Proclivity for Procrastination and Risk Aversion Framework

A Theoretical Approach to Equity Premium Puzzle

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Abstract

This paper extends a non-standard model investigating the tendency for procrastination on the basis of conventional asset pricing models, attempting to delineate the temporal variation in individuals' risk aversion coefficients. For the sake of intuitive illustration, the entire model structure is bi-periodic and omits uncertainty. Intuitively, incorporating the temporal variation of risk aversion coefficients should facilitate elucidating the conundrum of risk premium without necessitating any non-standard adjustments in asset pricing. Building upon the intuitive explanation, this paper also conducts rudimentary experimental validation and parameter estimation; however, due to model pricing constraints and sample size, it does not present specific estimates of equity premiums.

Keywords: Behavioral Finance, Asset Pricing, Equity Premium Puzzle

Proclivity for Procrastination and Risk Aversion Framework

Lucas (1978) formulated a general asset pricing theory in an exchange economy, incorporating consumption optimization based on the consumption-portfolio choice problem. This gave rise to the concept of rational expectations equilibrium, initiating the rational expectations revolution and preliminarily encapsulating the framework of Stochastic Discount Factor (SDF) pricing. Rational expectations constitute an exceptionally perfect and ideal model, thus serving as the fundamental framework for numerous theoretical research papers on asset pricing.

However, this favorable situation did not last long, as the rational expectations model's pricing was soon confronted with counterexamples from empirical evidence. A direct assault on this theory came from Mehra and Prescott (1985), who, using a simplistic Lucas parametric model, proposed the renowned equity premium puzzle. The essence of this enigma lies in the absence of a Relative Risk Aversion (RRA) coefficient that satisfies both the risk-return SDF equation and the risk-free return SDF equation simultaneously. Successive generations proposed various explanations for this conundrum. Models with relatively evident explanatory effects include those discussing the psychological impact of wealth by Bakshi and Chen (1996) and Barberis, Huang, and Santos (1999); those examining the influence of consumption habits by Campbell and Cochrane (1999); and those exploring subjective sentiment parameter fluctuations by Mehra and Sah (2002).

Among these, works discussing the psychological impact of wealth should be discarded from an economic research methodology perspective: one cannot directly incorporate a variable into a utility function solely for discussing it. Although studies on consumption habits have achieved better results, they generate a new conundrum surrounding risk-free interest rate volatility, rendering them less than ideal. Although examining subjective sentiment parameter fluctuations is a simplistic approach, it has proven effective, and this paper considers it a suitable starting point.

While volatility has been previously addressed, the temporal variation of subjective sentiment parameters, particularly the core RRA, has not been discussed. This paper aims to introduce the temporal variation of RRA from a reasonable perspective and examine the intuitive plausibility of this variation in explicating the equity premium puzzle.

Two-Period Model and Intuition

Asset Pricing Model

The asset pricing model considered here is entirely based on the standard framework discussed by Lucas (1978). Building upon the foundation of the infinite-period model, we simplify it into a two-period model for the sake of clarity. The notation and analysis are essentially consistent with those in Xu Gao (2019) Lecture 7, in which we explore consumer optimization problems within the context of a Lucas Tree:

$$\max_{c_1,c_2} u^I(c_1) + \beta u^{II}(c_2)$$

$$\text{s.t.} \begin{cases} c_1 + v_1 s_1 = d_1 s_0 + v_1 s_0 \\ c_2 = d_2 s_1 \end{cases}$$

The setup entails that residents obtain endowments solely through holding shares of the Lucas Tree, without considering labor allocation. At the beginning of the first period, each resident possesses a share $s_0 = 1$, and the dividends generated by the Lucas Tree in each period are d_i $(i = 1,2)$. After the payment of dividends in the first period, residents can choose to trade their shares at a real price of v_1. Both the measure of people and dividends are standardized to 1, rendering all real quantities as values between $(0,1)$. It is worth noting that, compared to the standard model, I adopt distinct utility functions for each period u^{I}, u^{II}. In other words, we do not assume that agents' evaluation of consumption utility is identical across periods; this is, in fact, a more general assumption.

The solution to this model is straightforward, and we focus solely on the asset pricing equation:

$$v_1 = \beta \frac{u^{II'}(c_2)}{u^{I'}(c_1)} d_2 \tag{1}$$

Observe that in equation (1) $\beta \frac{u'(c_2)}{u'(c_1)}$ represents the Stochastic Discount Factor (SDF) within the Lucas framework. Since consumers face a price of v_1 per unit of assets in period 1 and receive dividends of d_2 per unit of assets in period 2, the total return is $1 + R_2 = d_2/v_1$. Consequently, we can rewrite equation (1) as follows:

$$1 = \beta \frac{u^{II'}(c_2)}{u^{I'}(c_1)} (1 + R_2) \tag{2}$$

Compared to equation (1), equation (2) merely transforms the absolute price into a return rate.

Procrastination Tendency Model

Up to this point, the model we have considered remains quite standard. We will now discuss a less conventional model. In order to depict the risk aversion structure through procrastination tendencies, we consider a "leisure-labor" allocation problem that corresponds to our realistic understanding of this issue.

Consider that an agent is simultaneously making allocations for leisure (e) and labor (w). Each period's time (endowment) is fixed; we standardize it as 1. Additionally, the agent faces an exogenous total labor requirement $\bar{w}$. Thus, their allocation problem can be simply characterized as:

$$\max_{e_1,e_2} u^I(e_1) + \beta u^{II}(e_2)$$

$$\text{s.t.} \begin{cases} e_t + w_t = 1, \qquad t = 1,2 \\ w_1 + w_2 = \bar{w} \\ e_t \geq 0, \qquad w_t \geq 0 \end{cases}$$

To render the problem standard, we set each period's utility to adopt a simple CRRA (Constant Relative Risk Aversion) form:

$$u^I(e_1) = \frac{e_1^{1-\gamma_1} - 1}{1-\gamma_1}$$

$$u^{II}(e_2) = \frac{e_2^{1-\gamma_2} - 1}{1-\gamma_2}$$

Where $\gamma_1, \gamma_2 \in (0, +\infty)$. To make the results intriguing, we consider cases where $\bar{w} < 1$, that is, we examine whether it is rational to procrastinate all tasks that can be completed within a day until the last period or to distribute them evenly across both periods. If allocated, how is the ratio influenced by the parameters?

We can assert that if the solution to this model is an interior point, the equilibrium distribution of labor and leisure should be characterized by the following three equations:

$$\begin{cases} \beta(1-w_2)^{-\gamma_2} = (1-w_1)^{-\gamma_1} \\ w_1 + w_2 = \bar{w} \\ e_t + w_t = 1, \qquad t = 1,2 \end{cases} \tag{3}$$

At this point, labor is positively allocated across both periods, but the proportion is still determined by the parameters.

Characterization of Procrastination Tendency on Risk Aversion Structure

Lemma: When $\gamma_1, \gamma_2 \neq 1$, and β is relatively large (specifically, $\beta > (1-\bar{w})^{\gamma_2}$), the solution of the procrastination tendency model is an interior solution, meaning the equilibrium is characterized by (3).

The lemma suggests that, under normal circumstances, rational decision-making should involve appropriate allocation of labor in each period, rather than piling it all up in the last period.

Proposition 1: Given β, the intertemporal allocation of labor is a binary function of $(\gamma_1, \gamma_2/\gamma_1)$, and roughly speaking, if $w_2/w_1 = f(\gamma_2/\gamma_1)$, then f is a decreasing function.

Proposition 1 indicates that the intertemporal allocation of labor is determined by the relative size of the intertemporal RRA, and this trend is intuitive: the more risk-averse in the second period, the more concave the utility function, and people will try to minimize procrastination and increase leisure, as reducing leisure a little may lead to a significant marginal utility loss, which is not cost-effective

Proposition 2: Given β, when $\gamma_2/\gamma_1 = 1$, w_2/w_1 is slightly greater than 1.

Proposition 2 illustrates that risk-averse people have a natural slight tendency to procrastinate. This is also intuitive, as allocating the same labor will reduce the same utility, and when future discounting exists, more labor will be allocated in the future.

However, procrastination does not stem from risk aversion; rather, as Propositions 3 and 4 elaborate, one source of procrastination is "non-risk aversion," and another source is "short-sightedness."

Proposition 3: When $\gamma_1 = \gamma_2 = \gamma$, if $\gamma \to \infty$, then $w_2/w_1 \to 1$; if $\gamma \to 0$, then $w_2/w_1 \gg 1$.

Proposition 3 demonstrates that the more risk-neutral individuals are, the more they dare to procrastinate. $\gamma \to \infty$ expresses extreme risk aversion, while $\gamma \to 0$ expresses risk neutrality. This is intuitive, as when people are highly risk-averse, the utility function becomes extremely concave, leading to the pursuit of extreme smoothing and an even distribution of labor.

Proposition 4: Given γ_2/γ_1, if we roughly write the intertemporal allocation of labor as $w_2/w_1 = g(\beta)$, then g is a decreasing function.

Proposition 4 suggests that the more shortsighted a person is, the more they dare to procrastinate. This is also intuitive, as the less future utility is discounted to the present, the more labor will be allocated in the future.

Proofs of the above lemma and propositions can be found in the appendix.

An intuitive explanation for the equity premium puzzle

In the standard CRRA model, we consider the case of $\gamma_1 = \gamma_2 = \gamma$. In this case, the specific form of equation (2) is:

$$1 = \beta \left(\frac{c_1}{c_2}\right)^{\gamma} (1 + R_2) \tag{4}$$

According to *Proposition 3*, if people in reality exhibit a strong propensity for procrastination, it implies that γ is quite small. This is consistent with the experimental results of Prescott and others, leading to a lower equity premium and thus giving rise to the equity premium puzzle.

Now, by introducing intertemporal heterogeneity in risk aversion, the specific form of Equation (2) becomes:

$$1 = \beta \frac{c_1^{\gamma_1}}{c_2^{\gamma_2}} (1 + R_2) \tag{5}$$

Under the standardization assumption, $c_1, c_2 \in (0,1)$. Based on *Propositions 4* and *1*, a stronger tendency for procrastination implies a smaller β, a relatively reduced γ_2, or a relatively increased γ_1. When Equation (5) holds, all three of these factors result in a larger R_2, thereby generating a higher equity premium.

Relaxing the non-time-varying assumption of the risk aversion structure and discussing within a more general framework, the intuition is as follows: although people's overall degree of risk aversion may be relatively low, the degree of risk aversion in specific time periods varies. This can be evidenced by the prevalent procrastination tendency in reality. This time-varying risk aversion can yield a higher equity premium.

Experiment and Model Calibration

Experiment Design

As a small sample experiment, we only aimed to investigate the extent of procrastination in a specific scenario. The scenario designed was as follows: "If you receive an assignment on Wednesday this week, which is due next Wednesday, and you are unaware of how many assignments you will receive in other subjects by next Wednesday, and completing this assignment will take approximately 1/2/3 days, how much do you tend to work on it this week? (All tasks are considered 100, fill in the integer)" There were 3 questions in total, and based on the participants' answers, we obtained the w_2/w_1 data corresponding to $\overline{w} = 1/3.5\,,2/3.5\,,3/3/5$.

It should be noted that the design of this experiment was very rough, and different scenario designs may correspond to different types of procrastination, which in turn may lead to different expected w_2/w_1 results. Additionally, different samples may correspond to different levels/preference groups, leading to differentw_2/w_1 results.

Here, we used a sample of students from the fall semester 2023 MFin program at University of California, Irvine (53 valid questionnaires). We will perform one calibration of the model based on the results of this survey and explain the results, but this does not necessarily represent the most representative results of this sample. It should be emphasized that the core content of this article is the construction of the model and the intuitive explanation, rather than the performance of this empirical part.

Model Calibration Results

We obtained 3 sample points:

$\overline{w}$	$Average\ w_2/w_1$
0.2857	0.6358
0.5714	0.5247
0.8571	0.3151

Using these 3 sample points, we performed nonlinear least squares regression on the mapping of $\overline{w} \to w_2/w_1$ and obtained the estimated model parameters:

$\hat{\beta}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
0.9590	0.1455	0.5854

Based on this estimate, we can immediately obtain the risk-free interest rate:

$$\hat{R}_f = 1/\hat{\beta} - 1 = 4.275\%$$

If we need to obtain a 6% equity risk premium and consider consumption as a random variable c in steady state, then we should have:

$$1 = \hat{\beta}\mathbb{E}\left[c^{\hat{\gamma}_1 - \hat{\gamma}_2}\right]\left(1 + \mathbb{E}\left[\hat{R}_e\right]\right)$$

Solving for w_1, we get:

$$\mathbb{E}[c^{0.4399}] = 0.9248$$

Validation of this point cannot be performed in the two-period model (because there is no so-called steady state), so we will not discuss it here.

Conclusion

In this article, we started with the intuition that "people in reality have a certain tendency to procrastinate" and used a "leisure-labor" allocation model to characterize the time-varying structure of the rational individual's risk aversion coefficient. If this intuition is well confirmed by experiments, it will increase the possibility of explaining the Equity Premium Puzzle compared to the standard model.

The results of the parameter estimation obtained through actual experiments show that, on the one hand, the parameters all fall within a reasonable range, and on the other hand, the estimate of the risk-free interest rate that can be calculated in the two-period model is also reasonable. This indicates that it is also possible to verify the equity risk premium through an infinite-period model of consumption sequences. Due to space limitations and research level, this study ends here. By improving the rigor of experimental design and the completeness of the model, more effective results can be obtained.

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Appendix

Proof of Lemma 1

Substituting the constraint $w_1 + w_1 = \overline{w}$ into the original value function, we get:

$$V = \frac{(1-w_1)^{1-\gamma_1} - 1}{(1-\gamma_1)} + \beta \frac{[(1-(\overline{w}-w_1)]^{1-\gamma_2} - 1}{1-\gamma_2}$$

We denote the right-hand side as $f(w_1)$, then we have:

$$\frac{d}{dw_1} f(w_1) = -(1-w_1)^{-\gamma_1} + \beta[(1-(\overline{w}-w_1)]^{-\gamma_2}$$

$$\frac{d^2}{dw_1^2} f(w_1) = -\gamma_1(1-w_1)^{-\gamma_1-1} - \gamma_2\beta[(1-(\overline{w}-w_1)]^{-\gamma_2-1} < 0$$

Thus, the internal solution of the original model is equivalent to:

$$\begin{cases} \dfrac{d}{dw_1} f(0) > 0 \\ \dfrac{d}{dw_1} f(\overline{w}) < 0 \end{cases} \Leftrightarrow \beta \in ((1-\overline{w})^{\gamma_2}, (1-\overline{w})^{-\gamma_1})$$

Due to the fact that $(1-\overline{w})^{-\gamma_1} > 1$, the upper bound of β naturally holds. Also, since $(1-\overline{w})^{\gamma_2} < 1$, as long as β is not lower than this lower bound, the solution of the model is an internal solution. At this time, $f'(w_1) = 0$, so we get:

$$\beta(1-w_2)^{-\gamma_2} = (1-w_1)^{-\gamma_1}$$

which gives the equilibrium solution by the system of equations (3).

Proof of Proposition 1

Given any $\overline{w} < 1$, equation $(3-1)$ implies:

$$\ln \beta^{-\frac{1}{\gamma_1}} + \frac{\gamma_2}{\gamma_1} \ln[1-(\overline{w}-w_1)] = \ln(1-w_1)$$

It can be seen that the main parameters that determine the solution (w_1, w_2) are$(\gamma_1, \gamma_2/\gamma_1)$, although in fact, they are γ_1 and γ_2. However, according to the numerical simulation results, the "absolute level" of RRA is less important, and we generally pay more attention to how the "relative level" performs in w_2/w_1.

Proof of Proposition 2

When $\gamma_2/\gamma_1 = 1$, equation $(3-1)$ can be written as:

$$-\frac{1}{\gamma_1}\ln\beta = \ln\frac{1-w_1}{1-w_2}$$

Since the LHS is greater than 0, then RHS is positive. Therefore, $w_1 < w_2$.

Proof of Proposition 3

Similar to the proof of Proposition 2, we can write equation $(3-1)$ as:

$$-\frac{1}{\gamma}\ln\beta = \ln\frac{1-w_1}{1-w_2}$$

Obviously, $\gamma \to \infty$ means that LHS$\to 0$, resulting in $(1-w_1)/(1-w_2) \to 1$, which means that $w_2/w_1 \to 1$. Conversely, as $\gamma \to 0$, the LHS$\to +\infty$, resulting in $(1-w_1)/(1-w_2) \to +\infty$, which means that $w_2/w_1 \to +\infty$.

Proof of Proposition 4

Given any $\bar{w} < 1$, using the same reasoning as in the proof of Proposition 1, we can write equation $(3-1)$ as:

$$\ln\beta^{-\frac{1}{\gamma_1}} + \frac{\gamma_2}{\gamma_1}\ln[1-(\bar{w}-w_1)] = \ln(1-w_1)$$

Implicitly differentiating this equation, we obtain:

$$\frac{d}{d\beta}w_1 = \left[\frac{\gamma_2/\gamma_1}{1-(\bar{w}-w_1)} + \frac{1}{1-w_1}\right]^{-1}\frac{1}{\gamma_1\beta} > 0$$

Thus, given $\bar{w}$ the greater the value of β, the larger the value of w_1, and the smaller the value of w_2/w_1.

Table 1:

Obtained Sample Points

$\bar{w}$	$Average\ w_2/w_1$
0.2857	0.6358
0.5714	0.5247
0.8571	0.3151

Table 2:

Obtained the Estimated Model Parameters

$\hat{\beta}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
0.9590	0.1455	0.5854