

# Fundamental Notes.

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Particle	spin	charge	type of field	real/complex
$\pi^0$ meson		neutral		real
$\pi^+/\pi^-$ meson		positive/negative		complex
Higgs boson	0.	neutral	Scalar field (rank-0 tensor field)	real
$K^+/K^-$ meson		positive/negative		complex
$K^0$ meson / anti- $K^0$ meson		neutral		complex.
<del>phot</del> electron		negative		
proton		positive	Spinor field (rank-1/2 tensor field)	complex.
neutron	$\frac{1}{2}$	neutral		
$\mu^-$ (muon)		negative		
neutrino		neutral		
photon		neutral	Vector field	real
$W^+/W^-$ boson	1	positive/negative	(rank-1 tensor field)	complex
Z boson		neutral		real
gluon		neutral		real
Graviton	2	neutral	Symmetric (rank-2 tensor field)	real.

# Common Field Lagrangian Densities in QFT

## ① Free scalar field.

spin: 0.

charge: neutral.

field: real-valued  $\phi$ ,

Lagrangian:  $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$

(derived to Klein-Gordon equation).

(examples of particles described:  $\pi^0$  meson, Higgs boson,

## ② Charged scalar field.

spin: 0.

charge: charged.

field: complex-valued  $\phi$ .

Lagrangian:  $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2$

(The complex nature allows for the inclusion of interactions with electromagnetic fields).

(examples of particles described:  $\pi^+/\pi^-$  meson,  
 $K^+/K^-$  meson).

## ③ Dirac spinor field.

spin:  $\frac{1}{2}$ .

charge: neutral / charged.

field:  $\psi$  (a spinor with four complex-valued components).

Lagrangian:  $\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$ .

(derived to Dirac equation).

(examples of particles described: electron, muon, tau lepton, quark).

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## ④ Electromagnetic Field.

spin: 1

charge: neutral.

field: the electromagnetic four-potential  $A^\mu$   
(with four real-valued components).

Lagrangian:  $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$  with  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$   
(particle described: photon).

## ⑤ Yang-Mills Field.

spin: 1

field: Non-Abelian gauge group  $A_\mu^P(x)$ .

Lagrangian:  $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^P F^{\mu\nu}$

where  $F_{\mu\nu}^P = \partial_\mu A_\nu^P - \partial_\nu A_\mu^P + g f^{abc} A_\mu^b A_\nu^c$ .

## ⑥ Gravitational Field.

spin: 2

charge: neutral.

field: metric tensor  $g_{\mu\nu}$  (with real-valued components).

Lagrangian:  $\mathcal{L} = \frac{R}{2\kappa}$ , where  $\kappa = 8\pi G$  and

$R$  is the Ricci scalar  $R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} R^\rho_{\mu\rho\nu}$

$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$  Christoffel  
and  $R^\sigma_{\mu\nu\rho} = \partial_\nu \Gamma^\sigma_{\mu\rho} - \partial_\mu \Gamma^\sigma_{\nu\rho} + \Gamma^\gamma_{\mu\rho} \Gamma^\sigma_{\nu\gamma}$  symbol

$- \Gamma^\gamma_{\nu\rho} \Gamma^\sigma_{\mu\gamma}$ .