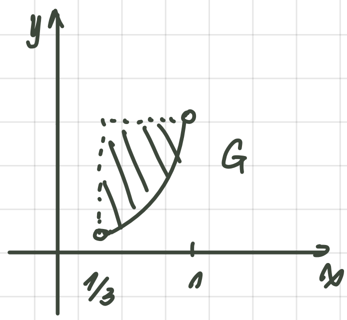
Hausaufgaben 05

Ana II Ing
Gruppe: Nico 6

Aufgabe 5.1

$\frac{\partial f}{\partial x}(x,y) = \frac{3x^2}{y}$ $\frac{\partial f}{\partial y}(x,y) = -\frac{x^3}{y^2}$



Diese Funktionen sind auf D stetig

D.h. ist f stetig partiell differenzierbar.

$\left|\frac{\partial f}{\partial x}(x,y)\right| = \left|\frac{3x^2}{y}\right| \leq \left|\frac{3y}{y}\right| = 3 =: M_1 \quad | \; x^2 < y$

$\left|\frac{\partial f}{\partial y}(x,y)\right| = \left|-\frac{x^3}{y^2}\right| \leq \left|\frac{x^3}{x^4}\right| = \left|\frac{1}{x}\right| \quad | \; y < 1 \rightarrow y^2 < 1 \rightarrow \frac{1}{y^2} > 1$

$\leq 3 : M_2$

G ist konvex, weil die Verbindungsstrecke mit 2 Punkten aus G jeweils zu G gehört

Mit Fehlerschrankensatz folgt:

$$|f(\vec{a}) - f(\vec{b}) \leq M_1 |a_1 - b_1| + M_2 |a_2 - b_2| \leq 3(|a_1 - b_1| + |a_2 - b_2|)$$

für alle $\vec{a} = (a_1, a_2) \in G$, $\vec{b} = (b_1, b_2) \in G$

Aufgabe 5.3

(i) $\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h - 0} = \lim_{h \to 0} \frac{\frac{h^4}{3h^2} - 0}{h} = \lim_{h \to 0} \frac{h^4}{3h^3} = \lim_{h \to 0} \frac{h}{3} = 0$

$\frac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h} = 0$

Die Funktion f ist in (0,0) partiell diff bar.

f ist auf $\mathbb{R}^2 \setminus \{(0,0)\}$ zweimal stetig partiell diffbar.

$\frac{\partial f}{\partial x}(x,y) = \frac{4x^3}{3x^2 + y^2} - \frac{6x^5}{(3x^2 + y^2)^2}$ $\quad (x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

$\frac{\partial f}{\partial y}(x,y) = -\frac{2x^4 y}{(3x^2 + y^2)^2}$ $\quad (x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$

(ii) $\frac{\partial f}{\partial x}(0,y) = 0 - 0 = 0 \qquad \frac{\partial f}{\partial y}(x,0) = 0$

$$\frac{\partial f}{\partial x}(x,0) = \frac{4}{3}x - \frac{6x^5}{3x^4} = \frac{4}{3}x - \frac{2}{3}x = \frac{2}{3}x$$

$$\frac{\partial f}{\partial y}(0,y) = 0$$

$$\frac{\partial^2 f}{\partial x^2}(0,0) = \lim_{h\to 0} \frac{\frac{\partial f}{\partial x}(h,0) - \frac{\partial f}{\partial x}(0,0)}{h-0} = \lim_{h\to 0} \frac{\frac{2}{3}h - 0}{h} = \frac{2}{3}$$

$$\frac{\partial^2 f}{\partial y^2}(0,0) = \lim_{h\to 0} \frac{\frac{\partial f}{\partial y}(0,h) - \frac{\partial f}{\partial y}(0,0)}{h-0} = 0$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = \lim_{h\to 0} \frac{\frac{\partial f}{\partial y}(h,0) - \frac{\partial f}{\partial y}(0,0)}{h-0} = 0$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \lim_{h\to 0} \frac{\frac{\partial f}{\partial x}(0,h) - \frac{\partial f}{\partial x}(0,0)}{h-0} = 0$$