Hausaufgaben 04

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Gruppe: Nico 6

Anfgabe 4.1

(i)

$$\vec{f}'(x,y,z) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x\cos y & -x^2 \sin y \\ y\cos x & \sin x \\ y & x \end{pmatrix}$$

$$\vec{g}'(x,y,z) = \begin{pmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \end{pmatrix} = \begin{pmatrix} 3e^{3x-5y+z^2} & -5e^{3x-5y+z^2} & 2ze^{3x-5y+z^2} \end{pmatrix}$$

(ii)

$$(\vec{g} \circ \vec{f})(x,y) = \vec{g}(\vec{f}(x,y)) = e^{3f_1(x,y,z) - 5f_2(x,y,z) + f_3^2(x,y,z)}$$

$$= e^{3x^2\cos y - 5y\sin x + x^2y^2}$$

$$(\vec{g} \circ \vec{f})'(x,y) = \begin{pmatrix} \frac{\partial(\vec{g}\circ\vec{f})}{\partial x} & \frac{\partial(\vec{g}\circ\vec{f})}{\partial y} \end{pmatrix}$$

$$= \begin{pmatrix} (6x\cos y - 5y\cos x + 2xy^2)e^{3x^2\cos y - 5y\sin x + x^2y^2} & (-3x^2\sin y - 5\sin x + 2x^2y)e^{3x^2\cos y - 5y\sin x + x^2y^2} \end{pmatrix}$$

(iii)

$$(\vec{g} \circ \vec{f})'(x,y) = \vec{g}'(\vec{f}(x,y)) \cdot \vec{f}'(x,y)$$

$$= \begin{pmatrix} 3e^{3x^2\cos y - 5y\sin x + x^2y^2} & -5e^{3x^2\cos y - 5y\sin x + x^2y^2} & 2xye^{3x^2\cos y - 5y\sin x + x^2y^2} \end{pmatrix}$$

$$\cdot \begin{pmatrix} 2x\cos y & -x^2\sin y \\ y\cos x & \sin x \\ y & x \end{pmatrix}$$

$$= \begin{pmatrix} (6x\cos y - 5y\cos x + 2xy^2)e^{3x^2\cos y - 5y\sin x + x^2y^2} & (-3x^2\sin y - 5\sin x + 2x^2y)e^{3x^2\cos y - 5y\sin x + x^2y^2} \end{pmatrix}$$

Aufgabe 4.2

(i) $\rho^2 = x^2 + y^2 \in [4, 9] \Rightarrow \rho \in [2, 3]$

$|y| \leq -x \Rightarrow 0 \leq |\rho \sin\varphi| \leq -\rho\cos\varphi$

$\Rightarrow \cos\varphi \leq 0 \Rightarrow \varphi \in [\frac{\pi}{2}, \frac{3}{2}\pi]$ ① wegen $\varphi \in [0, 2\pi]$

$|\rho\sin\varphi| \leq -\rho\cos\varphi \Rightarrow |\tan\varphi| \leq 1 \Rightarrow \varphi \in [0, \frac{\pi}{4}] \cup [\frac{3}{4}\pi, \frac{5}{4}\pi] \cup [\frac{7}{4}\pi, 2\pi]$ ②

Aus ① ② $\Rightarrow \varphi \in [\frac{3}{4}\pi, \frac{5}{4}\pi]$

$A := \{(\rho\cos\varphi, \rho\sin\varphi) \in \mathbb{R}^2 : \rho \in [2,3], \varphi \in [\frac{3}{4}\pi, \frac{5}{4}\pi]$

(ii) $x^2 + y^2 + \frac{z^2}{16} = \rho^2 + \frac{z^2}{16} \leq 1 \Rightarrow 0 \leq z^2 \leq 16(1-\rho^2) \Rightarrow \rho^2 \leq 1$
$\Rightarrow \rho \in [0, 1]$

wegen $z \leq 0$, haben wir $z \in [-4\sqrt{1-\rho^2}, 0]$

$B := \{(\rho\cos\varphi, \rho\sin\varphi, z) \in \mathbb{R}^3 ; \rho \in [0,1], \varphi \in [0, 2\pi], z \in [-4\sqrt{1-\rho^2}, 0]\}$

(iii) $r \in [0,3] \Rightarrow x^2 + y^2 + z^2 = r^2 \in [0, 9]$

$\theta \in [0, \pi/4] \Rightarrow \tan\theta = \frac{\sqrt{x^2+y^2}}{z} \in [0,1] \Rightarrow 0 \leq x^2 + y^2 \leq z^2$

$z = r\cos\theta \geq 0$ (wegen $r \geq 0$, $\cos\theta \geq 0$)

$C := \{(x,y,z) \in \mathbb{R}^3 : 0 \leq x^2+y^2+z^2 \leq 9,\ 0 \leq x^2+y^2 \leq z^2,\ z \geq 0\}$

(ii) $z^2 \leq 16 \Rightarrow |z| \leq 4 \Rightarrow z \in [-4, 0]$

$\rho^2 = x^2 + y^2 \leq 1 - \frac{z^2}{16} \Rightarrow \rho \leq \sqrt{1 - \frac{z^2}{16}} = \frac{1}{4}\sqrt{16 - z^2}$

$B := \{(\rho\cos\varphi, \rho\sin\varphi, z) : z \in [-4, 0], \varphi \in [0, 2\pi], \rho \in [0, \frac{1}{4}\sqrt{16-z^2}]\}$