

x_i = change in FED interest rate

y_i = change in inflation

w_i = weight

$$y = bx_i + c$$

$$\text{Total Cost} = \sum_{i=1}^n w_i (y_i - (y))^2$$

$$\text{Total Cost} = \sum_{i=1}^n w_i (y_i - (bx_i + c))^2$$

* Goal is to find values of b and c that minimize the Total Cost the most. Do this by setting partial derivatives with respect to b and c and set to zero.

$$\frac{\partial \text{Cost}}{\partial c} = \sum_{i=1}^n -2w_i (y_i - bx_i - c) = 0$$

* -2 cancels out

$$\frac{\partial \text{Cost}}{\partial c} = \sum_{i=1}^n w_i y_i - b \sum_{i=1}^n w_i x_i - c \sum_{i=1}^n w_i = 0$$

$$= \left[c \sum_{i=1}^n w_i + b \sum_{i=1}^n w_i x_i = \sum_{i=1}^n w_i y_i \right] \quad (1)$$

$$\frac{\partial \text{Cost}}{\partial b} = \sum_{i=1}^n -2w_i x_i (y_i - bx_i - c) = 0$$

$$= \sum_{i=1}^n w_i x_i y_i - b \sum_{i=1}^n w_i x_i^2 - c \sum_{i=1}^n w_i x_i = 0$$

$$= b \sum_{i=1}^n w_i x_i^2 + c \sum_{i=1}^n w_i x_i = \sum_{i=1}^n w_i x_i y_i \quad (2)$$

* Simplifying some terms

$$W = \sum_{i=1}^n w_i, \quad S_x = \sum_{i=1}^n w_i x_i, \quad S_y = \sum_{i=1}^n w_i y_i,$$

$$S_{xx} = \sum_{i=1}^n w_i x_i^2, \quad S_{xy} = \sum_{i=1}^n w_i x_i y_i$$

$$(1) cW = S_y - bS_x$$

$$(2) bS_{xx} + cS_x = S_{xy}$$

* Solve for c in (1) and plug into (2) for b .

$$(1) cW = S_y - bS_x$$

$$c = \frac{S_y - bS_x}{W}$$

$$(2) \quad bS_{xx} + \cancel{bx} cS_x = S_{xy}$$

$$bS_{xx} + S_x \left(\frac{S_y - bS_x}{W} \right) = S_{xy}$$

$$bS_{xx} + \frac{S_x S_y}{W} - b \frac{S_x^2}{W} = S_{xy}$$

$$b \left(S_{xx} - \frac{S_x^2}{W} \right) + \frac{S_x S_y}{W} = S_{xy}$$

$$b \left(S_{xx} - \frac{S_x^2}{W} \right) = S_{xy} - \frac{S_x S_y}{W}$$

$$b = \frac{S_{xy} - \frac{S_x S_y}{W}}{S_{xx} - \frac{S_x^2}{W}}$$