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Rechenübung Signale & Systeme (WiSe 2023/2024)

Lineare Systeme im Zeitbereich (5. Termin)

20.11 - 26.11.2023

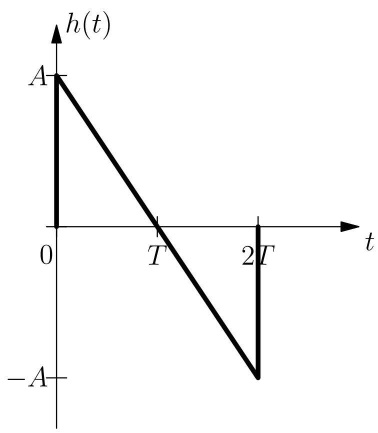
Hinweise

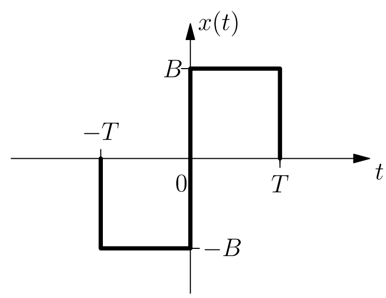
- Die Aufgabenblätter zur Rechenübung stehen jeweils vor dem jeweiligen Termin auf dem ISIS-Portal zum Download bereit.
- Aufgaben, die mit **[HA]** bzw. **[AK]** beginnen, sind Hausaufgaben bzw. alte Klausuraufgaben, die als Hausaufgabe bearbeitet werden sollen. Diese werden zusätzlich in den freiwilligen Tutorien vorgerechnet bzw. besprochen.

1 Impulsantwort und Faltung

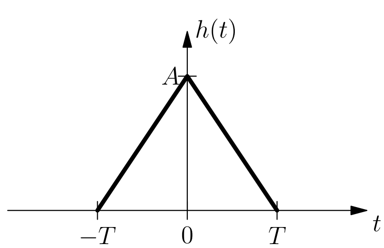
1.1 Bestimme das Ausgangssignal des Filters für die gegebenen Paare aus Impulsantwort $h(t)$ und Eingangssignal $x(t)$.

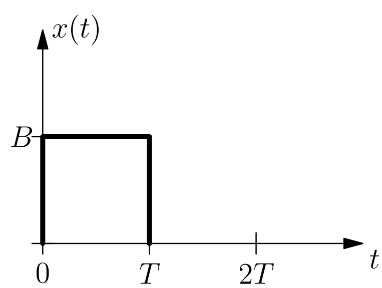
a)





b) **[AK]:**



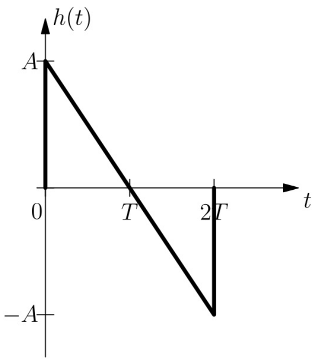


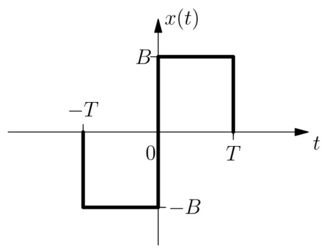
$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$ $\qquad y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)\,d\tau$

1 Impulsantwort und Faltung

1.1 Bestimme das Ausgangssignal des Filters für die gegebenen Paare aus Impulsantwort $h(t)$ und Eingangssignal $x(t)$.

a)



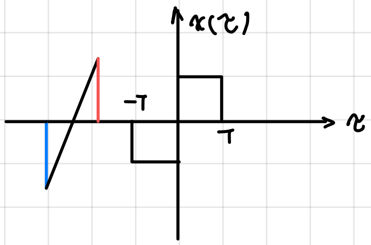


$h(t) = -\frac{A}{T}(t-T)\cdot \Pi_{2T}(t-T)$ $\qquad x(t) = -B\,\Pi_T(t+T/2) + B\,\Pi_T(t-T/2)$

Grenz: (I) $0 = t-\tau \iff \tau = t$ $\qquad$ (II) $2T = t-\tau \iff \tau = t-2T$

1. Fall $\quad t < -T$

$y(t) = 0$



2. Fall $\quad t<0 \wedge t \geq -T$

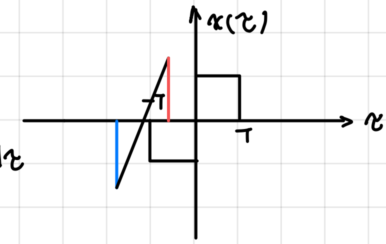
$\Rightarrow -T \leq t < 0$

$y(t) = \int_{-T}^{t} (-B)(-\frac{A}{T})(t-\tau-T)\,d\tau$

$= \frac{AB}{T}(t\tau - \frac{1}{2}\tau^2 - T\tau)\Big|_{-T}^{t}$

$= \frac{AB}{T}(t(\cancel{t}+\cancel{t}) - \frac{1}{2}(t^2 - T^2) - T(\cancel{t}+T))$

$= \frac{AB}{T}(\frac{1}{2}t^2 - \frac{1}{2}T^2)$



3. Fall

$t \geq 0 \wedge t < T$

$\Rightarrow 0 \leq t < T$

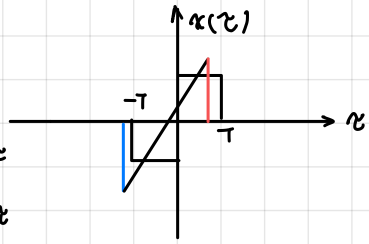
$y(t) = \int_{-T}^{0}(-B)(-\frac{A}{T}(t-\tau-T))d\tau$

$+ \int_{0}^{t} B(-\frac{A}{T}(t-\tau-T))d\tau$

$= \frac{AB}{T}(t\tau - \frac{1}{2}\tau^2 - T\tau)\Big|_{-T}^{0} - \frac{AB}{T}(t\tau - \frac{1}{2}\tau^2 - T\tau)\Big|_{0}^{t}$

$= \frac{AB}{T}\cdot(-(-tT - \frac{1}{2}T^2 + T^2) - (t^2 - \frac{1}{2}t^2 - Tt))$

$= \frac{AB}{T}(2tT - \frac{1}{2}T^2 - \frac{1}{2}t^2)$



4. Fall

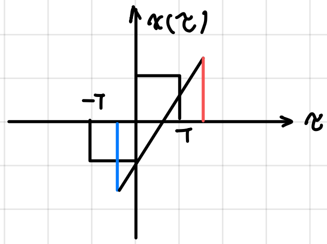
$t \geq T,\ 0 > t-2T$

$\Rightarrow T \leq t < 2T$

$y(t) = \int_{t-2T}^{0}(-B)(-\frac{A}{T}(t-\tau-T))d\tau$

$+ \int_{0}^{T} B(-\frac{A}{T}(t-\tau-T))d\tau$

$= \frac{AB}{T}(-\frac{t^2}{2} + \frac{3}{2}T^2)$



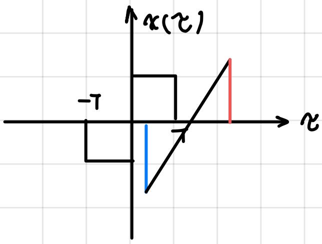
5. Fall

$t-2T \geq 0 \wedge t-2T < T$

$\Rightarrow 2T \leq t < 3T$

$y(t) = \int_{t-2T}^{T} B(-\frac{A}{T}(t-\tau-T))d\tau$

$= \frac{AB}{T}(\frac{1}{2}t^2 - 2tT + \frac{3}{2}T^2)$

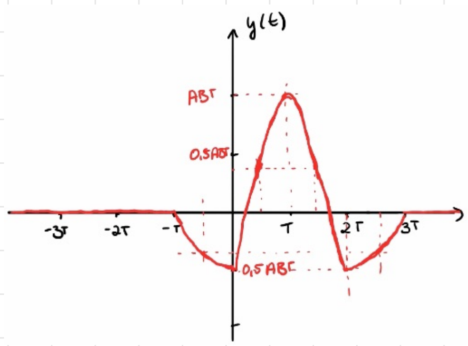


6. Fall $\quad t-2T \geq T$

$\Rightarrow t \geq 3T$

$y(t) = 0$

$$y(t) = \begin{cases} 0 & ; -\infty < t < \\ \frac{AB}{T}(\frac{1}{2}t^2 - \frac{1}{2}T^2) & ; -T \leq t < 0 \\ \frac{AB}{T}(-\frac{t^2}{2} + 2tT - \frac{T^2}{2}) & ; 0 \leq t < T \\ \frac{AB}{T}(-\frac{t^2}{2} + \frac{3}{2}T^2) & ; T \leq t < 2T \\ \frac{AB}{T}(\frac{1}{2}t^2 - 2tT + \frac{3}{2}T^2) & ; 2T \leq t < 3T \\ 0 & ; 3T \leq t < +\infty \end{cases}$$



$y(0) = -\frac{1}{2}ABT$

$y(\frac{T}{2}) = \frac{3}{8}ABT$

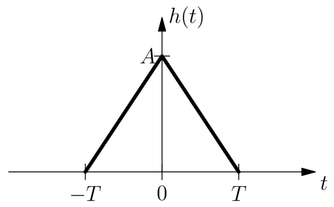
$y(T) = ABT$

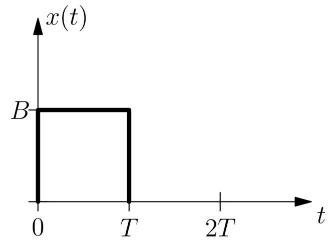
$y(\frac{3}{2}T) = \frac{3}{8}ABT$

$y(2T) = -0{,}5ABT$

$y(2{,}5T) = -\frac{3}{8}ABT$

b) **[AK]:**





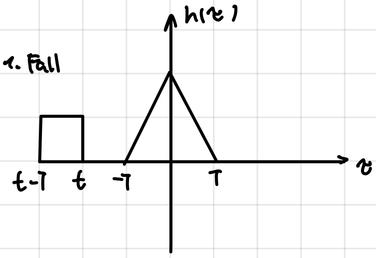
$h(t) = (\frac{A}{T} t + A)\, \Pi_T(t + \frac{1}{2}T) + (-\frac{A}{T} t + A)\, \Pi_T(t - \frac{1}{2}T)$

$x(t) = B\, \Pi_T(t - \frac{1}{2}T)$

$y = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$ mit $h(\tau) = (\frac{A}{T}\tau + A)\, \Pi_T(\tau + \frac{1}{2}T) + (-\frac{A}{T}\tau + A)\, \Pi_T(\tau - \frac{1}{2}T)$

Grenze: ① $t - \tau = 0 \Rightarrow \tau = t$ ② $t - \tau = T \Rightarrow \tau = t - T$

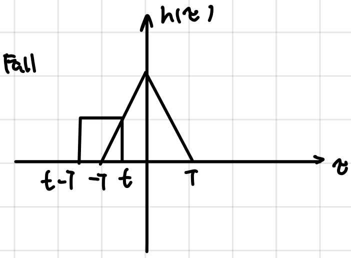
1. Fall



$t < -T$

$y(t) = 0$

2. Fall



$-T \leq t < 0$

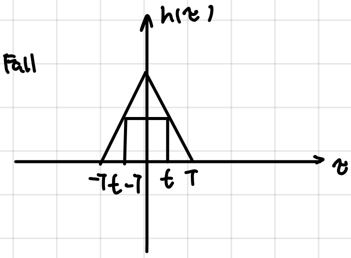
$y(t) = \int_{-T}^{t} (\frac{A}{T}\tau + A) \cdot B\, d\tau$

$= \frac{A}{2T}\tau^2 + A\tau \Big|_{-T}^{t}$

$= \frac{A}{2T}(t^2 - T^2) + At + AT$

$= \frac{A}{T}(\frac{t^2}{2} + \frac{T^2}{2} + tT)$

3. Fall



$0 \leq t < T$

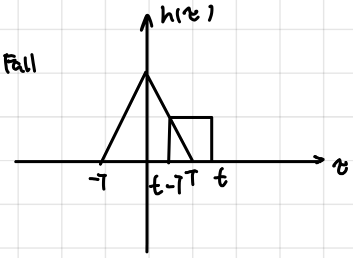
$y(t) = \int_{t-T}^{0} (\frac{A}{T}\tau + A) \cdot B\, d\tau + \int_{0}^{t} (-\frac{A}{T}\tau + A) \cdot B\, d\tau$

$= AB \cdot (\frac{1}{T} \cdot \frac{1}{2} \cdot \tau^2 + \tau) \Big|_{t-T}^{0} + AB\, (-\frac{1}{T} \cdot \frac{1}{2} \cdot \tau^2 + \tau) \Big|_{0}^{t}$

$= AB \cdot (\frac{1}{2T} \cdot (-t^2 + 2tT - T^2) + T - t) + AB\, (-\frac{t^2}{2T} + t)$

$= AB\, (\frac{t^2}{T} + t + \frac{1}{2}T)$

4. Fall



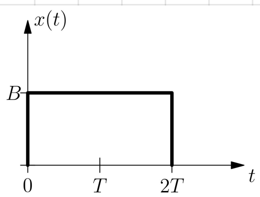
$t > T \wedge t - T < T$

$\Rightarrow T \leq t < 2T$

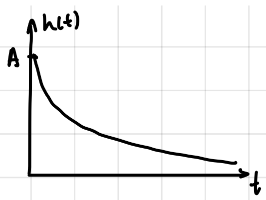
$y(t) = \int_{t-T}^{T} (-\frac{A}{T}\tau + A) B\, d\tau = \frac{AB}{T}(\frac{t^2}{2} - 2Tt + 2T^2)$

5. Fall $t \geq 2T$

$y(t) = 0$



c) **[AK]:** $h(t) = \begin{cases} A \cdot e^{-\frac{t}{2T}}, & t \geq 0 \\ 0, & t < 0 \end{cases}$



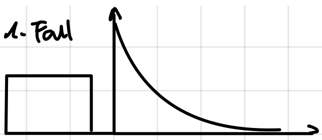
$x(t) = \Pi_{2T}(t - T)$

Grenze:

i) $t - \tau = 0 \Rightarrow \tau = t$

ii) $t - \tau = 2T \Rightarrow \tau = t - 2T$

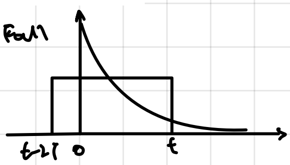
1. Fall



$t < 0$

$y(t) = 0$

2. Fall



$0 \leq t < 2T$

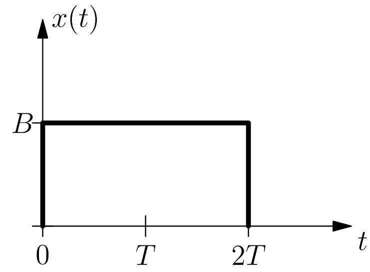
$y(t) = \int_{0}^{t} A e^{-\frac{\tau}{2T}} \cdot B\, d\tau$

$= AB\, e^{-\frac{\tau}{2T}} \cdot (-2T) \Big|_{0}^{t} = 2TAB\, (1 - e^{-\frac{t}{2T}})$

3. Fall

$t \geq 2T$

$y(t) = \int_{t-2T}^{t} A e^{-\frac{\tau}{2T}} B\, d\tau = 2TAB\, (e^{-\frac{t-2T}{2T}} - e^{-\frac{t}{2T}})$

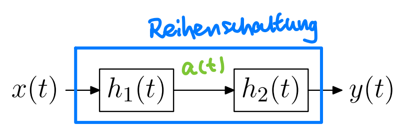


c) **[AK]:** $h(t) = \begin{cases} A \cdot e^{-\frac{t}{2T}}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

2 Verknüpfung von Filtern im Zeitbereich

2.1 Gib die Gesamtimpulsantwort $h_{ges}(t)$ für die unten skizzierten Netzwerke in Abhängigkeit von den Einzelimpulsantworten $h_i(t)$ an.

a)

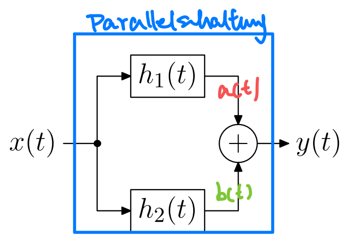


$a(t) = x(t) * h_1(t)$

$y(t) = a(t) * h_2(t)$

$y(t) = x(t) * h_1(t) * h_2(t)$

b)



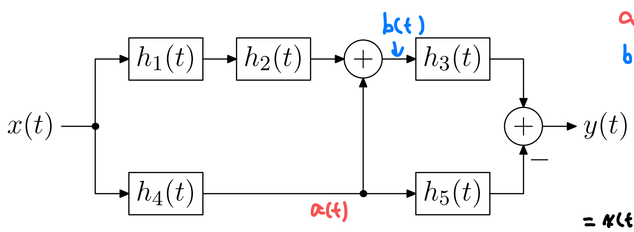
$a(t) = x(t) * h_1(t)$

$b(t) = x(t) * h_2(t)$

$y(t) = a(t) + b(t)$

$= x(t) * (h_1(t) + h_2(t))$

c)



$a(t) = x(t) * h_4(t)$

$b(t) = x(t) * h_1(t) * h_2(t) + a(t)$

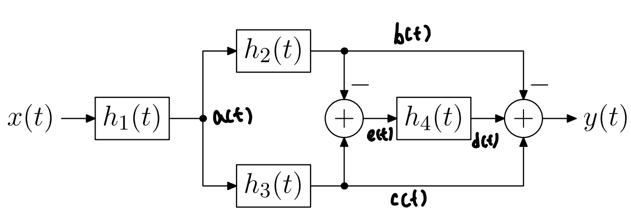
$y(t) = b(t) * h_3(t) - a(t) * h_5(t)$

$= (x(t) * h_1(t) * h_2(t) + x(t) * h_4(t)) * h_3(t)$

$- x(t) * h_4(t) * h_5(t)$

$= x(t) * \underbrace{[h_1(t) * h_2(t) * h_3(t) + h_4(t) * h_3(t) - h_4(t) * h_5(t)]}_{h_{gs}}$

d) **[AK]:**



e) **[HA]:**

$a(t) = x(t) * h_1(t)$

$b(t) = a(t) * h_2(t) = x(t) * h_1(t) * h_2(t)$

$c(t) = a(t) * h_3(t) = x(t) * h_1(t) * h_3(t)$

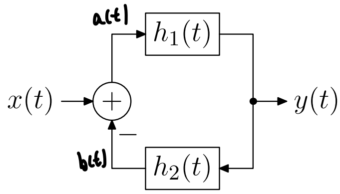
$e(t) = c(t) - b(t)$

$d(t) = e(t) * h_4(t) = (-b(t) + c(t)) * h_4(t)$

$y(t) = -b(t) + d(t) + c(t) = x(t) * h_1(t) * (h_3(t) - h_2(t)) * h_4(t)$

$+ x(t) * h_1(t) * (h_3(t) - h_2(t))$

$= x(t) * h_1(t) * (h_3(t) - h_2(t) + h_3(t) * h_4(t) - h_2(t) * h_4(t))$



$y(t) = x(t) * h_1(t)$

$a(t) = x(t) - b(t) = x(t) - y(t) * h_2(t)$

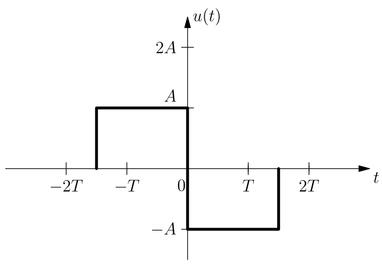
$b(t) = y(t) * h_2(t)$

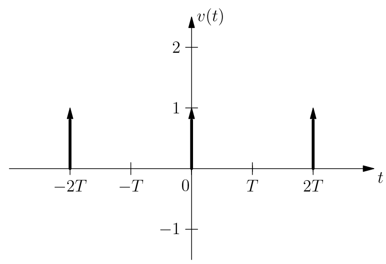
$\Rightarrow y(t) = (x(t) - y(t) * h_2(t)) * h_1(t)$

$= x(t) * h_1(t) - y(t) * h_2(t) * h_1(t)$

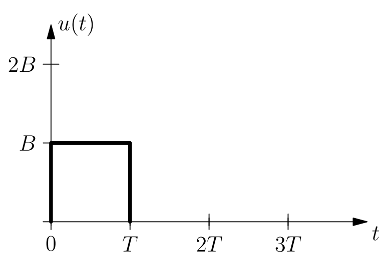
3 [AK]: Bestimme jeweils die Faltung der folgenden Signalpaare.

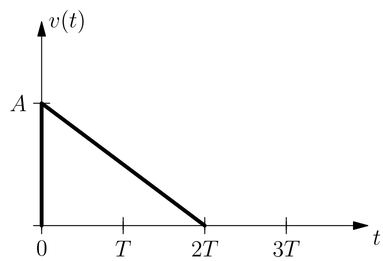
a)



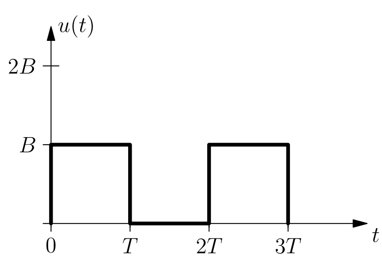


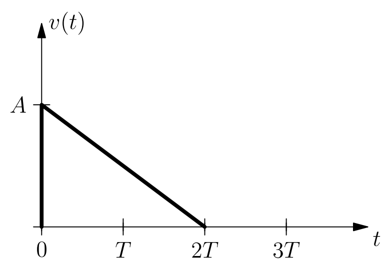
b)



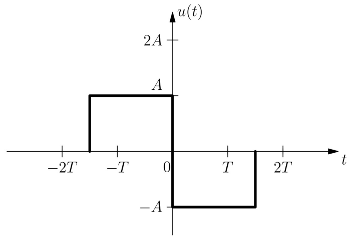


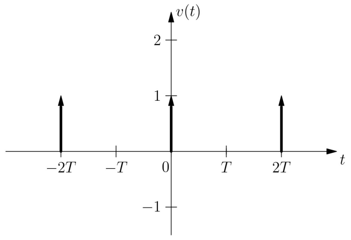
c)





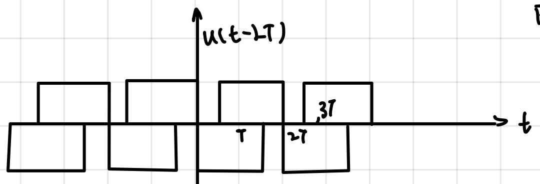
a)

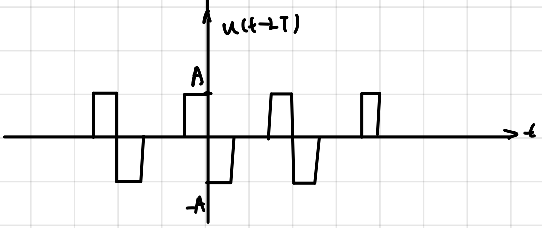




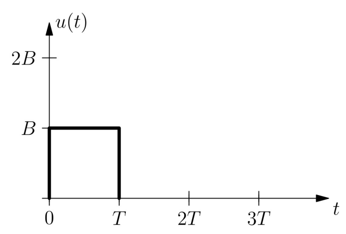
$v(t) * u(t) = \delta(t-2T) * u(t) = \underline{u(t-2T)}$

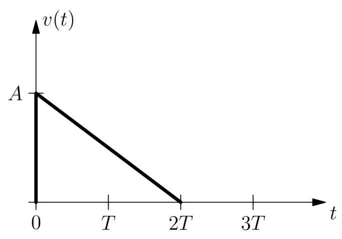
periodische Signal mit $T_p = 2T$





b)



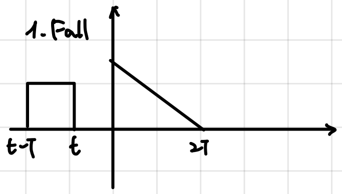


$u(t) = B\,\Pi(t - \frac{1}{2}T)$ $\quad$ $v(t) = (-\frac{A}{2T}t + A)\cdot \Pi_{2T}(t-T)$

Grenz: $t-\tau = 0 \Rightarrow \tau = t$

$t - \tau = T \Rightarrow \tau = t - T$

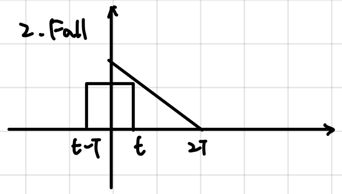
1. Fall



$t < 0$

$y(t) = 0$

2. Fall



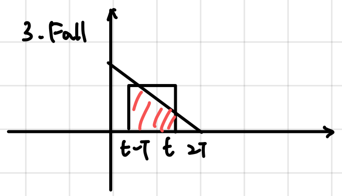
$0 \le y < T$

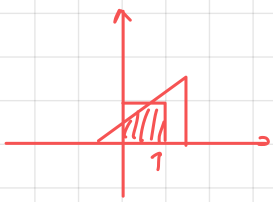
$$y(t) = \int_0^t \left(-\frac{A}{2T}\tau + A\right) B\, d\tau$$

$$= \left(-\frac{A}{2T}\cdot\frac{1}{2}\tau^2 + A\tau\right)B\Big|_0^t$$

$$= -\frac{At^2B}{4T} + AtB$$

3. Fall





$\int_0^T$

$T \le t < 2T$

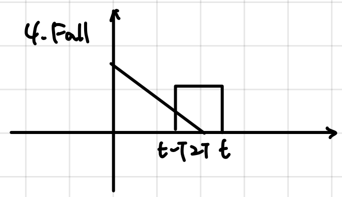
$$y(t) = \int_{t-T}^{t} \left(-\frac{A}{2T}\tau + A\right) B\, d\tau$$

$$= -AB\left(\frac{\tau^2}{4T} - \tau\right)\Big|_{t-T}^{t}$$

$$= -AB\left(\frac{1}{4T}\cdot(\cancel{t^2} - \cancel{t^2} + 2tT - T^2) - T \quad \right)$$

$$= -AB\left(\frac{1}{2}t + \frac{5}{4}T\right)$$

4. Fall



$2T \le t < 3t$

$$y(t) = \int_{t-T}^{2T} \left(-\frac{A}{2T}\tau + A\right) B\, d\tau$$

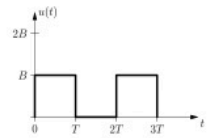
$$= -AB\left(\frac{\tau^2}{4T} - \tau\right)\Big|_{t-T}^{2T} = -AB\left(\frac{1}{4T}\cdot(4T^2 - t^2 + 2tT - T^2) - 3T + t\right)$$

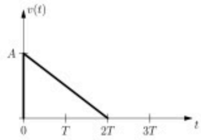
$$= -AB\cdot\left(-\frac{9}{4}T - \frac{t^2}{4T} + \frac{3}{2}t\right)$$

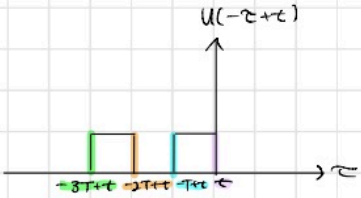
5. $t \geq 3T$

$y(t) = 0$

3. c)

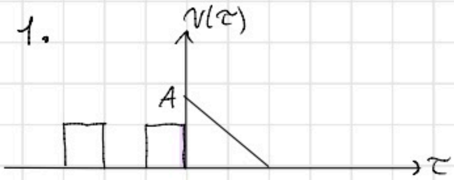






$u(t) = B\{\Pi_T(t-\frac{T}{2}) + \Pi_T(t-\frac{5T}{2})\}$ $\quad$ $v(t) = (-\frac{A}{2T}t + A)\cdot \Pi_{2T}(t-T)$

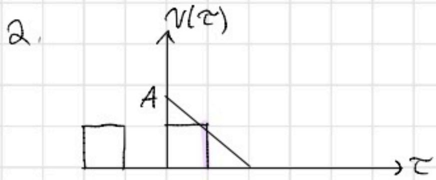
1.



$t < 0$

$y = 0$

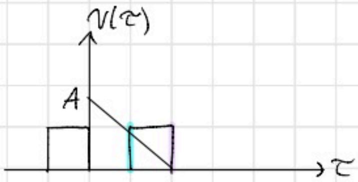
2.



$0 \le t < T$

$\int_0^t B\cdot(-\frac{A}{2T}\tau + A)\, d\tau$

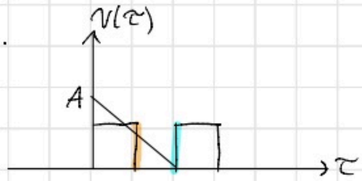
3.



$T \le t < 2T$

$\int_{t-T}^{t} B\cdot(-\frac{A}{2T}\tau + A)\, d\tau$

4.

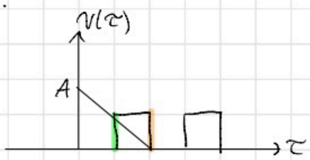


$T \le t-T < 2T$

$\Leftrightarrow 2T \le t < 3T$

$\int_0^{t-2T} B\cdot(-\frac{A}{2T}\tau + A)\, d\tau + \int_{t-T}^{2T} B\cdot(-\frac{A}{2T}\tau + A)\, d\tau$

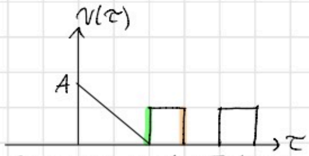
5.



$0 \le t-3T < T \Leftrightarrow 3T \le t < 4T$

$\int_{t-3T}^{t-2T} B\cdot(-\frac{A}{2T}\tau + A)\, d\tau$

6.



$2T \le t-2T \wedge t-3T < 2T$

$\Leftrightarrow 4T \le t < 5T$

$\int_{t-3T}^{2T} B\cdot(-\frac{A}{2T}\tau + A)\, d\tau$

7.

$t-3T \ge 2T$

$\Leftrightarrow t \ge 5T$

$y(t) = 0$

1. Für die Faltungsoperation gilt das ..

- ○ Assoziativgesetz.
- ○ Kommutativgesetz.
- ○ Distributivgesetz.

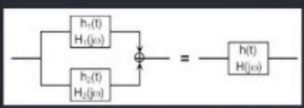
2. Gegeben sei ein lineares System mit der Impulsantwort h(t) . Welche Systemantwort ergibt sich für das Eingangssignal u(t) = δ(t) ?

- ○ $y(t)=\delta(t)$
- ○ $y(t)=\delta(t)\cdot h(t)$
- ○ $y(t)=h(t)$

3. Die Impulsantwort ist die Antwort des Systems auf einen ...

- ☐ Deltaimpuls.
- ☐ Rechteckimpuls.
- ☐ Dreiecksimpuls.

4. Für die Parallelschaltung zweier LTI-Systeme gilt:



- ☐ Für das Ausgangssignal des Gesamtsystems gilt: y(t) = u (t) * (h_1(t) + h_2(t))
- ☐ Die Gesamtimpulsantwort ist das Produkt der Einzelimpulsantworten.
- ☐ Die Einzelimpulsantworten addieren sich.

Lösung : 1 – Kommutativgesetz
2 – $y(t) = h(t)$
3 – Deltaimpuls
4 – $y(t) = u(t) * (h_1 + h_2)$,
Einzelimpulsantworten addieren sich