**Question 1**

Show that is valid, where is the largest

number of x and y. [3 marks]

{⊤}

if (x > y){

{⊤ ∧ *x* > *y*} If-statement

{*x* = max(*x*,*y*)} Implied

z = x;

{*z* = max(*x*,*y*)} Assignment

} else {

{⊤ ∧ ¬(*x* < *y*)} If-statement

{*y* = max(*x*,*y*)} Implied

z = y;

{*z* = max(*x*,*y*)} Assignment

}

Explanation:

First, If branch, by the Assignment rule we can prove

From by the Implied rule we can prove

the else branch is similar, so we can show

**Question 2**

Show that is valid

1. Write down a proper loop invariant which is useful for constructing the correctness proof. [2 marks]
2. Write down a proper variant which is useful for proving the termination of the program. [1 mark]
3. Provide the full proof using proof rules. [4 marks]
4. Justify the correct uses of the implied rule in three places of the proof in English. [3 marks]

(1). If a > 0

{0 ≤ x}

{ 1\*x!= x! ∧ 0 ≤ x} Imply

a = x;

{ 1\*a! = x! ∧ 0 ≤ a} Assignment

y = 1;

{ y\*a! = x! ∧ 0 ≤ a} Assignment

while (a > 0){

{y\*a! = x! ∧ a > 0 ∧ 0 ≤ a = E0} Invariant Hyp. and guard

{y \* a \*(a-1)! = x! ∧ 0 ≤ a -1 < E0} Implied

y = y ∗ a;

{y\*(a-1)! = x! ∧ 0 ≤ a -1 < E0} Assignment

a = a – 1;

{y\*a! = x! ∧ 0 ≤ a < E0} Assignment

}

{y\*a! = x! ∧ ¬(a > 0)} Total-while

{y = x!} Implied

(2). If a = 0

{0 ≤ x} Implied

a = x = 0;

y = 1;

while (a > 0){

y = y ∗ a;

a = a – 1;

}

{y = 0! = x!} Implied

If a = x = 0, so that we can imply x is greater than or equal to 0. Also, it will not enter the while loop, so it will always terminate. Thus, we can imply y = 1 = 0! = x!.

1. (1) First, from the Invariant Hyp. and guard , as a! equals to , so that y\*a! = x! can imply .

Also, as a is greater than 0, known from the condition of the while loop, and , so that a – 1 is greater than or equal to 0 and also less than E0.

Thus, can imply

(2) From , as is a tautology, so that T ∧ 0 ≤ x can imply the pre-condition .

(3) From , as and , so that a is equal to 0. Because 0! is equal to 1, so that . Thus, can imply .