Main Idea: Ownership Matrix

Empirical Models of Pricing in Industries with Differentiated-Products

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Fall 2018

Empirical Models of Pricing with Differentiated-Products

Up to now we assumed that the products were homogenous (at least as an approximation).

Bresnahan, JIE 87, "Competition and Collusion in the American Automobile Industry: 1955 Price War"

Q: In 1955 quantities of autos sold were higher and prices were lower, relative to 54 and 56. Why? Was this due to a price war/breakdown of collusion?

1st Bresnahan note: Need Demand to rotate

Bresnahan, *JIE* 87

- Basic idea: use variation in demand to learn about model of competition (like 1st Bresnahan note). However, now the variation is across products (and not between markets).
- Treat location in characteristic space as fixed; Given location, markups will differ depending on ownership of nearby products. Ask which supply model best fits the data. (**See graph**)
- This is essentially like using the characteristics of other products as IV.

"X" is a highway

- impose some MC curve

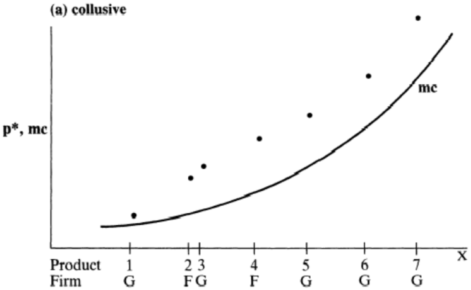


Figure 2(a)

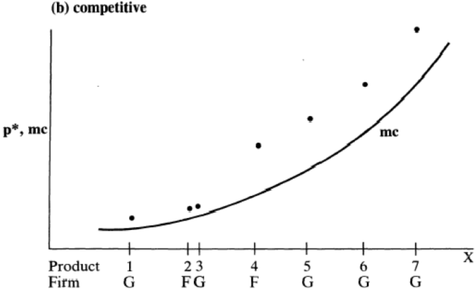


Figure 2(b)

Model

Supply (static multi-product pricing)

$f = 1, ..., F$ firms; $j = 1, ..., J$ products.
Each firms produces some subset, $\mathscr{F}_f$, of the J products.

Cost of production: $C(\overset{\text{quality}}{x}, \overset{\text{quantity}}{q}) = \overset{\text{fixed}}{A(x)} + mc(x)q$
where q is the quantity produced and x is the quality of the product;

Let $mc(x) = \mu e^x$; No economies of scale

The profits of firm f are

$$\Pi_f = \sum_{j \in \mathscr{F}_f} \underbrace{(p_j - mc_j)}_{\text{markup}} \overbrace{\underbrace{M}_{\text{size of mkt}} \underbrace{s_j(p)}_{\text{share}}}^{\text{Think as } q_j} - C_f$$

Assuming: (1) existence of a pure-strategy Bertrand-Nash equilibrium in prices; (2) prices that support it are strictly positive. The first order conditions are

$$s_j(p) + \sum_{r \in \mathscr{F}_f} (p_r - mc_r) \frac{\partial s_r(p)}{\partial p_j} = 0 \quad j = 1, \ldots, J$$

Define $S_{jr} = -\partial s_r / \partial p_j, \quad j, r = 1, \ldots, J$, and an "ownership" structure defined by

$$\underbrace{H_{jr}}_{j \times j} = \begin{cases} 1, & if \exists f : \{r, j\} \subset \mathscr{F}_f; \\ 0, & otherwise \end{cases}$$

single product firms
H is Identity

Multiproduct Firms
H has blocks of 1s

and let $\Omega_{jr} = H_{jr} * S_{jr}$.
elt by elt

Monopoly!
Mx of 1s

substitutes: upward pricing pressure

complements: lowers the price

Then the first order conditions become

$$s(p) - \Omega(p - mc) = 0$$

which implies a pricing equation

can we est this in nonpar, ML way?

$$p - mc = \underbrace{\Omega^{-1} s(p)}$$

⊗ See price, estimate demand, back out MC

for single product firms:

$$p = mc - \frac{s(p)}{\partial s_j / \partial p_j}$$

can add $(p_k - mc_k)\frac{\partial s_k}{\partial p_j}$

$$\frac{p - mc}{p} = -\frac{1}{\eta}$$ η = elasticity

Lerner index

Therefore by:

(1) assuming a model of conduct; and

(2) using estimates of the demand substitution;

Heart of ID

we are able to:

(1) measure PCM (without using cost data);

(2) compute these margins under different “ownership” structures (i.e., different Ω^*).

Note: we assumed away any cost synergies across products and across time.

Model

Demand (vertical differentiation)

consumers agree on the ranking of the quality of the product

Let:

- v - measure consumer taste (WTP for quality);
  $v \sim U[0, V_{max}]$ *with density* δ can treat as $1/V_{max}$
- x - auto quality;
- y - consumer income;
- p - price of the auto;

The indirect utility of consumer (v, y) from auto (x, p) is

$$vx + y - p$$

and if no auto is bought it is

$$v\underbrace{\gamma}_{\text{quality}} + y - \underbrace{E}_{\text{price of some outside good.}}$$

where γ and E are parameters to be estimated.

Assume each consumer chooses exactly one of the $J + 1$ options; Therefore the demand for product $j = 0, ..., J$ is

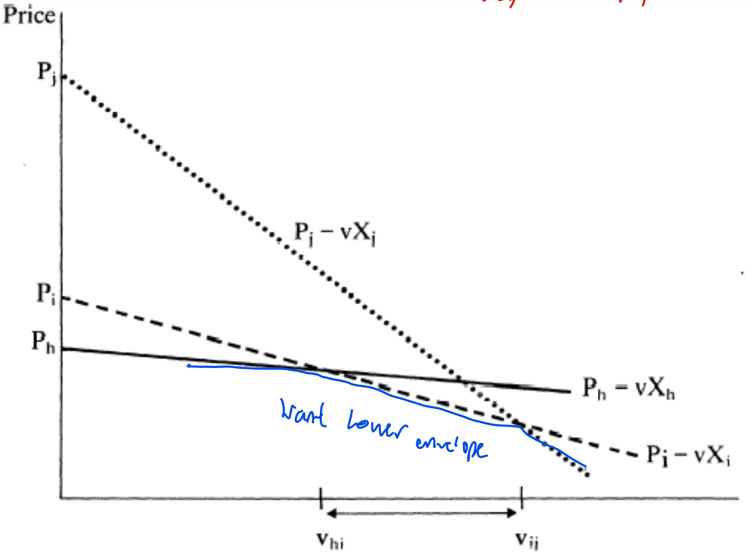
$$q_j = \delta[v_{j+1} - v_j]$$

where the cutoff points, v_j, are computed by the consumer who is just indifferent between two options. These are given by

$$v_j = \begin{cases} 0 & j = 0 \\ \frac{p_{j-1} - p_j}{x_{j-1} - x_j} & j = 1, ..., J \\ V_{max} & j = J + 1 \end{cases}$$

where $p_0 = E$ and $x_0 = \gamma$

(very used graph)



Want lower envelope

cutoffs where lines intersect

Figure 1

The price derivatives are

$$\frac{\partial q_j}{\partial p_j} = \delta \Big[\frac{1}{x_j - x_{j+1}} + \frac{1}{x_{j-1} - x_j} \Big]$$

and

$$\frac{\partial q_j}{\partial p_k} = \begin{cases} \delta \left[\frac{1}{x_j - x_k} \right] & k = j-1, j+1 \\ 0 & otherwise \end{cases}$$

Conditional on the x's we could estimate demand and compute the implied markups for different demand structures.

Quality is assumed $x_j = \sqrt{\beta_0 + \sum_k \beta_k z_{jk}}$;
where z_{jk} is a characteristic of product j and β's are parameters to be estimated.

Data
Prices: list prices
quantities: quantity produced
characteristics

Econometrics

1) Let $P^*(z; H, \beta, \gamma, V_{max}, \delta, \mu)$ and $Q^*(z; H, \beta, \gamma, V_{max}, \delta, \mu)$ represent the equilibrium prices and quantities predicted by the model.

   Assume: $P_j = P_j^* + \epsilon_j^p$ and $q_j = q_j^* + \epsilon_j^p$ where ϵ_j^p and ϵ_j^q are iid zero mean normally distributed shocks with variance σ_p^2 and σ_q^2 Then the likelihood function is given by

$$\prod_{j=1}^{J} \frac{1}{2\pi\sigma_p^2} exp\Big[-\frac{(P_j - P_j^*)^2}{2\sigma_p^2}\Big] \frac{1}{2\pi\sigma_q^2} exp\Big[-\frac{(q_j - q_j^*)^2}{2\sigma_q^2}\Big]$$

   Note: the likelihood function is not well behaved (the ranking of the cars will change with the values of the parameters).

2) 4 models are estimated:

Collusive: the ownership matrix is a matrix of 1's;

Nash (multi-product pricing): the ownership matrix is blocks of 1's;

Products (single-product pricing): the ownership matrix is identity;

Hedonic: $P_j^* = exp[\alpha_0 + \sum_k \alpha_k z_{jk}]$ and $q_j^* = exp[\lambda_0 + \lambda_1(P_j - P_j^*)]$

3) The models are estimated separately for each year. Identification is coming from cross-product variation.

4) Testing:
   (a) Cox test of non-nested alternatives (LR of the null and the alternative is the central statistic, the mean and variance are computed under the null and used to compute a test statistic that is distributed standard normal)
   (b) Informal: compares estimates across years. Either structural parameters are unstable or competition changes.

Results

Table 3: 54, 56 only collusive model not rejected; 55 only Nash model not rejected;

Table 4: structural parameters do not change under maintained assumption;

Table 5: structural parameters vary between 55 and 54/56 if competition model is held fixed;

Table III
Cox Test Statistics

Hypotheses	C	N–C	'p'	H
a—1954				
Collusion	—	0.8951	0.9464	−1.934
Nash-Competition	−2.325	—	−0.8878	−2.819
"Products"	−3.978	3.029	—	−1.604
Hedonic	−12.37	−10.94	−13.02	—
b—1955				
Collusion	—	−10.36	−9.884	−13.36
Nash-Competition	−1.594	—	1.260	0.6341
"Products"	−0.7598	−4.379	—	−1.527
Hedonic	−3.353	−8.221	−5.950	—
c—1956				
Collusion	—	1.227	0.8263	1.629
Nash-Competition	−2.426	—	−4.586	0.8314
"Products"	−3.153	0.9951	—	4.731
Hedonic	−5.437	−9.671	−11.58	—

collusion not really rejected by any of the 3

collusion model borderline rejected by Hedonic

collusion rejected by all 3

Same as 54

Row denotes the null (i.e., the model assumed true), while column denote the alternative. Values of the test statistic (asymptotically a standard normal) sig different from zero lead to rejection. The intuition of the test is as follows: if the residuals under the null can be explained by the alternative then the null is rejected.

TABLE IV
PARAMETER ESTIMATES 1954–56, MAINTAINED SPECIFICATION

Parameters	*1954*[a]	*1955*[b]	*1956*[a]
Physical Characteristics			
Quality Proxies			
Constant	47.91	48.28	50.87
	(32.8)	(43.2)	(29.4)
Weight #/1000	0.3805	0.5946	0.5694
	(0.332)	(0.145)	(0.374)
Length "/1000	0.1819	0.1461	0.1507
	(0.128)	(0.059)	(0.146)
Horsepower/100	2.665	3.350	3.248
	(0.692)	(0.535)	(0.620)
Cylinders	−0.7387	−0.9375	−0.9639
	(0.205)	(0.115)	(0.186)
Hardtop Dummy	0.9445	0.4531	0.4311
	(0.379)	(0.312)	(0.401)
Demand/Supply			
μ—*Marginal Cost*	0.1753	0.1747	0.1880
	(0.024)	(0.020)	(0.035)
γ—*Lower Endpoint*	4.593	3.911	4.441
	(1.49)	(2.08)	(1.46)
$V_{\max}$—*Upper Endpoint*	$1.92E+7$	$2.41E+7$	$2.83E+7$
	$(8.44E+6)$	$(9.21E+6)$	$(7.98E+6)$
δ *Taste Density*	0.4108	0.4024	0.4075
	(0.138)	(0.184)	(0.159)

Notes: Figures in parentheses are asymptotic standard errors.
[a] Estimated using the Collusion specification.
[b] Estimated using the Nash-Competition specification.

TABLE V(i)
PARAMETER ESTIMATES 1954–56, COLLUSIVE SPECIFICATION

Parameters	*1954*	*1955*	*1956*
Constant	47.91	−23.37	50.87
	(32.8)	(24.5)	(29.4)
Weight	0.3805	0.0103	0.5694
	(0.332)	(5.43E−2)	(0.374)
Length	0.1819	−2.88E−3	0.1507
	(0.128)	(0.102)	(0.146)
Horsepower	2.665	0.1165	3.248
	(0.692)	(0.106)	(0.620)
Cylinders	−0.7387	−1.309	−0.9639
	(0.205)	(1.52)	(0.186)
Hardtop	0.9445	1.468	0.4311
	(0.379)	(1.08)	(0.401)
μ	0.1753	1.344	0.1880
	(0.024)	(0.151)	(0.035)
γ	4.593	1.604	4.441
	(1.49)	(4.83)	(1.46)
V_{max}	1.92E+7	1.46E+8	2.83E+7
	(8.44E+6)	(6.74E+6)	(7.98E+6)
δ	0.4108	5.75E−2	0.4075
	(0.138)	(8.28E−2)	(0.159)

Note: Figures in parentheses are asymptotic standard errors.

consumers suddenly change preferences for these char?
⇒ prob not!

Assumes that collusive for all 3 years not just 54 and 56.

TABLE V(ii)

PARAMETER ESTIMATES 1954–56, BERTRAND SPECIFICATION

Parameters	*1954*	*1955*	*1956*
Constant	31.64	48.28	33.23
	(29.9)	(43.2)	(17.8)
Weight	0.9311	0.5946	$6.23E-3$
	(0.210)	(0.145)	$(8.73E-4)$
Length	0.1474	0.1461	0.1605
	(0.038)	(0.059)	(0.149)
Horsepower	4.962	3.350	$2.972E-2$
	(0.676)	(0.535)	$(1.47E-2)$
Cylinders	−0.8846	−0.9375	−0.9078
	(0.194)	(0.115)	(0.256)
Hardtop	−0.2474	0.4531	0.5282
	(0.464)	(0.312)	(0.249)
μ	0.2518	0.1747	0.2902
	(0.074)	(0.312)	(0.249)
γ	6.352	3.911	1.204
	(3.54)	(2.08)	(3.19)
V_{max}	$9.81E+5$	$2.41E+7$	$1.03E+6$
	$(8.78E+6)$	$(9.21E+6)$	$(8.90E+6)$
δ	5.04	0.4024	7.334
	(1.21)	(0.184)	(2.46)

Note: Figures in parentheses are asymptotic standard errors.

Comments

1) The formal test requires that at least one of the alternatives be true. The test proposed by Voung (*EMA*, 88) does not require this. It is applied by Gasmi, Laffont and Voung (*JEMS*, 92) to testing models of collusion in the soft-drink market.
2) The test for collusion relies critically on getting the demand estimates right. The demand model is very restrictive in several ways:

   Assumes only competes w/ next door neighbors

   (a) The model imposes very restrictive substitution patterns. Even in this market it is not clear that the vertical model is a good approximation;
   (b) No error in quality measures;

3) The implicit assumption is that the locations, i.e., characteristics, are exogenous (pre-determined). Is this a reasonable assumption?

4) The model ignores dynamics on both the producer and consumer side.