Probability Of Queueing Example

- 1 Mb/s link, each user sends at 100 Kb/s when "active"
  - model this as each user active 10% of the time (at random time)
    - or, at any time, probability of a user being active is $p = 0.1$
    - further, assume *i.i.d.* (*i.e., users are independent and identically distributed*)
      - recall that $P(X \wedge Y) = P(X) \cdot P(Y)$ if X and Y are independent
  - *no queueing* if ≤ 10 simultaneous users present
- Want to show (i.e., prove) that with 35 users, the probability of > 10 simultaneous active users is < 0.0004
  - i.e., *probability of queueing* is < 0.0004
- Use a smaller example to understand the solution
  - if there are 3 possible users (let’s call them user A, B, and C), what can you *expect* at any random time?

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^{3}$
0	0	1	$(0.9)^{2} \times (0.1)^{1}$
0	1	0	$(0.9)^{2} \times (0.1)^{1}$
0	1	1	$(0.9)^{1} \times (0.1)^{2}$
1	0	0	$(0.9)^{2} \times (0.1)^{1}$
1	0	1	$(0.9)^{1} \times (0.1)^{2}$
1	1	0	$(0.9)^{1} \times (0.1)^{2}$
1	1	1	$(0.1)^{3}$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly *1* user is present is: $3 \times (0.9)^2 \times (0.1)^1$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly *2* users are present is: $3 \times (0.9)^1 \times (0.1)^2$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is *present* and *0* to represent that a particular user is *not present*

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly *0* users are present is: $1 \times (0.9)^3$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly *3* users are present is: $1 \times (0.1)^3$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly *0* users are present is: $1 \times (0.9)^3$

Probability that exactly *1* user is present is: $3 \times (0.9)^2 \times (0.1)^1$

Probability that exactly *2* users are present is: $3 \times (0.9)^1 \times (0.1)^2$

Probability that exactly *3* users are present is: $1 \times (0.1)^3$

Probability Of Queueing Example

- Let's use *1* to represent that a particular user is ***present*** and *0* to represent that a particular user is ***not present***

A	B	C	Event Probability
0	0	0	$(0.9)^3$
0	0	1	$(0.9)^2 \times (0.1)^1$
0	1	0	$(0.9)^2 \times (0.1)^1$
0	1	1	$(0.9)^1 \times (0.1)^2$
1	0	0	$(0.9)^2 \times (0.1)^1$
1	0	1	$(0.9)^1 \times (0.1)^2$
1	1	0	$(0.9)^1 \times (0.1)^2$
1	1	1	$(0.1)^3$

Probability that exactly *0* users are present is: $\binom{3}{0} \times (0.9)^3$

Probability that exactly *1* user is present is: $\binom{3}{1} \times (0.9)^2 \times (0.1)^1$

Probability that exactly *2* users are present is: $\binom{3}{2} \times (0.9)^1 \times (0.1)^2$

Probability that exactly *3* users are present is: $\binom{3}{3} \times (0.1)^3$

Pascal triangle

Probability Of Queueing Example

- For a population of n independent users, each with probability p of being present, the probability that exactly k users are present is:

$$\binom{n}{k} \times (1-p)^{n-k} \times (p)^{k}$$

- For a population of 35 independent users, each with probability 0.1 of being present, the probability that > 10 users are present is:

$$\sum_{k>10}^{n=35} \binom{n}{k} \times (1-p)^{n-k} \times (p)^{k}$$

- Binomial expansion:

$$(x+y)^{n} = \sum_{k=0}^{n} \binom{n}{k} \times (x)^{n-k} \times (y)^{k}$$

Probability Of Queueing Example

- **Binomial expansion:**

$$(x + y)^{n} = \sum_{k=0}^{n} \binom{n}{k} \times (x)^{n-k} \times (y)^{k}$$

$$(p + (1-p))^{n} = \sum_{k=0}^{n} \binom{n}{k} \times (x)^{n-k} \times (y)^{k}$$

$$1 = \sum_{k=0}^{n} \binom{n}{k} \times (x)^{n-k} \times (y)^{k}$$

$$1 = \sum_{k=0}^{10} \binom{35}{k} \times (1-p)^{35-k} \times (p)^{k} + \sum_{k>10}^{n=35} \binom{n}{k} \times (1-p)^{n-k} \times (p)^{k}$$

$$1 - \sum_{k=0}^{10} \binom{35}{k} \times (1-p)^{35-k} \times (p)^{k} = \sum_{k>10}^{n=35} \binom{n}{k} \times (1-p)^{n-k} \times (p)^{k}$$