UCI Paul Merage School of Business

Leadership for a Digitally Driven World™

MFIN 290: Financial Econometrics

Lecture 5-2

Q&A/Review

- What have we covered in this course? List…
- Forecasting with Time Series, Prediction
- Cointegration/Stationarity
- Heteroskedasticity
- Time Series: MA models, AR models, ARMA models, Yule Walker Equations, ARCH
- Moments (first, second)
- Least Squares Assumptions, consequences of violating homoscedasticity and exogeneity
- OLS, GLS, FGLS
- Hypothesis Testing

This time

VAR models
Lagged Dependent Variables with time series effects (clever tricks part 1)

Time for final projects – confirm ideas with me, pull down data, make sure you know what you are looking at, etc.

VAR models

Imagine that we have a *system* of variables related to one another:

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

e.g., y_t is GDP, x_t is unemployment or prices (or both.. we can have multiple dimensions)

Changes in each variable affect future values that they take as well as future values of OTHER variables in the system.

VAR models

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

Known as a "Vector Autoregression" (VAR) as there is now a vector of variables that has an autocorrelated relationship.

This example is shown with a single lag – we can generalize this as well – so we call it a VAR(1) model.

VAR models

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

If X and Y are stationary I(0) variables, we can estimate each of these through OLS. If they are I(1) – or I(N) – we have to appropriately difference the data first.

The effects of such dynamic systems can be very interesting...

VAR models

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

For example, if y is GDP and x is unemployment, a large value for y (GDP)in one period will likely lead to a larger GDP reading the following period (assuming β_{11}>0), and may lead to a lower unemployment value in the future as well (that is, β_{21} should be negative).

Two periods out, there are two effects on y_{t+2}: the lingering effect of the larger y_t value, and the effect that the induced change in x_{t+1} has on y_{t+2} through β_{12}.

VAR models

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

Not obvious this will not spiral out of control!

We would like the system to “dampen” over time: things should return to equilibrium after all. This translates to placing restrictions on the joint coefficients… has to have negative divergence (this is a linear function…)

VAR models

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} x_{t-1} + v_t^y$$

$$x_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} x_{t-1} + v_t^x$$

We can express this as a matrix as follows:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} v_t^y \\ v_t^x \end{bmatrix}$$

$$Y_T = \Gamma Y_{t-1} + V_t$$

VAR models

$$Y_T = \Gamma Y_{t-1} + V_t$$

Dynamics of this system are governed by properties of Γ.

If all of the eigenvalues of Γ are less than one in absolute value, the system will have the desired convergence properties. Note that if y_t and x_t have finite variance and their autocovariance decays to zero over time, then the system must as well!

VAR models

$$Y_T = \Gamma Y_{t-1} + V_t$$

Can get a sense of the dynamics of these systems by looking at how all of the variables react over time to a sudden shock in one variable.

This is called an "Impulse Response"

VAR example

Phillips Curve:

Relationship between inflation and (un)employment: we know unemployment changes are autocorrelated, and inflation changes are autocorrelated, but how do they affect one another?

That is, does a shock to one tend to precede a shock to the other (but not vice versa)?

If we can boost employment with inflation, that's great!

If we THINK we can, and we can't, that's terrible!

VAR example

Stata example

```
use "phillips.dta"
gen obsno = _n
tsset obsno
* 1961q1 - 2005q4
* dp = change in inflation in percentage points
* du = change in unemp in percentage points

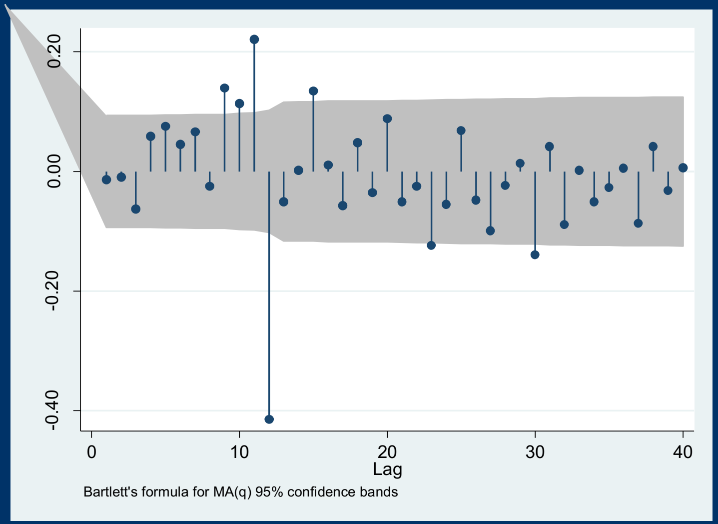
summ du dp, det

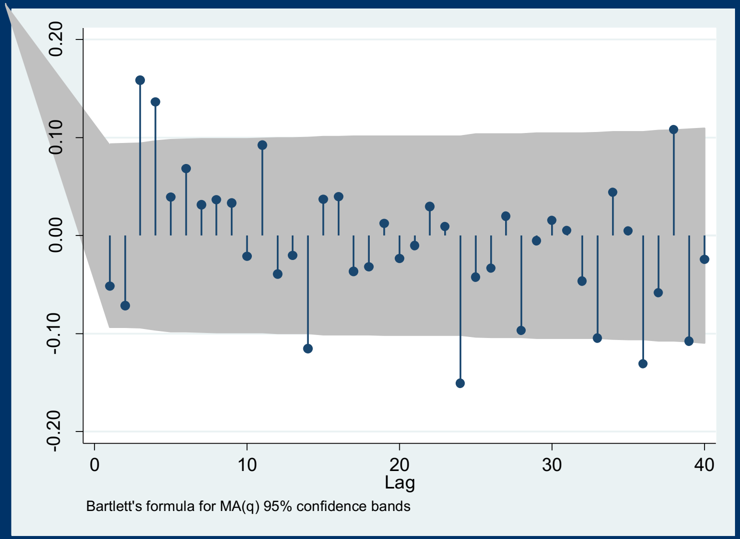
* Estimate VAR(1-2), store residual estimates for each equation
var du dp, lags(1/2)
predict e_hat, resid eq(du)
predict u_hat, resid eq(dp)
```

VAR example

Stata example

```
*residuals shouldn't be too autocorrelated (or we need a different VAR structure!)
ac u_hat
ac e_hat
```





VAR example

Stata example

```
*estimate VAR
*set up impulse response functions, graph
var du dp, lags(1/2)
capture irf drop phillips
irf set phillips
capture irf create phillips
irf graph irf
```

VAR example

Stata example

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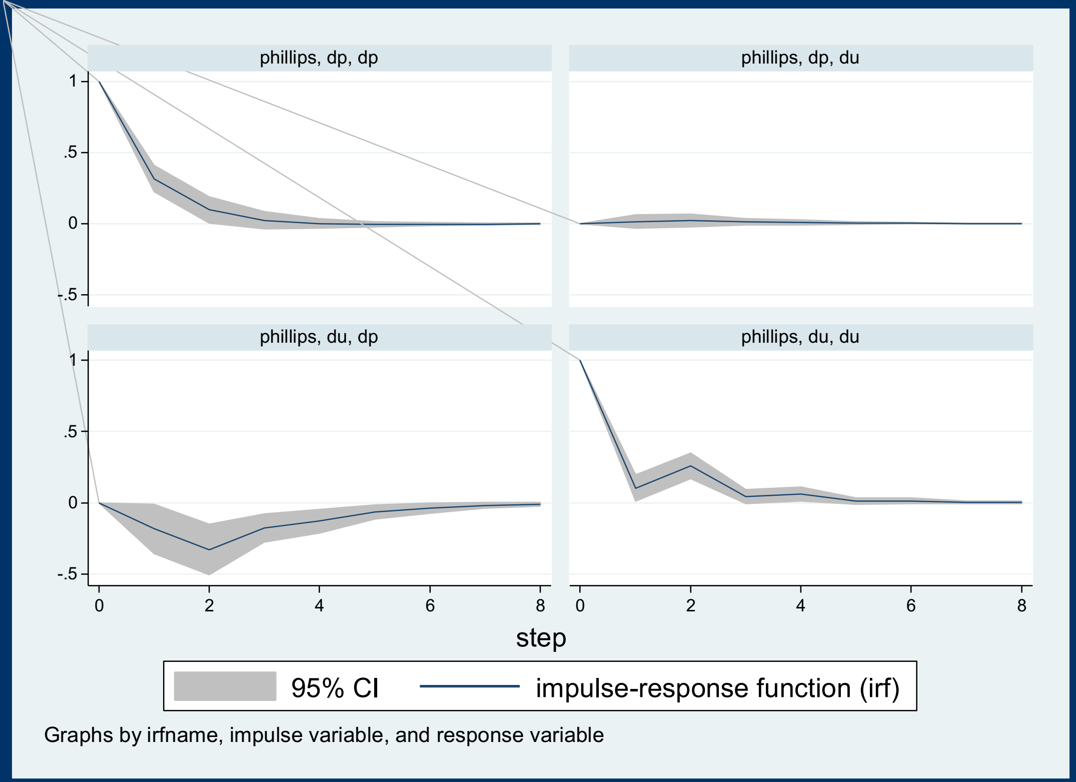

Shocks to prices tend to precede
- Future shocks to prices (top left)

Shocks to unemployment tend to precede:
- Future shocks to unemployment (bottom right)
- Price declines(bottom left)

VAR example

Stata example

```
*estimate VAR
*set up impulse response functions, graph
var du dp, lags(1/2)
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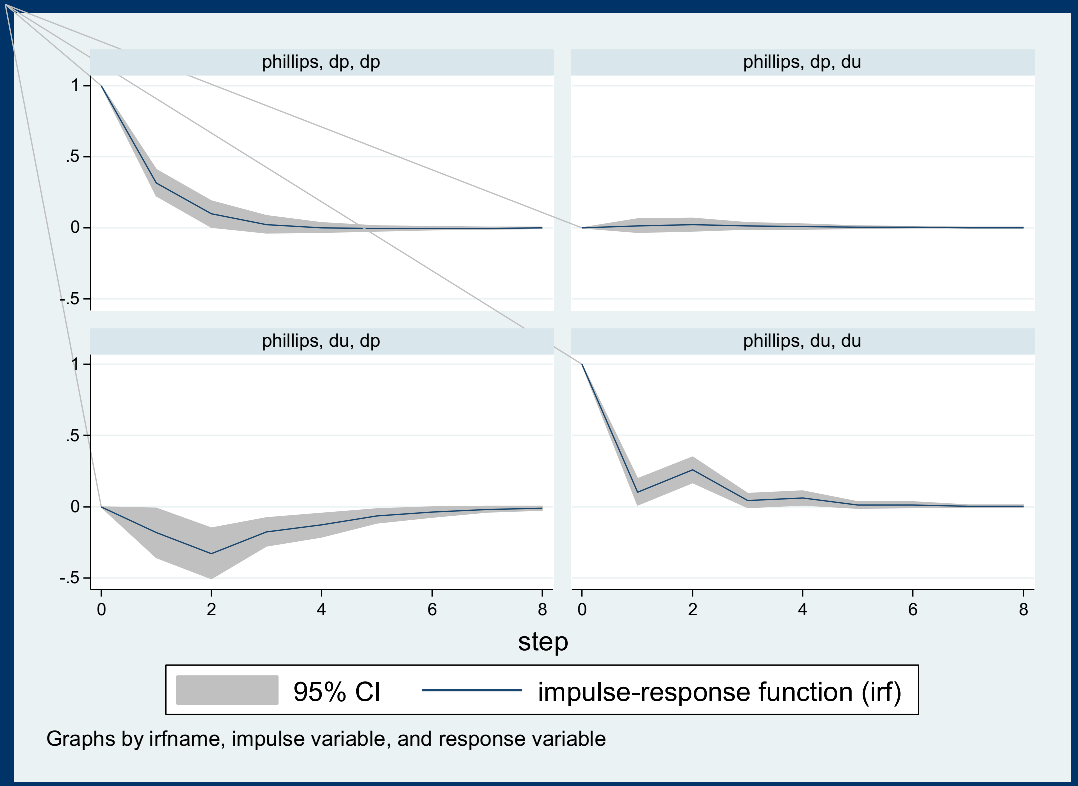

But it doesn’t look like I can manipulate prices to change unemployment (top right)

Lagged Dependent Variables

You will occasionally run into estimations where the model should rightly control for information last period: default rates, profits, etc. These sorts of estimations can either be completely fine, or very problematic.

We now have the background to dig in and find out why.

Consider

$$y_t = \beta y_{t-1} + \varepsilon_t$$

where the residual ε_t is an AR(1) process with parameter ρ.

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

Lagged Dependent Variables

Consider

$$y_t = \beta y_{t-1} + \varepsilon_t$$

where the residual ε_t is an AR(1) process with parameter ρ.

$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

What's the problem here?

Lagged Dependent Variables

Subtract ρy_{t-1} from each side…

$$y_t - \rho y_{t-1} = \beta y_{t-1} + \varepsilon_t - \rho y_{t-1}$$

$$y_t - \rho y_{t-1} = \beta y_{t-1} + \varepsilon_t - \rho[\beta y_{t-2} + \varepsilon_{t-1}]$$

Plug in $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$

$$y_t - \rho y_{t-1} = \beta y_{t-1} + \rho\varepsilon_{t-1} + u_t - \rho\beta y_{t-2} - \rho\varepsilon_{t-1}$$

Lagged Dependent Variables

$$y_t - \rho y_{t-1} = \beta y_{t-1} \cancel{+ \rho\varepsilon_{t-1}} + u_t - \rho\beta y_{t-2} \cancel{- \rho\varepsilon_{t-1}}$$

$$y_t - \rho y_{t-1} = \beta y_{t-1} + u_t - \rho\beta y_{t-2}$$

$$y_t = (\beta + \rho) y_{t-1} - \rho\beta y_{t-2} + u_t$$

We can actually estimate this regression!

Lagged Dependent Variables

$$y_t = (\beta + \rho)y_{t-1} - \rho\beta y_{t-2} + u_t$$

Alternatively, Yule Walker Equations (multiply each side by $(y_i - \mu)$ and take expectations):

$$\gamma_0 = (\beta + \rho)\gamma_1 - \rho\beta\gamma_2 + {\sigma^2}_u$$

$$\gamma_1 = (\beta + \rho)\gamma_0 - \rho\beta\gamma_1$$

$$\gamma_2 = (\beta + \rho)\gamma_1 - \rho\beta\gamma_0$$

Lagged Dependent Variables

But why do people worry about this?

What happens if we regress y_t on y_{t-1} alone as originally specified?

Lagged Dependent Variables

But what happens if we regress y_t *on* y_{t-1} alone as originally specified?

What β will we recover with OLS?

There is no intercept here, so you'll get the covariance over the variance as the beta… as usual.

Which is $\frac{\gamma_1}{\gamma_0}$ …

Lagged Dependent Variables

But what happens if we regress y_t *on* y_{t-1} alone as originally specified?

What β will we recover with OLS?

You'll get the covariance over the variance as the beta… as usual.

Which is $\frac{\gamma_1}{\gamma_0}$ …

$$\gamma_1 = (\beta + \rho)\gamma_0 - \rho\beta\gamma_1$$

$$\frac{\gamma_1}{\gamma_0} = (\beta + \rho) - \rho\beta\frac{\gamma_1}{\gamma_0}$$

$$\frac{\gamma_1}{\gamma_0} = \frac{(\beta + \rho)}{(1 + \rho\beta)}$$

Lagged Dependent Variables

$$\frac{\gamma_1}{\gamma_0} = \frac{(\beta + \rho)}{(1 + \rho\beta)}$$

Will not be equal to $(\beta + \rho)$ unless $\beta = 0$ or $\rho = 0$!

We can't estimate Lagged Dependent Variable models with OLS if there are time series effects in the residuals.

This should make sense… we are violating exogeneity here (can you prove it?)

Possible Solution: Instrumenting

What if we have other variables in the regression, X_t

$$y_t = \gamma y_{t-1} + \beta X_t + \varepsilon_t$$
$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

We can't use OLS on the first equation directly to estimate β because of endogeneity as we just showed.

Would have to instrument for the y_{t-1} (as usual) when we have this problem.

To do this, we need something correlated with y_{t-1} but uncorrelated with ε_t

$$Z_t : Z_t{}' y_{t-1}! = 0; Z_t{}'\varepsilon_t = 0$$

Any ideas?

Instrumenting

What if we have

$$y_t = \gamma y_{t-1} + \beta X_t + \varepsilon_t$$
$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

Would have to instrument for the y_{t-1} (as usual) when we have this problem.
To do this, we need something correlated with y_{t-1} but uncorrelated with ε_t

Any ideas?

We can use the X_{t-1} as an instrument for y_{t-1}. Clearly correlated with y_{t-1}, free from the endogenity with ε_t.

This will give us consistent estimates of $\widehat{\varepsilon_t}$, which we can then use to estimate ρ!

Lagged Dep Var Summary

Not a problem on its own, but…

Causes bias if the residuals have time series effects….

That, if the model is true, can usually be solved with lags of other variables in the system.

Process just requires awareness and a few steps.