frac{R}{L} = A(s - s_{x2}) + B(s - s_{x1})$$

setzen $s = s_{x1}$ ein :

$$s_{x1} \cdot \frac{R}{L} = A \cdot (s_{x1} - s_{x2})$$

$$\left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right) \cdot \frac{R}{L} = A \cdot \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} + \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)$$

$$\left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right) \cdot \frac{R}{L} = 2A \cdot \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\Leftrightarrow A = \frac{\frac{R}{L}\left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)}{2\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}} = \frac{-\frac{R^2}{2L^2}}{2\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}} + \frac{R}{2L}$$

setzen $s = s_{x2}$ ein:

$$B = \frac{\frac{R^2}{2L^2}}{2\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}} + \frac{R}{2L}$$

$$\Rightarrow H(s) = A \cdot \frac{1}{s - s_{x1}} + B \cdot \frac{1}{s - s_{x2}}$$

Rücktransformation:

$$h(t) = A \cdot e^{s_{x1} \cdot t} + B \cdot e^{s_{x2} \cdot t} \quad , \forall t > 0$$

Aufgabe 2

Impedanzen, Spannungsteiler

$Z_L = s \cdot L$

$Z_C = \frac{1}{s \cdot C}$

$Z_R = R$

Reihenschaltung: $Z_{ges} = Z_1 + Z_2 + \ldots + Z_n$

Parallelschaltung: $Z_{ges} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}}$

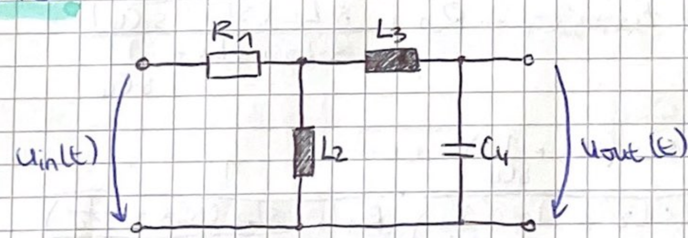
↳ 2 Bauteile: $Z_{ges} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$

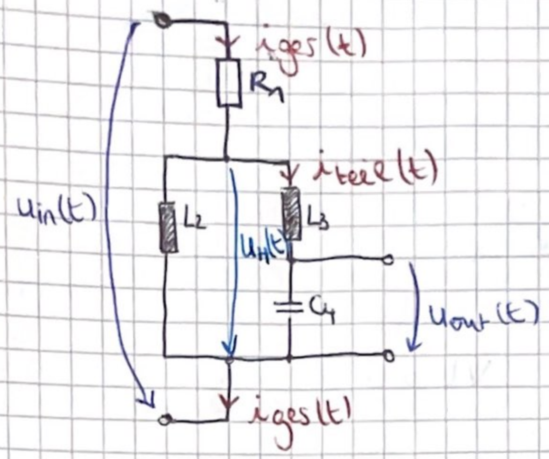
allgemein gilt: $u(t) = Z \cdot i(t)$

Spannungsteiler: $\frac{U_{teil}}{U_{ges}} = \frac{Z_{teil}}{Z_{ges}}$

→ WICHTIG: es muss gelten Teilstrom = Gesamtstrom

2.2.





a) $H(S) = \frac{U_{out}(S)}{U_{in}(S)} \cdot \frac{U_H(S)}{U_H(S)} = \underbrace{\frac{U_{out}(S)}{U_H(S)}} \cdot \underbrace{\frac{U_H(S)}{U_{in}(S)}}$

beides Spannungsteiler

$$\frac{U_{out}(S)}{U_H(S)} = \frac{Z_{C4} \cdot I_{teil}(S)}{Z_{L3,C4} \cdot I_{teil}(S)}$$

$$\frac{U_H(S)}{U_{in}(S)} = \frac{Z_{L2,L3,C4} \cdot I_{ges}(S)}{Z_{R1,L2,L3,C4} \cdot I_{ges}(S)}$$

$$Z_{C4} = \frac{1}{S \cdot C_4}$$

$$Z_{L3,C4} = S \cdot L_3 + \frac{1}{S \cdot C_4}$$

$$Z_{L2,L3,C4} = Z_{L2} // Z_{L3,C4} = \frac{Z_{L2} \cdot (Z_{L3} + Z_{C4})}{Z_{L2} + Z_{L3} + Z_{C4}} = \frac{S \cdot L_2 \left(S \cdot L_3 + \frac{1}{S \cdot C_4}\right)}{S \cdot L_2 + S \cdot L_3 + \frac{1}{S \cdot C_4}}$$

$$Z_{R1,L2,L3,C4} = Z_{R1} + Z_{L2,L3,C4} = R_1 + \frac{S \cdot L_2 \left(SL_3 + \frac{1}{S \cdot C_4}\right)}{S \cdot L_2 + S \cdot L_3 + \frac{1}{S \cdot C_4}}$$

$$H(S) = \frac{\frac{1}{S C_4}}{\left(S \cdot L_3 + \frac{1}{S \cdot C_4}\right)} \cdot \frac{S L_2 \left(S L_3 + \frac{1}{S C_4}\right)}{\left(S \cdot L_2 + S L_3 + \frac{1}{S \cdot C_4}\right)} \cdot \frac{1}{R_1 + \frac{S L_2 \left(S L_3 + \frac{1}{S C_4}\right)}{S L_2 + S L_3 + \frac{1}{S C_4}}}$$

$$H(S) = \frac{S L_2}{S^3 \cdot C_4 \cdot L_3 \cdot L_2 + S^2 \cdot C_4 \cdot R_1 (L_2 + L_3) + S \cdot L_2 + R_1}$$

b) gegeben: $R_1 = \frac{125}{2}\,\Omega$, $L_2 = 125\,H$, $L_3 = 25\,H$, $C_4 = 0{,}01\,F$

$$H(s) = \frac{s \cdot 125}{s^3 \cdot \frac{125}{4} + s^2 \cdot \frac{375}{4} + s \cdot 125 + \frac{125}{2}} \cdot \frac{\frac{4}{125}}{\frac{4}{125}}$$

$$H(s) = \frac{4s}{s^3 + 3s^2 + 4s + 2}$$

-> suchen Polstellen:

durch ausprobieren: $s_1 = -1$

$$s^3 + 3s^2 + 4s + 2 \div s+1 = s^2 + 2s + 2$$
$$\begin{array}{l} -(s^3 + s^2) \\ \quad 2s^2 + 4s + 2 \\ \quad -(2s^2 + 2s) \\ \qquad 2s + 2 \\ \qquad -(2s + 2) \\ \qquad\quad 0 \end{array}$$

$$\Rightarrow s^3 + 3s^2 + 4s + 2 = (s+1)(s^2 + 2s + 2)$$

$$s_{2/3} = -1 \pm \sqrt{1-2} = -1 \pm j$$

$$H(s) = \frac{4s}{(s+1)(s+1-j)(s+1+j)}$$

c) $$H(s) = \frac{4s}{(s+1)(s+1-j)(s+1+j)} = \frac{A}{s+1} + \frac{B}{s+1-j} + \frac{C}{s+1+j}$$

$$A = \frac{-4}{(-1+1-j)(-1+1+j)} = -4$$

$$B = \frac{4(-1+j)}{(-1+j+1)(-1+j+1+j)} = 2-2j$$

$$C = \frac{4(-1-j)}{(-1-j+1)(-1-j+1-j)} = 2+2j$$

$$H(s) = \frac{-4}{s+1} + \frac{2-2j}{s+1-j} + \frac{2+2j}{s+1+j}$$

$$\Rightarrow h(t) = -4 \cdot e^{-t} + (2-2j) \cdot \underbrace{e^{(-1+j) \cdot t}}_{= e^{-t} \cdot e^{jt}} + (2-2j) \cdot \underbrace{e^{(-1-j) \cdot t}}_{= e^{-t} \cdot e^{-jt}}$$

$$= e^{-t} \left(-4 + (2-2j) \cdot e^{jt} + (2+2j) \cdot e^{-jt}\right)$$

$$= e^{-t} \left(-4 + 2 \cdot \underbrace{(e^{jt} + e^{-jt})}_{= 2 \cdot \cos(t)} - 2j \cdot \underbrace{(e^{jt} - e^{-jt})}_{= 2j \cdot \sin(t)}\right)$$

$$h(t) = e^{-t} \left(-4 + 4 \cdot \cos(t) + 4 \cdot \sin(t)\right)$$

$$h(t_1 = 0{,}10s) = 0{,}343$$

$$h(t_2 = 1{,}92s) = -0{,}236$$