STAT 428 Spring 2017

- 1. Let the p.d.f. $f(x) = \lambda X^{\lambda-1}$ for $\lambda > 0$ and 0 < x < 1.
- a. Write an R function to draw random samples from f that takes as arguments the sample size n and the parameter λ .
- b. Choose a value of λ and draw a very large sample from f and plot the empirical c.d.f. on the same plot as the c.d.f. $F(x) = \int_0^x f(t)dt$.
- 2. Let $Z_{(k)}$ be the kth order statistic in a sample of size n from a $N(\mu, \sigma^2)$ distribution.
- a. Write an R function to draw samples of size m from the distribution of $Z_{(k)}$. The function should take m (number of replicates of $Z_{(k)}$), n(size of each normal sample), k, μ , and σ^2 as its arguments. You may use rnorm() within the function, and try to avoid using any loops if you can.
- b. For n=100, draw large samples of $Z_{(25)}$, $Z_{(50)}$ and $Z_{(75)}$ when $\mu=0$ and $\sigma^2=1$. Use density() to construct estimates of the p.d.f. of each, and plot them together using different colors to distinguish them.
- 3. Let X have probability mass function $f(x) = p(1-p)^{x-1}$, for $0 and <math>x = 1, 2, 3, \dots$
- a. Write an R function to obtain samples of size n from the p.m.f. f. The function should have n and p as its arguments. Do not use any existing R functions for random variable generation other than $\mathtt{runif}()$ within the function.
- b. Let Y denote the number of Bernoulli trials required to observe the kth success. What is the relationship between Y and X? Generalize part (a) to draw from the distribution of Y, using k as an additional argument.
- 4. Let (X,Y) be a random vector such that X|Y=y is N(y,1) and Y has marginal p.d.f $f(y)=e^{-y}$ for y>0.
- a. Write a function to obtain draws from the marginal p.d.f. of X.
- b. Use a large sample from X to estimate the mean of X (μ) and the variance of X (σ^2), and use density() to estimate the p.d.f of X. Plot the estimated p.d.f. of X alongside the density of a $N(\mu, \sigma^2)$ distribution. Comment on their similarities and differences.
- 5. Rizzo problem 3.3
- 6. Rizzo problem 3.4
- 7. Rizzo problem 3.5
- 8. Rizzo Problem 3.9

Bonus (multivariate extension): Write R code using acceptance-rejection sampling to draw from the p.d.f $f(x,y) = 60x^2y$ for 0 < x < 1, 0 < y < 1, x+y < 1.