Differentiation

Thursday, October 6, 2022 3:36 PM

Power Rule

$$\frac{d}{dx}(2x^{3-1}) = 3(2x^2)$$

$$\frac{d}{dx}(14x^{3-1} - 6x^2 + 2)$$
$$= 3(14)x^{3-1} - 6(2)x^{2-1} + 0$$
$$= 42x^2 - 12x \quad \#$$

Product rule

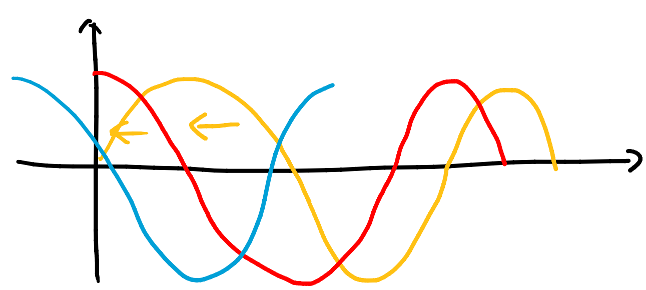
$$\frac{d}{dx}((x-1)(x+3))$$
$$= (x-1)\frac{d}{dx}(\cancel{x}^{1-1} + \cancel{3}^{0}) + (x+3)\frac{d}{dx}(x-1)$$
$$= (x-1)1 + (x+3)1$$
$$= x-1 + (x+3)$$
$$= 2x+2$$

$$\frac{d}{dx}e^{2x} = \overbrace{(2x)'}^{\text{derivative of } ax}e^{2x}$$
$$= 2e^{2x}$$

Eg. $2\boxed{\cos(x)}\boxed{e^{2x}}$

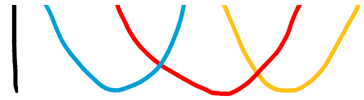
$$\frac{d}{dx}\underset{\text{coefficient}}{2}\boxed{\cos(x)}\boxed{e^{2x}} = 2\frac{d}{dx}\left[\cos(x)e^{2x}\right]$$
$$= 2\left[\cos(x)\frac{d}{dx}e^{2x} + e^{2x}\frac{d}{dx}\cos(x)\right]$$
$$= 2\left[\cos(x)(2e^{2x}) + e^{2x}(-\sin(x))\right]$$
$$= 2\left[2\cos(x)e^{2x} - e^{2x}\sin(x)\right]$$
$$= 4e^{2x}\cos(x) - 2e^{2x}\sin(x) \quad \#$$

$$\frac{d}{dx}\sin(x) = \cos x$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}-\sin(x) = -\cos(x)$$



$$\frac{d}{dx} -\sin(x) = -\cos(x)$$

$$\frac{d}{dx} -\cos(x) = -(-\sin(x)) = \sin(x)$$



Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

$$\frac{d}{dx}\left(\frac{\boxed{6x^2}}{\boxed{2-x}}\right) = \frac{(2-x)(6x^2)' - 6x^2(\overset{0}{\cancel{2}}-\overset{1}{\cancel{x}})'}{(2-x)^2}$$

$$= \frac{(2-x)(12x) - 6x^2(-1)}{(2-x)^2}$$

$$= \frac{24x - 12x^2 + 6x^2}{(2-x)^2} = \frac{24x - 6x^2}{(2-x)^2} \quad \#$$

If $f(x) = \dfrac{1+5x}{\ln 5x}$, Find derivative of $f(x)$.

$$\frac{d}{dx}\left(\frac{1+5x}{\ln 5x}\right) = \frac{(\ln 5x)\frac{d}{dx}(1+5x) - (1+5x)\frac{d}{dx}\ln 5x}{(\ln 5x)^2}$$

$$= \frac{(\ln 5x)5 - (1+5x)\frac{1}{x}}{(\ln 5x)^2}$$

$$\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)}(f(x))'$$

$$\frac{d}{dx}\ln\left(\boxed{5x}\right) = \frac{1}{\boxed{5x}}(5x)' = \frac{5}{5x} = \frac{1}{x}$$

Q3 (a)i. $y = (x^2+3x-4)^{3-1}$

$$\frac{dy}{dx} = 3(x^2+3x-4)^2 \frac{d}{dx}(x^2+3x-4)$$
$$= 3(x^2+3x-4)^2(2x+3) \quad \#$$

(a)ii. $y = \ln\left(\frac{2x+3}{x-5}\right)$

$$\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)}(f(x))'$$

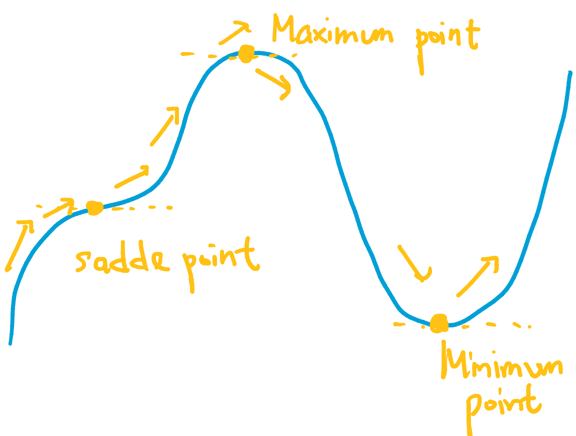
$$\frac{dy}{dx} = \frac{d}{dx}\ln\left(\frac{2x+3}{x-5}\right)$$
$$= \frac{1}{\left(\frac{2x+3}{x-5}\right)} \frac{d}{dx}\left(\frac{2x+3}{x-5}\right)$$
$$= \frac{x-5}{2x+3} \cdot \frac{(x-5)\frac{d}{dx}(2x+3) - (2x+3)\frac{d}{dx}(x-5)}{(x-5)^2}$$
$$= \frac{x-5}{2x+3} \cdot \frac{(x-5)2 - (2x+3)1}{(x-5)^2}$$
$$= \frac{\cancel{x-5}}{2x+3} \cdot \frac{\cancel{2x}-10-\cancel{2x}-3}{(x-5)^{\cancel{2}}}$$
$$= -\frac{13}{(2x+3)(x-5)} \quad \#$$

(a) iii.

(a) iii. (skip).

(b). $y = \frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x + 1$

$\frac{dy}{dx} = x^2 - 5x - 6$



Stationary point $\Rightarrow$ when $\frac{dy}{dx} = 0$

When $\frac{dy}{dx} = 0$, $x^2 - 5x - 6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ or $x = 2$ (Critical points)

$\frac{d^2y}{dx^2} = 2x - 5$

Max point, $\frac{d^2y}{dx^2} < 0$

Min point, $\frac{d^2y}{dx^2} > 0$

As $x = 3$, $\frac{d^2y}{dx^2} = 2(3) - 5 = 6 - 5 = 1 > 0$ (Minimum point)

As $x = 2$, $\frac{d^2y}{dx^2} = 2(2) - 5 = 4 - 5 = -1 < 0$ (Max point).

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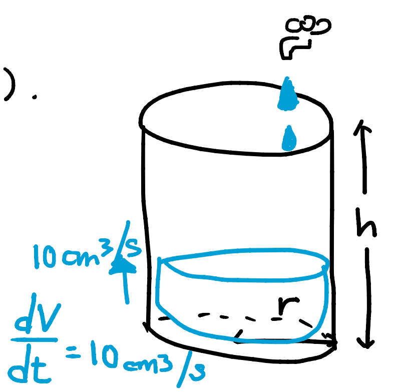
(c).



Base Area $= 25\pi$

Volume, V = Base Area $\times$ Height

$= 25\pi(h)$

$V = 25\pi h \Rightarrow$ Change in term of V

$h = \frac{V}{25\pi}$

$\frac{dh}{dV} = \frac{1}{25\pi}\frac{d}{dV}V$

$\frac{dV}{dt} = 10 \text{ cm}^3/s$

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$$\frac{dh}{dV} = \frac{1}{25\pi}\frac{d}{dV}V$$

$$\boxed{\frac{dh}{dV} = \frac{1}{25\pi}}$$

Find, $\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$ [Chain Rule]

$$= \frac{1}{25\pi} \cdot 10 = \frac{10}{25\pi} = \frac{2}{5\pi} \text{ cm/s}$$

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