KOLEJ UNIVERSITI TUNKU ABDUL RAHMAN

FACULTY OF COMPUTING AND INFORMATION TECHNOLOGY

ACADEMIC YEAR 2022/2023

Assignment

MATHEMATICS AAMS1623
CALCULUS AND ALGEBRA

STUDENT'S DECLARATION OF ORIGINALITY
By submitting this assignment, we declare that this submitted work is free from all forms of plagiarism and for all intents and purposes is our own properly derived work. We understand that we have to bear the consequences if we fail to do so.

Program: DFT1
Tutorial Group: 14

Role	Name	ID	Contribution	Signature
Leader	SEM HONG FAI	22WMD07154		hf
Member 1	PO YI MING	22WMD07176		YM
Member 2	PHUA QING HAO	22WMD07147		QH

No :
Date :
Subject :

AAMS1623 Calculus and Algebra.

$-1x^{-2}$

x^{-1}

Q1) a) $y = 2x^2 + \frac{1}{x} + e^3$

$\frac{dy}{dx} = 4x - x^{-2} + 3e^2$ #

b) $y = e^{3x^2} \ln(2x^2 + 3x)$

$u = e^{3x^2}$

$\frac{du}{dx} = e^{3x^2} \times 2(3x^{2-1})$

$= e^{3x^2} \times 6x$

$= 6xe^{3x^2}$

$v = \ln(2x^2 + 3x)$

$\frac{dv}{dx} = \frac{1}{2x^2 + 3x} \times (2(2x) + 1(3))$

$= \frac{1}{2x^2 + 3x} \times (4x + 3)$

$= \frac{4x + 3}{2x^2 + 3x}$

$\frac{dy}{dx} = \left(v \times \frac{du}{dx}\right) + \left(u \times \frac{dv}{dx}\right)$

$= \left(\ln(2x^2 + 3x) \times 6xe^{3x^2}\right) + \left(e^{3x^2} \times \frac{4x + 3}{2x^2 + 3x}\right)$

$= 6x\ln(2x^2 + 3x)e^{3x^2} + \frac{4xe^{3x^2} + 3e^{3x^2}}{2x^2 + 3x}$ #

c) $y = \frac{2x}{x^2 + 1}$

$u = 2x$ $\qquad$ $v = x^2 + 1$

$\frac{du}{dx} = 2$ $\qquad$ $\frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \frac{(v \times \frac{du}{dx}) - (u \times \frac{dv}{dx})}{v^2}$

$= \frac{(x^2 + 1)(2) - 2x(2x)}{(x^2 + 1)^2} = \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2} = \frac{-2x^2 + 2}{(x^2 + 1)^2}$ #

d) $y = (2ax^2 + c)^2 (bx - cx)^{-1}$

✓ $u = (2ax^2 + c)^2$ $\qquad$ ✓ $v = (bx - cx)^{-1}$

chain rule → $u = 2ax^2 + c$ $\quad$ $y = u^2$ $\qquad$ chain rule → $u = bx - cx$ $\quad$ $y = u^{-1}$

$\frac{du}{dx} = 2a \times 2x$ $\quad$ $\frac{dy}{du} = 2u$ $\qquad$ $\frac{du}{dx} = b - c$ $\quad$ $\frac{dy}{du} = -u^{-2}$

✓ $\frac{du}{dx} = 2(2ax^2 + c) \times 2a \times 2x$ $\qquad$ ✓ $\frac{dv}{dx} = -(bx - cx)^{-2}(b - c)$

Product Rule → $\frac{dy}{dx} = \left[v \times \frac{du}{dx}\right] + \left[u \frac{dv}{dx}\right]$

$= \left[(bx - cx)^{-1} \times 2(2ax^2 + c) \times 2a \times 2x\right] + \left[(2ax^2 + c)^2 \times (-(bx - cx)^{-2}(b - c))\right]$

e) $y = \dfrac{3bx + ac}{\sqrt{ax}}$

$u = 3bx + ac$ $\qquad v = (ax)^{\frac{1}{2}}$

$\dfrac{du}{dx} = 3b$ $\qquad \dfrac{dv}{dx} = \dfrac{1}{2}(ax)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = (ax)^{\frac{1}{2}}(3b) - (3bx + ac)\frac{1}{2}(ax)^{-\frac{1}{2}}(a)$$

$$= \frac{3b\sqrt{ax} - (3bx + ac)\frac{a}{2\sqrt{ax}}}{((ax)^{\frac{1}{2}})^2}$$

$$\frac{dy}{dx} = \frac{6abx - a(3bx - ac)}{2ax\sqrt{ax}}$$

#

f) $y = \sin^2(ax + b)$

$u = \sin(ax + b)$ $\qquad y = u^2$

$\dfrac{dy}{dx} = a(\cos)(ax + b)$ $\qquad \dfrac{dy}{dx} = 2u$

$\qquad\qquad = 2[\sin(ax + b)]$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= a\cos(ax + b) \times 2\sin(ax + b)$$

$$= \sin(2ax + b)\,a$$

#

2) $x = 2t \quad , y = t^4 + 1 \quad ; t = 1 \quad \Rightarrow x = 2 \quad , y = 2$

$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = 4t^3$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= 4t^3 \times \frac{1}{2}$

$= 2t^3$

$= 2(1)^3$

$= 2$

$y - 2 = 2(x - 2)$

$y - 2 = 2x - 4$

$y = 2x - 2$ equation of tangent.

$M_{nor} \cdot 2 = -1$

$M_{nor} = \frac{-1}{2}$

$y - 2 = \frac{-1}{2}(x - 2)$

$y - 2 = \frac{-1}{2}x + 1$

$y = -\frac{1}{2}x + 3$ equation of normal.

Q3) $y = x\sqrt{x+1}$

$= x(x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = x\left(\frac{1}{2\sqrt{x+1}}\right) + (x+1)^{\frac{1}{2}}$

Gradient of the curve $= 3\left(\frac{1}{2\sqrt{3+1}}\right) + (3+1)^{\frac{1}{2}}$

$= \frac{3}{4} + \frac{2}{1}$

$= \frac{11}{4}$

$\frac{dy}{dx} = 0$

$x\left(\frac{1}{2\sqrt{x+1}}\right) + \sqrt{x+1} = 0$

$\frac{x}{2\sqrt{x+1}} + \sqrt{x+1} = 0$

$x + 2(x+1) = 0$

$x + 2x + 2 = 0$

$3x = 2$

$x = \frac{2}{3}$ #

4) $y = (2x+1)e^{-2x}$

$u = 2x+1 \qquad v = e^{-2x}$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = -2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(2) + (2x+1)(-2e^{-2x})$

$= (2-4x-2)^{-2x}$

$\frac{dy}{dx} = -4xe^{-2x}$

Stationary point $= \frac{dy}{dx} = 0$

$-4xe^{-2x} = 0$

$x = 0$

$y = (2(0)+1)e^{-2(0)} = 1$

Determine $-4xe^{-2x}$

$u = -4x \qquad v = e^{-2x}$

$\frac{du}{dx} = -4 \qquad \frac{dv}{dx} = -2e^{-2x}$

$\frac{d^2y}{dx^2} = (e^{-2x})(-4) + (-4x)(-2e^{-2x})$

$= (-4+8x)e^{-2x}$

$= (-4+8(0))e^{-2(0)}$

$= -4$

$\therefore \frac{dy}{dx} = -4xe^{-2x}$, maximum $(0,1)$, $\frac{d^2y}{dx^2} < 0$ #

(Q5) $\frac{dr}{dt} = 1.5 \qquad r = 5$

$\frac{dv}{dt} = ? \qquad \frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2(1.5)$

$= 6\pi r^2$

$= 6\pi(5)^2$

$= 150\pi$ cm/s #

6) $400\pi = \pi r^2 h$

$$h = \frac{400\pi}{\pi r^2}$$

$$= \frac{400}{r^2}$$

$$A = \pi r^2 + 2\pi r\left(\frac{400}{r^2}\right)$$

$$= \pi r^2 + 800\pi r^{-1}$$

$$\frac{dA}{dr} = 2\pi r - 800\pi r^{-2}$$

$$0 = \frac{2\pi r^2 - 800\pi}{r^2}$$

$$2\pi r^3 - 800\pi = 0$$

$$800\pi = 2\pi r^3$$

$$r^3 = \frac{800\cancel{\pi}}{2\cancel{\pi}}$$

$$r = \sqrt[3]{400}$$

$$= 7.368$$

$$A = \pi(\sqrt[3]{400})^2 + 800\pi(\sqrt[3]{400})^{-1}$$

$$= \pi(20\sqrt[3]{20}) + \pi(40\sqrt[3]{20})$$

$$= 60\pi\sqrt[3]{20}$$

$$= 162.865\pi$$

$\therefore$ $r = 7.368$, Minimum surface area is 162.865π. #

7) Given : $\frac{dx}{dt} = 0.05$ cm/s

$V = x^2 h$

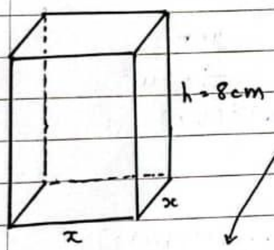
$V = 8x^2$

$\frac{dv}{dx} = 16x$

$\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$

$= 16x \times 0.05$

$= 0.8x$ cm³/s



Surface Area $= 2x^2 + 4xh$

$A = 2x^2 + 4(8)x$

$210 = 2x^2 + 32x$

$2x^2 + 32x - 210 = 0$

$(x-5)(x+21) = 0$

$x = 5 \quad / x = -21$ (ignore)

$\frac{dA}{dx} = 4x + 32$

$\frac{dA}{dx} = 4(5) + 32$

$= 52$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$= 52 \times 0.05$

$= 2.6$ cm²/s . #