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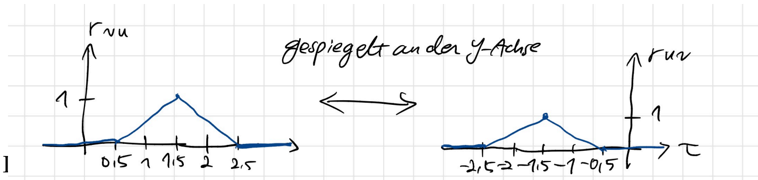
Rechenübung Signale & Systeme (WiSe 2023/2024)

# Korrelation (3. Termin)

6.11 - 12.11.2023




## Hinweise

- Die Aufgabenblätter zur I … al zum Download bereit.
- Aufgaben, die mit **[HA]** b … als Hausaufgabe bearbeitet v … ge- rechnet bzw. besprochen.

∴ Eigenschaft der Korrelation : $r_{uv}(\tau) = r_{vu}(-\tau)$

## 1 Eigenschaften von Korrelation und Fal

### 1.1 Beweise die folgenden Zusammenhänge.

a) $r_{uv}(\tau) = r_{vu}(-\tau)$

b) **[HA]:** $u(t) * v(t) = v(t) * u(t)$

a) $r_{uv}(\tau) = \int_{-\infty}^{\infty} u(t)\, v(t+\tau)\, dt$

$x = t+\tau$ ; $\frac{dx}{dt} = 1$

$\Rightarrow t = x - \tau$ ; $dx = dt$

$\int_{-\infty}^{\infty} u(x-\tau)\, v(x)\, dx = r_{vu}(-\tau)$

b. zu zeigen: $u(t) * v(t) = v(t) * u(t)$

$u(t) * v(t) = \int_{-\infty}^{\infty} u(\tau) \cdot v(t-\tau)\, d\tau$

→ Substitution: $x = t - \tau \Rightarrow \tau = t - x$

$d\tau = -dx$

obere Grenze: $\tau = \infty \rightarrow x = t - \tau = t - \infty = -\infty$

untere Grenze: $\tau = -\infty \rightarrow x = t - (-\infty) = \infty$

$u(t) * v(t) = -\int_{\infty}^{-\infty} u(t-x) \cdot v(t-(t-x))\, dx$

$= \int_{-\infty}^{\infty} u(t-x) \cdot v(x)\, dx$

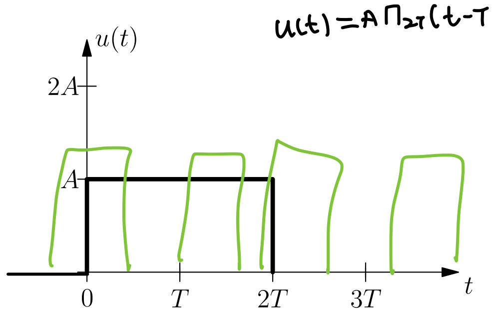
$\overset{x=\tau}{=} \int_{-\infty}^{\infty} u(t-\tau) \cdot v(\tau)\, d\tau = v(t) * u(t)$

## 2 Korrelation

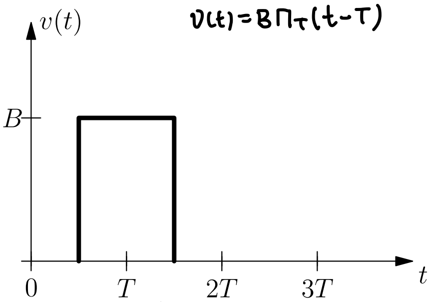
### 2.1 Bestimme die Kreuzkorrelationsfunktion $r_{uv}(\tau)$ für folgende Signalpaare.

a)

$u(t) = A\,\Pi_{2T}(t-T)$




$v(t) = B\,\Pi_{T}(t-T)$




b) **[HK]:**

$r_{uv}(\tau) = \int_{-\infty}^{\infty} u(t)\, v(t+\tau)\, dt$

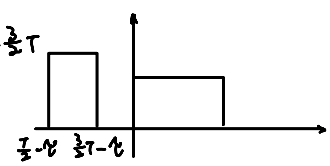
linke Grenze: $t + \tau = \frac{T}{2} \Leftrightarrow t = \frac{T}{2} - \tau$

rechte Grenze: $t + \tau = \frac{3}{2}T \Leftrightarrow t = \frac{3}{2}T - \tau$

1. Fall:

$\frac{3}{2}T - \tau < 0 \Leftrightarrow \tau > \frac{3}{2}T$

$r_{uv}(\tau) = 0$

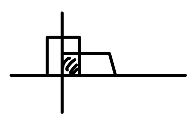



2. Fall $\frac{3}{2}T - \tau > 0 \wedge \frac{1}{2}T - \tau < 0$

$\Rightarrow \frac{3}{2}T \geq \tau > \frac{1}{2}T$

$r_{uv}(\tau) = \int_0^{\frac{3}{2}T-\tau} A \cdot B\, dt$

$= AB\left(\frac{3}{2}T - \tau\right)$

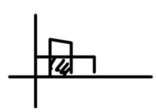


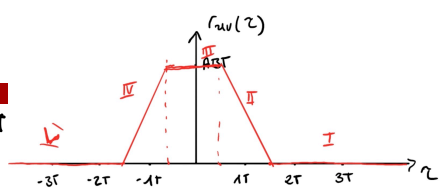
3. Fall $\frac{1}{2}T - \tau > 0 \wedge \frac{3}{2}T - \tau < 2T$

$\Rightarrow \frac{T}{2} \geq \tau > -\frac{1}{2}T$

$r_{uv}(\tau) = \int_{\frac{1}{2}T-\tau}^{\frac{3}{2}T-\tau} A \cdot B\, dt$

$= AB \cdot (T)$



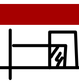





4. Fall $\frac{T}{2} - \tau < 2T \wedge \frac{3}{2}T - 2\tau > 2T$

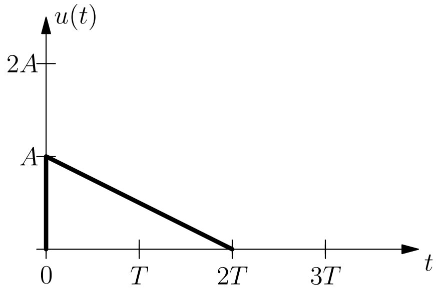
$\Rightarrow -\frac{T}{2} \geq \tau > -\frac{3}{2}T$

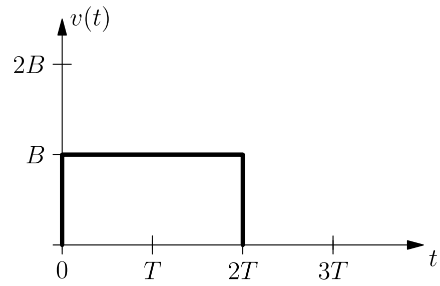
$r_{uv}(\tau) = \int_{\frac{T}{2}-\tau}^{2T} AB\, dt = AB \cdot \left(\frac{3}{2}T + \tau\right)$



5. Fall $\frac{T}{2} - \tau > 2T \Rightarrow \tau \leq -\frac{3}{2}T$

$r_{uv}(\tau) = 0$




## 2.2 Berechne die Autokorrelationsfunktion (AKF) $r_{uu}(\tau)$ für das nachfolgend skizzierte Signal. Gib weiterhin den Wert der normierten AKF $\rho_{uu}(\tau)$ an den Punkten $-2T$, $0$, $T$ an.

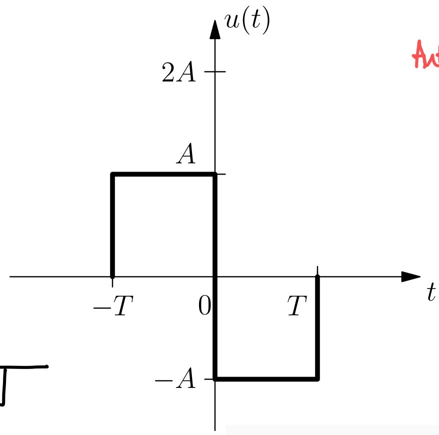
$r_{uu}(\tau) = \int_{-\infty}^{\infty} u(t)u(t+\tau)\,dt$

$u(t) = A\sqcap_T(t+T/2) - A\sqcap_T(t-T/2)$

linke Grenze: $t+\tau = -T \Rightarrow t = -T-\tau$

mittlere Grenze: $t+\tau = 0 \Rightarrow t = -\tau$

rechte Grenze: $t+\tau = T \Rightarrow t = T-\tau$




Autokorrelation immer symmetrisch

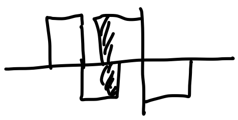
可以不计算 Fall 4. Fall 5.

Fall 1: $-T > T-\tau \Rightarrow \tau > 2T$

$r_{uu}(\tau) = 0$

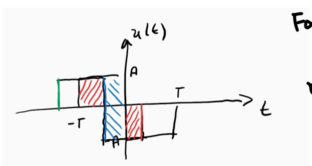
Fall 2: $-T < T-\tau \wedge -\tau < -T$

$\Rightarrow 2T \geq \tau > T$



$r_{uu}(\tau) = \int_{-T}^{T-\tau} (-A)A\,dt$

$= -A^2\cdot(2T-\tau)$



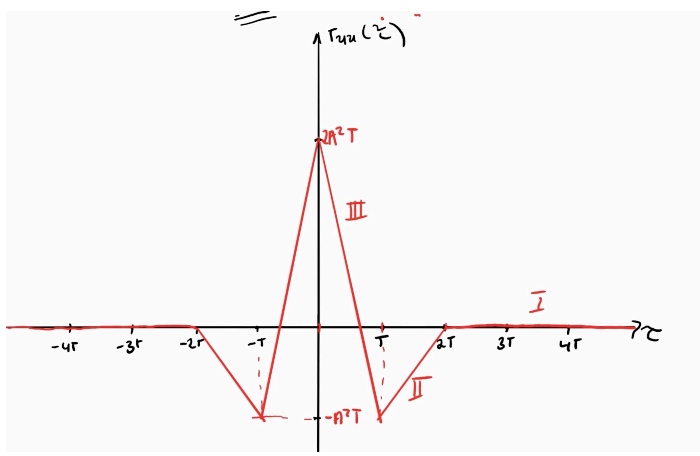

Fall 3. $-\tau < 0 \wedge -T < -\tau$

$\Rightarrow T \geq \tau > 0$

$r_{uu}(\tau) = \int_{-T}^{-\tau} A^2\,dt + \int_{-\tau}^{0} -A^2\,dt + \int_{0}^{T-\tau} A^2\,dt$

$= A^2(-\tau+T-\tau+T-\tau)$

$= A^2(2T-3\tau)$



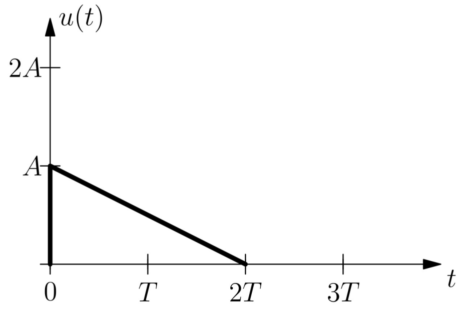

$\rho_{uu}(\tau) = \dfrac{r_{uu}(\tau)}{r_{uu}(0)}$

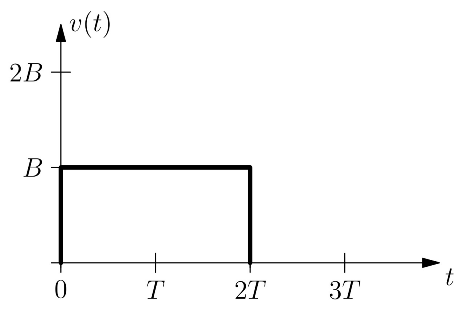
$\rho_{uu}(-2T) = \dfrac{0}{2A^2T} = 0$ //

$\rho_{uu}(0) = \dfrac{r_{uu}(0)}{r_{uu}(0)} = 1$ //

$\rho_{uu}(T) = \dfrac{-A^2T}{+2A^2T} = -1/2$

2.1.b.




$u(t) = \left(-\frac{A}{2T}\cdot t + A\right)\Pi_{2T}(t-T)$ $\qquad$ $v(t) = B\,\Pi_{2T}(t-T)$

$\int_{-\infty}^{\infty} u(t)\,v(t+\tau)\,dt$

links: $t+\tau = 0 \Rightarrow t = -\tau$

Rechts: $t+\tau = 2T \Rightarrow t = 2T-\tau$

1. Fall

$2T-\tau < 0 \Rightarrow \tau > 2T$

$r_{uv}(\tau) = 0$

2. Fall

$2T-\tau \geq 0 \wedge -\tau < 0$

$\Rightarrow 0 < \tau \leq 2T$

$r_{uv}(\tau) = \int_0^{2T-\tau} AB\cdot\left(-\frac{t}{2T}+1\right)dt$

$= -\frac{AB}{2T}\cdot\int_0^{2T-\tau}(t-2T)\,dt$

$= -\frac{AB}{2T}\cdot\left[\frac{1}{2}t^2 - 2Tt\,\Big|_0^{2T-\tau}\right]$

$= -\frac{AB}{2T}\cdot\left[\frac{1}{2}\cdot(\cancel{4T^2}-4T\tau+\tau^2) - \cancel{4T^2} + 2T\tau\right]$

$= -\frac{AB}{2T}\cdot\left[-2T^2+\frac{1}{2}\tau^2\right]$

3. Fall

$0 \leq -\tau \wedge -\tau < 2T$

$\Rightarrow -2T < \tau \leq 0$

$r_{uv}(\tau) = \int_{-\tau}^{2T} A\cdot B\cdot\left(-\frac{t}{2T}+1\right)dt$

$= -\frac{AB}{2T}\int_{-\tau}^{2T}(t-2T)\,dt$

$= -\frac{AB}{2T}\cdot\left(\frac{1}{2}t^2 - 2Tt\right)\Big|_{-\tau}^{2T}$

$= -\frac{AB}{2T}\left(2T^2 - \frac{1}{2}\tau^2 - 2T(2T-\tau)\right)$

$= -\frac{AB}{2T}\cdot\left(-2T^2 - \frac{1}{2}\tau^2 + 2\tau T\right)$

4. Fall $\quad -\tau \geq 2T \Rightarrow \tau \leq -2T$

$r_{uv}(\tau) = 0$