APPENDIX

Table VIII: First part of the group of quorums with all-pairs property for $p = 112$: This group of quorums are determined using heuristic method by decomposing by $112 = 7 \times 16$. Constructing first layer with $p_1 = 7, n_1 = 3$, an interest set $\{0, 1, 3\}$ and second layer with $p_2 = 16, n_2 = 5$, an interest set $\{0, 1, 2, 5, 8\}$, the final quorum size is $n = n_1 \times n_2 = 15$, which is a very competitive value as the quorum size of $p = 111$ is 12.

Qrorums
$S_0 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 15, 16, 17, 24, 25, 26\}$
$S_1 = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 18, 19, 20, 27, 28, 29\}$
$S_2 = \{6, 7, 8, 9, 10, 11, 12, 13, 14, 21, 22, 23, 30, 31, 48\}$
$S_3 = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 24, 25, 26, 49, 50, 51\}$
$S_4 = \{12, 13, 14, 15, 16, 17, 18, 19, 20, 27, 28, 29, 52, 53, 54\}$
$S_5 = \{15, 16, 17, 18, 19, 20, 21, 22, 23, 30, 31, 48, 55, 56, 57\}$
$S_6 = \{18, 19, 20, 21, 22, 23, 24, 25, 26, 49, 50, 51, 58, 59, 60\}$
$S_7 = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 52, 53, 54, 61, 62, 63\}$
$S_8 = \{24, 25, 26, 27, 28, 29, 30, 31, 48, 55, 56, 57, 0, 1, 2\}$
$S_9 = \{27, 28, 29, 30, 31, 48, 49, 50, 51, 58, 59, 60, 3, 4, 5\}$
$S_{10} = \{30, 31, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 6, 7, 8\}$
$S_{11} = \{49, 50, 51, 52, 53, 54, 55, 56, 57, 0, 1, 2, 9, 10, 11\}$
$S_{12} = \{52, 53, 54, 55, 56, 57, 58, 59, 60, 3, 4, 5, 12, 13, 14\}$
$S_{13} = \{55, 56, 57, 58, 59, 60, 61, 62, 63, 6, 7, 8, 15, 16, 17\}$
$S_{14} = \{58, 59, 60, 61, 62, 63, 0, 1, 2, 9, 10, 11, 18, 19, 20\}$
$S_{15} = \{61, 62, 63, 0, 1, 2, 3, 4, 5, 12, 13, 14, 21, 22, 23\}$
$S_{16} = \{16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 40, 41, 42\}$
$S_{17} = \{19, 20, 21, 22, 23, 24, 25, 26, 27, 34, 35, 36, 43, 44, 45\}$
$S_{18} = \{22, 23, 24, 25, 26, 27, 28, 29, 30, 37, 38, 39, 46, 47, 64\}$
$S_{19} = \{25, 26, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 65, 66, 67\}$
$S_{20} = \{28, 29, 30, 31, 32, 33, 34, 35, 36, 43, 44, 45, 68, 69, 70\}$
$S_{21} = \{31, 32, 33, 34, 35, 36, 37, 38, 39, 46, 47, 64, 71, 72, 73\}$
$S_{22} = \{34, 35, 36, 37, 38, 39, 40, 41, 42, 65, 66, 67, 74, 75, 76\}$
$S_{23} = \{37, 38, 39, 40, 41, 42, 43, 44, 45, 68, 69, 70, 77, 78, 79\}$
$S_{24} = \{40, 41, 42, 43, 44, 45, 46, 47, 64, 71, 72, 73, 16, 17, 18\}$
$S_{25} = \{43, 44, 45, 46, 47, 64, 65, 66, 67, 74, 75, 76, 19, 20, 21\}$
$S_{26} = \{46, 47, 64, 65, 66, 67, 68, 69, 70, 77, 78, 79, 22, 23, 24\}$
$S_{27} = \{65, 66, 67, 68, 69, 70, 71, 72, 73, 16, 17, 18, 25, 26, 27\}$
$S_{28} = \{68, 69, 70, 71, 72, 73, 74, 75, 76, 19, 20, 21, 28, 29, 30\}$
$S_{29} = \{71, 72, 73, 74, 75, 76, 77, 78, 79, 22, 23, 24, 31, 32, 33\}$
$S_{30} = \{74, 75, 76, 77, 78, 79, 16, 17, 18, 25, 26, 27, 34, 35, 36\}$
$S_{31} = \{77, 78, 79, 16, 17, 18, 19, 20, 21, 28, 29, 30, 37, 38, 39\}$
$S_{32} = \{32, 33, 34, 35, 36, 37, 38, 39, 40, 47, 48, 49, 56, 57, 58\}$
$S_{33} = \{35, 36, 37, 38, 39, 40, 41, 42, 43, 50, 51, 52, 59, 60, 61\}$
$S_{34} = \{38, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 62, 63, 80\}$
$S_{35} = \{41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 81, 82, 83\}$
$S_{36} = \{44, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 84, 85, 86\}$
$S_{37} = \{47, 48, 49, 50, 51, 52, 53, 54, 55, 62, 63, 80, 87, 88, 89\}$
$S_{38} = \{50, 51, 52, 53, 54, 55, 56, 57, 58, 81, 82, 83, 90, 91, 92\}$
$S_{39} = \{53, 54, 55, 56, 57, 58, 59, 60, 61, 84, 85, 86, 93, 94, 95\}$
$S_{40} = \{56, 57, 58, 59, 60, 61, 62, 63, 80, 87, 88, 89, 32, 33, 34\}$
$S_{41} = \{59, 60, 61, 62, 63, 80, 81, 82, 83, 90, 91, 92, 35, 36, 37\}$
$S_{42} = \{62, 63, 80, 81, 82, 83, 84, 85, 86, 93, 94, 95, 38, 39, 40\}$
$S_{43} = \{81, 82, 83, 84, 85, 86, 87, 88, 89, 32, 33, 34, 41, 42, 43\}$
$S_{44} = \{84, 85, 86, 87, 88, 89, 90, 91, 92, 35, 36, 37, 44, 45, 46\}$
$S_{45} = \{87, 88, 89, 90, 91, 92, 93, 94, 95, 38, 39, 40, 47, 48, 49\}$
$S_{46} = \{90, 91, 92, 93, 94, 95, 32, 33, 34, 41, 42, 43, 50, 51, 52\}$
$S_{47} = \{93, 94, 95, 32, 33, 34, 35, 36, 37, 44, 45, 46, 53, 54, 55\}$
$S_{48} = \{48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 72, 73, 74\}$
$S_{49} = \{51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 75, 76, 77\}$
$S_{50} = \{54, 55, 56, 57, 58, 59, 60, 61, 62, 69, 70, 71, 78, 79, 96\}$

Table IX: Second part of the group of quorums with all-pairs property for $p = 112$

Qrorums
$S_{51} = \{57, 58, 59, 60, 61, 62, 63, 64, 65, 72, 73, 74, 97, 98, 99\}$
$S_{52} = \{60, 61, 62, 63, 64, 65, 66, 67, 68, 75, 76, 77, 100, 101, 102\}$
$S_{53} = \{63, 64, 65, 66, 67, 68, 69, 70, 71, 78, 79, 96, 103, 104, 105\}$
$S_{54} = \{66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 106, 107, 108\}$
$S_{55} = \{69, 70, 71, 72, 73, 74, 75, 76, 77, 100, 101, 102, 109, 110, 111\}$
$S_{56} = \{72, 73, 74, 75, 76, 77, 78, 79, 96, 103, 104, 105, 48, 49, 50\}$
$S_{57} = \{75, 76, 77, 78, 79, 96, 97, 98, 99, 106, 107, 108, 51, 52, 53\}$
$S_{58} = \{78, 79, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 54, 55, 56\}$
$S_{59} = \{97, 98, 99, 100, 101, 102, 103, 104, 105, 48, 49, 50, 57, 58, 59\}$
$S_{60} = \{100, 101, 102, 103, 104, 105, 106, 107, 108, 51, 52, 53, 60, 61, 62\}$
$S_{61} = \{103, 104, 105, 106, 107, 108, 109, 110, 111, 54, 55, 56, 63, 64, 65\}$
$S_{62} = \{106, 107, 108, 109, 110, 111, 48, 49, 50, 57, 58, 59, 66, 67, 68\}$
$S_{63} = \{109, 110, 111, 48, 49, 50, 51, 52, 53, 60, 61, 62, 69, 70, 71\}$
$S_{64} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 15, 64, 65, 72, 73, 74\}$
$S_{65} = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 66, 67, 68, 75, 76, 77\}$
$S_{66} = \{6, 7, 8, 9, 10, 11, 12, 13, 14, 69, 70, 71, 78, 79, 80\}$
$S_{67} = \{9, 10, 11, 12, 13, 14, 15, 64, 65, 72, 73, 74, 81, 82, 83\}$
$S_{68} = \{12, 13, 14, 15, 64, 65, 66, 67, 68, 75, 76, 77, 84, 85, 86\}$
$S_{69} = \{15, 64, 65, 66, 67, 68, 69, 70, 71, 78, 79, 80, 87, 88, 89\}$
$S_{70} = \{66, 67, 68, 69, 70, 71, 72, 73, 74, 81, 82, 83, 90, 91, 92\}$
$S_{71} = \{69, 70, 71, 72, 73, 74, 75, 76, 77, 84, 85, 86, 93, 94, 95\}$
$S_{72} = \{72, 73, 74, 75, 76, 77, 78, 79, 80, 87, 88, 89, 0, 1, 2\}$
$S_{73} = \{75, 76, 77, 78, 79, 80, 81, 82, 83, 90, 91, 92, 3, 4, 5\}$
$S_{74} = \{78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 94, 95, 6, 7, 8\}$
$S_{75} = \{81, 82, 83, 84, 85, 86, 87, 88, 89, 0, 1, 2, 9, 10, 11\}$
$S_{76} = \{84, 85, 86, 87, 88, 89, 90, 91, 92, 3, 4, 5, 12, 13, 14\}$
$S_{77} = \{87, 88, 89, 90, 91, 92, 93, 94, 95, 6, 7, 8, 15, 64, 65\}$
$S_{78} = \{90, 91, 92, 93, 94, 95, 0, 1, 2, 9, 10, 11, 66, 67, 68\}$
$S_{79} = \{93, 94, 95, 0, 1, 2, 3, 4, 5, 12, 13, 14, 69, 70, 71\}$
$S_{80} = \{16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 80, 81, 88, 89, 90\}$
$S_{81} = \{19, 20, 21, 22, 23, 24, 25, 26, 27, 82, 83, 84, 91, 92, 93\}$
$S_{82} = \{22, 23, 24, 25, 26, 27, 28, 29, 30, 85, 86, 87, 94, 95, 96\}$
$S_{83} = \{25, 26, 27, 28, 29, 30, 31, 80, 81, 88, 89, 90, 97, 98, 99\}$
$S_{84} = \{28, 29, 30, 31, 80, 81, 82, 83, 84, 91, 92, 93, 100, 101, 102\}$
$S_{85} = \{31, 80, 81, 82, 83, 84, 85, 86, 87, 94, 95, 96, 103, 104, 105\}$
$S_{86} = \{82, 83, 84, 85, 86, 87, 88, 89, 90, 97, 98, 99, 106, 107, 108\}$
$S_{87} = \{85, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 109, 110, 111\}$
$S_{88} = \{88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 16, 17, 18\}$
$S_{89} = \{91, 92, 93, 94, 95, 96, 97, 98, 99, 106, 107, 108, 19, 20, 21\}$
$S_{90} = \{94, 95, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 22, 23, 24\}$
$S_{91} = \{97, 98, 99, 100, 101, 102, 103, 104, 105, 16, 17, 18, 25, 26, 27\}$
$S_{92} = \{100, 101, 102, 103, 104, 105, 106, 107, 108, 19, 20, 21, 28, 29, 30\}$
$S_{93} = \{103, 104, 105, 106, 107, 108, 109, 110, 111, 22, 23, 24, 31, 80, 81\}$
$S_{94} = \{106, 107, 108, 109, 110, 111, 16, 17, 18, 25, 26, 27, 82, 83, 84\}$
$S_{95} = \{109, 110, 111, 16, 17, 18, 19, 20, 21, 28, 29, 30, 85, 86, 87\}$
$S_{96} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 15, 32, 33, 40, 41, 42\}$
$S_{97} = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 34, 35, 36, 43, 44, 45\}$
$S_{98} = \{6, 7, 8, 9, 10, 11, 12, 13, 14, 37, 38, 39, 46, 47, 96\}$
$S_{99} = \{9, 10, 11, 12, 13, 14, 15, 32, 33, 40, 41, 42, 97, 98, 99\}$
$S_{100} = \{12, 13, 14, 15, 32, 33, 34, 35, 36, 43, 44, 45, 100, 101, 102\}$
$S_{101} = \{15, 32, 33, 34, 35, 36, 37, 38, 39, 46, 47, 96, 103, 104, 105\}$
$S_{102} = \{34, 35, 36, 37, 38, 39, 40, 41, 42, 97, 98, 99, 106, 107, 108\}$
$S_{103} = \{37, 38, 39, 40, 41, 42, 43, 44, 45, 100, 101, 102, 109, 110, 111\}$
$S_{104} = \{40, 41, 42, 43, 44, 45, 46, 47, 96, 103, 104, 105, 0, 1, 2\}$
$S_{105} = \{43, 44, 45, 46, 47, 96, 97, 98, 99, 106, 107, 108, 3, 4, 5\}$
$S_{106} = \{46, 47, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 6, 7, 8\}$
$S_{107} = \{97, 98, 99, 100, 101, 102, 103, 104, 105, 0, 1, 2, 9, 10, 11\}$
$S_{108} = \{100, 101, 102, 103, 104, 105, 106, 107, 108, 3, 4, 5, 12, 13, 14\}$
$S_{109} = \{103, 104, 105, 106, 107, 108, 109, 110, 111, 6, 7, 8, 15, 32, 33\}$
$S_{110} = \{106, 107, 108, 109, 110, 111, 0, 1, 2, 9, 10, 11, 34, 35, 36\}$
$S_{111} = \{109, 110, 111, 0, 1, 2, 3, 4, 5, 12, 13, 14, 37, 38, 39\}$

Table X: All feasible interest sets for $p = 4$ to $13, 21, 31$ and 57, $p*$ denotes special values that satisfy $p = n(n-1)+1$

$p = 4, n = 3$	
$\{0,1,2\}$	$\{0,1,3\}$
$p = 5, n = 3$	
$\{0,1,2\}$	$\{0,1,4\}$
$\{0,1,3\}$	$self$
$p = 6, n = 3$	
$\{0,1,3\}$	$\{0,1,4\}$
$p* = 7, n = 3$	
$\{0,1,3\}$	$\{0,1,5\}$
$p = 8, n = 4$	
$\{0,1,2,4\}$	$\{0,1,5,7\}$
$\{0,1,2,5\}$	$\{0,1,4,7\}$
$\{0,1,2,6\}$	$\{0,1,3,7\}$
$\{0,1,3,4\}$	$\{0,1,5,6\}$
$\{0,1,3,5\}$	$\{0,1,4,6\}$
$p = 9, n = 4$	
$\{0,1,2,4\}$	$\{0,1,6,8\}$
$\{0,1,2,5\}$	$\{0,1,5,8\}$
$\{0,1,2,6\}$	$\{0,1,4,8\}$
$\{0,1,2,7\}$	$\{0,1,3,8\}$
$\{0,1,3,4\}$	$\{0,1,6,7\}$
$\{0,1,3,5\}$	$\{0,1,5,7\}$
$\{0,1,3,6\}$	$\{0,1,4,7\}$
$\{0,1,3,7\}$	$self$
$\{0,1,4,6\}$	$self$
$p = 10, n = 4$	
$\{0,1,2,5\}$	$\{0,1,6,9\}$
$\{0,1,2,7\}$	$\{0,1,4,9\}$
$\{0,1,3,5\}$	$\{0,1,6,8\}$
$\{0,1,3,6\}$	$\{0,1,5,8\}$
$\{0,1,4,6\}$	$\{0,1,5,7\}$
$p = 11, n = 4$	
$\{0,1,2,5\}$	$\{0,1,7,10\}$
$\{0,1,2,8\}$	$\{0,1,4,10\}$
$\{0,1,3,5\}$	$\{0,1,7,9\}$
$\{0,1,3,7\}$	$\{0,1,5,9\}$
$\{0,1,3,8\}$	$\{0,1,4,9\}$
$\{0,1,4,6\}$	$\{0,1,6,8\}$
$p = 12, n = 4$	
$\{0,1,3,7\}$	$\{0,1,6,10\}$
$\{0,1,4,6\}$	$\{0,1,7,9\}$
$p* = 13, n = 4$	
$\{0,1,3,9\}$	$\{0,1,5,11\}$
$\{0,1,4,6\}$	$\{0,1,8,10\}$
$p* = 21, n = 5$	
$\{0,1,4,14,16\}$	$\{0,1,6,8,18\}$
$p* = 31, n = 6$	
$\{0,1,3,8,12,18\}$	$\{0,1,14,20,24,29\}$
$\{0,1,3,10,14,26\}$	$\{0,1,6,18,22,29\}$
$\{0,1,4,6,13,21\}$	$\{0,1,11,19,26,28\}$
$\{0,1,4,10,12,17\}$	$\{0,1,15,20,22,28\}$
$\{0,1,8,11,13,17\}$	$\{0,1,15,19,21,24\}$
$p* = 57, n = 8$	
$\{0,1,3,13,32,36,43,52\}$	$\{0,1,6,15,22,26,45,55\}$
$\{0,1,4,9,20,22,34,51\}$	$\{0,1,7,24,36,38,49,54\}$
$\{0,1,4,12,14,30,37,52\}$	$\{0,1,6,21,28,44,46,54\}$
$\{0,1,5,7,17,35,38,49\}$	$\{0,1,9,20,23,41,51,53\}$
$\{0,1,5,27,34,37,43,45\}$	$\{0,1,13,15,21,24,31,53\}$
$\{0,1,7,19,23,44,47,49\}$	$\{0,1,9,11,14,35,39,51\}$

Table XI: p values appeared in section V, their corresponding quorum size n and an interest set: for a faster reference

p	n	Interest set
7	3	$\{0,1,3\}$
8	4	$\{0,1,2,4\}$
10	4	$\{0,1,2,5\}$
27	6	$\{0,1,2,5,13,22\}$
30	7	$\{0,1,2,3,4,9,19\}$
47	8	$\{0,1,2,3,5,16,22,40\}$
49	8	$\{0,1,2,5,24,33,36,44\}$
57	8	$\{0,1,3,13,32,36,43,52\}$
64	9	$\{0,1,2,5,14,16,34,42,59\}$
72	10	$\{0,1,2,3,6,11,18,31,37,51\}$
81	11	$\{0,1,2,3,4,5,12,20,26,39,53\}$
86	11	$\{0,1,2,3,4,11,17,24,29,48,54\}$
91	10	$\{0,1,3,9,27,49,56,61,77,81\}$
100	12	$\{0,1,2,3,4,5,13,20,28,34,56,63\}$
111	12	$\{0,1,2,5,12,27,36,38,44,52,65,93\}$