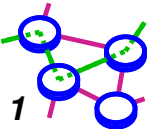


Probability Of Queueing Example

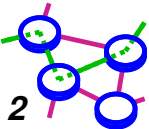
- ➡ 1 Mb/s link, each user sends at 100 Kb/s when "active"
 - ➡ model this as each user active 10% of the time (at random time)
 - or, at any time, probability of a user being active is $p = 0.1$
 - further, assume *i.i.d.* (i.e., users are *independent* and *identically distributed*)
 - ◆ recall that $P(X \wedge Y) = P(X) \cdot P(Y)$ if X and Y are independent
 - ➡ *no queueing* if ≤ 10 simultaneous users present
- ➡ Want to show (i.e., prove) that with 35 users, the probability of > 10 simultaneous active users is < 0.0004
 - ➡ i.e., *probability of queueing* is < 0.0004
- ➡ Use a smaller example to understand the solution
 - ➡ if there are 3 possible users (let's call them user A, B, and C), what can you *expect* at any random time?



Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is *present* and **0** to represent that a particular user is *not present*

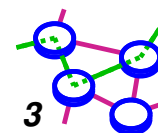
A	B	C	Event Probability
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is *present* and **0** to represent that a particular user is *not present*

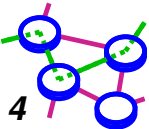
A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$



Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is *present* and **0** to represent that a particular user is *not present*

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

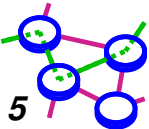


Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is **present** and **0** to represent that a particular user is **not present**

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly **1** user is present is: $3 \times (0.9)^2 \times (0.1)^1$

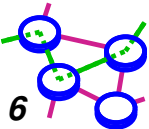


Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is **present** and **0** to represent that a particular user is **not present**

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

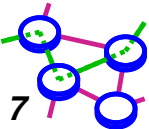
Probability that exactly **2** users are present is: $3 \times (0.9)^1 \times (0.1)^2$



Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is **present** and **0** to represent that a particular user is **not present**

A	B	C	Event Probability	
0	0	0	$(0.9)^3$	➡ Probability that exactly 0 users are present is: $1 \times (0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$	
0	1	0	$(0.9)^2 \times (0.1)^1$	
0	1	1	$(0.9)^1 \times (0.1)^2$	
1	0	0	$(0.9)^2 \times (0.1)^1$	
1	0	1	$(0.9)^1 \times (0.1)^2$	
1	1	0	$(0.9)^1 \times (0.1)^2$	
1	1	1	$(0.1)^3$	

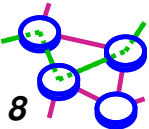


Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is **present** and **0** to represent that a particular user is **not present**

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

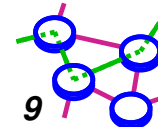
Probability that exactly **3** users are present is: $1 \times (0.1)^3$



Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is **present** and **0** to represent that a particular user is **not present**

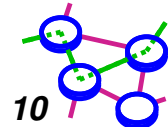
A	B	C	Event Probability	
0	0	0	$(0.9)^3$	➡ Probability that exactly 0 users are present is: $1 \times (0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$	➡ Probability that exactly 1 user is present is: $3 \times (0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$	
0	1	1	$(0.9)^1 \times (0.1)^2$	
1	0	0	$(0.9)^2 \times (0.1)^1$	➡ Probability that exactly 2 users are present is: $3 \times (0.9)^1 \times (0.1)^2$
1	0	1	$(0.9)^1 \times (0.1)^2$	
1	1	0	$(0.9)^1 \times (0.1)^2$	
1	1	1	$(0.1)^3$	➡ Probability that exactly 3 users are present is: $1 \times (0.1)^3$



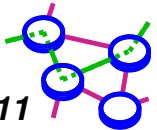
Probability Of Queueing Example

➡ Let's use **1** to represent that a particular user is **present** and **0** to represent that a particular user is **not present**

A	B	C	Event Probability	
0	0	0	$(0.9)^3$	➡ Probability that exactly 0 users are present is: $\binom{3}{0} \times (0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$	➡ Probability that exactly 1 user is present is: $\binom{3}{1} \times (0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$	
0	1	1	$(0.9)^1 \times (0.1)^2$	➡ Probability that exactly 2 users are present is: $\binom{3}{2} \times (0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$	
1	0	1	$(0.9)^1 \times (0.1)^2$	
1	1	0	$(0.9)^1 \times (0.1)^2$	➡ Probability that exactly 3 users are present is: $\binom{3}{3} \times (0.1)^3$
1	1	1	$(0.1)^3$	



Pascal triangle



Probability Of Queueing Example

- ➡ For a population of n independent users, each with probability p of being present, the probability that exactly k users are present is:

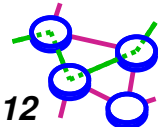
$$\binom{n}{k} \times (1-p)^{n-k} \times (p)^k$$

- ➡ For a population of 35 independent users, each with probability 0.1 of being present, the probability that > 10 users are present is:

$$\sum_{k=11}^{n=35} \binom{n}{k} \times (1-p)^{n-k} \times (p)^k$$

- ➡ Binomial expansion:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} \times (x)^{n-k} \times (y)^k$$



Probability Of Queueing Example

➡ Binomial expansion:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} \times (x)^{n-k} \times (y)^k$$

$$(p + (1-p))^n = \sum_{k=0}^n \binom{n}{k} \times (x)^{n-k} \times (y)^k$$

$$1 = \sum_{k=0}^n \binom{n}{k} \times (x)^{n-k} \times (y)^k$$

$$1 = \sum_{k=0}^{10} \binom{35}{k} \times (1-p)^{35-k} \times (p)^k + \sum_{k=11}^{35} \binom{n}{k} \times (1-p)^{n-k} \times (p)^k$$

$$1 - \sum_{k=0}^{10} \binom{35}{k} \times (1-p)^{35-k} \times (p)^k = \sum_{k=11}^{35} \binom{n}{k} \times (1-p)^{n-k} \times (p)^k$$

