# Empirical Models of Entry

Aviv Nevo

University of Pennsylvania and NBER

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## Goals and Empirical Approach

We'll be looking at a cross section of markets. In fact we won't see entry in the data, we'll be looking at market structure (which is a function of entry).

Goals:

1) Recover fixed costs
2) Recover parameters of the profit function
3) Learn about competition from general entry patterns

# Model

Enter or Not | Competition

- 2 stage game
- Complete information (mostly)
- Stage 1 : entry decision
  - sequential or simultaneous
  - 0-1 entry decision or a location choice. where to locate
  - focus on Pure Strategy Subgame Perfect
- Stage 2: price/quantity competition
- Dynamics not modeled explicitly
- Revealed preference approach: firms enter if and only if (reduced form) profits are above some threshold.

# Typical Data

The typical data consists of a cross-section of markets.

· Variation in mkt sizes, etc.

Exogenous variables include demographics and market characteristics.

· Size of mkt

Endogenous variables include the number of entrants, their identity (sometimes), prices and quantities (sometimes) and potentially other choices.

# Bresnahan-Reiss

A market is defined to be a small (pop. 50-10K) and isolated (50 miles away from closest town) town in the US

Focus on service/retail firms: auto dealers, tire dealers, plumbers, dentists, etc.

They focus on entry thresholds:

- What's the minimum town size that supports 1 firm? 2 firms? 3 firms?
- Comparing these thresholds we might be able to make inference on the mode of competition.
- E.g., linear thresholds are consistent with: CRS together with no competition *or* extreme collusion
- Key question: what can we learn from these thresholds?

$N_t$ is the number of firms in market $t$

$x_t$ are demand and variable cost shifters

$\pi(N, x)$ is the per-firm profit

Homogeneous firms

Assume $\pi_t(N_t, x_t) = v(N_t, x_t; \theta) - F_t$, where $v(\cdot)$ are reduced form variable profits coming from stage 2 and $F_t$ are fixed costs

$F_t \sim_{iid} \Phi(F|N_t, w_t)$, where $\Phi$ is the standard normal cdf are $w_t$ are market characteristics. Note $w_t$ affects only fixed costs.

# Bresnahan-Reiss

## Model

In equilibrium it must be that

$$\pi(N_t^*, x_t; \theta) \geq 0 \quad \text{and} \quad \pi(N_t^* + 1, x_t; \theta) \leq 0$$

or, equivalently

$$v(N_t^*, x_t; \theta) \geq F_t \geq v(N_t^* + 1, x_t; \theta)$$

Because $F_t$'s are iid across markets we can write the likelihood as

$$\begin{aligned}
\mathcal{L} &= \prod_{t=1}^{T} \mathbb{P}(N_t^* | x_t, w_t; \theta) \\
&= \prod_{t=1}^{T} \Phi(v(N_t^*, x_t; \theta)) - \Phi(v(N_t^* + 1, x_t; \theta))
\end{aligned}$$

Note the ordered model structure.

# Bresnahan-Reiss

Comments

1. With additional structure, we can translate $N^*$ into a reduced form profit function $\pi$
2. How do we relate $v(\cdot)$ to the strength of competition?

# Bresnahan-Reiss

Let the demand in market $t$ be given by

$$Q_t = S_t q(p_t)$$

$S_t$ is market size and $q(p_t)$ is the per capita demand at price $p_t$.

Let firm $i$'s cost be $C_i(q_i)$ and its marginal cost be $MC_i(q_i)$ .

In a Cournot equilibrium, firm $i$ chooses output such that

$$p_t = \frac{\eta_t}{\eta_t - s_i} MC_i$$

$\eta_t$ is demand elasticity (in absolute value) and $s_i$ is firm $i$'s share

# Bresnahan-Reiss

Using market shares as weights we can derive

$$p_t = \frac{\eta_t}{\eta_t - H} \bar{mc}^w$$

where $H = \sum_i s_i^2$ and $mc^w = \sum_i s_i mc_i$.

Just averaging we get

$$p_t = \frac{N\eta_t}{N\eta_t - 1} \bar{mc}$$

The two equations above relate $p_t$ to $N$, but to get $v(\cdot)$ we need more assumptions.

# Bresnahan-Reiss

Assume linear demand,: $Q = S(\alpha - \beta p)$ or, equivalently, $p = a - b\frac{Q}{S}$ where $a = \alpha/\beta$ and $b = 1/\beta$.

Assume quadratic costs: $C(q) = F + cq - dq^2$

Using these additional assumptions, one can show that

$$p^*(N) = a - \frac{N(a-c)}{N + 1 + 2s\frac{a}{b}}$$

This implies that

$$\pi = \frac{\theta_1^2 s(1 + \theta_2 s)}{(N + 1 + 2\theta_2 s)^2} - F$$

where $\theta_1 = \frac{a-c}{\sqrt{b}}$ and $\theta_2 = \frac{d}{b}$.

# Bresnahan-Reiss

With CRS ($d = 0$), this reduces to

$$\pi = \frac{\theta_1^2 s}{(N+1)^2} - F$$

Now define $S_N$ to be the $N$ firm entry threshold, i.e.

$$S_N := \min S \text{ s.t. } \pi(N, S) \geq 0$$

With constant returns to scale ($d = 0$), we get

$$s_N = \frac{(N+1)^2 F}{\theta_1^2}$$

# Bresnahan-Reiss

BR define

$$\Delta_N = \frac{S_N}{N}$$

Then

$$\frac{\Delta_{N+1}}{\Delta_N} = \frac{(N+2)^2 N}{(N+1)^3}$$

# Bresnahan-Reiss

BR take this to the data. They write

$$\pi(N, y, x, w) = S(y)v_n(x) - F_n(w) + \varepsilon$$

where

- $y$ is market size (nearby population, +growth and -growth, number of outside commuters)
- $x$ includes demographics, income and wealth
- $w$ includes retail wages + land values

$\varepsilon$ is the unobserved part of the fixed cost (at the market level)

# Bresnahan-Reiss

If $\varepsilon$ follows a standard normal distribution,

$$\mathbb{P}(N_t) = \Phi(-S(y)v_{N_t^*+1}(x) + F_{N_t^*+1}(w)) - \Phi(-S(y)v_{N_t^*}(x) + F_{N_t^*}(w))$$

They further assume specific functional forms for the various functions.

# ReStud (1990): Results

ReStud paper looks at auto dealers in 149 town

TABLE 2

*Distribution of town population by the number of dealers in town*

| | No entrants | Monopoly | Duopoly | Three or more entrants |
|---|---|---|---|---|
| Number of markets | 34 | 42 | 40 | 33 |
| Sample percentile | | | | |
| 25th | 488 | 970 | 1248 | 1814 |
| 30th | 545 | 1052 | 1417 | 2016 |
| 35th | 600 | 1070 | 1573 | 2115 |
| Sample mean | 817 | 1514 | 2282 | 2977 |
| 65th | 824 | 1492 | 2579 | 3130 |
| 70th | 896 | 1657 | 2949 | 3387 |
| 75th | 924 | 1752 | 3154 | 3616 |

*Notes.* The town population data come from the *1980 Census of Population and Housing* and Rand McNally's *Commercial Atlas and Marketing Guide*. Dealer counts come from R. L. Polk's dealer mailing lists. See the text for sample definitions and counting rules.

Table 2 suggests values for the thresholds

TABLE 5

*Ordered probit models of the number of dealers*

| Variable | Coefficient | Specification (1) | (2) | (3) | (4) |
|---|---|---|---|---|---|
| *OPOP10* | $\lambda_1$ | | 0·254 (1·26) | 0·196 (0·99) | 0·208 (1·15) |
| *NGRW70* | $\lambda_2$ | | 2·666 (3·45) | 2·682 (3·40) | 2·692 (3·41) |
| *PGRW70* | $\lambda_3$ | | −1·598 (−4·94) | −1·421 (−3·43) | −1·475 (−4·07) |
| *OCTY* | $\lambda_4$ | | −0·645 (−0·74) | −0·180 (−0·22) | −0·188 (−0·23) |
| *V-Monopoly* | $\theta^M$ | 0·933 (3·39) | 1·636 (4·96) | 1·310 (1·37) | 0·645 (1·50) |
| *V-Duopoly* | $\theta^M+\theta^D$ | 0·702 (5·02) | 0·965 (5·91) | 0·673 (0·71) | −0·037 (−0·12) |
| *V-Income* | $\theta_I$ | | | −0·008 (−0·09) | |
| *V-RETWAGE* | $\theta_W$ | | | −0·129 (−0·81) | |
| *V-LANDVAL* | $\theta_L$ | | | 0·321 (2·62) | 0·298 (2·60) |
| *V-FARMFRAC* | $\theta_F$ | | | 0·414 (1·71) | 0·444 (1·88) |
| *F-Monopoly* | $\gamma^M$ | 0·536 (1·79) | 0·975 (3·26) | 1·829 (1·78) | 1·235 (1·71) |
| *F-Duopoly* | $\gamma^M+\gamma^D$ | 1·277 (5·39) | 1·434 (5·11) | 2·384 (2·24) | 1·749 (2·42) |
| *F-RETWAGE* | $\gamma_W$ | | | −0·323 (−1·64) | −0·194 (−1·63) |
| *F-LANDVAL* | $\gamma_L$ | | | 0·367 (2·01) | 0·341 (1·96) |
| Log Likelihood | | −123·76 | −115·63 | −108·44 | −108·79 |

Variable profits fall from $M$ to $D$. Fixed costs also change.

# ReStud (1990): Results

Entry thresholds are computed from the parameter estimates. BR obtain

$$S_1 = 700 \text{ and } S_2 = 1400$$

which implies

$$\frac{\Delta_2}{\Delta_1} \approx 1$$

What does this tell us?

# JPE (1991): Results

TABLE 5

A. Entry Threshold Estimates

| Profession | Entry Thresholds (000's) $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | Per Firm Entry Threshold Ratios $s_2/s_1$ | $s_3/s_2$ | $s_4/s_3$ | $s_5/s_4$ |
|---|---|---|---|---|---|---|---|---|---|
| Doctors | .88 | 3.49 | 5.78 | 7.72 | 9.14 | 1.98 | 1.10 | 1.00 | .95 |
| Dentists | .71 | 2.54 | 4.18 | 5.43 | 6.41 | 1.78 | .79 | .97 | .94 |
| Druggists | .53 | 2.12 | 5.04 | 7.67 | 9.39 | 1.99 | 1.58 | 1.14 | .98 |
| Plumbers | 1.43 | 3.02 | 4.53 | 6.20 | 7.47 | 1.06 | 1.00 | 1.02 | .96 |
| Tire dealers | .49 | 1.78 | 3.41 | 4.74 | 6.10 | 1.81 | 1.28 | 1.04 | 1.03 |

B. Likelihood Ratio Tests for Threshold Proportionality

| Profession | Test for $s_4 = s_5$ | Test for $s_3 = s_4 = s_5$ | Test for $s_2 = s_3 = s_4 = s_5$ | Test for $s_1 = s_2 = s_3 = s_4 = s_5$ |
|---|---|---|---|---|
| Doctors | 1.12 (1) | 6.20 (3) | 8.33 (4) | 45.06* (6) |
| Dentists | 1.59 (1) | 12.30* (2) | 19.13* (4) | 36.67* (5) |
| Druggists | .43 (2) | 7.13 (4) | 65.28* (6) | 113.92* (8) |
| Plumbers | 1.99 (2) | 4.01 (4) | 12.07 (6) | 15.62* (7) |
| Tire dealers | 3.59 (2) | 4.24 (3) | 14.52* (5) | 20.89* (7) |

Note.—Estimates are based on the coefficient estimates in table 4. Numbers in parentheses in pt. B are degrees of freedom.
* Significant at the 5 percent level.

Market size thresholds and relative thresholds for different professions

Plumbers are different. What does this tell us?

# JPE (1991): Variation on the model

Consider a variation of the model (continue to assume homogeneous good, Cournot competition, CRS, symmetric firms:

Demand: $D = S\frac{K}{P}$ and costs: $c(q) = F$
Can show that

$$\pi_N(S) = \frac{KS}{N^2} - F$$

which implies that

$$S_N = \frac{N^2 F}{K}$$

This in turn yields

$$\Delta_N = \frac{FN}{K} \quad \text{and} \quad \frac{\Delta_{N+1}}{\Delta_N} = \frac{N+1}{N}$$

which is roughly consistent with what we see in table 5.

TABLE 11

TIRE PRICE REGRESSIONS ($N$ = 282)

| VARIABLE NAME | ORDINARY LEAST SQUARES (1) | ORDINARY LEAST SQUARES (2) | LEAST ABSOLUTE DEVIATIONS (3) |
|---|---|---|---|
| Constant term | 26.4 | 29.9 | 29.5 |
| | (4.69) | (4.87) | (4.43) |
| Monopoly market dummy | .88 | .26 | .54 |
| | (2.12) | (2.33) | (2.12) |
| Duopoly market dummy | 1.88 | −.62 | .96 |
| | | (2.42) | (2.30) |
| Triopoly market dummy | −1.80 | −2.60 | −2.12 |
| | (2.05) | (2.34) | (2.11) |
| Quadropoly market dummy | −1.80 | −3.36 | −2.53 |
| | | (2.21) | (2.01) |
| Quintopoly market dummy | −1.80 | −1.99 | −2.00 |
| | | (2.22) | (2.01) |
| Urban market dummy | −12.1 | −11.0 | −11.4 |
| | (2.62) | (2.62) | (2.38) |
| Mileage rating | .43 | .38 | .39 |
| | (.05) | (.05) | (.05) |
| County retail wage | 1.00 | .62 | .74 |
| | (.53) | (.53) | (.49) |
| Other dummy variables | Michelin brand | 11 brands | 11 brands |
| Regression $R^2$ | .43 | .51 | |
| $F$ or $\chi^2$ hypothesis tests: | | | |
| $\alpha_1 = \alpha_2$ | .01 | .01 | 1.1 |
| $\alpha_3 = \alpha_4 = \alpha_5$ | .68 | .70 | 2.3 |
| $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5$ | 2.82* | 2.86* | 448* |

## Berry & Waldfogel (RJE, 99)

Authors supplement entry with demand data. Why do this?

1) Recover fixed costs.
2) Endogenize the number of firms (could also think of endogenous "location").

Mankiw-Whinston result: competition could deliver *too much* entry (fixed costs higher than social benefit) from a social perspective.

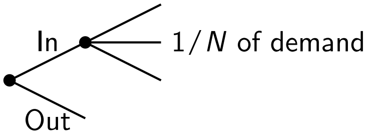
Goal of the paper: how much excess entry is there? I.e., is Mankiw-Whinston borne out by the data?

1) Entrants' fixed costs.
2) How much stations expand listening.
3) How much entry changes ad prices.

Radio stations, 135 MSAs. The number of stations is endogenous. Mean is 18.7.

# BW Model: Listener Demand

Stations are symmetric. Listener decides if to listen and which station to listen to. If there are $N$ stations in the market, each station gets $1/N$ of the agents that decide to listen




The share of each station is given by

$$s_j = \frac{1}{N}\left(\frac{N^{1-\sigma}}{v_0 + N^{1-\sigma}}\right) = S(N)$$

where $v_0$ is the value of the outside good (not listening) and $\sigma$ is the elasticity of substitution between in-market stations.

# BW Model: Listener Demand Cont.

Two particular cases:

1) $\sigma = 1$: entry is pure business stealing, there's no market expansion.
2) $\sigma = 0$: new entrant takes from outside good.

Share of outside good is given by

$$s_0 = 1 - NS(N) = \frac{v_0}{v_0 + N^{1-\sigma}}$$

Using the expressions above we obtain

$$\ln\left(\frac{S_N}{s_0}\right) = \ln v_0 - \sigma \ln N$$

and, letting $\ln v_0 = x\beta + \xi$,

$$\ln\left(\frac{s_N}{s_0}\right) = x_k\beta + \sigma \ln N + \xi_k$$

# BW Model Cont.

**Advertisers' inverse demand** (for listeners) is assumed to be given by

$$\ln p_k = x_k\gamma - \eta \ln(\underbrace{N_k s_k}_{1-s_0}) + \omega_k$$

**Entry Condition.**

Like in B-R, heterogeneity only in fixed costs across markets.

$N_k$ stations are observed if

$$\pi_k(N_k) \geq 0 \geq \pi_k(N_k + 1)$$

where profits are given by

$$\pi_k(N) = \underbrace{M_k}_{\text{market size}} \underbrace{p_k(N_k)S(N_k)}_{\text{from above}} - F_k$$

# BW Model Cont.

BW also impose

$$\ln F_k = x_k \cdot \mu + \lambda v_k$$

where $v_k \sim N(0,1)$

This leads to a ordered probit (as in B-R). Estimation can be done as a system or separately.

# BW Estimation Results

Parameter estimates

- $\hat{\sigma} \approx 0.8 \Rightarrow$ mostly business stealing
- $\hat{\eta} \approx 0.5 \Rightarrow$ elasticity $\approx 2$

Table 4: Comparison of Free Entry, Optimal and Monopoly in aggregate
Table 5: Simulation for selected markets

**TABLE 4** **Comparison of Free Entry, Optimality, and Monopoly**

| | Free Entry | Optimal | Monopoly |
|---|---|---|---|
| In-metro entry | 2,509 | 649 | 341 |
| | | (46) | (55) |
| Aggregate costs ($ millions) | 5,007 | 1,144 | 602 |
| | (3) | (92) | (101) |
| Aggregate revenue ($ millions) | 5,100 | 4,334 | 3,959 |
| | | (204) | (173) |
| Welfare ($ millions) | 5,331 | 7,640 | 7,422 |
| | (3,064) | (3,037) | (2,878) |
| Ad price | 277 | 326 | 375 |
| | | (11) | (48) |
| Listening share (%) | 12.91 | 9.28 | 7.53 |
| | | (.19) | (.50) |

The free-entry numbers without standard errors are calculated directly from data. The difference between free entry and optimal welfare has a standard error of 167.

**TABLE 5** **Simulation Results for Selected Markets**

| | Rockford | Jackson | Toledo | Charlotte | San Diego |
|---|---|---|---|---|---|
| Description of city | | | | | |
| Population (millions) | .2 | .3 | .5 | 1.0 | 2.2 |
| Population percentile | 10 | 25 | 50 | 75 | 90 |
| Outside stations | 11 | 0 | 8 | 4 | 4 |
| Number of in-metro stations | | | | | |
| Free entry | 9 | 17 | 15 | 20 | 31 |
| Optimal | 4 | 3 | 5 | 5 | 9 |
| | (.3) | (.5) | (.5) | (.4) | (.5) |
| % In-metro listening | | | | | |
| Free entry | 11.9 | 13.0 | 12.5 | 12.7 | 13.1 |
| Optimal | 8.7 | 9.5 | 8.6 | 8.9 | 9.5 |
| | (.3) | (.3) | (.3) | (.2) | (.2) |
| Revenue ($ millions) | | | | | |
| Free entry | 7.5 | 12.2 | 16.2 | 39.8 | 85.1 |
| Optimal | 6.4 | 10.4 | 13.4 | 33.4 | 72.6 |
| | (.4) | (.5) | (.7) | (1.6) | (3.5) |
| Costs ($ millions) | | | | | |
| Free entry | 7.2 | 11.9 | 15.7 | 38.9 | 83.9 |
| | (.1) | (.0) | (.0) | (.0) | (.0) |
| Optimal | 3.2 | 2.1 | 5.2 | 9.7 | 24.4 |
| | (.2) | (.3) | (.5) | (.9) | (1.4) |
| Welfare ($ millions) | | | | | |
| Free entry | 8.1 | 12.8 | 17.0 | 41.7 | 88.6 |
| | (4.5) | (7.3) | (9.7) | (23.8) | (50.8) |
| Optimal | 9.9 | 19.1 | 21.9 | 58.0 | 122.8 |
| | (4.5) | (7.2) | (9.5) | (23.5) | (50.3) |
| Ad price ($/listener-year) | | | | | |
| Free entry | 286.9 | 282.8 | 252.1 | 303.6 | 292.9 |
| Optimal | 336.1 | 331.6 | 305.8 | 363.3 | 344.8 |
| | (11.9) | (13.9) | (10.3) | (13.1) | (13.1) |

# BW Concluding comments

Bottom line: too many firms.

Are these numbers believable?

**Comments**: Demand structure.

- Also examine different structure: segments by format.
- Same $\sigma$: Could have allowed for different $\sigma$'s.

# Allowing for firm heterogeneity

Up to now, we assumed fixed costs only vary by market. What if fixed costs vary by firm?

$$D_i = \begin{cases} 1 & \text{if } i \text{ enters} \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2$.

$$\pi_i = \begin{cases} 0 & \text{if } D_i = 0 \\ \bar{\pi}_i^M(x, z_i) + \varepsilon_i & \text{if } D_i = 1 \text{ and } D_{3-i} = 0 \\ \bar{\pi}_i^D(x, z_i) + \underbrace{\varepsilon_i}_{\text{firm specific}} & \text{if } D_i = 1 \text{ and } D_{3-i} = 1 \end{cases}$$

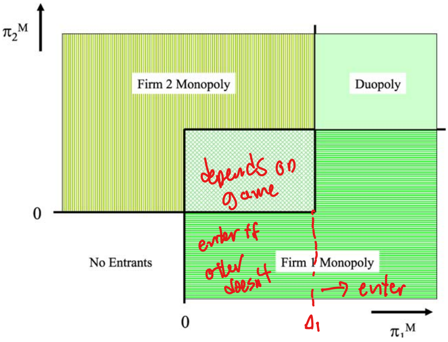
doesn't enter

if firm i enters not both

if both enter

# Firm heterogeneity

Define $\Delta_i := \pi_i^M - \pi_i^D$.




**Implications**

1) Where we are in the figure above depends on $\varepsilon$.
2) "Usual" probit will not work. Why? No game
3) Can compute likelihood but need to deal with the multiple equilibria area.

# Dealing with Multiple Equilibria

1) Focus on feature that is common to all equilibria. E.g.: number of firms.
2) Assume timing.
3) Efficient entry: split center bit.

Equilib. selection rule

4) Estimate selection rule:
   - Tamer (Restud, 03).
   - Bajari-Hong-Ryan.
   - Is the selection rule constant?
   - Identification?
5) Can identify parameters using only $(0, 0)$ and $(1, 1)$ markets.
6) Bounds approach: bound probabilities by allocating the square differently.
   - Tamer (Restud).
   - Ciliberto-Tamer.
7) Incomplete Information.

# Berry, Econometrica 1992

We'll discuss three of the solutions to the multiplicity problem

- We start with Berry (EMA, 1992).
- Focuses on a feature that is common to all equilbria

**Question:** How important is airport presence in determining the profit of operating in a given city pair (a market)?

# Berry, Econometrica 1992

- Heterogeneity in fixed costs only.
- If firm $i$ enters market $k$ it obtains

$$\pi_{ik} = \underbrace{\overset{\text{mkt char}}{x_k\beta} - \overset{\text{firms}}{\delta \ln N} + \overset{\text{mkt level shock}}{\rho\mu_{0k}}}_{v_k(N):\ \text{variable profits}} + \underbrace{z_{ik}\alpha + \overset{\text{unobs}}{\sigma\mu_{ik}}}_{\phi_{ik}:\ \text{Fixed costs}}$$

similar to bres-reese

- Normalize $\rho^2 + \sigma^2 = 1 \Rightarrow \sigma = \sqrt{1-\rho^2}$ (relative importance of the market level and the firm level shock)
- Note that
  (1) Only $N$ matters *not* identity (no differentiation)
  (2) $\delta > 0$. competition decr profits
  (3) Entry is independent across markets.
- Special cases
  (1) $\underset{\text{no middle area}}{\delta = 0}$ and $\underset{\text{no mkt effect}}{\rho = 0} \Rightarrow$ Probit
  (2) $\rho = 0$ and $\delta \neq 0$
  (3) $\rho = 1 \Rightarrow$ ordered probit (B-R).

# Berry, Econometrica 1992

**Claim:** $N_k^*$ is unique.

This is the unique feature used for estimation (go back to figure)

To figure out who enters, order firms according to $\pi_{ik}(N_k^*)$.

(1) Entry according to profitability

(2) Entry according to incumbency.

# Estimation

Slide needs cleaning
"notation is a disaster"

- Estimation based on moment of equilibrium $N_k^*$

$$E[N_k^* - E(N_k^*|w_i, \theta)|data, \ \theta = \theta^*] = 0$$

where

$$E(N_k^*|w_i, \theta) \equiv \bar{N} = \sum_{n=1}^{K_i} nPr(N^* = n)$$

- The main difficulty is to compute the computing integration region (go back to figure)

→ region of int. problem

- Use simulation:
  - draw $\hat{u} \Rightarrow$ get $\hat{\pi} \Rightarrow$ get $\hat{N}$
  - repeat $\tau$ times and average to get $\bar{N}$

# Data

- A market is an city pair
  - Focus on 50 largest metro areas: $50 \cdot 49/2 = 1225$ potential markets.
  - Focus on pairs with non-stop services $\Rightarrow$ 1219 markets
- US 1980 Q3 (Q1 defines incumbents).
- Data source: Origin and destination survey. 10% sample of all tickets sold in the US.
- Market characteristics, $x_k$: population (product of pops), distance
- $z_{ik}$: (city2, city share), where city2 is equal to 1 if firm $i$ serves both cities, zero otherwise.
- Entry.
  - Include connecting service if at least 90 passengers in the route in the quarter.
  - Look at change between Q1 and Q3 for entry/exit.

TABLE I

THE JOINT FREQUENCY DISTRIBUTION OF ENTRY AND EXIT, IN PERCENT OF TOTAL MARKETS SERVED

| | | Number of Exits, as % of Total Markets in the Sample: 0 | 1 | 2 | 3 + | Total |
|---|---|---|---|---|---|---|
| Number | 0 | 68.50 | 10.01 | 1.07 | 0.00 | 79.57 |
| of | 1 | 15.09 | 2.63 | 0.41 | 0.00 | 18.13 |
| Entrants | 2 | 1.96 | 0.25 | 0.00 | 0.00 | 2.05 |
| (as %) | 3 + | 0.16 | 0.08 | 0.00 | 0.00 | 0.24 |
| | Total | 85.56 | 12.96 | 1.48 | 0.00 | 100.00 |

TABLE II

NUMBER AND PERCENTAGE OF MARKETS ENTERED AND EXITED IN THE LARGE CITY SAMPLE, BY AIRLINE

| | Airline | # of Markets Served | # of Markets Entered | # of Markets Exited | % of Markets Entered | % of Markets Exited |
|---|---|---|---|---|---|---|
| 1 | Delta | 281 | 43 | 28 | 15.3 | 10.0 |
| 2 | Eastern | 257 | 33 | 36 | 12.8 | 14.0 |
| 3 | United | 231 | 36 | 10 | 15.6 | 4.3 |
| 4 | American | 207 | 22 | 12 | 10.6 | 5.8 |
| 5 | USAir | 201 | 20 | 17 | 10.0 | 8.5 |
| 6 | TWA | 174 | 22 | 23 | 12.6 | 13.2 |
| 7 | Braniff | 112 | 10 | 20 | 8.9 | 17.9 |
| 8 | Northwest | 75 | 6 | 7 | 8.0 | 9.3 |
| 9 | Republic | 69 | 9 | 6 | 13.0 | 8.7 |
| 10 | Continental | 62 | 9 | 5 | 14.5 | 8.1 |
| 11 | Piedmont | 61 | 14 | 2 | 23.0 | 3.3 |
| 12 | Western | 51 | 6 | 7 | 11.8 | 13.7 |
| 13 | Pan Am | 45 | 1 | 1 | 2.2 | 2.2 |
| 14 | Ozark | 28 | 18 | 4 | 64.3 | 14.3 |
| 15 | Texas Int'l | 27 | 3 | 6 | 11.1 | 22.2 |

## TABLE III

NUMBER OF POTENTIAL ENTRANTS, BY NUMBER OF CITIES SERVED WITHIN A CITY PAIR, WITH NUMBER AND PERCENTAGE ENTERING

| Number of Cities Served | Total # of Potential Entrants | # Entering | % Entering |
|---|---|---|---|
| 0 | 47600 | 4 | 0.01% |
| 1 | 12650 | 45 | 0.36 |
| 2 | 3590 | 232 | 6.46 |

## TABLE IV

### Regression Results for Number of Firms

| Var | Est Parm (Std. Error) | Mean Value of Var. (Std. Dev.) |
|---|---|---|
| $N$ | | 1.629 (1.393) |
| Const | −0.727 (0.097) | — |
| Pop | 2.729 (0.255) | 0.558 (0.114) |
| Dist | −1.591 (0.827) | 1.149 (0.093) |
| $\text{Dist}^2$ | 0.337 (1.850) | 0.022 (0.039) |
| Tourist | 0.134 (0.089) | 0.116 (0.320) |
| City $N2$ | 0.338 (0.011) | 4.574 (2.684) |
| City $N+$ | 0.084 (0.009) | 10.377 (3.656) |

$R$-squared is: 0.612

TABLE V

PROBIT RESULTS[a]

| Variable: | (1) | (2) | (3) |
|---|---|---|---|
| Const | −3.53 | −3.23 | −3.36 |
| | (0.069) | (0.101) | (0.064) |
| Pop | 1.21 | 0.18 | 1.21 |
| | (0.082) | (0.100) | (0.082) |
| Dist | 0.03 | 0.010 | 1.18 |
| | (0.004) | (0.005) | (0.169) |
| $\text{Dist}^2$ | −0.001 | −0.001 | — |
| | (0.0001) | (0.001) | — |
| Tourist | 0.12 | −0.071 | — |
| | (0.044) | (0.050) | — |
| City2 | 2.05 | 1.83 | 2.06 |
| | (0.054) | (0.054) | (0.054) |
| City share | 5.58 | — | 5.47 |
| | (0.164) | — | (0.162) |
| # Routes | — | 0.732 | — |
| | — | (0.018) | — |
| −2 log-likelihood: | 6799 | 6220 | 6834 |

[a]Observations are 18218 firm/market combinations. Standard errors are in parentheses.

TABLE VI

MAXIMUM LIKELIHOOD RESULTS[a]

| Variable | No Heterogeneity | Only Observed Heterogeneity | No Correlation |
|---|---|---|---|
| Constant | 1.00 | −0.973 | −1.54 |
| | (0.056) | (0.485) | (0.815) |
| Population | 4.33 | 4.16 | 4.32 |
| | (0.102) | (0.180) | (0.059) |
| Dist | −0.184 | −0.841 | −0.903 |
| | (0.034) | (0.070) | (0.112) |
| City2 | — | 1.68 | 1.43 |
| | | (0.479) | (0.524) |
| City share | — | 1.20 | −2.94 |
| | | (0.118) | (0.070) |
| $\delta$ | 1.81 | 1.66 | 0.252 |
| | (0.050) | (0.470) | (1.92) |
| −2 log-likelihood: | 3715 | 3619 | 1732 |

Bres-Reese

[a]Observations are 1219 markets. Standard errors are in parentheses.

## TABLE VII

### Simulation Estimates[a]

| Variable | Most Profitable Move First | Incumbents Move First |
| --- | --- | --- |
| Constant | −5.32 | −3.20 |
| | (0.354) | (0.258) |
| Population | 1.36 | 5.28 |
| | (0.239) | (0.343) |
| Dist | 1.72 | −1.45 |
| | (0.265) | (0.401) |
| City2 | 4.89 | 5.91 |
| | (0.295) | (0.149) |
| City Share | 4.73 | 5.41 |
| | (0.449) | (0.206) |
| $\delta$ | 0.527 | 4.90 |
| | (0.119) | (0.206) |
| Weight on umbs → $\rho$ | 0.802 | 0.050 |
| | (0.105) | (0.048) |
| Value of the objective fn: | 33.3 | 26.2 |

[a]Observations are 1219 markets. Standard errors are in parentheses.

prediction is similar

But counterfactual much better

- fine for structural models

TABLE VIII

PREDICTIONS FROM THE MODELS
ACTUAL MEAN NUMBER OF FIRMS: 1.629

| | | Predicted Mean Number of Firms | | |
|---|---|---|---|---|
| | | Probit | No Heterogeneity (Ordered Probit) | Full Model |
| 1. | Actual exog. data | 1.326 | 1.887 | 1.699 |
| 2. | .5 Mil. popl. increase | 10.78 | 2.308 | 1.809 |
| 3. | 500 extra miles dist. | 0.6710 | 1.770 | 1.406 |
| 4. | All entrants "Well-Qual" | 2.724 | — | 2.045 |
| 5. | All firms "Well-Qual" | 6.334 | — | 2.080 |
| 6. | 10% incr. in City Shr | 2.178 | — | 1.697 |
| | | % Correctly Predicted Entrants | | |
| 7. | | 90.29 | — | 92.75 |
| | | Mean Squared Error in the Prediction of $N$: | | |
| 8. | | 1.257 | 2.072 | 1.299 |

# Incomplete Information Models

Up to now we assumed complete information

To (partially) deal with the multiple equilibrium problem we will look at models of incomplete information.

Ex-post profits can be represented as

$$\pi_1(D_1, D_2) = D_1(\pi_1^M + D_2\delta_1 - \varepsilon_1)$$
$$\pi_2(D_1, D_2) = D_1(\pi_2^M + D_1\delta_2 - \varepsilon_2)$$

where $D_i$ is firm $i$'s entry decision.

# Incomplete Information Models

To introduce private information assume beliefs $F_i(\varepsilon_j)$.

$$\begin{aligned} D_1 = 1 \Leftrightarrow D_1(\pi_1^M + p_2^1\Delta_1 - \varepsilon_1) > 0 \\ D_2 = 1 \Leftrightarrow D_2(\pi_2^M + p_1^2\Delta_2 - \varepsilon_2) > 0 \end{aligned}$$

where

$$\begin{aligned} p_2^1 = \mathbb{E}_1[D_2] = F_1(\pi_2^M + p_1^2\Delta_2) \\ p_1^2 = \mathbb{E}_2[D_1] = F_2(\pi_1^M + p_2^1\Delta_1) \end{aligned}$$

are firms' beliefs about its competitor's entry probability.

# Incomplete Information Models

The solution to the system of equations above need not be unique, but it seems that it is with sufficient with enough uncertainty.

**Seim (2006)**

- Models location choices using the model above.
- $N$ entrants deciding which of $L$ locations to locate in.
- Application: video rental stores; locations are Census tracts.

Let

$$\pi_{il}(\bar{n}^i, x_l) = x_l \cdot \beta + \theta_{ll} \sum_{j \neq i}^{N} D_{jl} + \sum_{h \neq l} \theta_{lh} \sum_{k \neq i} D_{kh} + v_{il}$$

where $D_{kh} = 1$ if $k$ enters location $h$ and zero otherwise and $\bar{n}^i = (n_0^i, \ldots, n_L^i)$ denotes the number of competitors in each location ($h_0^i$ denotes no entry).

# Seim (2006)

Because of asymmetric information, a store treats competitors decisions as RVs:

$$\mathbb{E}_D[\pi_{il}] = \underbrace{x_l \cdot \beta + \theta_{ll}(N-1)p_l + \sum_{j \neq l} \theta_{lj}(N-1)p_j}_{\bar{\pi}_l} + v_{il}$$

Assume $v_{il}$ are "logit" error terms $\Rightarrow$ the location choice becomes a logit problem.

# Seim (2006)

To compute the NE:

$$
\begin{aligned}
p_0 &= \frac{1}{1+\sum_{l=1}^{L}\exp(\bar{\pi}_l)} \\
p_1 &= \frac{\exp(\bar{\pi}_1)}{1+\sum_{l=1}^{L}\exp(\bar{\pi}_l)} \\
\vdots &\quad \vdots \\
p_L &= \frac{\exp(\bar{\pi}_L)}{1+\sum_{l=1}^{L}\exp(\bar{\pi}_l)}
\end{aligned}
$$

Look for a fixed point in the probabilities.

**Estimation**

$$
\mathbb{P}(n_0,\ldots,n_L) = N!\prod_{j=0}^{L}\frac{p_j^{n_j}}{n_j!}
$$

Feed this into GMM estimation.

# Moment Inequality Approach

Recall: the problem we had was that there was a region of multiple equilibria $\Rightarrow$ we didn't know where to attribute its mass

Can we proceed without specifying exactly where this mass goes?

Tamer(03) + Ciliberto & Tamer (09) rely on moment inequalities/bounds/set identification to propose the following

# Moment Inequality Approach

$$\mathbb{P}(\underbrace{y}_{\text{structure}}|x) = \int \mathbb{P}(y|x,\varepsilon)dF(\varepsilon)$$

$$= \int_{\underbrace{R_1(\theta,x)}_{\text{y as the unique eqm}}} \mathbb{P}(y|x,\varepsilon)dF(\varepsilon) + \int_{\underbrace{R_2(\theta,x)}_{\text{multiple eqm}}} \mathbb{P}(y|x,\varepsilon)dF(\varepsilon)$$

$$= \int_{R_1(\theta,x)} dF(\varepsilon) + \int_{R_2(\theta,x)} \mathbb{P}(y|x,\varepsilon)dF(\varepsilon)$$

Note that $y$ in the last integral depends on equilibrium selection. It follows that

$$\int_{R_1(\theta,x)} dF(\varepsilon) \leq \mathbb{P}(y|x) \leq \int_{R_1(\theta,x)} dF(\varepsilon) + \int_{R_2(\theta,x)} dF(\varepsilon)$$

# Moment Inequality Approach

From the previous slide,

$$H_1(\theta, x) \leq \mathbb{P}(y|x) \leq H_2(\theta, x) \tag{1}$$

Let the *identified set* be defined by

$$\Theta_I := \{\theta | \text{ condition } (1) \text{ holds } \}$$

Note we're talking about set estimates/identification.

$$\hat{\Theta}_I = \{\theta | nQ_n(\theta) \leq v_n\}$$

where $v_n \to \infty$ and $\frac{v_n}{n} \to 0$ (e.g, $v_n = \ln n$).

# Moment Inequality Approach: Estimation

Define the estimator

$$\hat{\Theta}_I = \{\theta | nQ_n(\theta) \leq v_n\}$$

where $v_n \to \infty$, $\frac{v_n}{n} \to 0$ (e.g, $v_n = \ln n$) and

$$Q_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} [\|(P_n(x_i) - H_1(x_i, \theta))_- \| - \|(P_n(x_i) - H_2(x_i, \theta))_+ \|]$$

where $\| \cdot \|$ is the Euclidean norm and

$$A_- = (a_1 \mathbf{1}(a_1 \leq 0), \ldots, a_k \mathbf{1}(a_k \leq 0))$$
$$A_+ = (a_1 \mathbf{1}(a_1 \geq 0), \ldots, a_k \mathbf{1}(a_k \geq 0))$$

Basically square the violations of condition (1).

## Moment Inequality Approach: Estimation

The goal: choose $\theta$ that sets the value of violations below a certain level

$\hat{\Theta}_I \equiv$ identified set

In many cases $\hat{\Theta}_I \equiv$ will be a singleton.

There's a long econometric literature of inference in these models.