Task 1

1. $w_{+1} = \frac{1}{2}(x_3 + x_4) = \frac{1}{2}\left(\begin{bmatrix}4\\2\end{bmatrix} + \begin{bmatrix}2\\2\end{bmatrix}\right) = \begin{bmatrix}3\\2\end{bmatrix}$

$w_{-1} = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}\left(\begin{bmatrix}1\\-2\end{bmatrix} + \begin{bmatrix}1\\4\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}$

2. $w = w_{+1} - w_{-1} = \begin{bmatrix}3\\2\end{bmatrix} - \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$

$\beta = \frac{1}{2}(w_{+1}^T w_{+1} - w_{-1}^T w_{-1}) = \frac{1}{2}\left([3 \;\; 2]\begin{bmatrix}3\\2\end{bmatrix} - [1 \;\; 1]\begin{bmatrix}1\\1\end{bmatrix}\right)$

$= \frac{1}{2}[9 + 4 - 1 - 1] = \frac{11}{2}$

$\Rightarrow [2 \;\; 1]x - \frac{11}{2} = 0$

3. $f_{w,\beta}(x) = w^T x - \beta = [2 \;\; 1]x - \frac{11}{2} \overset{!}{=} \begin{cases} > 0 & \Rightarrow \text{class } +1 \\ < 0 & \Rightarrow \text{class } -1 \end{cases}$

for x_1: $f_{w,\beta}(x_1) = [2, 1]\begin{bmatrix}1\\-2\end{bmatrix} - \frac{11}{2} = -\frac{11}{2} < 0$ $\Rightarrow$ class -1

$\Rightarrow$ correctly classified

for x_2: $f_{w,\beta}(x_2) = [2 \;\; 1]\begin{bmatrix}1\\4\end{bmatrix} - \frac{11}{2} = \frac{1}{2} > 0$ $\Rightarrow$ class $+1$

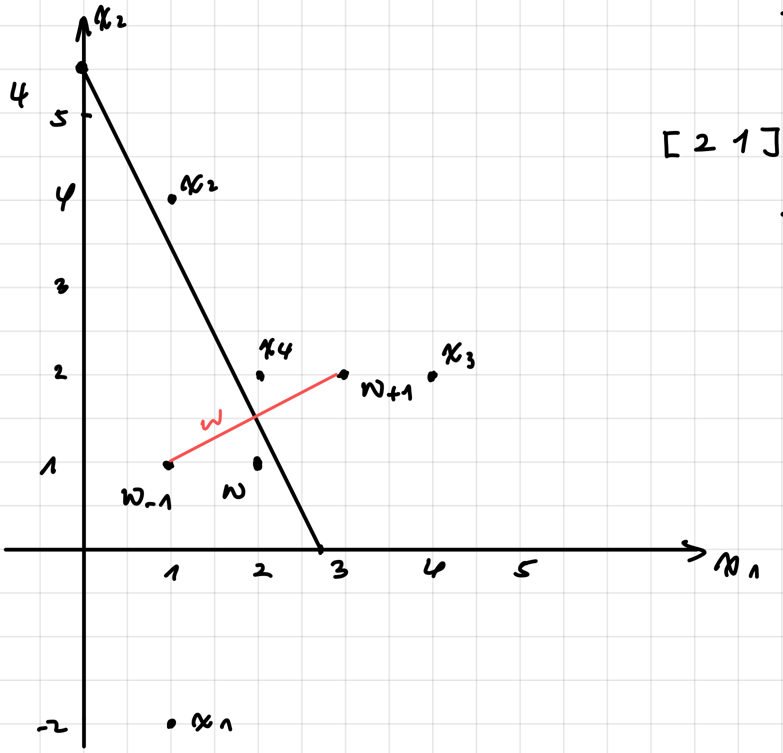
$\Rightarrow$ wrong classified because x_2 belong to class -1

for x_3: $f_{w,\beta}(x_3) = [2 \;\; 1]\begin{bmatrix}4\\2\end{bmatrix} - \frac{11}{2} = \frac{9}{2} > 0$ $\Rightarrow$ class $+1$

$\Rightarrow$ correctly classified

for x_4: $f_{w,\beta}(x_4) = [2 \;\; 1]\begin{bmatrix}2\\2\end{bmatrix} - \frac{11}{2} = \frac{1}{2} > 0$ $\Rightarrow$ class $+1$

$\Rightarrow$ correctly classified



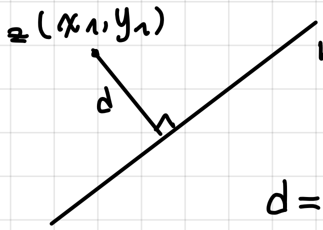
$[2 \;\; 1]\begin{bmatrix}x_1\\x_2\end{bmatrix} - \frac{11}{2} = 0$

$\Rightarrow 2x_1 + x_2 - \frac{11}{2} = 0$

$\Rightarrow x_2 = -2x_1 + \frac{11}{2}$

Task 2

$$[w_1\ w_2]\begin{bmatrix} x \\ y \end{bmatrix} - \beta = w_1 x + w_2 y - \beta$$



$$w_1 x + w_2 y - \beta = 0$$

$$d = \frac{|w_1 x_1 + w_2 y_2 - \beta|}{\sqrt{w_1^2 + w_2^2}}$$

1. $\beta = 0$, $\|w\| = \sqrt{w_1^2 + w_2^2} = 1 \Rightarrow d = \frac{|w_1 x_1 + w_2 y_2 - 0|}{1} = |w^T z|$

2. $\|w\| = 1$, $\beta \neq 0 \Rightarrow d = |w^T z - \beta|$

3. $d = \frac{|w^T z - \beta|}{\|w\|}$

4. $d' = \frac{|w_1 \cdot 0 + w_2 \cdot 0 - \beta|}{\sqrt{w_1^2 + w_2^2}} = \frac{\beta}{\|w\|}$

5. No.

Suppose $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$w^T \begin{bmatrix} 0 \\ y' \end{bmatrix} - \beta = 0 \Rightarrow 2y' = \beta \Rightarrow y' = \frac{\beta}{2}$

$\Rightarrow$ here we find the intercept of the classification boundary with x_2-axis is $\frac{\beta}{2}$, not β