1. (Logistic regression): Suppose we have data in pairs (x_i, y_i) for $i = 1, 2, ..., 25$. Conditional on x_i, y_i is Bernoulli with success probability

$$p_i = P[y_i = 1|x_i] = exp(\beta_0 + \beta_1 x_i)/(1 + exp(\beta_0 + \beta_1 x_i))$$

The aim is to compute the maximum likelihood estimate $\hat{\boldsymbol{\beta}}$ of the parameter vector $\boldsymbol{\beta} = (\beta_0, \beta_1)^T$.

The log-likelihood is

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n} [y_i log(p_i) + (1 - y_i) log(1 - p_i)]$$

The data are given below:

```
X values:
1.34 -1.38 -0.19 -0.44  1.90 -0.80  0.91
 0.26  1.37 -1.62 -0.96  1.90
0.99  1.96 -1.57  1.51 -1.61 -1.02 -0.92
 -1.87  1.73 -1.23 -1.24  0.22
1.42

Y values:

 1 0 0 1 1 0 1 1 1 0
 1 1 1 1 1 0 0 0 0 0
 1 0 0 1 0
```

a. Use the function `optim()` to compute $\hat{\boldsymbol{\beta}}$ using initial value (.25,.75).
b. Again, starting with (.25,.75) find the next value when using the Newton-Raphson algorithm.
c. Assume that $\beta_0 = 0$, and plot the likelihood function $L(\beta_1)$ as a function of β_1.
d. Again, assume $\beta_0 = 0$ and compute $\hat{\beta}_1$ using `uniroot()`, a grid search, and by the Newton-Raphson algorithm. You can use the plot in part (c) to find a good initial value.

2. Recall the Poisson changepoint example from section 9.5 of the textbook. The dataset records the number of coal mine explosions that resulted in 10 or more fatalities each year from March 15, 1851 to March 22,1962. It appears that after a certain point, the rate of disasters decreased.

Let Y_i be the number of disasters in year i, where 1851 is year 1. Assume
$Y_i \sim Poisson(\mu)$ for $i = 1, 2, ..., k$
$Y_i \sim Poisson(\lambda)$ for $i = k + 1, ..., n$.
The likelihood function is

$$L(k, \mu, \lambda) = (\prod_{i=1}^{k} \frac{\mu^{y_i} e^{-\mu}}{y_i!})(\prod_{i=k+1}^{n} \frac{\lambda^{y_i} e^{-\lambda}}{y_i!})$$

Compute the maximum likelihood estimate of the parameter vector (k, μ, λ). To get the data in the shape you need, run the following code.

```
library(boot)
y=floor(coal[[1]])
y=tabulate(y)
y=y[1851:length(y)]
```