Dynamic Games

Heavily based on slides by Ulrich Doraszelski

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What are Dynamic Games?

- A tool for analyzing dynamic strategic interactions.
  - dynamic: forward-looking players optimize over time;
  - strategic: each player recognizes that its actions impact other players.
- We will use it to track evolution of oligopolistic industries.
  - Applied (numerical) theory
  - Estimation

Why use Dynamic Games?

- Key findings of empirical literature on industry evolution (Mueller1986, Dunne, Roberts, and Samuelson 1988, Davis and Halti- wanger 1992):
  - Entry and exit occur simultaneously.
  - Heterogeneity among firms evolves endogenously in re-sponse to random occurrences.
  - Heterogeneity among firms persists over long stretches of time.

Why use Dynamic Games?

- Game theory revolution in economics: emphasis on analytically tractable models
  - End effects
  - Transitional dynamics
  - Inherently dynamic phenomena
- :imit to where we can go with analytical models

Overview

We'll basically extend the single-agent framework, but some new issues arise:

(1) State space and setup:

- Observables: dimension can easily explode (since we, in principle, have to keep track of long history)
- Unobservables: need to decide what players know and when

Solutions:

- Markov Perfect Equilibrium.
- Independent Private Values.

(2) Estimation:
  - Nested Fixed Point (Rust): not realistic even for small problems.
  - Convert to static single agent problem using CCP.
    - Treat others + future self as given (i.e., from data) and compute best response
  - Recursive: extend Aguirregabiria-Mira.
    - Nice results from the single agent case don't extend to games.

(3) Some of the results we relied on in the single agent case do not hold.
  - No contraction mapping.
  - Recursive algorithm is not guaranteed to work.

Key Papers for "Empirical" Side

Setup: Ericson-Pakes, Pakes-McGuire.

Early work: Gowrisankaran, Benkard.

Estimation: Berry, Ostrovsky and Pakes; Bajari-Benkard-Levin (**BBL**); Pesendorfer and Schmidt-Dengler, Aguirregabiria-Mira.

Applications: Ryan, Collard-Wexler.

Agenda

- From dynamic programming to dynamic games.
- Application: Quality ladder model without entry/exit.
- Application: Capacity accumulation.
- Markov-perfect industry dynamics.
- Multiple equilibria.
- Computational burden.
- Open questions.

From Dynamic Programming...

- Time is discrete. The horizon is infinite.
- The state space $\Omega = \{1, 2, \ldots, L\}$ is finite.
- The state in period t is $\omega_t \in \Omega$. The law of motion is a controlled discrete-time, finite-state, first-order Markov process, where
$$\Pr(\omega_{t+1}|\omega_t, x_t)$$
is the probability that the state transits from ω_t to ω_{t+1} if the control is $x_t \in D(\omega_t)$ and $D(\omega_t)$ is the nonempty set of feasible controls in state ω_t.
- The objective is to maximize the expected NPV of payoffs
$$E\left\{\sum_{t=0}^{\infty} \beta^t \pi(\omega_t, x_t)\right\},$$
where $\beta \in [0,1)$ is the discount factor and $\pi(\omega_t, x_t)$ is the per-period payoff in state ω_t if the control is x_t.
- The value function $V(\omega)$ is the maximum expected NPV of present and future payoffs if the current state is ω. It satisfies the Bellman equation
$$V(\omega) = \max_{x \in D(\omega)} \pi(\omega, x) + \beta \sum_{\omega'=1}^{L} V(\omega')\Pr(\omega'|\omega, x) \tag{1}$$
and the optimal policy function $X(\omega)$ satisfies
$$X(\omega) \in \arg\max_{x \in D(\omega)} \pi(\omega, x) + \beta \sum_{\omega'=1}^{L} V(\omega')\Pr(\omega'|\omega, x).$$
- The collection of equation (1) for all states $\omega \in \Omega$ defines a system of nonlinear equations. The contraction mapping theorem ensures existence and uniqueness of a solution.

. . . to Dynamic Games

- N players.

- The law of motion is a controlled discrete-time, finite-state, first-order Markov process, where
$$\Pr(\omega_{t+1}|\omega_t, x_t)$$
is the probability that the state transits from ω_t to ω_{t+1} if the controls are $x_t = (x_{1t}, \ldots, x_{Nt}) \in \times_{n=1}^{N} D_n(\omega_t)$ and $D_n(\omega_t)$ is the nonempty set of feasible controls of player n in state ω_t.

- $\pi_n(\omega_t, x_t)$ is the per-period payoff of player n in state ω_t if the controls are x_t.

- The value function $V_n(\omega)$ of player n satisfies the Bellman equation
$$V_n(\omega) = \max_{x_n \in D_n(\omega)} \pi_n(\omega, x_n, X_{-n}(\omega)) + \beta \sum_{\omega'=1}^{L} V_n(\omega')\Pr(\omega'|\omega, x_n, X_{-n}(\omega)) \tag{2}$$
and his optimal policy function $X_n(\omega)$ satisfies
$$X_n(\omega) \in \arg\max_{x_n \in D_n(\omega)} \pi_n(\omega, x_n, X_{-n}(\omega)) + \beta \sum_{\omega'=1}^{L} V_n(\omega')\Pr(\omega'|\omega, x_n, X_{-n}(\omega)). \tag{3}$$

- The collection of equations (2) and (3) for all states $\omega \in \Omega$ and all players $n = 1, \ldots, N$ defines a Markov-perfect equilibrium. The contraction mapping theorem does not apply and neither existence nor uniqueness of a MPE is guaranteed.

. . . to Dynamic Games

- Special case: ω is a vector partitioned into
$$(\omega_1, \ldots, \omega_N),$$
where ω_n denotes the (one or more) coordinates of the state that describe player n.
Examples: Production capacity, marginal cost, product quality.
Nomenclature:
  - $\omega_n \in \Omega_n = \{1, 2, \ldots, L_n\}$ is the state of player n;
  - $\omega \in \times_{n=1}^{N} \Omega_n$ is the state of the game.

Equations (2) and (3) can be written as
$$V_n(\omega) = \max_{x_n \in D_n(\omega)} \pi_n(\omega, x_n, X_{-n}(\omega)) + \beta \sum_{\omega'_1=1}^{L_1} \ldots \sum_{\omega'_N=1}^{L_N} V_n(\omega') \text{Pr}(\omega'|\omega, x_n, X_{-n}(\omega)),$$
$$X_n(\omega) \in \arg \max_{x_n \in D_n(\omega)} \pi_n(\omega, x_n, X_{-n}(\omega)) + \beta \sum_{\omega'_1=1}^{L_1} \ldots \sum_{\omega'_N=1}^{L_N} V_n(\omega') \text{Pr}(\omega'|\omega, x_n, X_{-n}(\omega)).$$

- Even more special case: Transitions in player n's state are controlled by player n's actions and are independent of the actions of other players and transitions in their states, i.e.,
$$\text{Pr}\left(\omega'|\omega, x\right) = \prod_{n=1}^{N} \text{Pr}_n\left(\omega'_n|\omega_n, x_n\right).$$

Quality Ladder Model without Entry/Exit

- Pakes, A. & McGuire, P. (1994) "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model."

- Borkovsky, R., Doraszelski, U. & Kryukov, Y. (2010) "A User's Guide to Solving Dynamic Stochastic Games Using the Homotopy Continuation Method."

- Discrete time, infinite horizon.

- Two firms with potentially different product qualities

$$\omega = (\omega_1, \omega_2) \in \{1, \ldots, L\}^2 = \Omega.$$

- In each period, the timing is as follows:
  - Firms choose investments in quality improvements.
  - Product market competition takes place.
  - Investment outcomes and depreciation shocks are realized.

Demand

Setup for differentiated products:

$$u_{ij} = \nu_j - p_j^* + \varepsilon_{ij}, \quad i = 1, \ldots, I; \quad j = 1, \ldots, J$$

The consumer chooses option j only if

$$\begin{aligned} \epsilon_{ij} - \epsilon_{ik} &\geq (\nu_k - \nu_j) + (p_j^* - p_k^*) \\ &= (\nu_k - \nu_0) - (\nu_j - \nu_0) + (p_j^* - p_0) - (p_k^* - p_0) \\ &= g(\omega_k) - g(\omega_j) + p_j - p_k, \end{aligned}$$

ω: efficiency relative to outside option.
Assuming that ε_{ij} is iid extreme value $\Rightarrow$

$$\sigma(\omega_j; p, s) = \frac{exp(g(\omega_j) - p_j)}{1 + \sum_k exp(g(\omega_k) - p_k)}$$

Note this is the logit model; we can generalize this a bit (but not much) and still deal with what follows

Price Competition

Fixed marginal costs, no fixed costs, single-product firms, Bertrand competition.

$$p_j = mc_j + \frac{\sigma_j}{\frac{\partial \sigma_j}{\partial p_j}} = mc_j + \frac{1}{1 - \sigma_j(p)}$$

The equation above defines $p_j(\omega, s)$, which then gives us

$$\pi(\omega_j, s) = [p_j(\omega, s) - mc_j] M \sigma(\omega, s)$$

This could be somewhat generalized, but it is the first thing we need.

Product Market Competition

- Firm n's demand is
$$D_n(p_1,p_2;\omega) = M\frac{\exp\left(g(\omega_n)-p_n\right)}{1+\sum_{k=1}^{2}\exp\left(g(\omega_k)-p_k\right)},$$
where $M>0$ is market size and
$$g(\omega_n) = \begin{cases} 3\omega_n - 4 & \text{if} \quad \omega_n \leq 5, \\ 12+\ln\left(2-\exp\left(16-3\omega_n\right)\right) & \text{if} \quad \omega_n > 5 \end{cases}$$
maps product quality into consumers' valuations.

- Firm n solves
$$\max_{p_n \geq 0} D_n(p_1,p_2;\omega)(p_n - c),$$
where c is marginal cost of production.

- FOC:
$$0 = 1 - \frac{1+\exp\left(g(\omega_{-n})-p_{-n}\right)}{1+\exp\left(g(\omega_n)-p_n\right)+\exp\left(g(\omega_{-n})-p_{-n}\right)}(p_n - c), \quad n \neq -n.$$

- Compute Nash equilibrium $(p_1(\omega), p_2(\omega))$ by solving system of FOCs.

- Firm n's profit is
$$\pi_n(\omega) = D_n(p_1(\omega),p_2(\omega);\omega)(p_n(\omega)-c).$$

Dynamics

Define $\tau_{t+1} := \omega_{t+1} - \omega_t$.

ω_{t+1} can

- Go down, because of improvements in outside good, depreciation, ...
- Stay the same.
- Go up, possibly as a result of investment.

Dynamics

(1) Let $\tau = \nu_1 - \nu$ (the increment depends on advances in the firms efficiency relative to the outside good)
(2) Let $x \geq 0$ denote investment in quality improvement

$$v_1 = \begin{cases} 1 & \text{with prob. } \frac{ax}{1+ax} \\ 0 & \text{otherwise} \end{cases}$$

$$v_0 = \begin{cases} 1 & \text{with prob. } \delta \\ 0 & \text{otherwise} \end{cases}$$

Thus,

$$\tau_{t+1} = \begin{cases} 1 & \text{with prob. } \frac{ax}{1+ax}(1-\delta) \\ 0 & \text{with prob. } \frac{1}{1+ax}(1-\delta) + \frac{ax}{1+ax}\delta \\ -1 & \text{with prob. } \frac{1}{1+ax}\delta \end{cases}$$

Bellman Equation

- Let $V_n(\omega)$ denote the expected NPV to firm n if the current state is ω.

- Firm n's Bellman equation is

$$V_n(\omega) = \max_{x_n \geq 0} \pi_n(\omega) - x_n + \beta \sum_{\omega_n'=1}^{L} W_n(\omega_n'; \omega_{-n}, x_{-n}(\omega)) \mathrm{Pr}(\omega_n' | \omega_n, x_n),$$

  where

  - the expectation (with respect to its rival's successor state) of firm n's continuation value in state ω_n' is

  $$W_n(\omega_n'; \omega_{-n}, x_{-n}(\omega)) = \sum_{\omega_{-n}'=1}^{L} V_n(\omega') \mathrm{Pr}(\omega_{-n}' | \omega_{-n}, x_{-n}(\omega));$$

  - $x_{-n}(\omega)$ is the rival's investment strategy;

  - $\beta \in [0, 1)$ is the discount factor.

Investment Strategy

- Firm n's investment strategy is

$$x_n(\omega) = \arg\max_{x_n \geq 0} \pi_n(\omega) - x_n + \beta \sum_{\omega_n'=1}^{L} W_n(\omega_n') \Pr(\omega_n' | \omega_n, x_n),$$

  where $W_n(\omega_n')$ is shorthand for $W_n(\omega_n'; \omega_{-n}, x_{-n}(\omega))$.

- If $\omega_n \in \{2, \ldots, L-1\}$, then

$$x_n(\omega) = \frac{-1 + \sqrt{\max\{1, \beta\alpha\left((1-\delta)(W_n(\omega_n+1) - W_n(\omega_n)) + \delta(W_n(\omega_n) - W_n(\omega_n - 1))\right)\}}}{\alpha}.$$

  If $\omega_n \in \{1, L\}$, then

$$x_n(\omega) = \frac{-1 + \sqrt{\max\{1, \beta\alpha(1-\delta)\left(W_n(2) - W_n(1)\right)\}}}{\alpha},$$

$$x_n(\omega) = \frac{-1 + \sqrt{\max\{1, \beta\alpha\delta\left(W_n(L) - W_n(L-1)\right)\}}}{\alpha}.$$

Equilibrium

- Profits from product market competition are symmetric:

  $$\pi_1(\omega_1, \omega_2) = \pi_2(\omega_2, \omega_1).$$

  The remaining primitives are also symmetric.

- Symmetric Markov perfect equilibrium (MPE):
  - Value function $V_1(\omega_1, \omega_2) = V(\omega_1, \omega_2)$ and $V_2(\omega_1, \omega_2) = V(\omega_2, \omega_1)$.
  - Policy function $x_1(\omega_1, \omega_2) = x(\omega_1, \omega_2)$ and $x_2(\omega_1, \omega_2) = x(\omega_2, \omega_1)$.

- Existence in pure strategies is guaranteed (Doraszelski & Satterthwaite 2010), uniqueness is not.

- The goal is to compute the value and policy functions (or, more precisely, $L \times L$ matrices) $\mathbf{V}$ and $\mathbf{x}$.

Computation: Pakes & McGuire (1994) Algorithm

1. Make initial guesses $\mathbf{V}^0$ and $\mathbf{x}^0$, choose a stopping criterion $\epsilon > 0$, and initialize the iteration counter to $k = 1$.

2. For all states $\omega \in \Omega$ compute

$$x^{k+1}(\omega) = \arg\max_{x_1 \geq 0} \pi_1(\omega) - x_1 + \beta \sum_{\omega_1'=1}^{L} W^k(\omega_1')\mathrm{Pr}(\omega_1'|\omega_1, x_1)$$

   and

$$V^{k+1}(\omega) = \pi_1(\omega) - x^{k+1}(\omega) + \beta \sum_{\omega_1'=1}^{L} W^k(\omega_1')\mathrm{Pr}(\omega_1'|\omega_1, x^{k+1}(\omega)),$$

   where

$$W^k(\omega_1') = \sum_{\omega_2'=1}^{L} V^k(\omega')\mathrm{Pr}(\omega_2'|\omega_2, x^k(\omega_2, \omega_1)).$$

3. If

$$\max_{\omega \in \Omega} \left| \frac{V^{k+1}(\omega) - V^k(\omega)}{1 + |V^{k+1}(\omega)|} \right| < \epsilon \quad \wedge \quad \max_{\omega \in \Omega} \left| \frac{x^{k+1}(\omega) - x^k(\omega)}{1 + |x^{k+1}(\omega)|} \right| < \epsilon$$

   then stop; else increment the iteration counter k by one and go to step 2.

Application to Capacity Accumulation

- Besanko, D. & Doraszelski, U. (2004) "Capacity Dynamics and Endogenous Asymmetries in Firm Size."
- Substantial and persistent differences in firm sizes despite idiosyncratic shocks (Gort 1963, Mueller 1986, McGahan & Porter 1997).
- Size differences can arise endogenously in asymmetric equilibria of two- or three-stage models of capacity choice (Saloner 1987, Maggi 1996, Reynolds & Wilson 2000).
- But: What happens if firms are subject to idiosyncratic shocks? What about feedback effects?
- Dynamic models of capacity accumulation:
  - Steady-state analysis (Spence 1979, Fudenberg & Tirole 1983).
  - Linear-quadratic games (Hanig 1985, Reynolds 1987, 1991, Dockner 1992).

Relationship to Quality Ladder Model

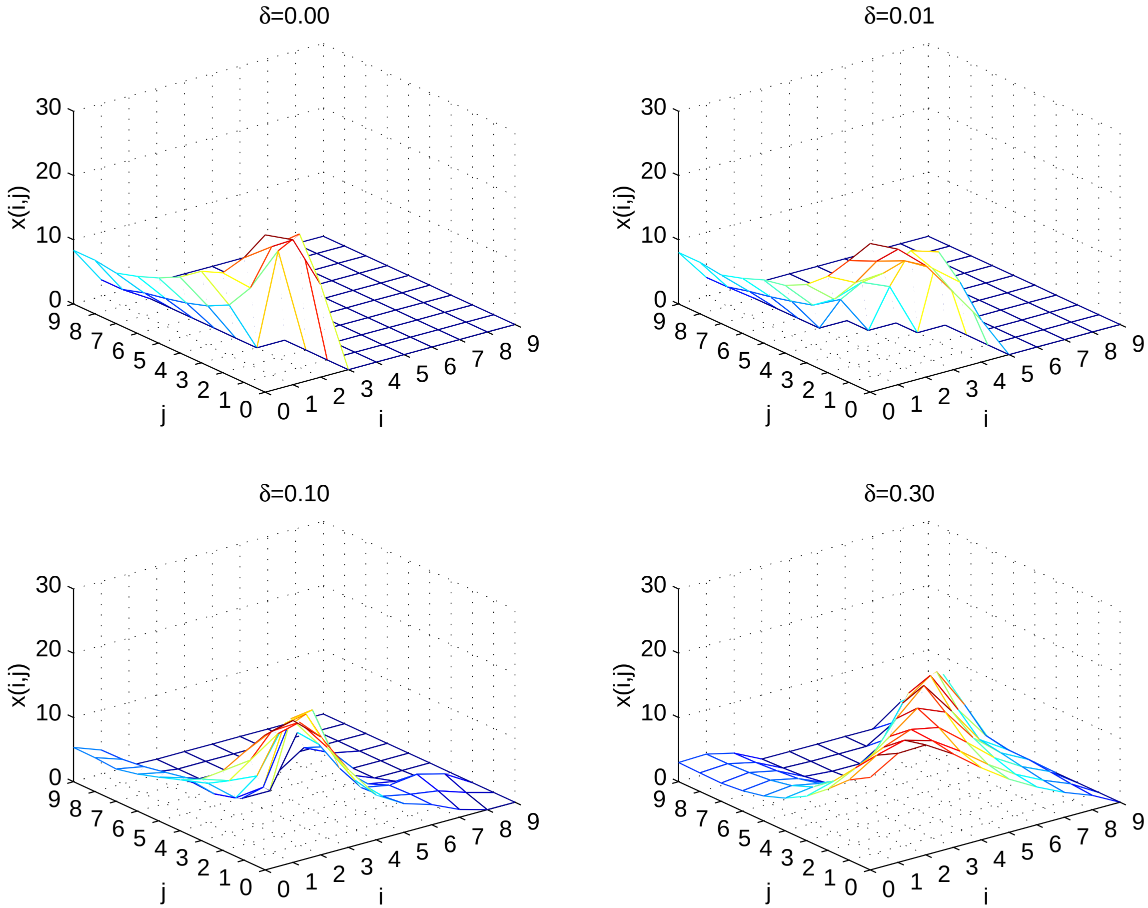
- State variables $\omega = (\omega_1, \omega_2)$ are capacities of firms 1 and 2.
- Firms invest in capacity. Capacity may depreciate.
- Product market competition:
  - Quantity competition subject to capacity constraints.
  - Price competition subject to capacity constraints.

Substantial and Persistent Differences in Firm Sizes

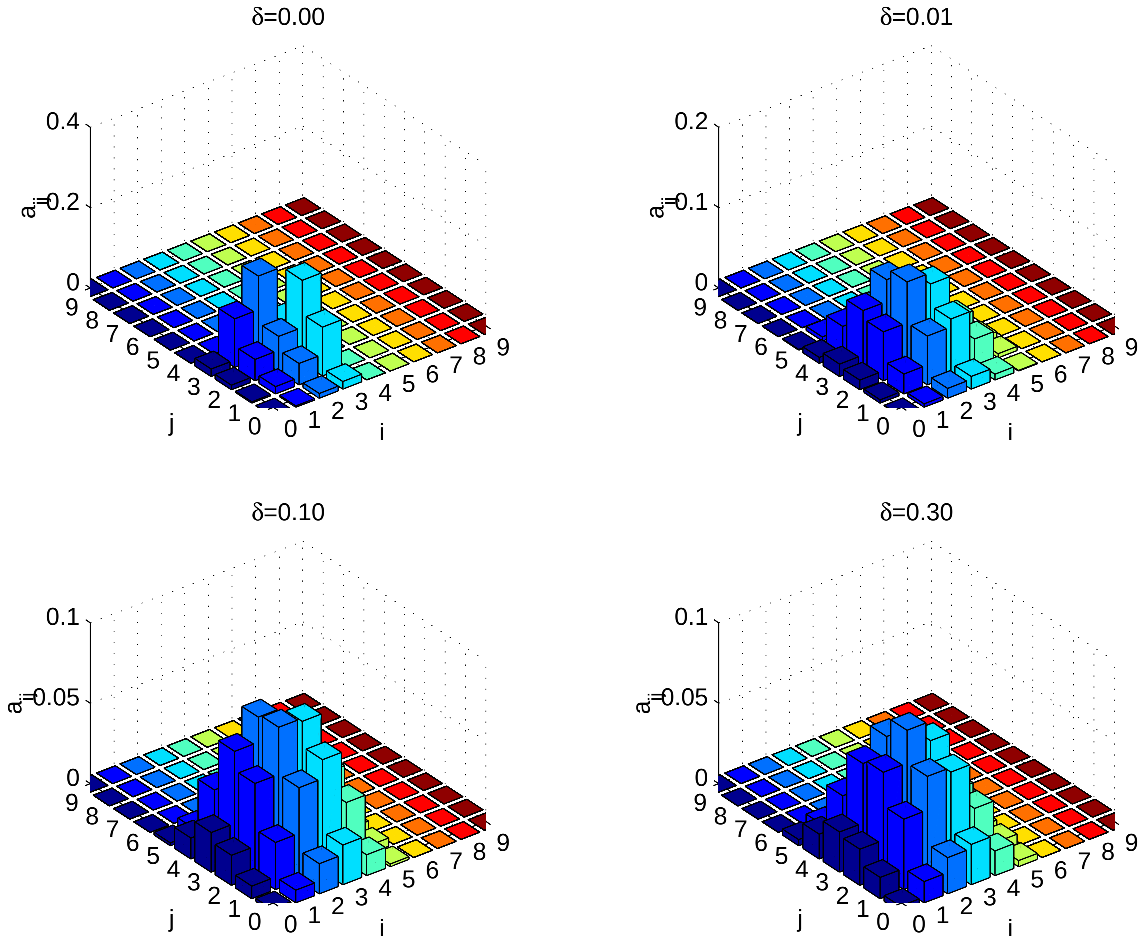
	quantity competition	**price competition**
irreversible investment ($\delta = 0$)	symmetric firms	slightly asymmetric firms
reversible investment ($\delta > 0$)	symmetric firms	hugely asymmetric firms

Investment Reversibility and Preemption Races

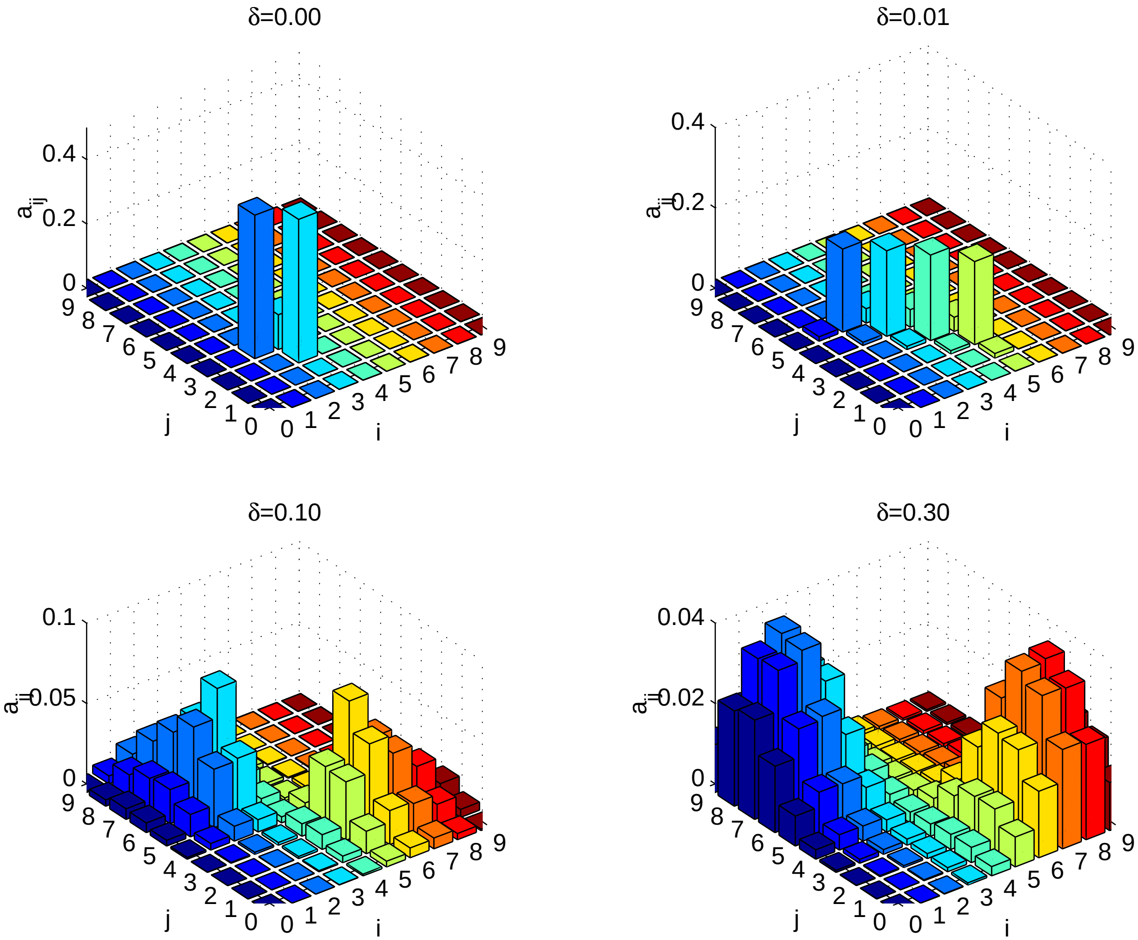
- Policy function $x(i,j)$. Price competition with $\delta \in \{0, 0.01, 0.1, 0.3\}$.



- Transient distribution after $T = 5$ with $i_0 = j_0 = 1$. Price competition with $\delta \in \{0, 0.01, 0.1, 0.3\}$.

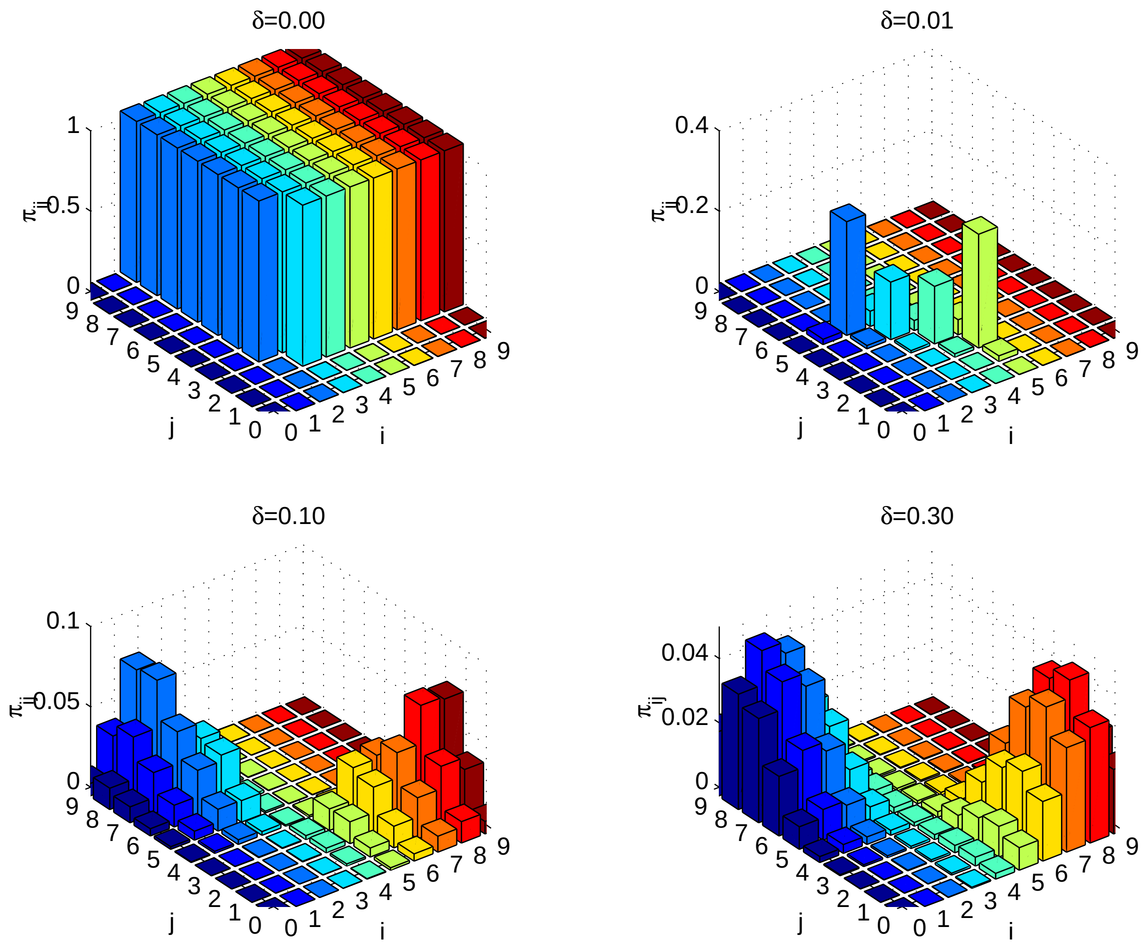


- Transient distribution after $T = 25$ with $i_0 = j_0 = 1$. Price competition with $\delta \in \{0, 0.01, 0.1, 0.3\}$.



Investment Reversibility and Preemption Races

- Limiting distribution. Price competition with $\delta \in \{0, 0.01, 0.1, 0.3\}$.



Investment Reversibility and Preemption Races

- "An open issue (...) is the behavior of investment in the industry when capital depreciates. Intuition suggests that capital ought to lose some of its commitment value and that the steady-state levels of capital should be less sensitive to the initial head start of one of the firms." (Tirole 1988, p. 345)

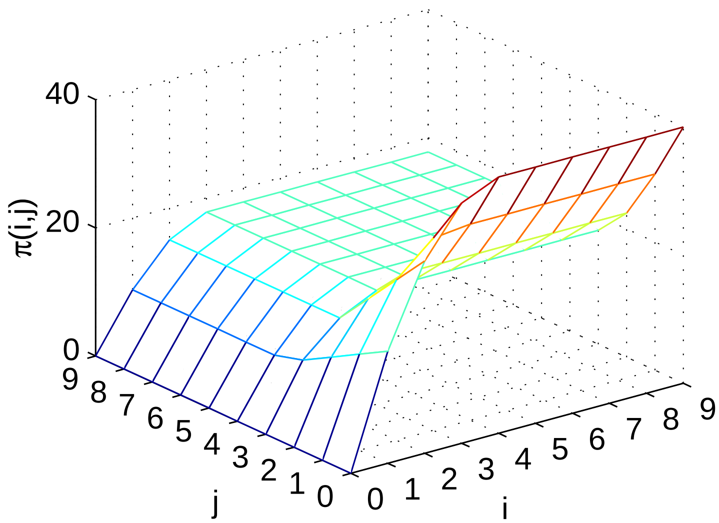
- This paper: Investment reversibility may make preemption races *more* attractive.

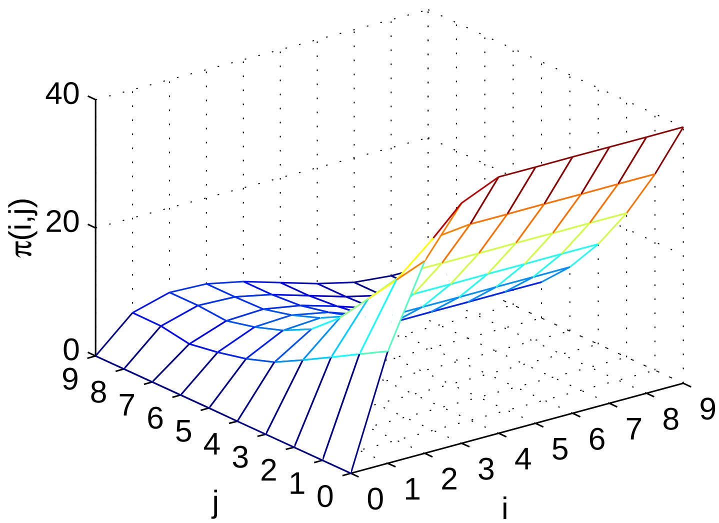
Investment Reversibility and Preemption Races

- What is the main difference between the two modes of product market competition?

  - Capacity-constrained quantity competition: a firm's profit *plateaus* in own capacity.

  - Capacity-constrained price competition: a firm's profit *peaks* in own capacity (provided rival has sufficient capacity).





Investment Reversibility and Preemption Races

- Under price competition, it is in the self-interest of a not-too-small firm to withdraw from the race once its rival has gained a size advantage over it.

- By building up its capacity, a firm hopes to gain an initial edge over its rival and to decide the race in its favor.

- A firm anticipates that once it gains an edge over its rival, its rival will withdraw capacity.

- It is easier to withdraw capacity if the rate of depreciation is high. Conversely, it is impossible to withdraw capacity if the rate of depreciation is zero.

Markov-Perfect Industry Dynamics

- Ericson, R. & Pakes, A. (1995) "Markov-Perfect Industry Dynamics: A Framework for Empirical Work."
- EP model tracks evolution of oligopolistic industries.
- Special case of dynamic game:
  - Entry, exit, and investment decisions.
  - Product market competition.
- Captures key findings of empirical literature on industry evolution:
  - Entry and exit occur simultaneously.
  - Heterogeneity among firms evolves endogenously and persists.

Applications in IO and Other Fields

- Advertising (Doraszelski & Markovich 2007).
- Capacity accumulation (Besanko & Doraszelski 2004, Chen 2009, Ryan 2012, Besanko, Doraszelski, Lu & Satterthwaite 2010a, 2010b, Wilson 2012).
- Collusion (Fershtman & Pakes 2000, 2005, de Roos 2004).
- Competitive convergence (Langohr 2003).
- Consumer learning (Ching 2010).
- Corporate reputation (Abito, Besanko & Diermeier 2012).
- Learning by doing (Benkard 2004, Besanko, Doraszelski, Kryukov & Satterthwaite 2010, Besanko, Doraszelski & Kryukov 2014, Besanko, Doraszelski & Kryukov 2016).
- Mergers (Berry & Pakes 1993, Gowrisankaran 1999, Mermelstein, Nocke, Satterthwaite & Whinston 2013).
- Network effects (Jenkins, Liu, Matzkin & McFadden 2004, Markovich 2004, Markovich & Moenius 2005, Chen, Doraszelski & Harrington 2009).
- Productivity growth (Laincz 2005).
- R&D (Gowrisankaran & Town 1997, Auerswald 2001, Song 2011).
- Switching costs (Chen 2011).
- Technology adoption (Schivardi & Schneider 2005).
- International trade (Erdem & Tybout 2003).
- Finance (Goettler, Parlour & Rajan 2004).

Quality Ladder Model with Entry/Exit

- Pakes, A. & McGuire, P. (1994) "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model."

- Borkovsky, R., Doraszelski, U. & Kryukov, Y. (2012) "A Dynamic Quality Ladder Model with Entry and Exit: Exploring the Equilibrium Correspondence Using the Homotopy Method."

- Incumbent firms (i.e., active firms) and potential entrants (i.e., inactive firms).

- Two firms that can be either a potential entrant or an incumbent firm with potentially different product qualities

$$\omega = (\omega_1, \omega_2) \in \{ \underbrace{1, \ldots, L}_{\text{active firm}}, \underbrace{L+1}_{\text{inactive firm}} \}^2 = \Omega.$$

- Exit is a transition from state $\omega_n \neq L+1$ to state $\omega_n' = L+1$.

- Entry is a transition from state $\omega_n = L+1$ to state $\omega_n' = \omega^e \neq L+1$, where ω^e is an exogenously given initial product quality.

Quality Ladder Model with Entry/Exit

- Let $\xi_n(\omega) \in [0,1]$ be firm n's probability of remaining in (if $\omega_n \neq L+1$) or entering into (if $\omega_n = L+1$) the industry.

- Transition probability: If $\omega_n \in \{2,\ldots,L-1\}$, then

$$\Pr(\omega_n'|\omega_n,\xi_n,x_n) = \begin{cases} \xi_n \frac{(1-\delta)\alpha x_n}{1+\alpha x_n} & \text{if} \quad \omega_n' = \omega_n + 1, \\ \xi_n \frac{1-\delta+\delta\alpha x_n}{1+\alpha x_n} & \text{if} \quad \omega_n' = \omega_n, \\ \xi_n \frac{\delta}{1+\alpha x_n} & \text{if} \quad \omega_n' = \omega_n - 1, \\ 1-\xi_n & \text{if} \quad \omega_n' = L+1, \end{cases}$$

  etc. If $\omega_n = L+1$, then

$$\Pr(\omega_n'|\omega_n,\xi_n) = \begin{cases} \xi_n & \text{if} \quad \omega_n' = \omega^e, \\ 1-\xi_n & \text{if} \quad \omega_n' = L+1. \end{cases}$$

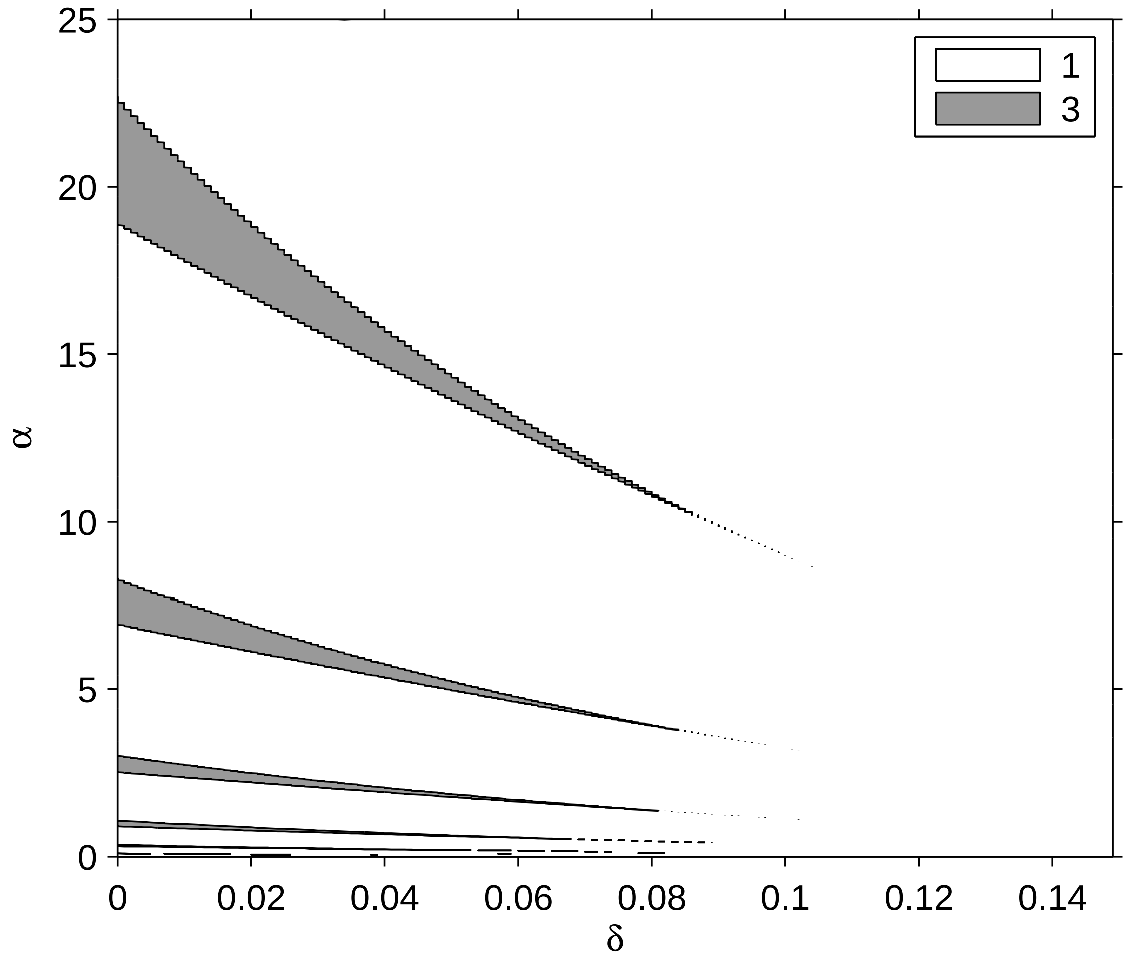
Quality Ladder Model with Entry/Exit

- Firm n is assigned a random scrap value $\phi_n \sim F$ (if $\omega_n \neq L+1$) or a random setup cost $\phi_n^e \sim F^e$ (if $\omega_n = L+1$).
  - Scrap values/setup costs are private information.
  - Scrap values/setup costs are independent across firms and periods.
- Because scrap values and setup costs are private to a firm, its rivals perceive the firm *as if* it is mixing.
- In each period the timing is as follows:
  - Incumbent firms learn their scrap value and decide on exit and investment. Potential entrants learn their setup cost and decide on entry and investment.
  - Incumbent firms compete in the product market.
  - Exit and entry decisions are implemented.
  - The investment decisions of the remaining incumbents and new entrants are carried out and their uncertain outcomes are realized.

Multiple Equilibria

. . . we have experimented quite a bit with the core version of the algorithm, and we never found two sets of equilibrium policies for a given set of primitives (we frequently run the algorithm several times using different initial conditions or different orderings of points looking for other equilibria that might exist). We should emphasize here that the core version, and indeed most other versions that have been used, all use quite simple functional forms for the primitives of the problem, and multiplicity of equilibrium may well be more likely when more complicated functional forms are used. Of course, most applied work suffices with quite simple functional forms. (Pakes 2000, pp. 18–19)

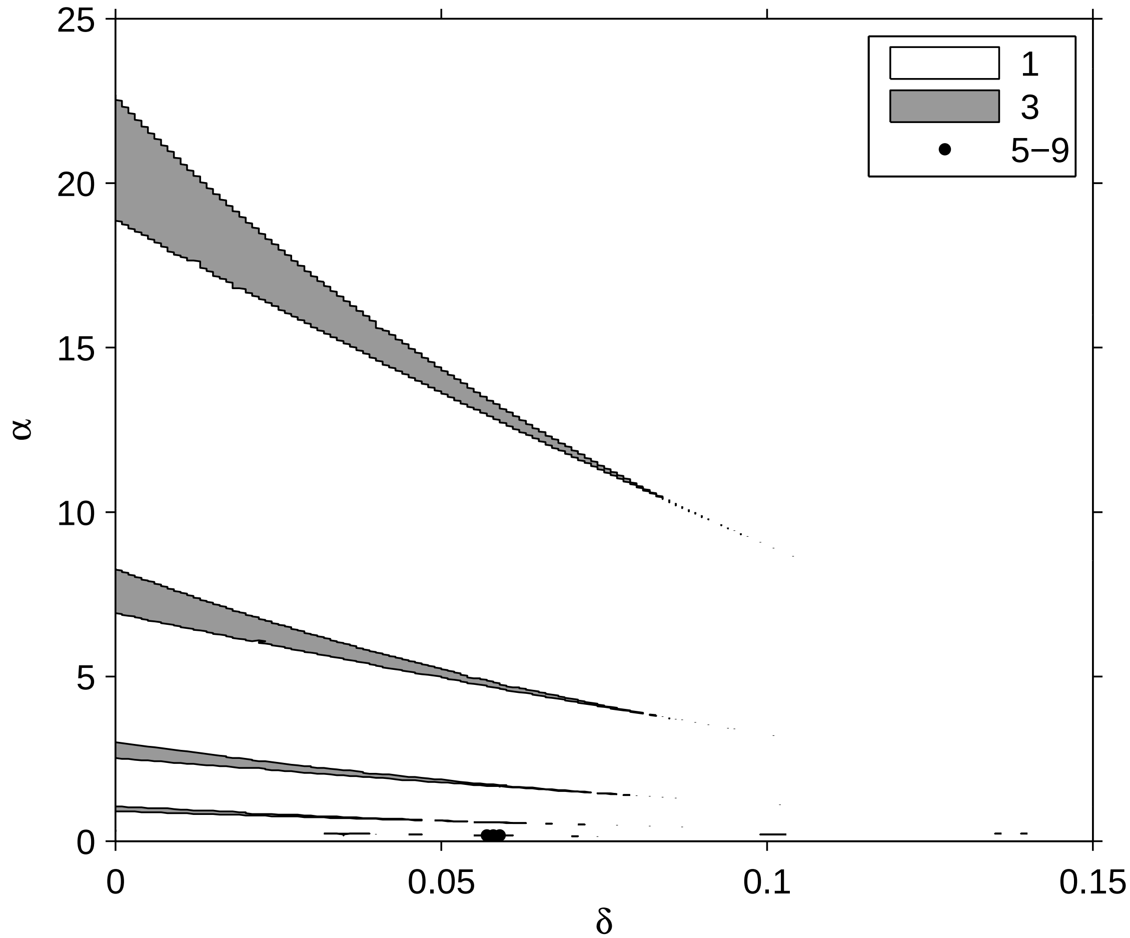
Multiple Equilibria: Quality Ladder Model without Entry/Exit



Number of equilibria in the Pakes & McGuire (1994) quality ladder model without entry/exit.

Source: Borkovsky, Doraszelski & Kryukov (2010).

Multiple Equilibria: Quality Ladder Model with Entry/Exit



Number of equilibria in the Pakes & McGuire (1994) quality ladder model with entry/exit.
Source: Borkovsky, Doraszelski & Kryukov (2012).

Multiple Equilibria

...I should note that virtually all Markov Perfect Models have multiple equilibria... (anonymous referee, 2013)

Computing all Equilibria

- Homotopy method:
  - Besanko, D., Doraszelski, U., Kryukov, S. & Satterthwaite, M. (2010) "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics."
  - Borkovsky, R., Doraszelski, U., & Kryukov, Y. (2010) "A User's Guide to Solving Dynamic Stochastic Games Using the Homotopy Method."

- If the system of equations is polynomial, then...
  - Judd, K., Renner, P. & Schmedders, K. (2012) "Finding all Pure-Strategy Equilibria in Games with Continuous Strategies."
  - Kubler, F, Schmedders, K. & Renner, P. (2013) "Computing all Solutions to Polynomial Equations in Economics."

- If movements through the state space are undirectional, then...
  - Judd, K. & Schmedders, K. (2004) "A Computational Approach to Proving Uniqueness in Dynamic Games."
  - Judd, K., Schmedders, K. & Yeltekin, S. (2012) "Optimal Rules for Patent Races."
  - Iskhakov, F., Rust, J. & Schjerning, B. (2016) "Recursive Lexicographical Search: Finding all Markov Perfect Equilibria of Finite State Directional Dynamic Games."
  - Iskhakov, F., Rust, J. & Schjerning, B. (2014) "The Dynamics of Bertrand Price Competition with Cost-Reducing Investments."

Sources of Computational Burden

- State space:
  - Suppose that each of N players can be at one of L states.
  - State space has L^N elements.
  - Symmetry reduces exponential to polynomial growth.

- Successor states:
  - Suppose that each of N players can move to one of K states from one period to the next.
  - Expectation over successor states involves K^N terms.

Alleviating the Computational Burden

System of equations:

- Ferris, M., Judd, K. & Schmedders, K. (2007) "Solving Dynamic Games with Newton's Method."

Ergodic set:

- Pakes, A. & McGuire, P. (2001) "Stochastic Algorithms, Symmetric Markov Perfect Equilibrium, and the 'Curse' of Dimensionality."

- Judd, K., Maliar, L. & Maliar, S. (2012) "Merging Simulation and Projection Approaches to Solve High-Dimensional Problems."

State aggregation and interpolation methods:

- Farias, V., Saure, D. & Weintraub, G. (2012) "An Approximate Dynamic Programming Algorithm to Solving Dynamic Oligopoly Models"

- Santos, C. (2012) "An Aggregation Method to Solve Dynamic Games"

- Arcidiacono, P., Bayer, P., Bugni, F. & James, J. (2011) "Sieve Value Function Iteration for Large State Space Dynamic Games."

- Aguirregabiria, V. and Vincentini, G. (2012) "Dynamic Spatial Competition Between Multi-Store Firms."

Alleviating the Computational Burden

Oblivious equilibrium and its extensions:

- Weintraub, G., Benkard, L. & Van Roy, B. (2008) "Markov Perfect Industry Dynamics with Many Firms."
- Weintraub, G., Benkard, L. & Van Roy, B. (2010) "Computational Methods for Oblivious Equilibrium."
- Weintraub, G., Benkard, L. Jeziorski, P. & Van Roy, B. (2008) "Nonstationary Oblivious Equilibrium."
- Benkard, L., Jeziorski, P. & Weintraub, G., (2015) "Oblivious Equilibrium for Concentrated Industries."
- Ifrach, B. and Weintraub, G. (2016) "A Framework for Dynamic Oligopoly in Concentrated Industries."

Alleviating the Computational Burden

Continuous-time stochastic games:

- Doraszelski, U. & Judd, K. (2011) "Avoiding the Curse of Dimensionality in Dynamic Stochastic Games."

- Arcidiacono, P. Bayer, P. Blevins, J. & Ellickson (2016) "Estimation of Dynamic Discrete Choice Models in Continuous Time with an Application to Retail Competition."

Discrete-time stochastic games with alternating moves:

- Doraszelski, U. & Judd, K. (2007) "Dynamic Stochastic Games with Sequential State-to-State Transitions."

- Doraszelski, U. & Escobar, J. (2016) "Protocol Invariance and the Timing of Decisions in Dynamic Games."

Open Questions

How can we deal with multiple equilibria in counterfactual analysis?

- Lee, R. & Pakes, A. (2009) "Multiple Equilibria and Selection by Learning in an Applied Setting."

- Doraszelski, U. & Escobar, J. (2010) "A Theory of Regular Markov Perfect Equilibria in Dynamic Stochastic Games: Genericity, Stability, and Purification."

- Aguirregabiria, V. (2012) "A Method for Implementing Counterfactual Experiments in Models with Multiple Equilibria."

- Doraszelski, U., Lewis, G. & Pakes, A. (2015) "Just Starting Out: Learning and Equilibrium in a New Market."

Open Questions

What do we know about the general properties of the set of equilibria?

- Doraszelski, U. & Escobar, J. (2010) "A Theory of Regular Markov Perfect Equilibria in Dynamic Stochastic Games: Genericity, Stability, and Purification."

What types of behaviors can arise?

- Besanko, D., Doraszelski, U., Kryukov, Y. & Satterthwaite, M. (2010) "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics."

- Yeltekin, S, Chai, Y. & Judd, K. (2016) "Computing Equilibria of Dynamic Games."

- Doraszelski, U. & Escobar, J. (2012) "Restricted Feedback in Long Term Relationships."

- Balbus, L., Reffett, K. & Wozny, L. (2010) "A Constructive Study of Markov Equilibria in Stochastic Games with Strategic Complementarities."

Open Questions

How can we deal with persistent asymmetric information?

- Fershtman, C. & Pakes, A. (2012) "Dynamic Games With Asymmetric Information: A Framework For Empirical Work."

- Asker, J., Fershtman, C., Jeon, J. & Pakes, A. (2016) "The Competitive Effects of Information Sharing."

- Bernhardt, D. & Taub, B. (2012) "Oligopoly Learning Dynamics."