

# An Innovative Function-tree Building Method Based on Similarity Theory and Extension Theory

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## Abstract

*Product design is a goal-directed problem-solving activity. And function is the basic element for conceptual design which is the most creative activity in the process of product design. In function solving phase, the main method of target function solving is to find the similitude functions, but the similitude functions' searching is depending on the designers' experience. In this paper, the sensuous association methods are classified based on association characteristics. Then by using similarity theory to calculate the similarity degree of function matter-elements, an evaluation system of similarity is set up among the functions which could help the designers to search the best suitable similitude functions for solving the target functions. At last, a method of function-tree's rebuilding was given by adding the similarity functions to the original function-tree. With this method, the solution space of design problems was enlarged, and the designers could get more innovation ideas supporting by more design information.*

**Keywords:** Conceptual design, Similarity degree, Similarity function, Function matter-element.

## 1. Introduction

Innovation is the most important impetus in modern society, and it has a great significance to study how to innovate. In the process of product design, the product innovation mainly comes from the conceptual design which decides the product's functions, principles, form, and structure. And the functions of the product are the key point and the basic element of the conceptual design.

As one of the design methods, function analysis is widely used in the conceptual design process of many new products. Carla Schwartz and Richard Gran<sup>[1]</sup>

described function analysis using MATLAB, and uses computer-aided design tools to develop a describing function analysis of a pendulum clock. D.D.Xia and J. Bai, Fellow<sup>[2]</sup>, established a new mathematical model of the micro-axial blood pumps.

Function decomposing is one of the function analysis problems. J. J. Lou and J. A. Brzozowski<sup>[3]</sup> formalize Shestakov's method using blanket algebra and extend it to the multi-valued case to solve the problem of function decomposing. Remis Balaniuk, Emmanuel Mazer and Pierre Bessiere<sup>[4]</sup> propose a method to calculate function values using only selected information, and decomposing the functions according to the function values.

Obviously, after decomposing the functions, the common function analysis is focused on how to solve the sub-functions. But in conceptual design, the more information could be got, the more innovation ideas could be made. So it's important to enlarge the solution space for the problems.

In this paper, firstly the formula of the similarity degree was modified to calculate the similarity among the functions based on the similarity theory in order to get similarity functions of sub-functions. Then an innovation product design method is proposed which integrates the function analysis with the extension method and the similarity theory. By this method, the way of searching similarity functions and rebuilding of new function-tree with similarity functions are given. Thus, though the rebuilding functions tree, the solution space was enlarged, and more innovation solutions could be found by getting more information for the problems.

## 2. Background issues

### 2.1 Similarity theory

Similarity theory<sup>[5, 6]</sup> is a systematic methodology

that utilizes a suit of methods to measure comparability between two things. It considers that the system is composed of sub-systems or elements, and that the element is composed of some characteristics which have the weighed value.

As for a similar characteristic  $j$  of two elements (or sub-systems), the weighed value is  $U_j(a)$  and  $U_j(b)$  in turn, so the value of  $r_{ij}$  which reflects the similarity degree of characteristics is calculated as following:

$$r_{ij} = \frac{\min\{U_j(a), U_j(b)\}}{\max\{U_j(a), U_j(b)\}} \quad (1)$$

$$0 \leq r_{ij} \leq 1 \quad i=1,2,\dots,n \quad j=1,2,\dots,m$$

Given  $k$  is the number of characteristics in similarity unit  $a_i$ ,  $l$  is the number of characteristics in similarity unit  $b_i$ ;  $n$  is the number of same characteristics between  $a_i$  and  $b_i$ ; and  $r_{ij}$  is the similarity degree of characteristic,  $d_j$  is the weighted coefficient which the characteristic affects the unit's similarity, so it can use the value of similarity unit to reflect the similarity degree of similarity units:

$$q(u_i) = \frac{n}{k+l-n} \sum_{j=0}^n d_j r_{ij} \quad (2)$$

Similarly, the formula of which the similarity of system is determined by sub-systems or elements can be described as: ( $q(u_i)$  is the similarity degree of sub-systems or elements,  $D_i$  is the homologous weighted coefficient.)

$$Q = \frac{N}{K+L-N} \sum_{i=0}^n D_i q(u_i) \quad (3)$$

## 2.2. Extension theory

Extension theory<sup>[7]</sup> is another systematic method to provide users with some unique ways to solve problems efficiently. It was published by a Chinese researcher, Cai Wen<sup>[8,9]</sup>. Up to now its applications and research have already extended the reach of business management engineering, economics, sociology, strategy, etc<sup>[10]</sup>. The extension method could make people resolve problems separately by decomposing them, thinking a series of clues, recombining the problems, and searching for the feasible solutions.

In extension theory, everything is described by matter-element which is composed of its name, characteristic and value. So matter-element could be defined a three-dimensional group as follows:

$$R = \{N, C, V\}$$

Where  $R$  is the symbol of this matter-element,  $N$  is its name,  $C$  is characteristic, and  $V$  is value.

In conceptual design, the functions could be presented to the matter-elements.  $R$  is the symbol of the function,  $N$  is the function's name,  $C$  is the set of function's characteristics and  $V$  is the characteristics' value. The matter-element which presents the function could be called function matter-element.

## 3. Similarity functions

### 3.1 Classification of the similarity functions

According to the human sensuous association, the similitude functions may be classified in three types. Details are represented as following:

- The common similarity functions

This type of functions is the mainly similarity functions that the designers need. The common similarity functions could do the same work as the target functions. And their characteristics are mostly the same. And the designers could judge the similarity degree by their characteristics' numbers and the value of the characteristic.

- The cause-result similarity functions

It means that the function  $f_1$  could lead to the function  $f_2$ . So it can be expressed as: if  $f_1$  then  $f_2$ , just like rules in reasoning process. And supported by the knowledge-base and the reasoning technology, this type of similarity functions could easily be found.

- The contrast similarity functions

Some functions have the different characteristics, but some of these characteristics are just contrary to the others, such as hot and cool, left and right and so on. And many design's target is to get a function which the original product doesn't have. In order to deal with this type of similarity functions, we could import negative values. So when contrast characteristics are met, one characteristic's value could change to the negative value in order to transfer it to its contrast characteristic.

### 3.2 Formula of similarity degree

In the process of design, some shortcomings are met to the original similarity degree calculated formula.

- The way to deal with the negative value

In the similarity theory, the characteristics' values

are positive numbers, and absolute value is adopted when negative number is met. But some characteristics could have negative value. And in order to solve the contrast similarity functions, the characteristics' value is transformed into negative in order to get the same characteristics. So the similarity degree calculated formula should be modified to calculate the negative value.

- The same distance with different similarity degree

In similarity theory, the distance could be defined as the absolute value of the two characteristics' value minus. But with the original formula, the same distance may get the different similarity degree.

For example, there are three function matter-elements with the same characteristic  $x$  and the characteristic's value is different.

$$f_1 = (x, a - i);$$

$$f_2 = (x, a);$$

$$f_3 = (x, a + i);$$

$$0 \leq a - i \leq a \leq a + i$$

According to the original similarity formula, the similarity degree between  $f_1$  and  $f_2$  is  $r_{12} = \frac{\min\{a - i, a\}}{\max\{a - i, a\}} = \frac{a - i}{a}$ , and the similarity degree between  $f_2$  and  $f_3$  is  $r_{23} = \frac{\min\{a, a + i\}}{\max\{a, a + i\}} = \frac{a}{a + i}$ .

Obviously, the distance between  $f_1$  and  $f_2$  is the same to the distance between  $f_2$  and  $f_3$ , but the similarity degrees are different.

So the original similarity formula should be modified, and some improvements could be made as following:

$$r_{ij} = \frac{\min\{e^{v_i}, e^{v_j}\}}{\max\{e^{v_i}, e^{v_j}\}}$$

Thus, the value of  $r_{ij}$  keeps in the range of  $(0, +\infty)$ , but the value of characteristic extends to the range of  $(-\infty, +\infty)$ .

On the other hand, the similarity between two characteristics could be expressed as:

$$r_{ij} = \frac{1}{e^{|v_i - v_j|}};$$

So, with the same distance, the similarity degree

is obviously the same.

Also, comparing with the original similarity formula of characteristics, the new formula has a shortcoming. Because the coordinate switching, the new formula could not keep the linear relations, the similarity degree may change slowly when the values of characteristics in the range of  $(0, 1)$  and change greatly when the values of characteristics in the range of  $(1, +\infty)$ .

#### 4. Process of an innovation product design method

After giving some modify of similarity theory, an innovation product design method is proposed which approach integrates the function analysis with the extension method and the similarity theory, and detailed steps are presented as follows.

##### Step1 Functions summarization and representation

Firstly, that the product needs improving and refining could be given by the customers. Collectively the collected information and the market product disassembly will conduce to testing the functions of each sub-system or component. The design object is generated after sufficient information gathering.

Then transform these information into the functions which the product's should offered, and represent those functions by the matter-elements of extension theory.

##### Step2 Decomposing the function matter-elements and setting up the function-tree

After function matter-elements established, two ways could be adopted to decompose those functions.

**Definition 1**(group-decomposing transformation): if  $R = (N, C, V)$ ,  $R = R_1 * R_2 * \dots * R_n$ . Then  $R$  could be decomposing as the following:

$$TG_R(R) = \{R_1, R_2, \dots, R_n\}.$$

$TG_R$  is named as group-decomposing transformation.

**Definition 2**(characteristic-decomposing transformation) : if  $R = (N, C, V)$ ,  $C = C_1 * C_2 * \dots * C_n$ , then  $R$  could be decomposing as the following:

$$TC_R(R) = \{R_1, R_2, \dots, R_n\}$$

$$R_1 = (N_1, C_1, V_1), R_2 = (N_2, C_2, V_2), \dots, R_n = (N_n, C_n, V_n)$$

$TC_R$  is named as characteristic-decomposing transformation.

With the extension theory of matter-element mode, the process of decomposing functions could make easily,

and after decomposing, the function tree of product could be set up.

### Step3 Searching the similarity functions to the sub-functions and rebuilding the function-tree

After decomposing the functions, similarity functions could be searching in the database. And supported by the similarity theory, the similarity degree could be calculated between the two function matter-elements.

After similarity functions found, the designers should add the similarity functions to the function-tree in order to expand the designers' thinking space.

Supposing function SF is a similarity function to the function F. In order to add the SF to function-tree, add SF to the tree as F's brother node with the operation "or". This sub-tree could be named ST1. But this operation could cause some characteristics lost. So a new node (marked as C) that contains the characteristics which belong to F and not belongs to SF is added to the function-tree as the brother node of node "A" with the operation "and". Thus a node "F" of function-tree expanded to a sub-function-tree with the similarity function. The process of function-tree's rebuilding could be explained by the Figure 1.

### Step4 Combination the solutions of sub-functions

With the technologies of reasoning and searching, the solutions to the sub-functions could be got by the designers, and then the designers could combine those methods to get target functions' solutions.

The process flow could be presented by the following as Figure 2

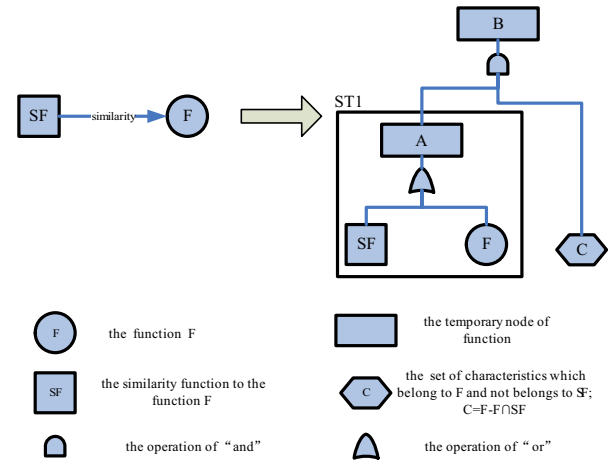


Figure 1 adding a similarity function to the function-tree

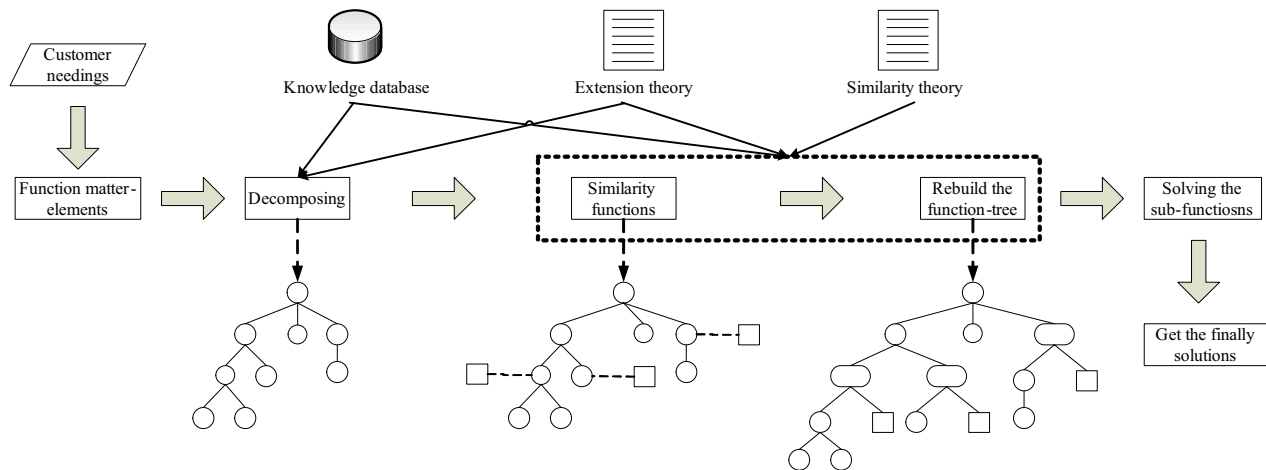


Figure 2 the process of innovation product method based on similarity theory and extension theory

As showing in figure 2, two steps were added in the functions analysis process. Though those steps, the similarity functions were added to the original function-tree. Obviously, the size of the function-tree was enlarged, but with the more problems' information, the larger solution space could be got, and the more innovation ideas could be got.

## 5. An example of design process

Here we have taken the design process of a travel cup as an instance. The cup for travelers should provide two main functions. Firstly the cup should be carried conveniently and hard enough to avoid breaking. And to the Chinese people, the cup is often used to drink tea, so it should deal with the tea-leaves easily.

After getting the customers' requirements, the designers could transform the requirements to the function matter-elements as following.

$TF=\{F_1, F_2\}$ ;

$F_1=\{\text{travel cup, carry conveniently, } 8\}$ ;

$F_2=\{\text{travel cup, drink tea, } 7\}$

$F_{11}=\{\text{carry conveniently, with handle, } 8\}$

$F_{12}=\{\text{carry conveniently, with cup tie, } 5\}$

$F_{13}=\{\text{carry conveniently, with cup ear, } 5\}$

$F_{21}=\{\text{drink tea, drink water, } 10\}$

$F_{22}=\{\text{drink tea, clean up, } 10\}$

$F_{23}=\{\text{drink tea, dissolved tea-leaves, } 8\}$

$F_{24}=\{\text{drink tea, separation, } 8\}$

Then the designers could get the function-tree as

Figure 3:

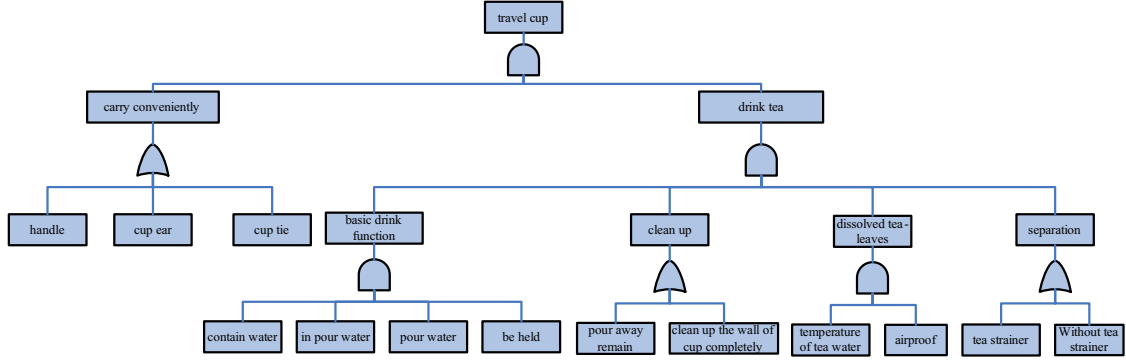


Figure 3 the original function-tree

And with the supporting of knowledge-base, we could search two similarity functions. They are presented by the function matter-elements as follow:

$SF_1=\{SF_{11}, SF_{12}, SF_{13}\}$

$SF_2=\{SF_{21}\}$

$SF_{11}=\{\text{drink milk powder, drink water, } 10\}$

$SF_{12}=\{\text{drink milk powder, clean up, } 8\}$

$SF_{13}=\{\text{drink milk powder, dissolved milk powder, } 7\}$

$SF_{21}=\{\text{drink wine, drink water, } 10\}$

Then we calculated the similarity degree to find the one which is more similarity to the target function. Firstly, Supposing 0.4, 0.2, 0.2, 0.2 are the weighted values of the characteristics to the function of “basic drink function”, and the similarity degree of the function “drink water” is:

$$q_1 = \frac{n}{k+l-n} \sum d_j r_j$$

$$= \frac{4}{4+4-4} (0.4 * e^{-1} + 0.2 * e^0 + 0.2 * e^0 + 0.2 * e^{-2})$$

$$= 0.5742$$

So the similarity degree between  $F_2$  and  $SF_2$  is:

$$Q_{F_2, SF_2} = \frac{n}{k+l-n} \sum d_j q_j$$

$$= \frac{1}{4+1-1} (1 * 0.5742) = 0.144$$

Because there are 3 same characteristics between

$F_2$  and  $SF_1$ , we could define the weighed value of characteristics as 0.5, 0.2, and 0.3. And there are some characteristics with different value, the similarity degree of characteristic is not 1, and we could use the characteristic similarity degree formula to calculate. Then the similarity degree between  $F_2$  and  $SF_1$  can be calculated by the same way. And the result is:

$$Q_{F_2, SF_1} = \frac{n}{k+l-n} \sum d_j q_j = 0.527$$

So  $SF_1$  is more similar to  $F_2$  than  $SF_2$  by the similarity degree, and with the different threshold value, the designers could decide the number of similarity functions should be considered in the process of design. Here we choose 0.5 as the threshold, so the  $SF_2$  is abandoned, and the new function tree could be got as

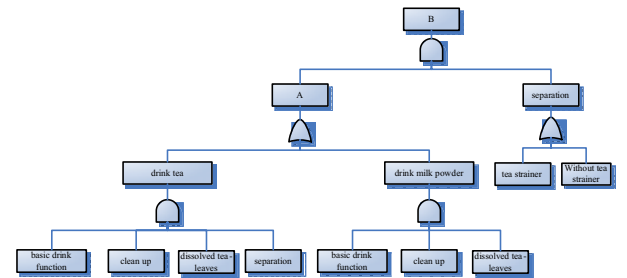


Figure 4 expand a node with similarity functions

After the similarity functions are added to the function-tree, the design space for the designers is determined. The new larger design space could help the designers to get similarity functions of target functions which could help the designers to solve the similarity functions instead of solving the original functions and giving the new way to thinking of the original functions.

## 6. Conclusion

The extension method and the similarity theory can play auxiliary roles in assisting designers in developing their products. In this paper, the extension theory is used to help the designers to analysis the customers' requirements and help the designers to decompose the functions easily. Then by the supporting of the similarity theory, an evaluation of the similarity degree could be got between the two functions, and a method of rebuild the function-tree with similarity functions is given. At last, based on the extension theory and the similarity theory, the process of a new innovation product design method is provided. On the next step, different solutions to the different classifies of similarity functions and the rules to the composing of the sub-solutions call for further research on the methodology of this design method.

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