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Quantifying Shot Quality in the NBA

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Abstract

In basketball, efficiency is primarily determined by the value from shooting the ball. Effective field goal percentage (EFG) is the advanced metric currently used to evaluate shooting. The problem is that EFG confounds two different properties: the quality of a shot and the ability to make that shot. In this paper, we introduce and propose methodologies for deriving and evaluating two new metrics: (1) Effective Shot Quality (ESQ) and (2) EFG+, which is EFG minus ESQ, a measure of shooting ability above expectation. We discuss how this recharacterizes performance for teams and players, and how this type of analysis can affect analysis beyond shooting.

1 Introduction

The primary challenge for sports teams on offense is to maximize the value of each opportunity when they possess the ball and equivalently to minimize that value when their opponent possesses it. In basketball, while generating free throws, not turning the ball over and rebounding are significant, efficiency is primarily determined by the value from shooting the ball as the vast majority of possessions end in field goal attempts. Dean Oliver in his seminal work categorized shooting the most significant of the Four Factors [1]. Effective field goal percentage (EFG) is the advanced metric currently used to evaluate shooting. It is the rate that field goals are made modified to account for the increased value of a made-three pointer. The problem is that EFG confounds two different properties: the quality of a shot and the ability to make that shot. Consider the following examples.

Is an NBA player who has an effective field goal percentage of 50% a good shooter? If the same shots were taken by an average NBA player and went in at 40%, it would tell one story. If the same shots were taken by an average NBA player and went in at 60% it would tell a different story. This notional "average NBA player" expectation represents what viewers of the game identify as *shot quality*. Intuitively, we can read a play-by-play line saying "Player X misses 20-foot jumper" but understand that not all 20-footers are not created the same. If the player is standing still with no one within 10 feet, that is an entirely different shot than if the player is fading off the dribble with two defenders in his face. NBA staffs understand this as well. Offenses and plays are designed to get "good shots". Coaches can be heard during broadcasts encouraging teams to continue taking the shots they are getting even though they aren't going in or conversely, warning them that they are not generating good opportunities even though their EFG might be high. The challenge is to devise a methodology to appropriately quantify shot quality.

In this paper, we introduce Shot Quality (SQ) and Effective Shot Quality (**ESQ**) as analogues to field goal percentage and effective field goal percentage, which is the value of a shot given that is taken by an "average NBA player". We discuss both methodologies to derive and evaluate these metrics. The methodologies are based on player tracking data from the NBA which has recently adopted the STATS SportVU system league-wide which enables us to consider many more properties of a shot such as shot distance, shot angle, defender distance, defender angle, player speed, player velocity angle and many many more. We leverage a variety of machine learning methods in combination with a mean-square-error / Briar-Score penalty function to quantitatively obtain the best predictors for the likelihood of a shot going in. We can use the results to see which players and teams take the easiest or toughest shots under a variety of contexts.

In addition, we introduce the notion of **EFG+**, which is EFG minus ESQ. This captures how much better than expectation that a player or team shoots. We propose that this is a better evaluation of shooting skill than EFG as it accounts for shot quality. Fundamentally, we are separating shooting into two components: shot quality (ESQ) and shooting ability (EFG+). We first discuss various components that go into calculating shot quality, then discuss the approach by which we derive ESQ and EFG+ and then discuss how these rankings reflects the abilities of teams and players in terms of shooting. We end with discussion on individual player adjustments and how these metrics can be used to evaluate other aspects of the game beyond just shooting.

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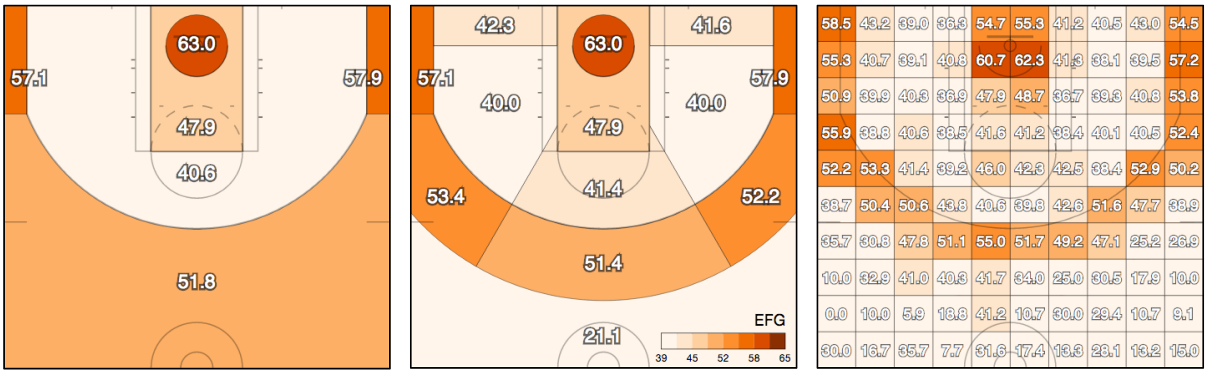
2 Components of Shot Quality

Here we take an initial look at some of the variables that we use to develop Effective Shot Quality (**ESQ**). Effective Field Goal percentage (EFG) essentially makes a simple classification separating shots by their potential value (three pointer vs. two pointer). An obvious first variable is the location of the shot. Below, we show three partitions of court space and show how EFG varies according to various cells within these partition.



The first is a relatively common partition of space into corner-threes, non-corner threes, mid-range, the paint and the circle. The second splits the non-corner three and the mid-range into additional cells and the third is a grid view with square cells. An important note is that the three pictures represent three different models which balance coarseness of the partitions with the sample sizes of shots available for each cell. Finding a way to choose between models is a key problem which we will discuss later. These figures clearly illustrates that shooting abilities of mid-range shooters and three-point shooters should not be judged on the same scale (by EFG) as the baseline levels of performance are significantly different. Why and whether they should be taking those shots is a different question. Researchers such as Kirk Goldsberry have attempted to build shooting metrics around the location of the shot [2].

As we discussed in the introduction, shots from the same location can have significantly different contexts. Let us next consider the distance of the nearest defender when the shot is taken. Sandy Weil did some early work on defender distance with a significantly smaller data set that indicated the value of tight defense [3]. We are now able to look at defender distance in a more fine-grained manner. The figure below shows the EFG of shots by shot distance (vertical axis) and closest defender distance (horizontal axis).

The heatmap indicates the significant and non-linear effects that defender distance can have on the resulting EFG. Let us consider three sets of shot conditions:

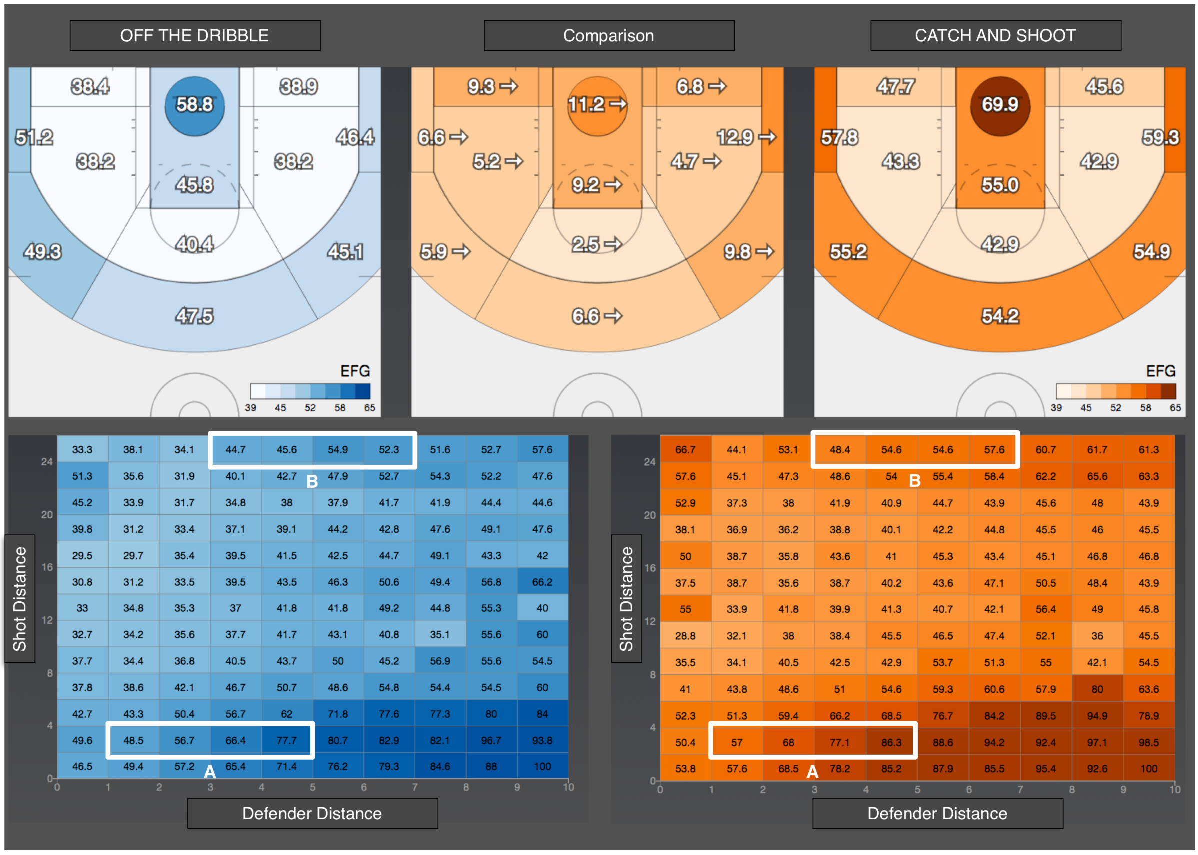
A. Shots close to the basket (2-4 feet) with defenders between 1 and 5 feet away. The marginal value of a foot of defender distance is 9.5% of EFG.

B. Three-pointers with defenders from 3 to 7 feet away. The marginal value of a foot of defender distance is 3.2% of EFG.

C. Mid-range shots from 12-14 feet with defenders from 1 to 7 feet away. The marginal value of a foot of defender distance is 2.0% of EFG.

The sets were chosen simply to illustrate how defender distance can have varying and significant impact at various parts of the court. This also indicates that it is not simply a matter of a shot being "contested" or not but that there is significant marginal value in every foot of space between the shooter and the closest defender.

Finally, we take a look at whether a shot was taken off the dribble or without dribbling, i.e., a catch-and-shoot.



In the first row of the figure, we see that if we compare off-the-dribble to catch-and-shoot by shot location, there is a significant difference in favor of catch-and-shoot at all areas of the court. One hypothesis might be that catch-and-shoots simply incorporate defender distance as they might be taken with more space than shots off the dribble. The second row of the figure above shows that when we compare off-the-dribble (Left-hand side) and catch-and-shoot (Right-hand side) while conditioning on both shot distance and defender distance, catch-and-shoot shots still have a significant advantage. Considering the sets we used earlier, for shots between 2 and 4 feet with defender distance between 1 and 4 feet, catch-and-shoot provides an average margin of 9.8% and for the three-point shot set, with defenders between 3 and 7 feet, it provides an average margin of 4.4%. The results above were illustrative to show how several variables have significant impact on the prediction of a likelihood of a shot being made. With player tracking data, we have the ability to include additional variables such as shot angle, defender angle, player speed, player velocity angle, defender velocity, defender velocity angle and many more. The key challenge is evaluating various models and their ability to appropriately capture shot quality.

3 Shot Quality Model Evaluation

Here, we discuss (1) the model evaluation methodology, (2) how error rates in the evaluation of shot quality impact the ability to analyze shooting and (3) what the error rates tell us in general about the predictability of shooting in general. We present a high-level view here. Details are found in the Appendix. A model **M** takes a feature vector $\mathbf{v_i}$ for a shot **i** and produces a prediction $\mathbf{p_i}$ that is the probability that the shot will go in, i.e., $\mathbf{p_i = M(v_i)}$. The model is learned using a training set and evaluated on a separate test set. We use 10-fold cross-validation in the results presented below. The loss function we use is the mean squared error (MSE), which is also referred to as a Brier Score depending on your academic community [4,5]. The reason for this is that the MSE is minimized when the prediction is the actual likelihood of occurrence. This is not the case for loss functions such as mean absolute error for which predicting the most likely outcome (e.g., a miss for every shot) would lead to the same error as predicting the actual likelihood. The model **M*** that minimizes the MSE using data from all player shots gives us the best estimator of Shot Quality (**SQ**). By making the appropriate adjustment for three-point shots, we can arrive at an Expected Shot Quality (**ESQ**) model.

The models that we considered are several generally applied in the machine learning communities including decision trees (using ID3, M5P), logistic regression, Gaussian process regression among others [6,7,8]. The focus here is not the modeling type as we expect that models will be improved with time but instead the methodology and its consequences. The table below shows various models and their MSE values:

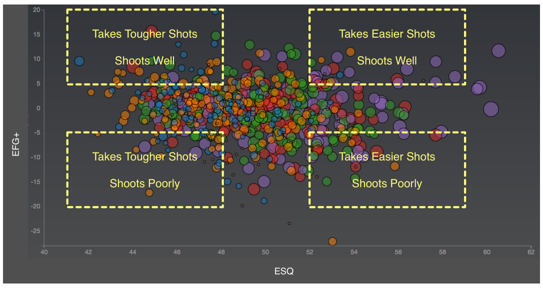
Model	Mean Error	ESQ Error Reduction	Predictability
M1: 0.500 for all shots	0.2500	--	0.500
M2: 0.466 for all shots	0.2488	0.035	0.534
M3: Only using shot distance	0.2373	0.107	0.613
M4: Only using shot distance and defender distance	0.2330	0.066	0.630
M5: All variables	0.2322	0.028	0.633

The models include the static predictions of 0.50 for all shots (M1), and the shot likelihood of the test set, e.g., the league average field goal percentage when using all the data as the test set (M2). The other models are the ones that performed the best with limitations on available variables identified in the table. When going from M4 to M5, the MSE is reduced by 0.0008. The critical question to be answered is: **why should someone care that an improved model can reduce MSE by 0.0008?**

To understand the impact on determining ESQ, consider the following. In the NBA, the range of offensive and defensive efficiencies (points per 100 possessions) is approximately 10 points, with the best teams around 110 and the worst around 100, in the last five years. If one is trying to estimate the shot quality and consequently the efficiency of various strategies, every 1.0 percentage point of error in shot quality estimation is approximately 2.0 points of error in efficiency estimation. A 2.5 percentage point error in shot quality estimation is shifting an average team to the best (or worst) team in the league. Small shifts in ESQ can have big impacts in efficiency estimation.
How does this relate to MSE? Consider the case where the true distribution is estimated incorrectly by some error. This error in estimation corresponds to a change of that error squared in MSE (see Appendix). So a 1.0 percentage point error in estimation (0.01) leads to a 0.0001 change in MSE. A 2.0 percentage point error in estimation (0.02) leads to a 0.0004 change in MSE. Small improvements in MSE correspond to big improvements in the value of the model. Given NBA efficiency ranges, even an improvement of 0.0001 in MSE is significant. Let us go back to the table above and see how the models improved our ability to predict. The "ESQ Error Reduction" column shows how much the model in that row reduced error in ESQ compared to the model in the row above it. Adding the shot distance had a big impact as did adding defender distance. We see the difference between the full model (M5) and (M4) is a 2.8 percentage point improvement. So why should care than an improved model can reduce MSE by 0.0008? **It is the difference in confusing an average offense with the best or worst offense in the league**.

The analysis above outlines an approach to generating and evaluating models for ESQ and demonstrates why seemingly trivial improvements have significant impact on evaluation. We can also evaluate the predictability of shooting in general using this approach. The "Predictability" column in the table above is the value of the Bernoulli distribution that yields the same MSE as the model when predicted optimally. Intuitively, it is the field goal percentage of a collection of shots, with no information other than make/miss, that is equivalently predictable as a given model. We see that the predictability goes from 53.4% for an informed static model (M2) to 63.3% for M5. We surmise that it will be difficult to get a model with predictability above 65%. This seems intuitive because while there are open layups and last-second heaves, very few shots in the NBA go in at higher than a 65% rate or lower than a 35% rate. Even if this is the case, that still leaves a lot of room to improve Effective Shot Quality (**ESQ**).

Given ESQ, we can then calculate **EFG+ = EFG – ESQ**, which is a measure of how well a player or team shoots above or below expectation as determined by the quality of shots that they are taking. By decomposing shooting into ESQ and EFG+, we now have a way of quantifying (1) the quality of shots that a team or player is generating and (2) their skill in hitting those shots, which were previously confounded in EFG. When the number of samples is sufficiently high, EFG+ indicate that a player or team has a particular skill in being able to shoot a particular type of shot beyond an average NBA player who takes those types of shot. The figure on the right shows that this decomposition yields a wide spread for players in terms of shot quality and ability even across positions (indicated by circle color).



4 ESQ and EFG+ for NBA Teams

We applied the full model (M5) to shots taken by NBA teams in the 2013-2014 NBA season through early January 2014. The tables below show values for offense on the left and defense on the right. The first, second and third row of tables show the top ten teams ranked by EFG, ESQ and EFG+, respectively.

OFFENSE

	EFG	ESQ	EFG+
Heat	56.5	51.1	5.4
Spurs	53.9	50.5	3.5
Rockets	53.5	51.5	2.0
Mavericks	52.0	48.8	3.2
Warriors	52.0	49.2	2.8
Suns	51.8	49.9	1.9
Trail Blazers	51.7	48.5	3.2
Thunder	51.5	48.6	2.9
Clippers	51.3	49.7	1.6
Hawks	51.2	50.0	1.2

	EFG	ESQ	EFG+
Rockets	53.5	51.5	2.0
Heat	56.5	51.1	5.4
Spurs	53.9	50.5	3.5
Hawks	51.2	50.0	1.2
Timberwolves	48.3	50.0	-1.7
76ers	48.9	50.0	-1.1
Nuggets	49.7	49.9	-0.2
Suns	51.8	49.9	1.9
Clippers	51.3	49.7	1.6
Pistons	48.8	49.5	-0.8

	EFG	ESQ	EFG+
Heat	56.5	51.1	5.4
Spurs	53.9	50.5	3.5
Trail Blazers	51.7	48.5	3.2
Mavericks	52.0	48.8	3.2
Thunder	51.5	48.6	2.9
Warriors	52.0	49.2	2.8
Rockets	53.5	51.5	2.0
Suns	51.8	49.9	1.9
Nets	49.4	47.7	1.7
Clippers	51.3	49.7	1.6

DEFENSE

	EFG	ESQ	EFG+
Pacers	44.5	47.2	-2.8
Warriors	46.8	48.2	-1.4
Bulls	47.1	47.7	-0.5
Thunder	47.1	49.5	-2.4
Nuggets	48.4	48.6	-0.2
Rockets	48.5	49.0	-0.5
Suns	48.5	49.1	-0.6
Celtics	48.9	49.2	-0.3
Clippers	48.9	49.2	-0.4
Bobcats	48.9	47.6	1.3

	EFG	ESQ	EFG+
Pacers	44.5	47.2	-2.8
Spurs	49.0	47.4	1.6
Bobcats	48.9	47.6	1.3
Bulls	47.1	47.7	-0.5
Magic	49.7	48.1	1.5
Warriors	46.8	48.2	-1.4
Hawks	50.2	48.4	1.9
Trail Blazers	49.4	48.5	0.9
Nuggets	48.4	48.6	-0.2
Wizards	51.4	48.8	2.5

	EFG	ESQ	EFG+
Pacers	44.5	47.2	-2.8
Thunder	47.1	49.5	-2.4
Warriors	46.8	48.2	-1.4
Lakers	49.7	50.5	-0.8
Suns	48.5	49.1	-0.6
Rockets	48.5	49.0	-0.5
Bulls	47.1	47.7	-0.5
Clippers	48.9	49.2	-0.4
Celtics	48.9	49.2	-0.3
Nuggets	48.4	48.6	-0.2

We see that on offense, the Heat, Spurs and Rockets form the top tier in EFG. This is because they are the top three in ESQ and also top teams in EFG+, with the Heat and Spurs being the top two EFG+ teams in the league. The Rockets are 7th in EFG+ but are the best in the league in terms of ESQ. These teams both take good shots and have the players who can make those shots better than average NBA players taking those shots. There is another cluster of four teams (Trail Blazers, Mavericks, Thunder and Warriors) who have EFG+ between 2.8 and 3.2 indicating that they have talented shooters on their team, however, they take worse shots than the first tier. There is another cluster of two teams (Suns and Clippers) who achieve good EFG with a balance of taking good shots and making them at slightly higher than league average rates. We also see from the ESQ Offensive rankings that the Timberwolves and 76ers are generating good shots but are converting them at a poor rate. On defense, we see that the Pacers are in a class by themselves as they give up the toughest shots and also keep offenses below expectation to the largest degree. The Thunder are almost as good at the Pacers at keeping offenses shooting below expectation (EFG+ = -2.4), however they give up much easier shots, ranking in the lower half of the league with a defensive ESQ of 49.5. The Warriors and Bulls are also solid in both dimensions ranking in the top 7 in terms of the quality of shots that they give up and keeping offenses from shooting under expectation. The Spurs, Bobcats and Magic are top 5 teams in terms of the quality of shots that they give up but teams shoot above expectation when they play them. Interestingly, the Lakers are the fourth best team in terms of keeping shooters below expectation but are hurt by giving up high quality shots. Their 50.4 ESQ on defense is worst in the league.

The analysis shows the value of the decomposition as the rankings for EFG on both offense and defense is composed of teams that can be of various combinations of success in shot quality and ability. This decomposition also helps move towards identifying areas for improvement as ESQ would seem to correlate more with scheme and decision-making and EFG+ more with talent, but these are questions that need significantly more research before conclusions can be drawn.

4 ESQ and EFG+ for NBA Players

Below are tables for the top five players with at least 100 attempts in various categories of shots ranked by EFG+.

	All Shots		
	EFG	ESQ	EFG+
Kyle Korver	63.3	48.6	14.7
Jose Calderon	60.7	47.7	13.1
LeBron James	62.9	50.5	12.5
Andre Iguodala	65.7	53.9	11.8
Marco Belinelli	61.6	49.9	11.7

	Three Pointers		
	EFG	ESQ	EFG+
Kyle Korver	69.6	53.1	16.5
Jose Calderon	67.1	52.7	14.4
Damian Lillard	64.4	50.4	14.0
Spencer Hawes	68.7	56.1	12.5
Kevin Durant	62.1	50.1	12.1

	Mid-Range		
	EFG	ESQ	EFG+
Gerald Green	59.8	42.0	17.8
Kyle Korver	55.9	41.7	14.3
Isaiah Thomas	54.0	41.1	12.9
Marco Belinelli	54.9	42.0	12.9
Damian Lillard	50.0	38.6	11.4

	Paint		
	EFG	ESQ	EFG+
LeBron James	75.6	56.7	19.0
Manu Ginobili	68.1	54.1	14.0
Kevin Durant	62.7	51.6	11.2
Boris Diaw	67.2	56.0	11.2
Al Horford	67.4	56.4	11.0

While EFG+ is highly correlated with EFG, the adjustments made due to ESQ do reveal interesting results. When considering all shots, ESQ values range from 41.9 to 60.3 and EFG+ ranges from -19.8 to 14.7. In the All-Shots list, the big difference from an EFG list is the removal of DeAndre Jordan and Chris Anderson who were #2 and #3. This is because they are #1 and #2 in ESQ – they take the easiest shots in the league. Nevertheless, they both actually shoot well above expectation (5.2 and 6.0, respectively). In the Three-Pointer list, we see that Spencer Hawes and Kevin Durant are performing at roughly the same level above expectation even though Hawes has an EFG that is 6.6 percentage points higher. ESQ shows that Spencer Hawes takes significantly easier three-pointers than Kevin Durant does. In the Mid-Range top five, we see that the 4.9 percentage point difference between Marco Belinelli and Damian Lillard primarily due to the difficulty in the shots that they are taking. In the Paint, we see Kevin Durant appear as #3 in EFG+ as his shots are much tougher than those around him. ESQ and EFG+ also validate Lebron James' domination in the paint, as his high EFG is not explained purely by the ease of shots but mostly by his ability to convert them significantly above expectation.

This type of approach where we separate shooting into shot quality and shooting ability also allows us to more properly value players whose EFGs are undercut by shot quality. A great example of this is Dirk Nowitzki, who ranked 64th in our list with an EFG of 53.9. However, Dirk had the 6th lowest ESQ (44.0) meaning he has been taking very difficult shots. Adjusting for this, Dirk has an EFG+ of 10.0, which is 6th in the league and more reflective of his elite shooting ability.

5 Conclusion

In this paper, we introduce the notions of Effective Shot Quality (ESQ) and EFG+ to separately characterize the difficulty of shots and the ability to make them. We developed methodologies to evaluate the generation of models for these metrics and the impact that they have in shot evaluation. We've shown how teams and players with similar EFG can be decomposed in different ways to gain insight about how they achieve that performance. Key areas for future work include investigation of additional variables that could bring additional predictability to estimating shot quality. One area of great value would be to apply these techniques to build models for individual players. Early results have shown that we can indeed increase accuracy with player specific information, however most players do not have sample sizes sufficiently large to perform such analysis at a large-scale at the current time. The notion of ESQ can expand beyond shooting. ESQ is essentially the value of the instantaneous state of the basketball game assuming the player will shoot at that instant. Thus, ESQ can be used to measure the value of passing and dribbling by applying the metric to the states at the beginning and end of when players get and release the ball. ESQ and EFG+ represent initial forays of the basketball world into establishing baseline expectations for various aspects of the game and judging performance relative to that expectation as opposed to using absolute metrics. This approach has already permeated baseball but spatiotemporal sports such as basketball have much more complexity in establishing these expectation levels. With the NBA having league-wide adoption of player tracking, we anticipate that we are at a tipping point for the development of these expectation models and thus, fundamentally changing the way the sport is analyzed.

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Appendix

Models

Let $M : V \to [0 \quad 1]$ be a model that is a mapping from a feature vector space V to probability values. Thus, $p_i = M(v_i)$, $\forall v_i \in V$ where $p_i \in [0 \quad 1]$ and i denotes the i-th feature vector associated with the i-th shot attempt in a data set. Let $o_i \in \{0, 1\}$ be the outcome associated with the i-th shot attempt where 0 indicates that the shot was missed and 1 indicates that the shot was made.

Loss Function

We define loss function to be the mean square error of the model M with respect to the test set S as:

$$e(M) = \frac{1}{|S|} \sum_{i \in S} (M(v_i) - o_i)^2$$

The best model $M^* \in \mathcal{M}$ where $\mathcal{M}$ is the space of considered models is:

$$M^* = \arg \min_{M in \mathcal{M}}$$

MSE Change for Estimation Error

Let $x_0 = |S_1|/|S|$ where $S_1 = \{i \in S : o_i = 1\}$ be the proportion of the set S where the outcome is 1, indicating a made shot. Let $M_\Delta(v_i) = x_0 + \Delta, \forall i \in S$ and $M_0(v_i) = x_0, \forall i \in S$. Then,

$$\begin{aligned} e(M_\Delta) - e(M_0) &= \left(x_0(1 - (x_0 + \Delta))^2 + (1 - x_0)(x_0 + \Delta)^2\right) \\ &\quad - \left(x_0(1 - x_0)^2 + (1 - x_0)(x_0)^2\right) \\ &= x_0(-2\Delta(1 - x_0) + \Delta^2) + (1 - x_0)(2\Delta x_0 + \Delta^2) \\ &= x_0\Delta^2 + (1 - x_0)\Delta^2 \\ &= \Delta^2 \end{aligned}$$

Predictability Given MSE

Given an MSE value of e_0 we can derive predictability by obtaining the x_0 such that $e(M_0) = e_0$:

$$\begin{aligned} x_0(1-x_0)^2 + (1-x_0)(x_0)^2 &= e_0 \\ (1-x_0)\left(x_0(1-x_0) + x_0^2\right) &= e_0 \\ (1-x_0)x_0 &= e_0 \\ x_0^2 - x_0 + e_0 &= 0 \end{aligned}$$

This is a quadratic in x_0 which we can solve:

$$x_0 = 0.5 \cdot (1 + \sqrt{1-4e_0})$$