Learning METAPOST by Doing

Introduction

TeX is the well-known typographic programming language that allows its users to produce high-quality typesetting especially for mathematical text. METAPOST is the graphic companion of TeX. It is a graphic programming language developed by John Hobby that allows its user to produce high-quality graphics. It is based on Donald Knuth's METAFONT, but with PostScript output and facilities for including typeset text. This course is only meant as a short, hands-on introduction to METAPOST for newcomers who want to produce rather simple graphics. The main objective is to get students started with METAPOST on a UNIX platform[1]. A more thorough, but also much longer introduction is the Metafun manual of Hans Hagen [Hag02]. For complete descriptions we refer to the METAPOST Manual and the Introduction to METAPOST of its creator John Hobby [Hob92a, Hob92b].

We have followed a few didactical guidelines in writing the course. Learning is best done from examples, learning is done from practice. The examples are often formatted in two columns, as follows:[2]

[1] You can also run METAPOST on a windows platform, e.g., using MikTeX and the WinEdt shell.

[2] On the left is printed the graphic result of the METAPOST code on the right. Here, a square is drawn.

```
beginfig(1);
draw unitsquare scaled 1cm;
endfig;
```

The exercises give you the opportunity to practice METAPOST, instead of only reading about the program. Compare your answers with the ones in the section 'Solutions to the Exercises'.

A Simple Example

METAPOST is not a WYSIWYG drawing tool like `xfig` or `xpaint`. It is a graphic document preparation system. First, you write a plain text containing graphic formatting commands into a file by means of your favourite editor. Next, the METAPOST program converts this text into a PostScript document that you can preview and print. In this section we shall describe the basics of this process.

Running METAPOST

EXERCISE 1 Do the following steps:

1. Create a text file, say `example.mp`, that contains the following very simple METAPOST document:

```
beginfig(1);
draw (0,0)--(10,0)--(10,10)--(0,10)--(0,0);
endfig;

end;
```

   For example, you can use the editor XEmacs:

```
xemacs example.mp
```

   The above UNIX command starts the editor and creates the source file `example.mp`. It is common and useful practice to always give a METAPOST source file a name with extension `.mp`. This will make it easier for you to distinguish the source document from files with other extensions, which METAPOST will create during the formatting.

2. Generate from this file PostScript code. Here the METAPOST program does the job:

```
mpost example
```

   It is not necessary to give the filename extension here. METAPOST now creates some additional files:

   `example.1` a PostScript file that can be printed and previewed;
   `example.log` METAPOST's log file.

3. Check that the file `example.1` contains the following normal Encapsulated PostScript code:[3]

[3] Notice that the bounding box is larger than you might expect, due to the default width of the line drawing the square.

```
%!PS
%%BoundingBox: -1 -1 11 11
%%Creator: MetaPost
%%CreationDate: 2003.05.11:2203
%%Pages: 1
%%EndProlog
%%Page: 1 1
 0 0.5 dtransform truncate idtransform setlinewidth pop [] 0 setdash
 1 setlinecap 1 setlinejoin 10 setmiterlimit
newpath 0 0 moveto
10 0 lineto
10 10 lineto
0 10 lineto
0 10 lineto
0 0 lineto stroke
showpage
%%EOF
```

4. Preview the PostScript document on your computer screen, e.g., by typing:

   ```
   gs example.1
   ```

   You will notice that gs does not use the picture's bounding box; alternatively, try ggv instead of gs, or:

5. Convert the PostScript document into a printable PDF-document:

   ```
   epstopdf example.1
   ```

   It creates the file `example.pdf` that you can you can view on the computer screen with the Adobe Acrobat Reader by entering the command:

   ```
   acroread example.pdf
   ```

   You can print this file in the usual way. The picture should look like the following small square:

Using the Generated PostScript in a LaTeX Document

EXERCISE 2 Do the following steps:

1. Create a file, say `sample.tex`, that contains the following lines of LaTeX commands that will include the image:

```
\documentclass{article}
\usepackage{graphicx}
\DeclareGraphicsRule{*}{mps}{*}{}
\begin{document}
\includegraphics{example.1}
\end{document}
```

   Above, we use the extended `graphicx` package for including the external graphic file that was prepared by METAPOST. The `DeclareGraphicsRule` statement causes all file extensions that are not associated with a well-known graphic format to be treated as Encapsulated PostScript files.

2. Typeset the LaTeX-file:

```
pdflatex sample
```

When typesetting is successful, the device independent file `sample.pdf` is generated.

The Structure of a METAPOST Document

We shall use the above examples to explain the basic structure of a METAPOST document. We start with a closer look at the slightly modified METAPOST code in the file `example.mp` of our first example:

```
beginfig(1); % draw a square
draw (0,0)--(10,0)--(10,10)
  --(0,10)--(0,0);
endfig;
end;
```

This example illustrates the following list of general remarks about regular METAPOST files

- It is recommended to end each METAPOST program in a file with extension `mp` so that this part of the name can be omitted when invoking METAPOST.
- Each statement in a METAPOST program is ended by a semicolon. Only in cases where the statement has a clear endpoint, e.g., in the `end` and `endfig` statement, you may omit superfluous semicolons. We shall not do this in this tutorial. You can put two or more statements on one line as long as they are separated by semicolons. You may also stretch a statement across several lines of code and you may insert spaces for readability.
- You can add comments in the document by placing a percentage symbol % in front of the commentary.
- A METAPOST document normally contains a sequence of `beginfig` and `endfig` pairs with an `end` statement after the last one. The numeric argument to the `beginfig` macro determines the name of the output file that will contain the PostScript code generated by the next graphic statements before the corresponding `endfig` command. In the above case, the output of the `draw` statement between `beginfig(1)` en `endfig` is written in the file `example.1`. In general, a METAPOST document consists of one or more instances of

  `beginfig(`*figure number*`);`
  graphic commands
  `endfig;`

  followed by `end`.
- The `draw` statement with the points separated by two hyphens (`--`) draws straight lines that connect the neighbouring points. In the above case for example, the point `(0,0)` is connected by straight lines with the point `(10,0)` and `(0,10)`. The picture is a square with edges of size 10 units, where a unit is $\frac{1}{72}$ of an inch. We shall refer to this default unit as a 'PostScript point' or 'big point' (`bp`) to distinguish it from the 'standard printer's point' (`pt`), which is $\frac{1}{72.27}$ of an inch. Other units of measure include `in` for inches, `cm` for centimetres, and `mm` for millimetres. For example,

  ```
  draw (0,0)--(1cm,0)--(1cm,1cm)--(0,1cm)--(0,0);
  ```

  generates a square with edges of size 1cm. Here, `1cm` is shorthand for `1*cm`. You may use `0` instead of `0cm` because `cm` is just a conversion factor and `0cm` just multiplies the conversion factor by zero.

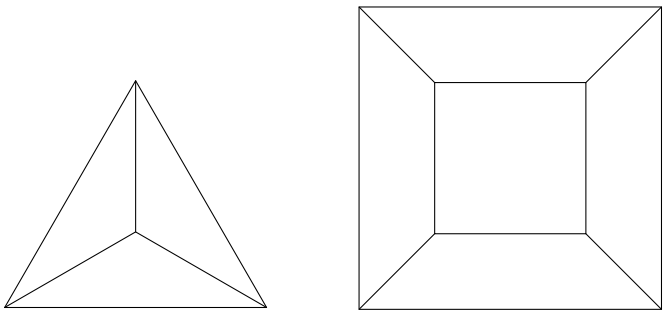
EXERCISE 3 Create a METAPOST file, say `exercise3.mp`, that generates a circle of diameter 2cm using the `fullcircle` graphic object.

EXERCISE 4

1. Create a METAPOST file, say `exercise4.mp`, that generates an equilateral triangle with edges of size 2cm.
2. Extend the METAPOST document such that it generates in a separate file the PostScript code of an equilateral triangle with edges of size 3cm.

EXERCISE 5 Define your own unit, say `0.5cm`, by the statement `u=0.5cm;` and use this unit `u` to generate a regular hexagon with edges of size 2 units.

EXERCISE 6 Create the following two pictures:



Numeric Quantities
Numeric quantities in METAPOST are represented in fixed point arithmetic as integer multiples of $\frac{1}{65536} = 2^{-16}$ and with absolute value less or equal to $4096 = 2^{12}$. Since METAPOST uses fixed point arithmetic, it does not understand exponential notation such as `1.23E4`. It would interpret this as the real number `1.23`, followed by the symbol `E`, followed by the number `4`. Assignment of numeric values can be done with the usual `:=` operator. Numeric values can be shown via the `show` command.

EXERCISE 7

1. Create a METAPOST file, say `exercise7.mp`, that contains the following code

```
numeric p,q,n;
n := 11;
p := 2**n;
q := 2**n+1;
show p,q;
end;
```

   Find out what the result is when you run the above METAPOST program.

2. Replace the value of `n` in the above METAPOST document by 12 and see what happens in this case (Hint: press Return to get processing as far as possible). Explain what goes wrong.
3. Insert at the top of the current METAPOST document the following line and see what happens now when you process the file.

```
warningcheck := 0;
```

The *numeric* data type is used so often that it is the default type of any non-declared variable. This explains why `n := 10;` is the same as `numeric n; n := 10;` and

why you cannot enter `p := (0,0);` nor `p = (0,0);` to define the point, but must use `pair p; p := (0,0);` or `pair p; p = (0,0);` .

If Processing Goes Wrong
If you make a mistake in the source file and METAPOST cannot process your document without any trouble, the code generation process is interrupted. In the following exercise, you will practice the identification and correction of errors.

EXERCISE 8 Deliberately make the following typographical error in the source file `example.mp`. Change the line

```
draw (0,0)--(10,0)--(10,10)--(0,10)--(0,0);
```

into the following two lines

```
draw (0,0)--(10,0)--(10,10)
draw (10,10)--(0,10)--(0,0);
```

1. Try to process the document. METAPOST will be unable to do this and the processing would be interrupted. The terminal window where you entered the `mpost` command looks like:

```
(example.mp
! Extra tokens will be flushed.
<to be read again>
                   addto
draw->addto
           .currentpicture.if.picture(EXPR0):also(EXPR0)else:doubl...
<to be read again>
                   ;
l.3 draw (10,10)--(0,10)--(0,0);

?
```

   In a rather obscure way, the METAPOST program notifies the location where it signals that something goes wrong, viz., at line number 3. However, this does not mean that the error is necessarily there.

2. There are several ways to proceed after the interrupt. Enter a question mark and you see your options:

```
? ?
Type <return> to proceed, S to scroll future error messages,
R to run without stopping, Q to run quietly,
I to insert something, E to edit your file,
1 or ... or 9 to ignore the next 1 to 9 tokens of input,
H for help, X to quit.
?
```

3. Press RETURN. METAPOST will continue processing and tries to make the best of it. Logging continues:

```
[1] )
1 output file written: example.1
Transcript written on example.log.
```

4. Verify that only the following path is generated:

```
newpath 0 0 moveto
10 0 lineto
10 10 lineto stroke
```

5. Format the METAPOST document again, but this time enter the character `e`.

Your default editor will be opened and the cursor will be at the location where METAPOST spotted the error. Correct the source file[4] by adding a semicolon at the right spot, and give the METAPOST processing another try.

Basic Graphical Primitives

In this section you will learn how to build up a picture from basic graphical primitives such as points, lines, and text objects.

Pair

The *pair* data type is represented as a pair of numeric quantities in METAPOST. On the one hand, you may think of a pair, say `(1,2)`, as a location in two-dimensional space. On the other hand, it represents a vector. From this viewpoint, it is clear that you can add or subtract two pairs, apply a scalar multiplication to a pair, and compute the dot product of two pairs.

You can render a point `(x,y)` as a dot at the specified location with the statement

```
draw (x,y);
```

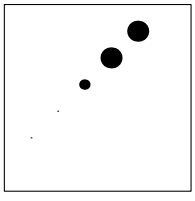
Because the drawing pen has by default a circular shape with a diameter of 1 PostScript point, a hardly visible point is rendered. You must explicitly scale the drawing pen to a more appropriate size, either locally in the current statement or globally for subsequent drawing statements. You can resize the pen for example with a scale factor 4 by

```
draw (x,y) withpen pencircle scaled 4; % temporary change of pen
```

or by

```
pickup pencircle scaled 4; % new drawing pen is chosen
draw (x,y);
```

EXERCISE 9 Explain the following result:



```
beginfig(1)
draw unitsquare scaled 70;
draw (10,20);
draw (10,15) scaled 2;
draw (30,40) withpen pencircle scaled 4;
pickup pencircle scaled 8;
draw (40,50);
draw (50,60);
endfig;
end;
```

Assignment of pairs is often not done with the usual `:=` operator, but with the equation symbol `=`. As a matter of fact, METAPOST allows you to use linear equations to define a pair in a versatile way. A few examples will do for the moment.

- Using a name that consists of the character `z` followed by a number, a statement such as `z0 = (1,2)` not only declares that the left-hand side is equal to the right-hand side, but it also implies that the variables `x0` and `y0` exist and are equal to `1` and `2`, respectively. Alternatively, you can assign values to the numeric variable `x1` and `y1` with the result that the pair `(x1,y1)` is defined and can be referred to by the name `z1`.
- A statement like

```
z1 = -z2 = (3,4);
```

[4] If you have not specified another editor in the EDITOR environment variable, then the `vi`-editor will be started. You can leave this editor by entering `ZZ`. In the c-shell you can add in the file `.cshrc` the line `setenv MPEDIT 'xemacs +%d %s'` so that XEMACS is used.

is equivalent to

```
z1 = (3,4);
z2 = -(3,4);
```

□ If two pairs, say `z1` and `z2`, are given, you can define the pair, say `z3`, right in the middle between these two points by the statement `z3 = 1/2[z1,z2]`.

□ When you have declared a pair, say `P`, then `xpart P` and `ypart P` refer to the first and second coordinate of `P`, respectively. For example,

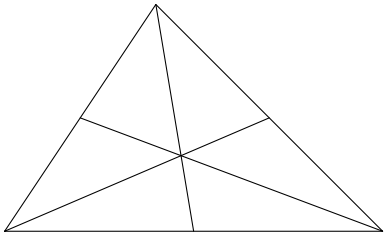
```
pair P;  P = (10,20);
```

is the same as

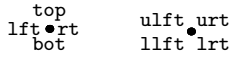
```
pair P;
xpart P = 10;
ypart P = 20;
```

EXERCISE 10 Verify that when you use a name that begins with `z.` followed by a sequence of alphabetic characters and/or numbers, a statement such as `z.P = (1,2)` not only declares that the left-hand side is equal to the right-hand side, but it also implies that the variables `x.P` and `y.P` exist and are equal to 1 and 2, respectively. Alternatively, you can assign values to the numeric variable `x.P` and `y.P`, with the result that the pair `(x.P,y.P)` is defined and can be referred to by the name `z.P`.

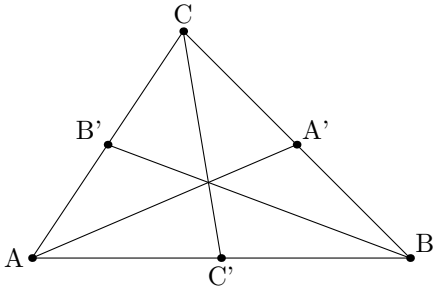
EXERCISE 11 1. Create the following geometrical picture of an acute-angled triangle together with its three medians[5]:



2. The `dotlabel` command allows you to mark a point with a dot and to position some text around it. For instance, `dotlabel.lft("A",(0,0));` generates a dot with the label `A` to the left of the point. Other `dotlabel` suffixes and their meanings are shown in the picture below:



Use the `dotlabel` command to put labels in the picture in part 1, so that it looks like

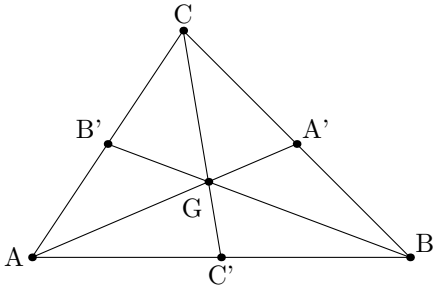


[5]The A-median of a triangle ABC is the line from A to the midpoint of the opposite edge BC.

3. Recall that `1/2[z1,z2]` denotes the point halfway between the points `z1` and `z2`. Similarly, `1/3[z1,z2]` denotes the point on the line connecting the points `z1` and `z2`, one-third away from `z1`. For a numeric variable, which is possibly unknown yet, `c[z1,z2]` is `c` times of the way from `z1` to `z2`. If you do not want to waste a name for a variable, use the special name `whatever` to specify a general point on a line connecting two given points:

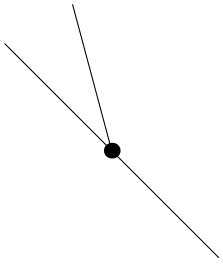
   ```
   whatever[z1,z2];
   ```

   denotes some point on the line connecting the points `z1` and `z2`. Use this feature to define the intersection point of the medians, also known as the centre of gravity, and extend the above picture to the one below. Use the `label` command, which is similar to the `dotlabel` command except that it does not draw a dot, to position the character `G` around the centre of gravity. If necessary, assign `labeloffset` another value so that the label is further away from the centre of gravity.



EXERCISE 12 The `dir` command is a simple way to define a point on the unit circle. For example, `dir(30);` generates the pair `(0.86603,0.5)` $\left(=(\frac{1}{2}\sqrt{3},\frac{1}{2})\right)$. Use the `dir` command to generate a regular pentagon.

EXERCISE 13 Use the `dir` command to draw a line in northwest direction through the point $(1,1)$ and a line segment through this point that makes an angle of 30 degrees with the line. Your picture should look like



Path

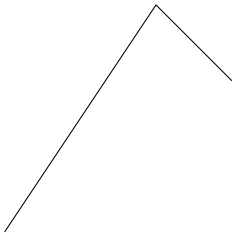
Open and Closed Curves. METAPOST can draw straight lines as well as curved ones. You have already seen that a `draw` statement with points separated by `--` draws straight lines connecting one point with another. For example, the result of

```
draw p0--p1--p2;
```

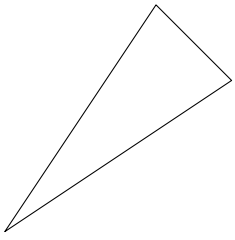
after defining three points by

```
pair p[];  p0 = (0,0);  p1 = (2cm,3cm);  p2 = (3cm,2cm);
```

is the following picture.



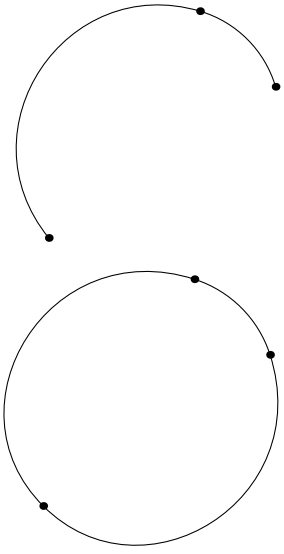
Closing the above path is done either by extending it with `--p0` or by connecting the first and last point via the `cycle` command. Thus, the path `p0--p1--p2--cycle`, when drawn, looks like



The difference between these two methods is that the path extension with the starting point only has the optical effect of closing the path. This means that only with the `cycle` extension it really becomes a closed path.

EXERCISE 14 Verify that you can only fill the interior of a closed curve with some colour or shade of gray, using the `fill` command, when the path is really closed with the `cycle` command. The gray shading is obtained by the directive `withcolor c*white`, where `c` is a number between 0 and 1.

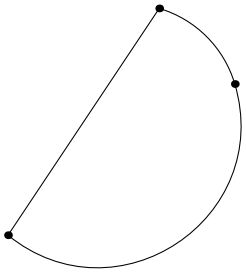
Straight and Curved Lines. Compare the pictures from the previous subsection with the following ones, which show curves[6] through the same points.



```
u := 1cm;
pair p[];
p0 = (0,0); p1 = (2u,3u); p2 = (3u,2u);

beginfig(1);
draw p0..p1..p2; % draw open curve
for i=0 upto 2:
  draw p[i] withpen pencircle scaled 3;
endfor; % draw defining points
endfig;

beginfig(2);
draw p0..p1..p2..cycle; % draw closed curve
for i=0 upto 2:
  draw p[i] withpen pencircle scaled 3;
endfor; % draw defining points
endfig;
end;
```

[6] the curves through the three points are a circle or a part of the circle

Just use -- where you want straight lines and . . where you want curves.

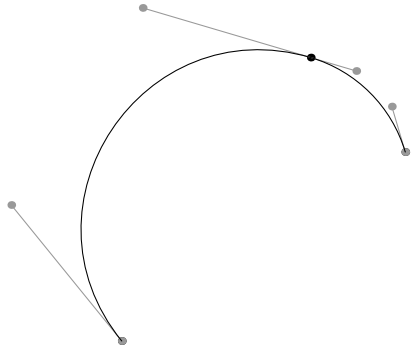


```
beginfig(1);
u := 1cm;
pair p[];
p0 = (0,0); p1 = (2u,3u); p2 = (3u,2u);
draw p0--p1..p2..cycle;
for i=0 upto 2:
  draw p[i] withpen pencircle scaled 3;
endfor;
endfig;
end;
```

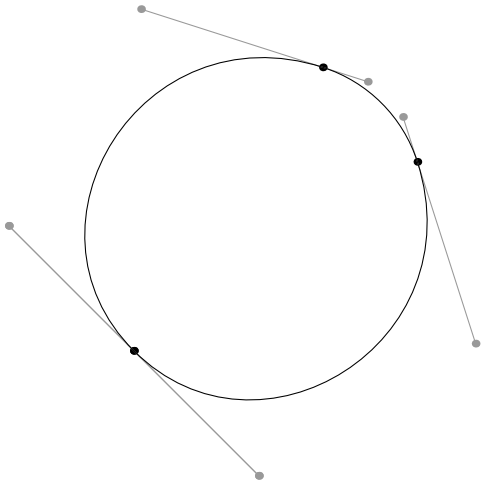
Construction of Curves. When METAPOST draws a smooth curve through a sequence of points, each pair of consecutive points is connected by a cubic Bézier curve, which needs, in order to be determined, two intermediate control points in addition to the end points. The points on the curved segment from points p_0 to p_1 with post control point c_0 and pre control point c_1 are determined by the formula

$$p(t) = (1-t)^3 p_0 + 3(1-t)^2 t c_0 + 3(1-t) t^2 c_1 + t^3 p_1 \,,$$

where $t \in [0,1]$. METAPOST automatically calculates the control points such that the segments have the same direction at the interior knots. In the figure below, the additional control points are drawn as gray dots and connected to their parent point with gray line segments. The curve moves from the starting point in the direction of the post control point, but possibly bends after a while in another direction. The further away the post control point is, the longer the curve keeps this direction. Similarly, the curve arrives at a point coming from the direction of the pre control point. The further away the pre control point is, the earlier the curve gets this direction. It is as if the control points pull their parent point in a certain direction and the further away a control point is, the stronger it pulls. By default in METAPOST, the incoming and outgoing direction at a point on the curve are the same so that the curve is smooth.



```
u := 1.25cm;
color gray; gray := 0.6white;
pair p[];
p0 = (0,0); p1 = (2u,3u); p2 = (3u,2u);
def drawpoint(expr z, c) = draw z
  withpen pencircle scaled 3 withcolor c;
enddef;

beginfig(1);
path q; q := p0..p1..p2;
for i=0 upto length(q):
  drawpoint(point i of q, black);
  p3 := precontrol i of q;
  p4 := postcontrol i of q;
  draw p3--p4 withcolor gray;
  drawpoint(p3, gray);
  drawpoint(p4, gray);
endfor;
draw q;
endfig;
```



```
beginfig(2);
path q; q := p0..p1..p2..cycle;
for i=0 upto length(q):
  drawpoint(point i of q, black);
  p3 := precontrol i of q;
  p4 := postcontrol i of q;
  draw p3--p4 withcolor gray;
  drawpoint(p3, gray);
  drawpoint(p4, gray);
endfor;
draw q;
endfig;

end;
```

Do not worry when you do not understand all details of the above METAPOST program. It contains features and programming constructs that will be dealt with later in the tutorial.

There are various ways of controlling curves:

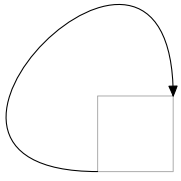
- Vary the angles at the start and end of the curve with one of the keywords `up`, `down`, `left`, and `right`, or with the `dir` command.
- Specify the requested control points manually.
- Vary the inflection of the curve with `tension` and `curl`. `tension` influences the curvature, whereas `curl` influences the approach of the starting and end points.

```
pair p[];  p0:=(0,0);  p1:=(1cm,1cm);
def drawsquare = draw unitsquare
    scaled 1cm withcolor 0.7white;
enddef;
```

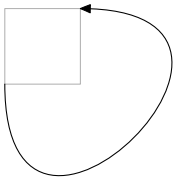
```
beginfig(1);
drawsquare;  drawarrow p0..p1;
endfig;
```

```
beginfig(2);
drawsquare;  drawarrow p0{right}..p1;
endfig;
```

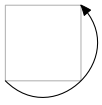
```
beginfig(3);
drawsquare;  drawarrow p0{up}..p1;
endfig;
```



```
beginfig(4);
drawsquare;  drawarrow p0{left}..p1;
endfig;
```

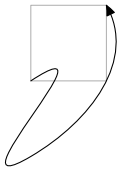


```
beginfig(5);
drawsquare;  drawarrow p0{down}..p1;
endfig;
```

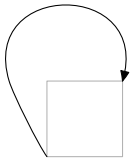


```
beginfig(6);
drawsquare;  drawarrow p0{dir(-45)}..p1;
endfig;
```

```
beginfig(7);
drawsquare;  drawarrow p0..
  controls (0,1cm) and (1cm,0) ..p1;
endfig;
```



```
beginfig(8);
drawsquare;  drawarrow p0{curl 80}..
  (0,-1cm)..{curl 8}p1;
endfig;
```



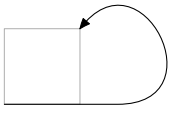
```
beginfig(9);
drawsquare;  drawarrow p0..tension(2)
  ..(0,1cm)..p1;
endfig;

end;
```

The METAPOST operators `--`, `---`, and `...` have been defined in terms of `curl` and `tension` directives as follows:

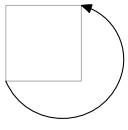
```
def --  = {curl 1}..{curl 1}      enddef;
def --- = .. tension infinity ..  enddef;
def ... = .. tension atleast 1 .. enddef;
```

The meaning of `...` is "choose an inflection-free path between the points unless the endpoint directions make this impossible". The meaning of `---` is "get a smooth connection between a straight line and the rest of the curve".

```
pair p[];  p0:=(0,0);  p1:=(1cm,1cm);
def drawsquare = draw unitsquare scaled 1cm
    withcolor 0.7white;
enddef;
```



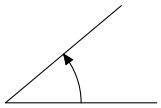
```
beginfig(1);
drawsquare;  drawarrow p0---(1.5cm,0)..p1;
endfig;
```



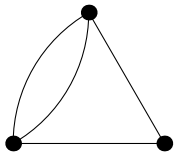
```
beginfig(2);
drawsquare;  drawarrow p0...(1.5cm,0)..p1;
endfig;

end;
```

The above examples were also meant to give you the impression that you can draw in METAPOST almost any curve you wish.

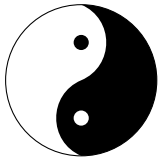
EXERCISE 15 Draw an angle of 40 degrees that looks like



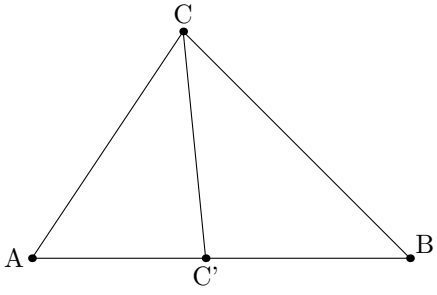
EXERCISE 16 Draw a graph that looks like



EXERCISE 17 Draw the Yin-Yang symbol[7] that looks like

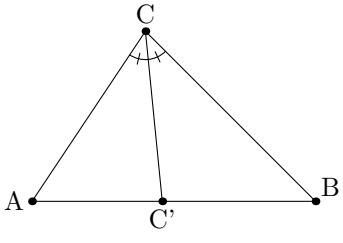


Angle and Direction Vector
In a previous exercise you have already seen that the `dir` command generates a pair that is a point on the unit circle at a given angle with the horizontal axis. The inverse of `dir` is `angle`, which takes a pair, interprets it as a vector, and computes the two-argument arctangent, i.e., it gives the angle corresponding with the vector. In the example below we use it to draw a bisector of a triangle.



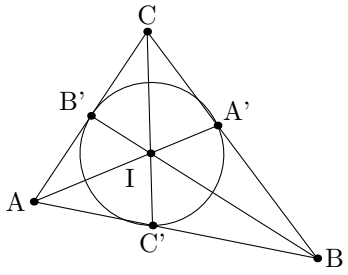

```
pair A, B, C, C';
u := 1cm; A=(0,0); B=(5u,0); C=(2u,3u);
C' = whatever[A,B] = C + whatever*dir(
       1/2*angle(A-C)+1/2*angle(B-C));
beginfig(1)
draw A--B--C--cycle;  draw C--C';
dotlabel.lft("A",A); dotlabel.urt("B",B);
dotlabel.top("C",C); dotlabel.bot("C'",C');
endfig;
end;
```

EXERCISE 18 Change the above picture to the following geometrical diagram, which illustrates better that a bisector is actually drawn for the acute-angled triangle.



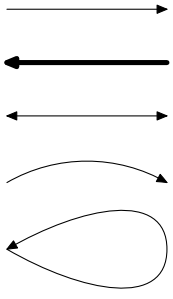
EXERCISE 19 Draw a picture that shows all the bisectors of a acute-angled triangle. Your picture should look like

[7] See `www.chinesefortunecalendar.com/YinYang.htm` for details about the symbol.



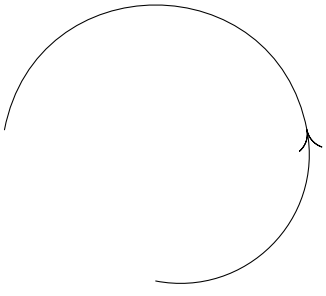
In this way, it illustrates that the bisectors of a triangle go through one point, the so-called incenter, which is the centre of the inner circle of the triangle.

Arrow
The `drawarrow` command draws the given path with an arrowhead at the end. For double-headed arrows, simply use the `drawdblarrow` command. A few examples:



```
beginfig(1);
drawarrow (0,0)--(60,0);
drawarrow reverse((0,-20)--(60,-20))
  withpen pencircle scaled 2;
drawdblarrow (0,-40)--(60,-40);
drawarrow (0,-65){dir(30)}..
  {dir(-30)}(60,-65);
drawarrow (0,-90){dir(-30)}..{up}(60,-90)..
  {dir(-150)}cycle;
endfig;
end;
```

If you want arrowheads of different size, you can change the arrowhead length through the variable `ahlength` (4bp by default) and you can control the angle at the tip of the arrowhead with the variable `ahangle` (45 degrees by default). You can also completely change the definition of the `arrowhead` procedure. In the example below, we draw a curve with an arrow symbol along the path. As a matter of fact, the path is drawn in separate pieces that are joined together with the `&` operator.

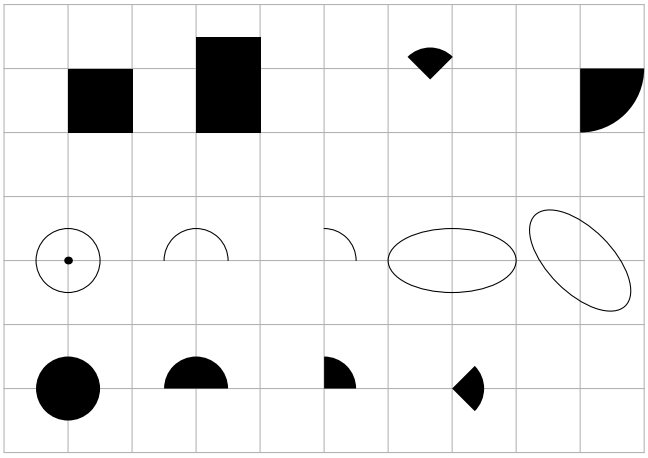


```
beginfig(1);
save arrowhead;
vardef arrowhead(expr p) =
  save A,u,a,b; pair A,u; path a,b;
  A := point length(p)/2 of p;
  u := unitvector(direction length(p)/2 of p);
  a := A{-u}..(A - ahlength*u rotated 30);
  b := A{-u}..(A - ahlength*u rotated -30);
  (a & reverse(a) & b & reverse(b))--cycle
enddef;
u:=2cm; ahlength:=0.3cm;
drawarrow (0,0)..(u,u)..(-u,u);
endfig; end;
```

Circle, Ellipse, Square, and Rectangle
You have already seen that you can draw a circle through three points `z0`, `z1`, and `z2`, that do not lie on a straight line with the statement `draw z0..z1..z2..cycle;`. But METAPOST also provides predefined paths to build circles and circular disks or parts of them. Similarly, you can draw a rectangle once the four corner points, say `z0`, `z1`, `z2`, and `z3`, are known with the statement `draw z0--z1--z2--z3--cycle;`. The path `(0,0)--(1,0)--(1,1)--(0,1)--cycle` is in METAPOST predefined as `unitsquare`.

Path	*Definition*
`fullcircle`	circle with diameter 1 and centre $(0, 0)$.
`halfcircle`	upper half of `fullcircle`
`quartercircle`	first quadrant of `fullcircle`
`unitsquare`	`(0,0)--(1,0)--(1,1)--cyle`

You can construct from these basic paths any circle, ellipse, square, or rectangle by rotating (`rotated` operator), by scaling (operators `scaled`, `xscaled`, and `yscaled`), and/or by translating the graphic object (`shifted` operator). Keep in mind that the ordering of operators has a strong influence on the final shape. But pictures say more than words. The diagram



is drawn with the following METAPOST code.

```
beginfig(1);
u := 24;  % 24 = 24bp = 1/3 inch
for i=-1 upto 9: draw (i*u,4u)--(i*u,-3u) withcolor 0.7white; endfor;
for i=-3 upto 4: draw (-u,i*u)--(9u,i*u)  withcolor 0.7white; endfor;
dotlabel("",origin);  % the grid with reference point (0,0) has been drawn
draw fullcircle scaled u;
draw halfcircle scaled u shifted (2u,0);
draw quartercircle scaled u shifted (4u,0);
draw fullcircle xscaled 2u yscaled u shifted (6u,0);
draw fullcircle xscaled 2u yscaled u rotated -45 shifted (8u,0);
fill fullcircle scaled u shifted (0,-2u);
fill halfcircle--cycle scaled u shifted (2u,-2u);
path quarterdisk;  quarterdisk := quartercircle--origin--cycle;
fill quarterdisk scaled u shifted (4u,-2u);
fill quarterdisk scaled u rotated -45 shifted (6u,-2u);
fill quarterdisk scaled u shifted (6u,-2u) rotated 45;
fill quarterdisk rotated -90 scaled 2u shifted (8u,3u);
fill unitsquare scaled u shifted (0,2u);
fill unitsquare xscaled u yscaled 3/2u shifted (2u,2u);
endfig;
end;
```

Text

You have already seen how the `dotlabel` command can be used to draw a dot and a label in the neighbourhood of the dot. If you do not want the dot, simply use the

`label` command:

`label.`*suffix*`(`*string expression*, *pair*`)` ;

It uses the same suffixes as the `dotlabel` command to position the label relative to the given pair. No suffix means that the label is printed with its centre at the specified location Available directives for the specification of the label relative to the given pair are (see also page 8):

`top` : *top*	`ulft` : *upper left*
`lft` : *left*	`urt` : *upper right*
`rt` : *right*	`lrt` : *lower right*
`bot` : *bot*	`llft` : *lower left*

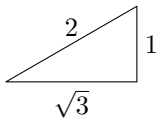
The distance from the pair to the label is set by the numeric variable `labeloffset`.

The commands `label` and `dotlabel` both use a string expression for the label text and typeset it in the default font, which is likely to be `"cmr10"` and which can changed through the variables `defaultfont` and `defaultscale`. For example,

```
defaultfont := "ptmr";
defaultscale := 12pt/fontsize(defaultfont);
```

makes labels come out as Adobe Times-Roman at about 12 points.

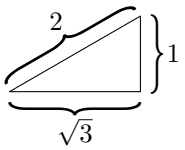
Until now the string expression in a text command has only been a string delimited by double quotes (optionally joined to another string via the concatenation operator `&`). But you can also bracket the text with `btex` and `etex` (do not put it in quotes this time) and pass it to TEX for typesetting. This allows you to use METAPOST in combination with TEX for building complex labels. Let us begin with a simple example:




```
beginfig(1);
z0 = (0,0);  z1 = (sqrt(3)*cm,0);
z2 = (sqrt(3)*cm,1cm);
draw z0--z1--z2--cycle;
label.bot(btex $\sqrt{3}$ etex, 1/2[z0,z1]);
label.rt(btex 1 etex, 1/2[z1,z2]);
label.top(btex 2 etex, 1/2[z0,z2]);
endfig;
end;
```

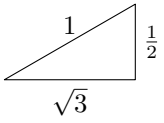
Whenever the METAPOST program encounters `btex` *typesetting commands* `etex`, it suspends the processing of the input in order to allow TEX to typeset the commands and the `dvitomp` preprocessor to translate the typeset material into a picture expression that can be used in a `label` or `dotlabel` statement. The generated low level METAPOST code is placed in a file with extension `.mpx`. Hereafter METAPOST resumes its work.

We speak about a picture expression that is created by typesetting commands because it is a graphic object to which you can apply transformation. This is illustrated by the following example, in which we use diagonal curly brackets and text.




```
beginfig(1);
z0 = (0,0);  z1 = (sqrt(3)*cm,0);
z2 = (sqrt(3)*cm,1cm);
draw z0--z1--z2--cycle;
label.bot(btex $\lbrace$ etex rotated 90
  xscaled 5 yscaled 1.4, 1/2[z0,z1]);
label.rt((btex $\rbrace$ etex) xscaled 1.3
  yscaled 3, 1/2[z1,z2]);
label(btex $\lbrace$ etex xscaled 1.5 yscaled 5.7
  rotated -60, 1/2[z0,z2] + dir(120)*2mm);
labeloffset:=3.5mm;
label.bot(btex $\sqrt{3}$ etex, 1/2[z0,z1]);
label.rt(btex 1 etex, 1/2[z1,z2]);
label(btex 2 etex, 1/2[z0,z2]+dir(120)*5mm);
endfig;
end;
```

Until now we have only used plain TEX commands. But what if you want to run another TEX-version? The following example shows how you can use a `verbatimtex.....etex` block to specify that LATEX is used and which style and/or packages are chosen.

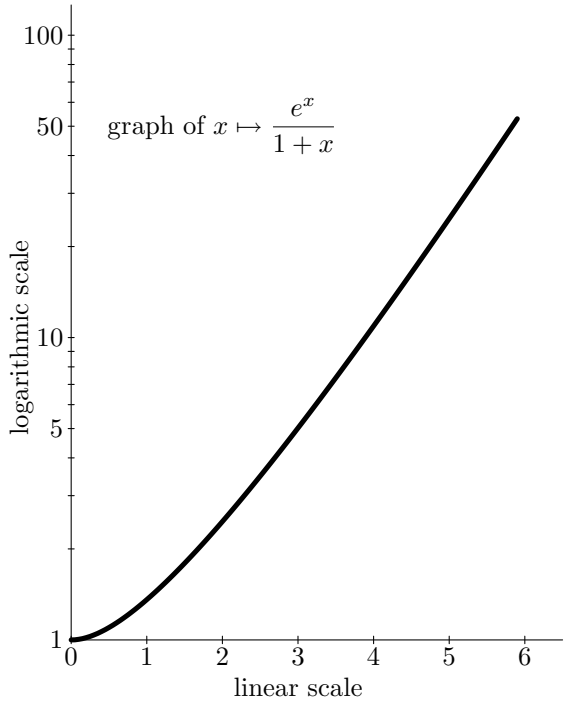



```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

beginfig(1);
z0 = (0,0); z1 = (sqrt(3)*cm,0);
z2 = (sqrt(3)*cm,1cm);
draw z0--z1--z2--cycle;
label.bot(btex $\sqrt{3}$ etex, 1/2[z0,z1]);
label.rt(btex $\frac{1}{2}$ etex, 1/2[z1,z2]);
label.top(btex 1 etex, 1/2[z0,z2]);
endfig;

end;
```

One last remark about using LATEX: Between `btex` and `etex`, you cannot use displayed math mode such as `$$\frac{x}{x+1}$$`. You must use `$\displaystyle \frac{x}{x+1}$` instead.

Let us use what we have learned so far in this chapter in a more practical example: drawing the graph of the function $x \mapsto \dfrac{e^x}{1+x}$ from 0 to 5 with the vertical axis in a logarithmic scale. The picture is generated by the following METAPOST code:

```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

% some function definitions
vardef exp(expr x) = (mexp(256)**x) enddef;
vardef ln(expr x) = (mlog(x)/256) enddef;
vardef log(expr x) = (ln(x)/ln(10)) enddef;
vardef f(expr x) = (exp(x)/(1+x)) enddef;
ux := 1cm; uy := 4cm;
```




```
beginfig(1)
numeric xmin, xmax, ymin, ymax;
xmin := 0; xmax := 6;
ymin := 0; ymax := 2;
% draw axes
draw (xmin,0)*ux -- (xmax+1/2,0)*ux;
draw (0,ymin)*uy -- (0,ymax+1/10)*uy;
% draw tickmarks and labels on horizontal axis
for i=0 upto xmax:
  draw (i,-0.05)*ux--(i,0.05)*ux;
  label.bot(decimal(i),(i,0)*ux);
endfor;
% draw tickmarks and labels on vertical axis
for i=2 upto 10:
  draw (-0.01,log(i))*uy--(0.01,log(i))*uy;
  draw (-0.01,log(10*i))*uy--(0.01,log(10*i))*uy;
endfor;
for i=0 upto 2: label.lft(decimal(10**i), (0,i)*uy); endfor;
for i=0 upto 1: label.lft(decimal(5*(10**i)),
  (0,log(5*(10**i)))*uy); endfor;
% compute and draw the graph of the function
xinc := 0.1;
path pts_f;
pts_f := (xmin*ux,log(f(xmin))*uy)
  for x=xmin+xinc step xinc until xmax:
    .. (x*ux,log(f(x))*uy)
  endfor;
draw pts_f withpen pencircle scaled 2;
% draw title
label(btex graph of $\displaystyle x\mapsto\frac{e^x}{1+x}$
  etex, (2ux,1.7uy));
% draw axis explanation
labeloffset := 0.5cm;
label.bot(btex linear scale etex, (3,0)*ux);
label.lft(btex logarithmic scale etex rotated(90),
  (0,1)*uy);
endfig;

end;
```

The above code needs some explanation.

First of all, METAPOST does not know about the exponential or logarithmic function. But you can easily define these functions with the help of the built-in functions `mexp(x)` $= \exp(x/256)$ and `mlog(x)` $= 256\ln x$. Note that we have reserved the name `log` for the logarithm with base 10 in the above program.

As you will see later in this tutorial, METAPOST has several repetition control structures. Here we apply the `for` loop to draw tick marks and labels on the axes and to compute the path of the graph. The basic form is:

`for` *counter* = *start* `step` *stepsize* `until` *finish* :
    loop text
`endfor;`

Instead of `step 1 until`, you may use the keyword `upto`. `downto` is another word for `step -1 until`.

In the following code snippet

```
for i=0 upto xmax:
  draw (i,-0.05)*ux--(i,0.05)*ux;
  label.bot(decimal(i),(i,0)*ux);
endfor;
```

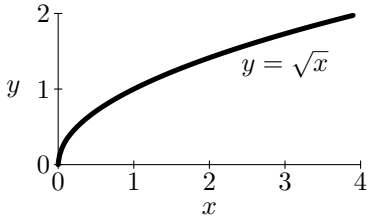
the input lines are two statements: one to draw tick marks and the other to put a label. We use the `decimal` command to convert the numeric variable `i` into a string

so that we can use it in the `label` statement. The following code snippet

```
pts_f := (xmin*ux,log(f(xmin))*uy)
    for x=xmin+xinc step xinc until xmax:
      .. (x*ux,log(f(x))*uy)
    endfor;
```

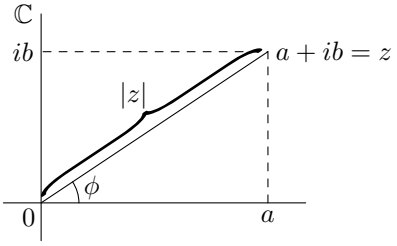
shows that you can also use the `for` loop to build up a single statement. The input lines within the `for` loop are pieces of a path definition. This mode of creating a statement may look strange at first sight, but it is an opportunity given by the fact that METAPOST consists more or less of two parts: a preprocessor and a PostScript generator. The preprocessor only reads from the input stream and prepares input for the PostScript generator.

EXERCISE 20 Draw the graph of the function $x \mapsto \sqrt{x}$ on the interval $(0, 2)$. Your picture should look like



Your METAPOST code should be such that only a minimal change in the code is required to draw the graph on a different domain, say $[0, 3]$.

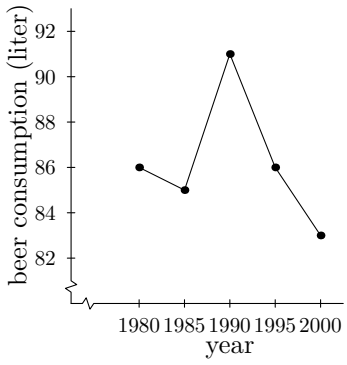
EXERCISE 21 Draw the following picture in METAPOST. The dashed lines can be drawn by adding `dashed evenly` at the end of the `draw` statement.



EXERCISE 22 The annual beer consumption in the Netherlands in the period 1980–2000 is listed below.

year	1980	1985	1990	1995	2000
litre	86	85	91	86	83

Draw the following graph in METAPOST.

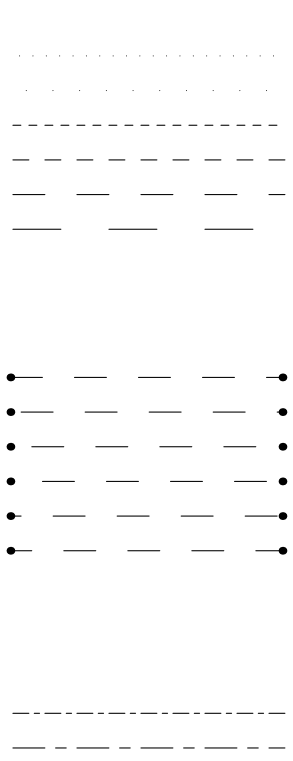


Style Directives

In this section we explain how you can alter the appearance of graphics primitives, e.g., allowing certain lines to be thicker and others to be dashed, using different colours, and changing the type of the drawing pen.

Dashing

Examples show you best how the specify a dash pattern when drawing a line or curve.



```
beginfig(1);
path p; p := (0,0)--(102,0);
def drawit (suffix p)(expr pattern) =
  draw p dashed pattern;
  p := p shifted (0,-13);
enddef;
drawit(p, withdots);
drawit(p, withdots scaled 2);
drawit(p, evenly);
drawit(p, evenly scaled 2);
drawit(p, evenly scaled 4);
drawit(p, evenly scaled 6);

p := (0,-150)--(102,-150);
def shiftit (suffix p)(expr s) =
  draw p dashed evenly scaled 4 shifted s;
  dotlabel("",point 0 of p);
  dotlabel("",point 1 of p);
  p := p shifted (0,-13);
enddef;
shiftit(p, (0,0));
shiftit(p, (4bp,0));
shiftit(p, (8bp,0));
shiftit(p, (12bp,0));
shiftit(p, (16bp,0));
shiftit(p, (20bp,0));

picture dd; dd :=
dashpattern(on 6bp off 2bp on 2bp off 2bp);
draw (0,-283)--(102,-283) dashed dd;
draw (0,-296)--(102,-296) dashed dd scaled 2;
endfig;

end;
```

In general, the syntax for dashing is

`draw` *path* `dashed` *dash pattern* `;`

You can define a dash pattern with the `dashpattern` function whose argument is a sequence of `on`/`off` distances. Predefined patterns are:

```
evenly   = dashpattern(on 3 off 3); % equal length dashes
withdots = dashpattern(off 2.5 on 0 off 2.5); % dotted lines
```

Colouring

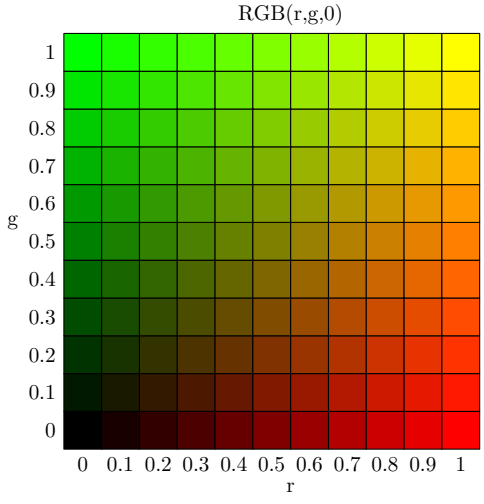
The `color` data type is represented as a triple (r, g, b) that specifies a colour in the RGB colour system. Each of r, g, and b must be a number between 0 and 1, inclusively, representing fractional intensity of red, green, or blue, respectively. Predefined colours are:

```
red   = (1,0,0);  green = (0,1,0);  blue = (0,0,1);
black = (0,0,0);  white = (1,1,1);
```

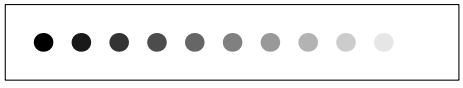
A shade of gray can be specified most conveniently by multiplying the `white` colour with some scalar between 0 and 1. The syntax of using a colour in a graphic statement is:

`withcolor` *colour expression*;

Let us draw two colour charts:




```
beginfig(1);
u := 1/2cm;
defaultscale := 10pt/fontsize(defaultfont);
beginfig(1);
path sqr; sqr := unitsquare scaled u;
for i=0 upto 10:
  label.bot(decimal(i/10), ((i+1/2)*u,0));
  label.lft(decimal(i/10), (0,(i+1/2)*u));
  for j=0 upto 10:
    fill sqr shifted (i*u,j*u)
         withcolor (i*0.1, j*0.1,0);
    draw sqr shifted (i*u,j*u); % draw grid
  endfor;
endfor;
label.bot("r",(6u,-2/3u));
label.lft("g",(-u,6u));
label.top("RGB(r,g,0)", (6u,11u));
endfig; end;
```



```
beginfig(2);
draw (-u,-u)--(11u,-u)--(11u,u)--(-u,u)
     --cycle; % bounding box
pickup pencircle scaled 1/2u;
for i=0 upto 10:
  draw (i*u,0) withcolor i*0.1*white;
endfor;
endfig;

end;
```

EXERCISE 23 Compare the linear conversion from colour to gray, defined by the function

$$(r, g, b) \mapsto \frac{(r + g + b)}{3} \times (1, 1, 1)$$

with the following conversion formula used in black and white television:

$$(r, g, b) \mapsto (0.30r + 0.59g + 0.11b) \times (1, 1, 1)\,.$$

Specifying the Pen
In METAPOST you can define your drawing pen for specifying the line thickness or for calligraphic effects. The statement

`draw` *path* `withpen` *pen expression*;

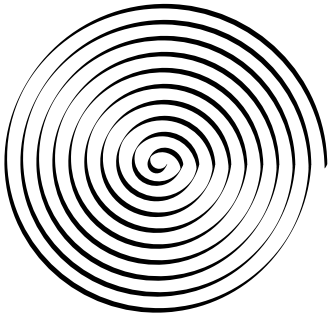
causes the chosen pen to be used to draw the specified path. This is only a temporary pen change. The statement

`pickup` *pen expression*;

causes the given pen to be used in subsequent `draw` statements. The default pen is circular with a diameter of 0.5 bp. If you want to change the line thickness, simply use the following pen expression:

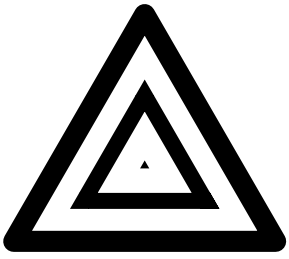
`pencircle scaled` *numeric expression*;

You can create an elliptically shaped and rotated pen by transforming the circular pen. An example:



```
beginfig(1);
pickup pencircle xscaled 2bp yscaled 0.25bp
  rotated 60 withcolor red;
for i=10 downto 1:
  draw 5(i,0)..5(0,i)..5(-i,0)
       ..5(0,-i+1)..5(i-1,0);
endfor;
endfig;
end;
```

In the following example we define a triangular shaped pen. It can be used to plot data points as triangles instead of dots. For comparison we draw a large triangle with both the triangular and the default circular pen.



```
beginfig(1);
path p; p := dir(-30)--dir(90)--dir(210)--cycle;
pen pentriangle;
pentriangle := makepen(p);
draw origin withpen pentriangle scaled 2;
draw (p scaled 1cm) withpen pentriangle scaled 4;
draw (p scaled 2cm) withpen pencircle scaled 8;
endfig;
end;
```

Setting Drawing Options
The function `drawoptions` allows you to change the default settings for drawing. For example, if you specify

```
drawoptions(dashed evenly withcolor red);
```

then all `draw` statements produce dashed lines in red colour, unless you overrule the drawing setting explicitly. To turn off `drawoptions` all together, just give an empty list:

```
drawoptions();
```

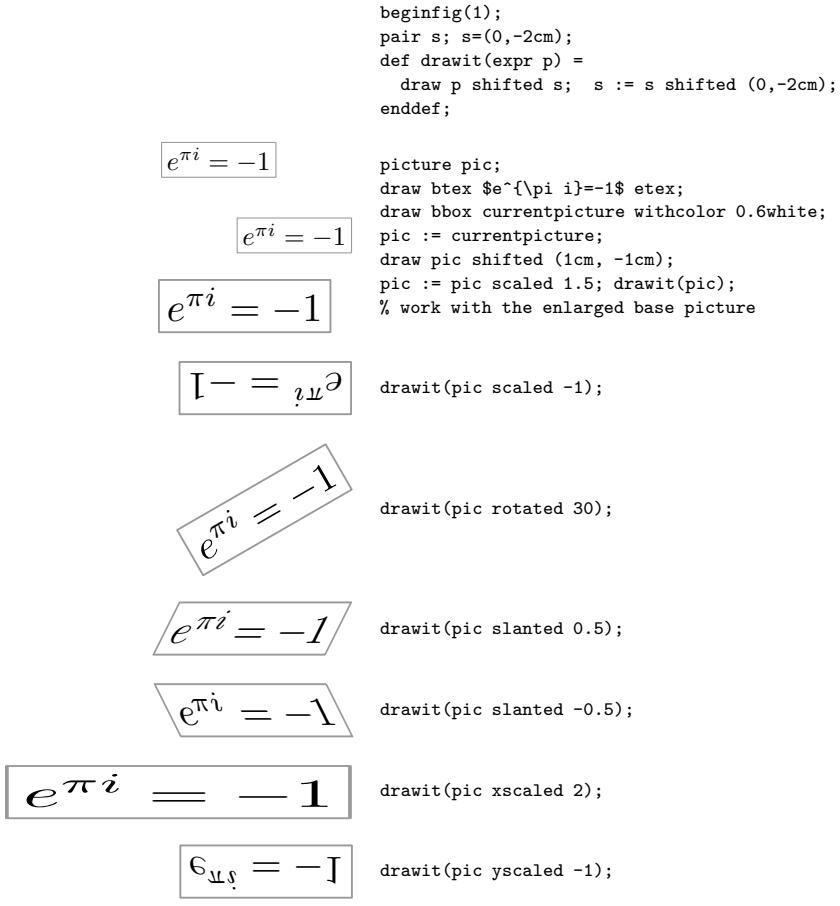
As a matter of fact, this is done automatically by the `beginfig` macro.

Transformations

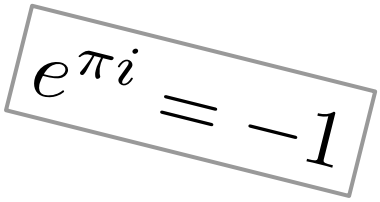
A very characteristic technique with METAPOST, which we applied already in many of the previous examples, is creating a graphic and then using it several times with different transformations. METAPOST has the following built-in operators for scaling, rotating, translating, reflecting, and slanting:

$$
\begin{aligned}
(x,y)\ \texttt{shifted}\ (a,b) &= (x+a, y+b)\,;\\
(x,y)\ \texttt{rotated}\ \theta &= (x\cos\theta - y\sin\theta,\, x\sin\theta + y\cos\theta)\,;\\
(x,y)\ \texttt{rotatedaround}\ \big((a,b),\theta\big) &= (x\cos\theta - y\sin\theta + a(1-\cos\theta) + b\sin\theta,\\
&\qquad x\sin\theta + y\cos\theta + b(1-\cos\theta) - a\sin\theta)\,;\\
(x,y)\ \texttt{slanted}\ a &= (x+ay, y)\,;\\
(x,y)\ \texttt{scaled}\ a &= (ax, ay)\,;\\
(x,y)\ \texttt{xscaled}\ a &= (ax, y)\,;\\
(x,y)\ \texttt{yscaled}\ a &= (x, ay)\,;\\
(x,y)\ \texttt{zscaled}\ (a,b) &= (ax-by, bx+ay)\,.
\end{aligned}
$$

The effect of the translation and most scaling operations is obvious. The following playful example, in which the formula $e^{\pi i} = -1$ is drawn in various shapes, serves as an illustration of most of the listed transformations.



```
beginfig(1);
pair s; s=(0,-2cm);
def drawit(expr p) =
  draw p shifted s;  s := s shifted (0,-2cm);
enddef;

picture pic;
draw btex $e^{\pi i}=-1$ etex;
draw bbox currentpicture withcolor 0.6white;
pic := currentpicture;
draw pic shifted (1cm, -1cm);
pic := pic scaled 1.5; drawit(pic);
% work with the enlarged base picture

drawit(pic scaled -1);

drawit(pic rotated 30);

drawit(pic slanted 0.5);

drawit(pic slanted -0.5);

drawit(pic xscaled 2);

drawit(pic yscaled -1);
```




```
drawit(pic zscaled (2, -0.5));
endfig;
end;
```

The effect of `rotated` θ is rotation of θ degrees about the origin counter-clockwise. The transformation `rotatedaround(p,`θ`)` rotates θ degrees counter-clockwise around point `p`. Accordingly, it is defined in METAPOST as follows:

```
def rotatedaround(expr p, theta) =  % rotates theta degrees around p
  shifted -p rotated theta shifted p enddef;
```

When you identify a point (x, y) with the 3-vector $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$, each of the above operations is described by an affine matrix. For example, the rotation of θ degrees around the origin counter-clockwise and the translation with (a, b) have the following matrices:

$$\texttt{rotated}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \texttt{translated}(a, b) = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}.$$

It is easy to verify that

$$\texttt{rotatedaround}\big((a, b), \theta\big) = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}.$$

The matrix of `zscaled(a,b)` is as follows:

$$\texttt{zscaled}(a, b) = \begin{pmatrix} a & -b & 0 \\ b & a & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Thus, the effect of `zscaled(a,b)` is to rotate and scale so as to map `(1,0)` into `(a,b)`. The operation `zscaled` can also be thought of as multiplication of complex numbers. The picture on the next page illustrates this.

The general form of an affine matrix T is

$$T = \begin{pmatrix} T_{xx} & T_{xy} & T_x \\ T_{yx} & T_{yy} & T_y \\ 0 & 0 & 1 \end{pmatrix}.$$

The corresponding transformation in the two-dimensional space is

$$(x, y) \mapsto (T_{xx}x + T_{xy}y + T_x, T_{yx}x + T_{yy}y + T_y).$$

This mapping is completely determined by the sextuple $(T_x, T_y, T_{xx}, T_{xy}, T_{yx}, T_{yy})$. The information about the mapping can be stored in a variable of data type `transform` and then be applied in a `transformed` statement. There are three ways to define a transform:

□ *In terms of basic transformations*. For example,

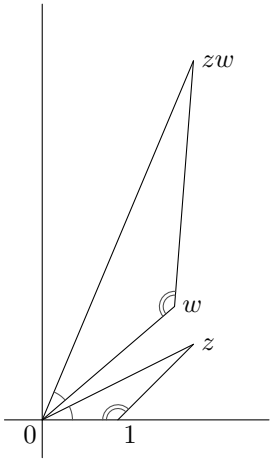
```
transform T; T = identity shifted (-1,0) rotated 60 shifted (1,0);
```

defines the transformation T as a composition of translating with vector $(-1, 0)$, rotating around the origin over 60 degrees, and translating with a vector $(1, 0)$.

□ *Specifying the sextuple* $(T_x, T_y, T_{xx}, T_{xy}, T_{yx}, T_{yy})$. The six parameters than define a transformation T can be referred to directly as `xpart T`, `ypart T`, `xxpart T`, `xypart T`, `yxpart T`, and `yypart T`. Thus,

```
transform T;
xpart T = ypart T = 1;
xxpart T = yypart T = 0;
xypart T = yxpart T = -1;
```

defines a transformation, viz., the reflection in the line through $(1, 0)$ and $(0, 1)$.




```
beginfig(1);
pair z; z := (2,1)*cm;
pair w; w := (7/4,3/2)*cm;
pair zw; zw := (z zscaled w) / cm;

draw (-0.5,0)*cm--(3,0)*cm;
draw (0,-0.5)*cm--(0,5.5)*cm;
draw (1,0)*cm--z; draw (0,0)--z; draw (0,0)--w;
draw (0,0)--zw; draw w--zw;

def drawangle(
   expr endofa, endofb, common, length) =
   save tn; tn :=
   turningnumber(common--endofa--endofb--cycle);
   draw (unitvector(endofa-common){(endofa-common)
   rotated (tn*90)} .. unitvector(endofb-common))
   scaled length shifted common withcolor 0.3white;
enddef;
drawangle((1,0)*cm, z, (0,0), 0.4cm);
drawangle(w, zw, (0,0), 0.4cm);
drawangle((0,0), z, (1,0)*cm, 0.2cm);
drawangle((0,0), z, (1,0)*cm, 0.15cm);
drawangle((0,0), zw, w, 0.2cm);
drawangle((0,0), zw, w, 0.15cm);
label.llft(btex $0$ etex,(0cm,0cm));
label.lrt(btex $1$ etex,(1cm,0cm));
label.rt(btex $z$ etex, z);
label.rt(btex $w$ etex, w);
label.rt(btex $zw$ etex, zw);
endfig;

end;
```

□ *Specifying the images of three points*. It is possible to apply an unknown transform to a known pair and use the result in a linear equation. For example,

```
transform T;
(1,0) transformed T = (1,0);
(0,1) transformed T = (0,1);
(0,0) transformed T = (1,1);
```

defines the reflection in the line through $(1, 0)$ and $(0, 1)$.

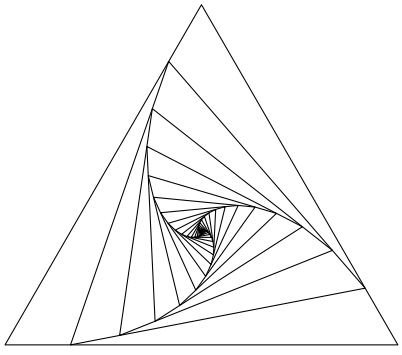
The built-in transformation `reflectedabout(p,q)`, which reflects about the line connecting the points `p` and `q`, is defined by a combination of the last two techniques:

```
def reflectedabout(expr p,q) = transformed
  begingroup
    transform T_;
```

```
    p transformed T_ = p;  q transformed T_ = q;
   xxpart T_ = -yypart T_; xypart T_ = yxpart T_; % T_ is a reflection
    T_
  endgroup
enddef;
```

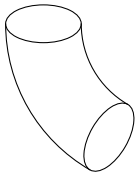
Given a transformation `T`, the inverse transformation is easily defined by `inverse(T)`.

We end with another playful example of an iterative graphic process.



```
beginfig(1);
pair A,B,C; u:=3cm;
A=u*dir(-30); B=u*dir(90); C=u*dir(210);

transform T;
A transformed T = 1/6[A,B];
B transformed T = 1/6[B,C];
C transformed T = 1/6[C,A];

path p; p = A--B--C--cycle;
for i=0 upto 60:
  draw p; p:= p transformed T;
endfor;
endfig;

end;
```

EXERCISE 24 Using transformations, construct the following picture:

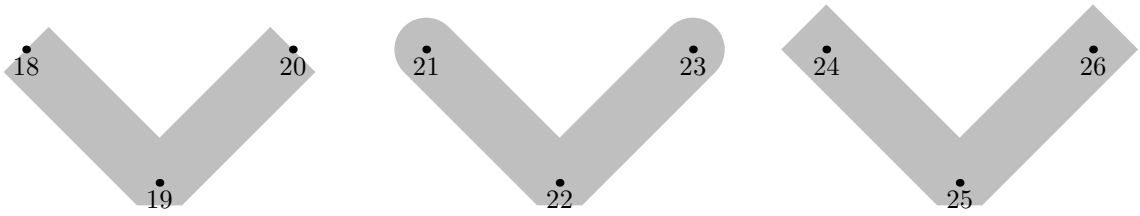


Advanced Graphics

In this section we deal with fine points of drawing lines and with more advanced graphics. This will allow you to create more professional-looking graphics and more complicated pictures.

Joining Lines

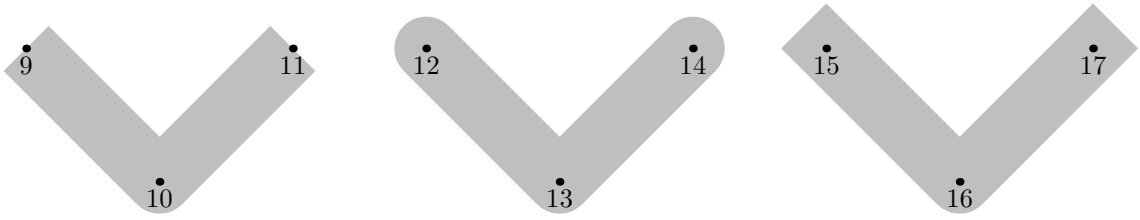
In the last example of the section on pen styles you may have noticed that lines are joined by default such that line joints are normally rounded. You can influence the appearances of the lines by the two internal variables `linejoin` and `linecap`. The picture below shows the possibilities.



linejoin=beveled
linecap=butt

linejoin=beveled
linecap=rounded

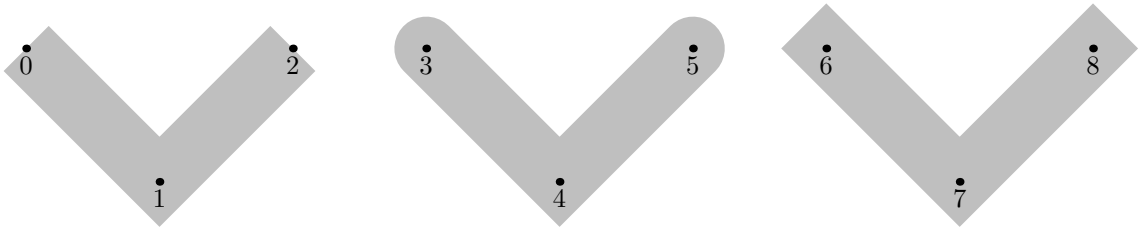
linejoin=beveled
linecap=squared



linejoin=rounded
linecap=butt

linejoin=rounded
linecap=rounded

linejoin=rounded
linecap=squared



linejoin=mitered
linecap=butt

linejoin=mitered
linecap=rounded

linejoin=mitered
linecap=squared

This picture can be produced by the following METAPOST code

```
beginfig(1);
for i=0 upto 2:
  for j=0 upto 2:
    z[3i+9j]=(150i, 150j);
    z[3i+9j+1]=(150i+50,150j-50);
    z[3i+9j+2]=(150i+100,150j);
  endfor;
endfor;
drawoptions(withpen pencircle scaled 24 withcolor 0.75white);
linejoin := mitered; linecap := butt;    draw z0--z1--z2;
linejoin := mitered; linecap := rounded; draw z3--z4--z5;
linejoin := mitered; linecap := squared; draw z6--z7--z8;
linejoin := rounded; linecap := butt;    draw z9--z10--z11;
linejoin := rounded; linecap := rounded; draw z12--z13--z14;
```

```
linejoin := rounded; linecap := squared; draw z15--z16--z17;
linejoin := beveled; linecap := butt;    draw z18--z19--z20;
linejoin := beveled; linecap := rounded; draw z21--z22--z23;
linejoin := beveled; linecap := squared; draw z24--z25--z26;
%
drawoptions();
for i=0 upto 26: dotlabel.bot(decimal(i), z[i]); endfor;
labeloffset := 25pt; label.bot("linejoin=mitered", z1);
labeloffset := 40pt; label.bot("linecap=butt",     z1);
labeloffset := 25pt; label.bot("linejoin=mitered", z4);
labeloffset := 40pt; label.bot("linecap=rounded",  z4);
labeloffset := 25pt; label.bot("linejoin=mitered", z7);
labeloffset := 40pt; label.bot("linecap=squared",  z7);
%
labeloffset := 25pt; label.bot("linejoin=rounded", z10);
labeloffset := 40pt; label.bot("linecap=butt",     z10);
labeloffset := 25pt; label.bot("linejoin=rounded", z13);
labeloffset := 40pt; label.bot("linecap=rounded",  z13);
labeloffset := 25pt; label.bot("linejoin=rounded", z16);
labeloffset := 40pt; label.bot("linecap=squared",  z16);
%
labeloffset := 25pt; label.bot("linejoin=beveled", z19);
labeloffset := 40pt; label.bot("linecap=butt",     z19);
labeloffset := 25pt; label.bot("linejoin=beveled", z22);
labeloffset := 40pt; label.bot("linecap=rounded",  z22);
labeloffset := 25pt; label.bot("linejoin=beveled", z25);
labeloffset := 40pt; label.bot("linecap=squared",  z25);
%
endfig;
end;
```

By setting the variable `miterlimit`, you can influence the mitering of joints. The next example demonstrates that the value of this variable acts as a trigger, i.e. when the value of miterlimit gets smaller than some threshhold value, then the mitered join is replaced by a beveled value.

```
beginfig(1);
for i=0 upto 2:
  z[3i]=(150i,0);  z[3i+1]=(150i+50,-50); z[3i+2]=(150i+100,0);
endfor;
drawoptions(withpen pencircle scaled 24pt);
labeloffset:= 25pt;
linejoin := mitered; linecap:=butt;
for i=0 upto 2:
  miterlimit := i;
  draw z[3i]--z[3i+1]--z[3i+2];
  label.bot("miterlimit=" & decimal(miterlimit), z[3i+1]);
endfor;
endfig;
end;
```

miterlimit=0 miterlimit=1 miterlimit=2

Building Cycles

In previous examples you have seen that intersection points of straight lines can be specified by linear equations. A more direct way to deal with path intersection is via the operator `intersectionpoint`. So, given four points `z1`, `z3`, `z3`, and `z4` in general position, you can specify the intersection point `z5` of the line between `z1` and `z2` and the line between `z3` and `z4` by

```
z5 = z1--z2 intersectionpoint z3--z4;
```

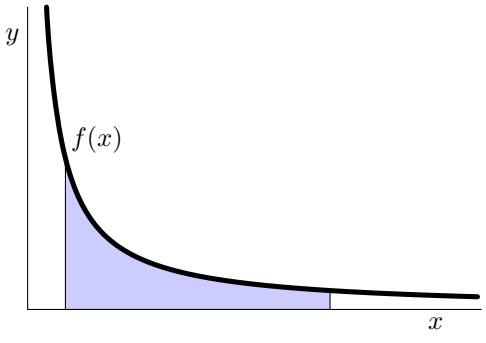
You do not need to rely on setting up linear equations with

```
z5 = whatever[z1,z2] = whatever[z3,z4];
```

The strength of the `intersection` operator is that it also works for curved lines. We use this operator in the next example of a filled area beneath the graph of a function. The closed curve that forms the border of the filled area is constructed with the `buildcycle` command. When given two or more paths, the `buildcycle` macro tries to piece them together so as to form a cyclic path. In case there are more intersection points between paths, the general rule is that

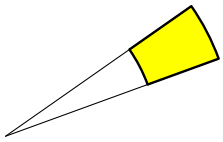
$$\texttt{buildcycle}(p_1, p_2, \ldots, p_n)$$

chooses the intersection between each p_i and p_{i+1} to be as late as possible on the path p_i and as early as possible on p_{i+1}. In practice, it is more convenient to choose the path arguments such that consecutive ones have a unique intersection.




```
beginfig(1);
numeric xmin, xmax, ymin, ymax;
xmin := 1/4; xmax := 6; ymax := 1/xmin; u := 1cm;
% compute the graph of the function
vardef f(expr x) = 1/x enddef;
xinc := 0.1;
path pts_f;
pts_f := (xmin,f(xmin))*u
    for x=xmin+xinc step xinc until xmax:
      .. (x,f(x))*u
    endfor;
path hline[], vline[];
hline0 = (0,0)*u -- (xmax,0)*u;
vline0 = (0,0)*u -- (0,ymax)*u;
vline0.5 = (0.5,0)*u -- (0.5,ymax)*u;
vline4 = (4,0)*u -- (4,ymax)*u;
fill buildcycle(hline0, vline0.5, pts_f, vline4)
  withcolor 0.8[blue,white];
draw hline0; draw vline0; % draw axes
draw (0.5,0)*u -- vline0.5 intersectionpoint pts_f;
draw (4,0)*u -- vline4 intersectionpoint pts_f;
draw pts_f withpen pencircle scaled 2;
label.bot(btex $x$ etex, (0.9xmax,0)*u);
label.lft(btex $y$ etex, (0,0.9ymax)*u);
label.urt(btex $f(x)$ etex, (0.5,f(0.5))*u);
endfig;

end;
```

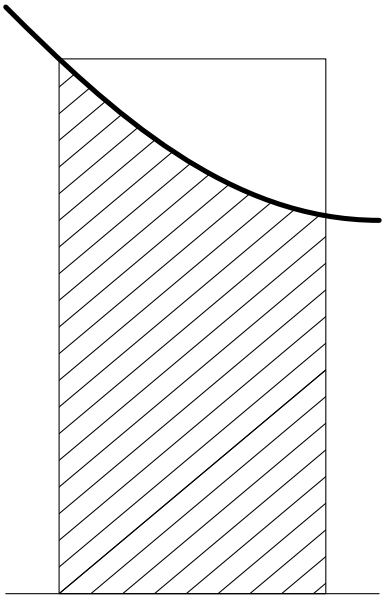
EXERCISE 25 Create the following picture:



Clipping
Clipping is a process to select just those parts of a picture that lie inside an area that is determined by a cyclic path and to discard the portion outside this is area. The command to do this in METAPOST is

`clip` *picture variable* `to` *path expression*;

You can use it to shade a picture element:



```
beginfig(1);
pair p[]; path c[];
c0 = -500*dir(40) -- 500*dir(40);
for i=0 upto 25:
  draw c0 shifted (0,10*i);
  draw c0 shifted (0,-10*i);
endfor;
p1 = (100,0);
c1 = (-20,0) -- (120,0);
c2 = p1--(100,infinity);
c3 = (-20,220){dir(-45)}..(120,140){right};
c4 = (0,0)--(0,infinity);
c5 = buildcycle(c1,c2,c3,c4);
clip currentpicture to c5;
p3 = c4 intersectionpoint c3;
p2 = (100, ypart p3);
draw (0,0)--p1--p2--p3--cycle;
draw c1; draw c3 withpen pencircle scaled 2;
endfig;

end;
```

Dealing with Paths Parametrically
In METAPOST, a path is a continuous curve that is composed of a chain of segments. Each segment is a cubic Bézier curve, which is determined by 4 control points. The points on the curved segment from points p_0 to p_1 with post control point c_0 and pre control point c_1 are determined by the formula

$$p(t) = (1-t)^3 p_0 + 3t(1-t)^2 c_0 + 3t^2(1-t)c_1 + t^3 p_1\,,$$

where $t \in [0,1]$. If the path consists of two arcs, i.e., consists of three points p_0, p_1, and p_2, then the time parameter t runs from 0 to 2. If the path consists of n curve segments, then t runs normally from 0 to n. At $t = 0$ it starts at point p_0 and at intermediate time $t = 1$ the second point p_1 is reached; a third point p_2 in the path, if present, is reached at $t = 2$, and so on. You can get the point on a *path* at any time t with the construction

`point` t `of` *path*;

For a cyclic path with n arcs through the points p_0, $p_1, \ldots, p_{n-1}$, the normal parameter range is $0 \leqslant t < n$, but `point` t `of` *path* can be computed for any t by first

reducing t modulo n. The number of arcs in a path is available through

`length(`*path*`);`

The correspondence between the time parameter and a point on the curve is also used to create a subpath of the curve. The command has the following syntax

`subpath` *pair expression* `of` *path expression*`;`

If the value of the pair expression is `(t1,t2)` and the path expression equals `p`, then the result is a path that follows `p` from `point t1 of p` to `point t2 of p`. If `t1` $>$ `t2`, then the subpath runs backward along `p`.

Based on the `subpath` operation are the binary operators `cutbefore` and `cutafter`. For intersecting paths `p1` and `p2`,

```
p1 cutbefore p2;
```

is equivalent to

```
subpath(xpart(p1 intersectiontimes p2), length(p1)) of p1;
```

except that it also sets the path variable `cuttings` to the parts of `p1` that gets cut off. With multiple intersections, it tries to cut off as little as possible. Similarly,

```
p1 cutafter p2;
```

tries to cut off the part of `p1` after its last intersection with `p2`.

We have seen that for a time parameter `t` we can find the corresponding point on the curve `p` by the statement `point t of p;` Another statement, of the general form

`direction t of` *path*`;`

allows you to obtain a direction vector at the point of the *path* that corresponds with time `t`. The magnitude of the direction vector is somewhat arbitrary. The `directiontime` operation is the inverse of the `direction of` operation. Given a direction vector (a pair) and a path,

`directiontime` *direction vector* `of` *path*`;`

return a numeric value that gives the first time `t` when the path has the indicated direction.

`directionpoint` *direction vector* `of` *path*`;`

returns the first point on the path where the given direction is achieved.

The more familiar concept of arc length is also provided for in METAPOST:

`arclength(`*path*`);`

returns the arc length of the given path. If `p` is a path and `a` is a number between 0 and `arclength(p)`, then

```
arctime a of p;
```

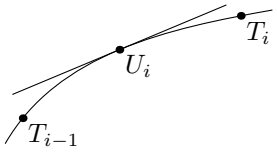
gives the time `t` such that

```
arclength(subpath(0,t) of p) = a;
```

A summary of the path operators is listed in the table below:

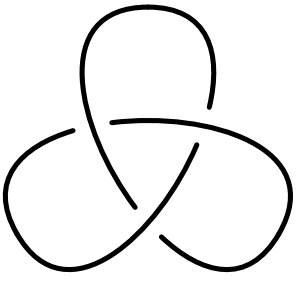
Name	*Arguments and Result*			*Meaning*
	left	right	result	
`arclength`	–	path	numeric	arc length of a path.
`arctime of`	numeric	path	numeric	time on a path where arc length from the start reaches a given value.
`cutafter`	path	path	path	left argument with part after the intersection dropped.
`cutbefore`	path	path	path	left argument with part before the intersection dropped.
`direction of`	numeric	path	path	the direction of a path at a given time.
`directionpoint of`	pair	path	pair	point where a path has a given direction.
`directiontime of`	pair	path	numeric	time when a path has a given direction.
`intersectionpoint`	path	path	pair	an intersection point.
`intersectiontimes`	path	path	numeric	times (t_1, t_2) on paths p_1 and p_2 when the paths intersect.
`length`	–	path	numeric	number of arcs in a path.
`point of`	numeric	path	pair	point on a path given a time value.
`subpath`	pair	path	path	portion of a path for given range of time values times.

Let us apply what we have learned in this subsection to a couple of examples. In the first example, we draw a tangent line at a point of a curve.



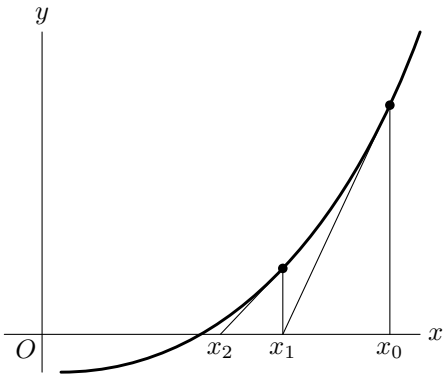

```
beginfig(1);
path c;
c = (0,0){dir(60)}..(100,50){dir(10)};
pair p[];
p0 = point(0.1) of c;
p1 = point(0.5) of c;
p2 = point(0.9) of c;
p3 = direction(0.5) of c;
dotlabel.lrt(btex $T_{i-1}$ etex, p0);
dotlabel.lrt(btex $U_i$ etex, p1);
dotlabel.lrt(btex $T_i$ etex, p2);
draw c;
draw (-p3 -- p3) scaled (40/xpart(p3))
  shifted p1;
endfig;
end;
```

The second example is the trefoil knot, i.e, the torusknot of type (1,3). The METAPOST code has been written such that assigning $n = 5$ and $n = 7$ draws the torusknot of type (1,5) and (1,7), respectively, in a nice way.



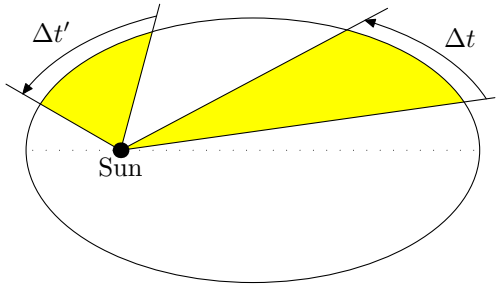
```
beginfig(1);
pair A,B; path p[];
numeric n; n:=3;
A = (0,2cm); B = A rotated (2*360/n);
p0 = A{dir(180)}..tension((n+1)/2)..
   B{dir(180+2*360/n)};
numeric a;
(a,whatever) = p0 intersectiontimes
   (p0 rotated (360/n));
p1 = subpath(0,a-.04) of p0;
p2 = subpath(a+.04,1) of p0;
drawoptions(withpen pencircle scaled 2);
for i=0 upto n-1:
  draw p1 rotated (i*360/n);
  draw p2 rotated (i*360/n);
endfor;
endfig;
end;
```

The third example shows a few steps in the Newton iterative method of finding zeros.




```
beginfig(1);
u := 1cm;
draw (-.5u,0)--(5u,0);
draw (0,-.5u)..(0,4u);
label.llft(btex $0$ etex, (0,0));
label.rt(btex $x$ etex, (5u,0));
label.top(btex $y$ etex, (0,4u));

path f;
f = (.25u,-.5u){right}..(5u,4u){dir(70)};
draw f withpen pencircle scaled 1.2pt;
x0 = 4.6*u;
numeric t[];
for i=0 upto 1:
  (t[i],whatever) = f intersectiontimes
    ((x[i],-infinity)--(x[i],infinity));
  z[i] = point t[i] of f;
  (x[i+1],0) = z[i] +
           whatever*direction t[i] of f;
  draw (x[i],0)--z[i]--(x[i+1],0);
  fill fullcircle scaled 3.6pt shifted z[i];
endfor;
label.bot(btex $x_0$ etex, (x0,0));
label.bot(btex $x_1$ etex, (x1,0));
label.bot(btex $x_2$ etex, (x2,0));
endfig;
end;
```

EXERCISE 26 Create the following picture, which has to do with Kepler's law of areas.



Control Structures

In this section we shall look at two commonly used control structures of imperative programming languages: condition and repetition.

Conditional Operations

The basic form of a conditional statement is

`if` *condition*`:` *balanced tokens* `else:` *balanced tokens* `fi`

where *condition* represents an expression that evaluates to a boolean value (i.e., true or false) and the *balanced tokens* normally represent any valid METAPOST statement or sequence of statements separated by semicolons. The `if` statement is used for branching: depending on some condition, one sequence of METAPOST statements is executed or another. The keyword `fi` is the reverse of `if` and marks the end of the conditional statement. The keyword `fi` separates the statement sequence of the preceding command clause so that the semicolon at the end of the last command in the `else:` part can be omitted. It also marks the end of the

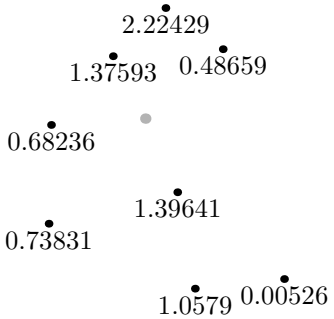
conditional statement so that you do not need a semicolon after the keyword to separate the conditional statement from then next statement.

Other forms of conditional statements are obtained from the basic form by:

- Omitting the `else:` part when there is nothing to be said.
- Nesting of conditional operations. The following shortcut can be used: `else: if` can be replaced by `elseif`, in which case the corresponding `fi` must be omitted. For example, nesting of two basic `if` operations looks as follows:

  `if` *1st condition*`:` *1st tokens* `elseif` *2nd condition*`:` *2nd tokens* `fi`

Let us give an example: computing the centre of gravity (also called barycentre) of a number of objects with randomly generated weight and position. The example contains many more programming constructs, some of which will be covered in sections later on in the tutorial; so you may ignore them if you wish. The nested conditional statement is easily found in the METAPOST code below. With the commands `numeric(x)` and `pair(x)` we test whether `x` is a number or a point, respectively.




```
beginfig(1);
vardef centerofgravity(text t) =
  save x, wght, G, X;
  pair G,X; numeric wght, w;
  G := origin; wght:=0;
  for x=t:
    if numeric(x):
       show("weight = "& decimal(x));
       G:= G + x*X;
       wght := wght + x;
    elseif pair(x):
       show("location = (" &
         decimal(xpart(x)) & ", " &
         decimal(ypart(x)) & ")");
       X:=x; % store pair
    else:
      errmessage("should not happen");
    fi;
  endfor;
  G/wght
enddef;

numeric w[]; pair A[];
n:=8;
for i=1 upto n:
  A[i] = 1.5cm*
    (normaldeviate, normaldeviate);
  w[i] = abs(normaldeviate);
  dotlabel.bot(decimal(w[i]), A[i]);
endfor;
draw centerofgravity(A[1],w[1]
  for i=2 upto n: ,A[i],w[i] endfor)
  withpen pencircle scaled 4bp
  withcolor 0.7white;
endfig;

end;
```

The `errmessage` command is for displaying an error message if something goes wrong and interrupting the program at this point. The `show` statement is used here for debugging purposes. When you run the METAPOST program from a shell, `show` puts its results on the standard output device. In our example, the shell window looked like:

```
(heck@remote 1) mpost barycenter
This is MetaPost, Version 0.641 (Web2C 7.3.1)
(barycenter.mp
>> "location = (-13.14597, -80.09227)"
>> "weight = 1.0579"
>> "location = (-19.7488, -43.93861)"
>> "weight = 1.39641"
>> "location = (-43.89838, 7.07126)"
>> "weight = 1.37593"
>> "location = (-2.69252, 9.70473)"
>> "weight = 0.48659"
>> "location = (-24.17944, 25.14096)"
>> "weight = 2.22429"
>> "location = (-67.98569, -55.73247)"
>> "weight = 0.73831"
>> "location = (20.28859, -76.48691)"
>> "weight = 0.00526"
>> "location = (-67.07672, -18.69904)"
>> "weight = 0.68236" [1] )
1 output file written: barycenter.1
Transcript written on barycenter.log.
(heck@remote 2)
```

The boolean expression that forms the condition can be built up with the following relational and logical operators.

Relational Operators	
Operator	*Meaning*
=	equal
<>	unequal
<	less than
<=	less than or equal
>	greater than
>=	greater than or equal
Logical Operators	
Operator	*Meaning*
`and`	test if all conditions hold
`or`	test if one of many conditions hold
`not`	negation of condition

One final remark on the use of semicolons in the conditional statement. Where the colons after the `if` and `else` part are obligatory, semicolons are optional, depending on the context. For example, the statement

```
if cycle(p): fill p;
elseif path(p): draw p;
else: errmessage("what?");
fi;
```

fills or draws a path depending on the path `p` being cyclic or not. You may omit whatever semicolon in this example and rewrite it even as

```
if cycle(p): fill p
elseif path(p): draw p
else: errmessage("what?")
fi
```

However, when you use the conditional clause to build up a single statement, then you must be more careful with placing or omitting semicolons. In

```
draw p withcolor if cycle(p): red else: blue fi withpen pensquare;
```

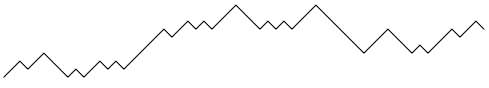
you cannot add semicolons after the colour specifications, nor omit the final semicolon that marks the end of the statement (unless it is a statement that is recognised as finished because of another keyword, e.g., `endfor`).

Repetition

Numerous examples in previous section have used the `for` loop of the form

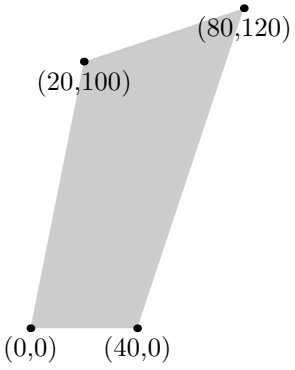
```
for counter = start step stepsize until finish :
    loop text
endfor;
```

where *counter* is a counting variable with initial value *start*. The counter is incremented at each step in the repetition by the value of *stepsize* until it passes the value of *finish*. Then the repetition stops and METAPOST continues with what comes after the `endfor` part. The loop text is usually any valid METAPOST statement or sequence of statements separated by semicolons that are executed at each step in the repetition. Instead of `step 1 until`, we can also use the keyword `upto`. `downto` is another word for `step -1 until`. This counted `for` loop is an example of an *unconditional repetition*, in which a predetermined set of actions are carried out. Below, we give another example of a counted `for` loop: generating a Bernoulli walk. We use the `normaldeviate` operator to generate a random number with the standard normal distribution (mean 0 and standard deviation 1).



```
beginfig(1);
n := 60; pair p[]; p0 = (0,0);
for i=1 upto n:
  p[i] = p[i-1] +
  (3,if normaldeviate>0: -3 else: 3 fi);
endfor;
draw p0 for i=1 upto n: --p[i] endfor;
endfig;
end;
```

The last example in the previous section, in which we computed the centre of gravity of randomly generated weighted points, contained another unconditional repetition, viz., the `for` loop over a sequence of zero or more expressions separated by commas. Another example of this kind is:

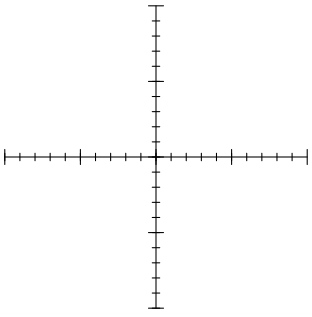



```
beginfig(1);
fill for p=(0,0),(40,0),(80,120),(20,100):
  p-- endfor cycle withcolor 0.8white;
for p=(0,0),(40,0),(80,120),(20,100):
  dotlabel.bot("(" & decimal(xpart(p)) &
    "," & decimal(ypart(p)) & ")", p);
endfor;
endfig;

end;
```

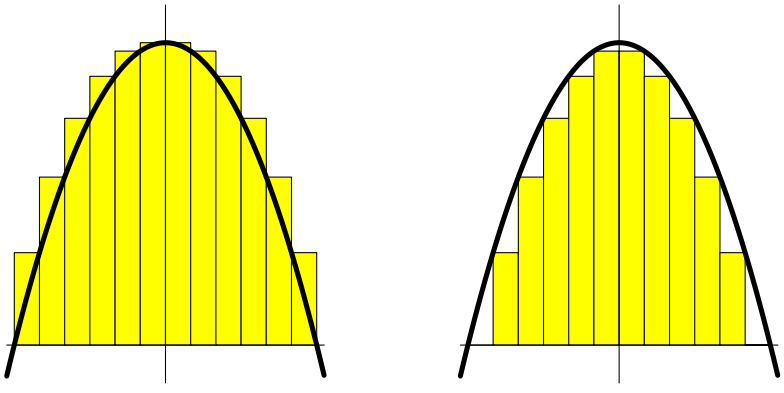
Nesting of counted `for` loops is of course possible, but there are no abbreviations: balancing with respect to `for` and `endfor` is obligatory. You must use both `endfor` keywords in

```
for i=0 upto 10:
  for j=0 upto 10:
    show("i = " & decimal(i) & ", j = " & decimal(j));
  endfor
endfor
```

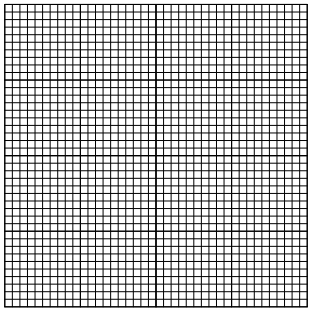
EXERCISE 27 Create the following picture:



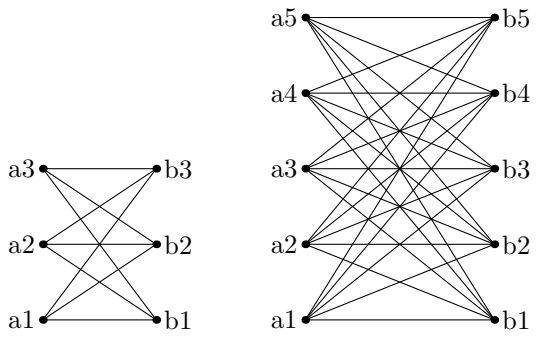
EXERCISE 28 Create the picture below, which illustrates the upper and lower Riemann sum for the area enclosed by the horizontal axis and the graph of the function $f(x) = 4 - x^2$.



EXERCISE 29 Create the following piece of millimetre paper.



EXERCISE 30 A graph is bipartite when its vertices can be partitioned into two disjoint sets A and B such that each of its edges has one endpoint in A and the other in B. The most famous bipartite graph is $K_{3,3}$ show below to the left. Write a program that draws the $K_{n,n}$ graph for any natural number $n > 1$. Show that your program indeed creates the graph $K_{5,5}$, which is shown below to the right.



Another popular type of repetition is the *conditional loop*. METAPOST does not have a pre- or post conditional loop (`while` loop or `until` loop) built in. You must create one by an endless loop and an explicit jump outside this loop. First the endless loop: this is created by

`forever:` *loop text* `endfor;`

To terminate such a loop when a boolean condition becomes true, use an exit clause:

`exitif` *boolean expression*`;`

When the exit clause is encountered, METAPOST evaluates the boolean expression and exits the current loop if the expression is true. Thus, METAPOST's version of a `until` loop is:

```
forever:
    loop text;
    exitif boolean expression;
endfor;
```

If it is more convenient to exit the loop when an expression becomes false, then use the predefined macro `exitunless` Thus, METAPOST's version of a `while` loop is:

```
forever: exitunless boolean expression;
    loop text
endfor
```

Macros

Defining Macros

In the section about the repetition control structure we introduced `upto` as a shortcut of `step 1 until`. This is also how it is is internally defined in METAPOST:

```
def upto = step 1 until enddef;
```

It is a definition of the form

`def` *name* = *replacement text* `enddef;`

It calls for a macro substitution of the simplest kind: subsequent occurrences of the token *name* will be replaced by the *replacement text*. The name in a macro is a variable name; the replacement text is arbitrary and may for example consist of a sequence of statements separated by semicolons.

It is also possible to define macros with arguments, so that the replacement text can be different for different calls of the macro. An example of a built-in, parametrised macro is:

```
def rotatedaround(expr z, d) =    % rotates d degrees around z
  shifted -z rotated d shifted z
enddef;
```

Although it looks like a function call, a use of `rotatedaround` expands into in-line code. The `expr` in this definition means that a formal parameter (here `z` or `d`) can be an arbitrary expression. Each occurrence of a formal parameter will be replaced by the corresponding actual argument (this is referred to as 'call-by-value'). Thus the line

```
rotatedaround(p+q, a+b);
```

will be replaced by the line

```
shifted -(p+q) rotated (a+b) shifted (p+q);
```

Macro parameters need not always be expressions. Another argument type is `text`, indicating that the parameters are just past as an arbitrary sequence of tokens.

Grouping and Local Variables

In METAPOST, all variables are global by default. But you may want to use in some piece of METAPOST code a variable that has temporarily inside that portion of code a value different from the one outside the program block. The general form of a program block is

`begingroup` *statements* `endgroup`

where *statements* is a sequence of one or more statements separated by semicolons. For example, the following piece of code

```
x := 1; y := 2;
begingroup x:=3; y:=x+1; show(x,y); endgroup
show(x,y);
```

will reveal that the `x` and `y` values are 3 and 4, respectively, inside the program block. But right after this program block, the values will be 1 and 2 as before.

The program block is used in the definition of the `hide` macro:

```
def hide(text t) = exitif numeric begingroup t; endgroup; enddef;
```

It takes a text parameter and interprets it as a sequence of statements while ultimately producing an empty replacement text. In other words, this command allows you to run code silently.

Grouping often occurs automatically in METAPOST. For example, the `beginfig` macro starts with `begingroup` and the replacement text for `endfig` ends with `endgroup`. `vardef` macros are always grouped automatically, too.

You may want to go one step further: not only treating values of a variable locally, but also its name. For example, in a macro definition you may want to use a so-called *local variable*, i.e., a variable that only has meaning inside that definition and does not interfere with existing variables. In general, variables are made local by the statement

`save` *name sequence*`;`

For example, the macro `whatever` has the replacement text[8]

```
begingroup save ?; ? endgroup
```

This macro returns an unknown. If the `save` statement is used outside of a group, the original values are simply discarded. This explains the following definition of the built-in macro `clearxy`:

[8] in fact, `save` is a `vardef` macro, which has the `begingroup` and `begingroup` automatically placed around the replacement text. Thus, the `begingroup` and `endgroup` are superfluous here.

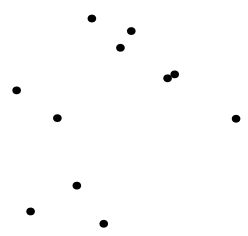
```
def clearxy = save x,y enddef
```

Vardef Definitions
Sometimes we want to return a value from a macro, as if it is a function or subroutine. In this case we want to make sure that the calculations inside the macro do not interfere with the expected use of the macro. This is the main purpose of the `vardef` definition of the form

`def` *name* = *replacement text*; *returned text* `enddef;`

By using `vardef` instead of `def` we hide the replacement text but the last statement, which returns a value.

Below we given an example of a macro that generates a random point in the region `[l,r]`$\times$`[b,u]`. We use the `uniformdeviate` to generate a random number with the uniform distribution between 0 and the given argument. A validity test on the actual arguments is carried out; in case a region is not defined properly, we use `errmessage` to display some text and exit from the macro call, returning the origin as default point when computing is continued.



```
beginfig(1);
vardef randompoint(expr l,r,b,u) =
  if (r<=l) or (u<=b):
     errmessage("not a proper region");
     origin
  else:

  numeric x, y;
  x = l+uniformdeviate(r-l);
  y = b+uniformdeviate(u-b);
  (x,y)
  fi
enddef;
for i=0 upto 10:
  dotlabel("",randompoint(10,100,10,100));
endfor;
endfig;
end;
```

Do not place a semicolon after `origin` or `(x,y)`. In that case, the statement becomes part of the hidden replacement text and an empty value is returned. This causes a runtime error.

Defining the Argument Syntax
In METAPOST, you can explicitly define the argument syntax and construct unary, binary, or tertiary operators. Let us look at the code of a predefined unary operator:

```
vardef unitvector primary z = z/abs z enddef;
```

As the example suggests, the keyword `primary` is enough to specify the macro as a unary `vardef` operator. Other keywords are `secondary` and `tertiary`. The advantage of specifying an n-ary operator is that you do not need to place brackets around arguments in compound statements; METAPOST will sort out which tokens are the arguments. For example

```
unitvector v rotated angle v;
```

is understood to be equivalent to

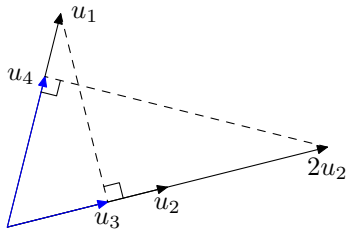
```
(unitvector(v)) rotated(angle(v));
```

You can also define a macro to play the role of an `of` operation. For example, the `direction of` macro is predefined by

```
vardef direction expr t of p =
  postcontrol t of p - precontrol t of p
enddef;
```

Precedence Rules of Binary Operators

METAPOST provides the classifiers `primarydef`, `secondarydef`, and `tertiarydef` (for `def` macros, not for `vardef` macros) to set the level of priority of a binary operator. In the example below, the orthogonal projection of a vector `v` along another vector `w` is defined as a secondary binary operator.

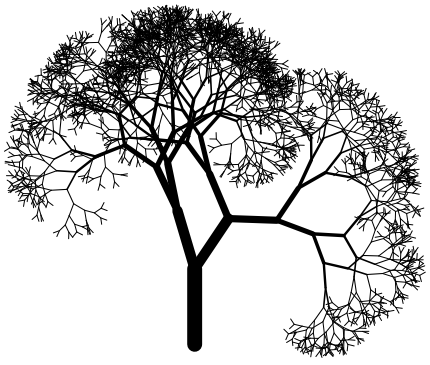



```
beginfig(1);
secondarydef v projectedalong w =
 if pair(v) and pair(w):
    (v dotprod w) / (w dotprod w) * w
 else:
    errmessage "arguments must be vectors"
 fi
enddef;
pair u[]; u1 = (20,80); u2 = (60,15);
drawarrow origin--u1;
drawarrow origin--u2;
drawarrow origin--2*u2;
u3 = u1 projectedalong u2;
u4 = 2*u2 projectedalong u1;
drawarrow origin--u3 withcolor blue;
draw u1--u3 dashed withdots;
draw ((1,0)--(1,1)--(0,1))
  zscaled (6pt*unitvector(u2)) shifted u3;
drawarrow origin--u4 withcolor blue;
draw 2*u2--u4 dashed withdots;
draw ((1,0)--(1,1)--(0,1))
  zscaled (6pt*unitvector(-u1)) shifted u4;
labeloffset := 4pt;
label.rt(btex $u_1$ etex, u1);
label.bot(btex $u_2$ etex, u2);
label.bot(btex $2u_2$ etex, 2*u2);
label.bot(btex $u_3$ etex, u3);
label.lft(btex $u_4$ etex, u4);
endfig;
end;
```

Recursion

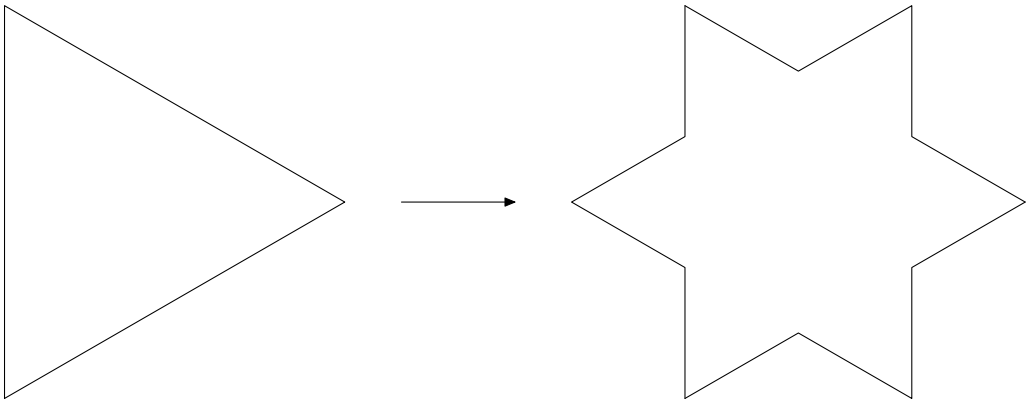
A macro is defined recursively if in its definition, it makes a call to itself. Recursive definition of a macro is possible in METAPOST. We shall illustrate this with the computation of a Pythagorean tree.

```
beginfig(1)
u:=1cm; branchrotation := 60;
offset := 180-branchrotation;
thinning := 0.7;
shortening := 0.8;
def drawit(expr p, linethickness) =
  draw p withpen pencircle scaled linethickness;
enddef;
vardef tree(expr A,B,n,size) =
    save C,D,thickness; pair C,D;
    thickness := size;
```

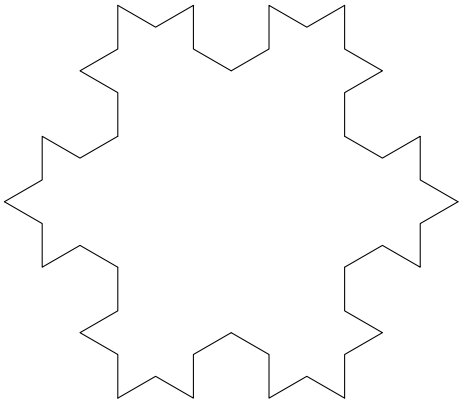


```
    C := shortening[B, A rotatedaround(B,
      offset+uniformdeviate(branchrotation))];
    D := shortening[B, A rotatedaround(B,
      -offset-uniformdeviate(branchrotation))];
    if n>0:
      drawit(A--B, thickness);
      thickness := thinning*thickness;
      tree(B, C, n-1, thickness);
      tree(B, D, n-1, thickness);
    else:
      drawit(A--B,thickness);
      thickness := thinning*thickness;
      drawit(B--C, thickness);
      drawit(B--D, thickness);
    fi;
enddef;
tree((0,0), (0,u), 10, 2mm);
endfig;
end;
```

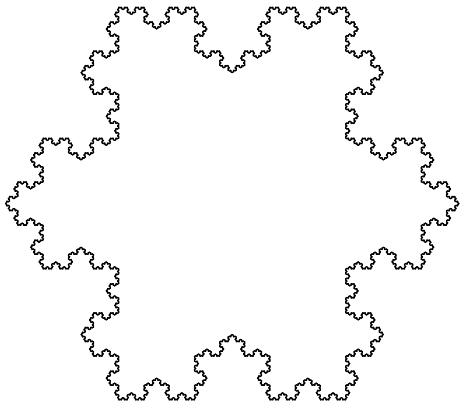
EXERCISE 31 The Koch snowflake is constructed as follows: start with an equilateral triangle. Break each edge into four straight pieces by adding a bump a shown below.



You can then repeat the process of breaking a line segment into four pieces of length one-fourth of the segment that is broken up. Below you see the next iteration.



Write a program that can compute the picture after n iterations. After six iteration, the Koch snowflake should look like



Using Macro Packages

METAPOST comes with built-in macro packages, which are nothing more than files containing macro definitions and constants. The most valuable macro package is `graph`, which contains high-level utility macros for easy drawing graphs and data plots. The `graph` package is loaded by the statement

```
input graph
```

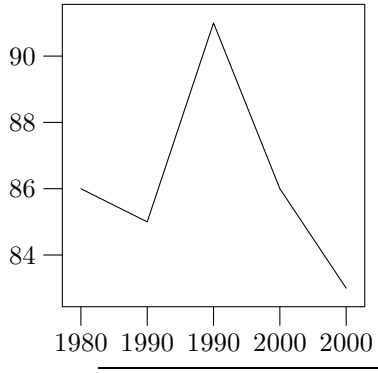
A detailed description, including examples, of the `graph` package can be found in [Hob92b, GRS94]. Here, we just show one example of its use. We represent the following data about the annual beer consumption in the Netherlands in the period 1980–2000 graphically[9].

year	1980	1985	1990	1995	2000
litre	86	85	91	86	83

Suppose that the data are stored columnwise in a file, say `consumption.dat`, as

```
1980 86
1985 85
1990 91
1995 86
2000 83
```

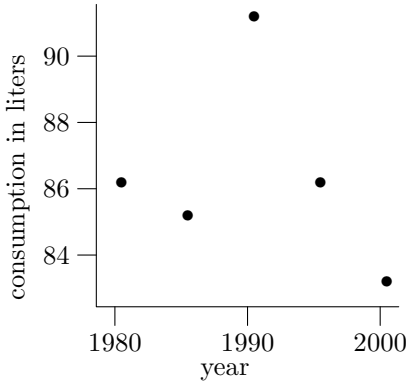
The following piece of code produces a line plot of the data. The only drawback of the `graph` package appears: years are by default marked by decade. This explains the densely populated labels.




```
input graph;
beginfig(1);
draw begingraph(4cm,4cm);
gdraw "consumption.dat";
endgraph;
endfig;

end;
```

[9] Compare this example with your own code in exercise 22.

A few changes in the code draws the data point as bullets, changes the frame style, limits the number of tick marks on the horizontal axis, and puts labels near the axes.




```
beginfig(1);
draw begingraph(4cm,4cm);
gdraw "consumption.dat" plot btex $\bullet$ etex;
Gmarks := 3; % limit number of ticks
frame.llft;
glabel.lft(btex consumption in litres etex
  rotated 90, OUT);
glabel.bot(btex year etex, OUT);
endgraph;
endfig;

end;
```

With the `graph` package, you can easily change in a plot the ticks, the scales, the grid used an/or displayed, the title of the plot, and so on.

Mathematical functions

The following METAPOST code defines some mathematical functions that are not built-in. Note that METAPOST contains two trigonometric functions, `sind` and `cosd`, but they expect their argument in degrees, not in radians.

```
vardef sqr  primary x = (x*x)                                    enddef;
vardef log  primary x = (if x=0: 0 else: mlog(x)/mlog(10) fi) enddef;
vardef ln   primary x = (if x=0: 0 else: mlog(x)/256 fi)      enddef;
vardef exp  primary x = ((mexp 256)**x)                          enddef;
vardef inv  primary x = (if x=0: 0 else: x**-1 fi)            enddef;
vardef pow (expr x,p) = (x**p)                                   enddef;
% trigonometric functions
numeric pi; pi := 3.1415926;
numeric radian;  radian := 180/pi; % 2pi*radian = 360 ;
vardef tand     primary x = (sind(x)/cosd(x))     enddef;
vardef cotd     primary x = (cosd(x)/sind(x))     enddef;
vardef sin      primary x = (sind(x*radian))      enddef;
vardef cos      primary x = (cosd(x*radian))      enddef;
vardef tan      primary x = (sin(x)/cos(x))       enddef;
vardef cot      primary x = (cos(x)/sin(x))       enddef;
% hyperbolic functions
vardef sinh     primary x = save xx ; xx = exp xx ; (xx-1/xx)/2 enddef ;
vardef cosh     primary x = save xx ; xx = exp xx ; (xx+1/xx)/2 enddef ;
vardef tanh     primary x = (sinh(x)/cosh(x))     enddef;
vardef coth     primary x = (cosh(x)/sinh(x))     enddef;
% inverse trigonometric and hyperbolic functions
vardef arcsind  primary x = angle((1+-+x,x))      enddef;
vardef arccosd  primary x = angle((x,1+-+x))      enddef;
vardef arcsin   primary x = ((arcsind(x))/radian) enddef;
vardef arccos   primary x = ((arccosd(x))/radian) enddef;
vardef arccosh  primary x = ln(x+(x+-+1))         enddef;
vardef arcsinh  primary x = ln(x+(x++1))          enddef;
```

Most definitions speak for themselves, except that you may not be familiar with Pythagorean addition (++) and subtraction (+−+):

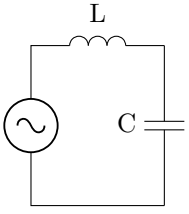
$$++(a,b) = \sqrt{a^2+b^2}, \quad +-+(a,b) = \sqrt{a^2-b^2}.$$

More Examples

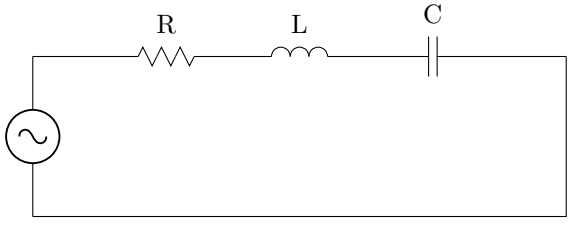
The examples in this section are meant to give you an idea of the strength of METAPOST.

Electronic Circuits

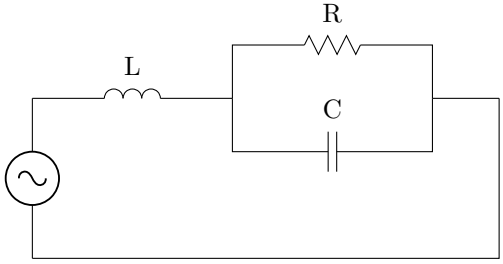
`mpcirc` is a macro package for drawing electronic circuits, developed by Tomasz Cholewo and downloadable from `http://ci.uofl.edu/tom/software/LaTeX/mpcirc/`. Let us use it to create some diagrams. We show the diagrams and the code to create them. The basic idea of `mpcirc` is that you have at your disposal a set of predefined electronic components such as resistor, capacity, diode, and so on. Each component has some connection points, referred to by `a`, `b`, ..., for wires. You place the elements, using predefined orientations, and then connect them with wires. This mode of operating with `mpcirc` is referred to as turtle-based. Another programming style for drawing diagrams is node-based. In this approach, the node locations are determined first and then the elements are put between them using `betw.x` macros. We shall use the turtle-mode in our examples and hope that the comments speak for themselves.




```
u:=10bp; % unit of length
input mpcirc;
beginfig(1);
prepare(L,C,Vac); % mention your elements
z0=(10u,10u);  % lower right node
ht:=6u;        % height of circuit
z1=z0+(0,ht);  % upper right node
C=.5[z0,z1];   % location of capacitor
L.t=T.r;       % use default orientation
C.t=Vac.t=T.u; % components rotated 90 degrees
% set the distance between Voltage and Capacitor
equally_spaced(5u,0) Vac, C;
L=z1-0.5(C-Vac); % location of spool
edraw; % draw components of the circuit
% draw wires connecting components
% the first ones rotated 90 degrees
wire.v(Vac.a,z0);
wire.v(Vac.b,L.a);
wire.v(L.b,z1);
wire(C.a,z0);
wire(C.b,z1);
endfig;
end;
```




```
u:=10bp; % unit of length
input mpcirc;
beginfig(1);
prepare (L,R,C,Vac); % mention your elements
z0=(0,0);      % lower left node
ht:=6u;        % height of circuit
z1=z0+(0,ht);  % upper left node
Vac=.5[z0,z1]; % location of voltage
Vac.t=T.u;     % rotated 90 degrees
L.t=R.t=C.t=T.r; % default orientation
% set equal distances
equally_spaced(5u,0) z1,R,L,C,z2;
```

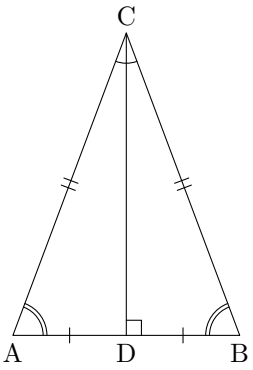



```
edraw; % draw elements of circuits
% draw wires connecting nodes
% the first ones rotated 90 degrees
wire.v(Vac.a,z0);
wire.v(Vac.b,z1);
wire.v(z2,z0);
wire(z1,R.a);
wire(R.b,L.a);
wire(L.b,C.a);
wire(C.b,z2);
endfig;
end;
```

```
u:=10bp; % unit of length
input mpcirc;
beginfig(1);
prepare (L,R,C,Vac); % mention your elements
z0=(0,0);      % lower left node
ht:=6u;        % height of circuit
z1=z0+(0,ht);  % upper left node
Vac=.5[z0,z1]; % location of voltage
Vac.t=T.u;     % rotated 90 degrees
L.t=R.t=C.t=T.r; % default orientation
% set equal distances
equally_spaced(7.5u,0) z1,z2,z3;
L=0.5[z1,z2]; % location of spool
C=0.5[z2,z3]-(0,2u);
R=0.5[z2,z3]+(0,2u);
z4 = z3+(2.5u,0);
edraw; % draw elements of circuits
% draw wires connecting nodes
wire.v(Vac.a,z0);
wire.v(Vac.b,z1);
wire(z1,L.a);
wire(L.b,z2);
wire.v(z2,C.a);
wire.v(z2,R.a);
wire.v(z3,C.b);
wire.v(z3,R.b);
wire(z3,z4);
wire.v(z4,z0);
endfig;
end;
```

Marking Angles and Lines

In geometric pictures, line segments of equal length are often marked by an equal number of ticks and equal angles are often marked the same, too. In the following example, the macros `tick`, `mark_angle`, and `mark_right_angle` mark lines and angles. When dealing with angles, we use the macro `turningnumber` to find the direction of a cyclic path: 1 means counter-clockwise, -1 means clockwise. We use it to make our macros `mark_angle`, and `mark_right_angle` independent of the order in which the non-common points of the angle are specified.

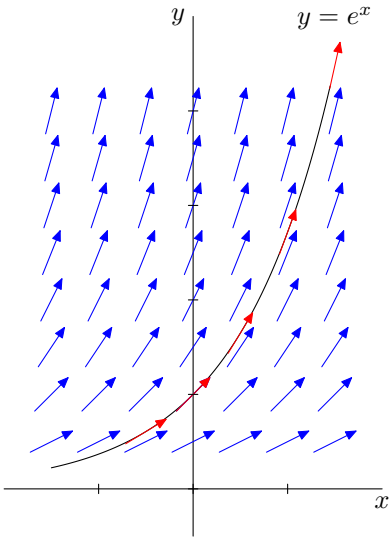



```
% set some user-adjustable constants
angle_radius := 4mm;
angle_delta := 0.5mm;
mark_size := 2mm;

def mark_angle(expr A, common, B, n) =
  % draw 1, 2, 3 or 4 arcs
  draw_angle(A, common, B, angle_radius);
  if n>1: draw_angle(A, common, B,
    angle_radius+angle_delta); fi;
  if n>2: draw_angle(A, common, B,
    angle_radius-angle_delta); fi;
  if n>3: draw_angle(A, common, B,
    angle_radius+2*angle_delta); fi;
enddef;

def draw_angle(expr endofa, common, endofb, r) =
   begingroup
   save tn;
   tn := turningnumber(common--endofa--endofb--cycle);
   draw (unitvector(endofa-common){(endofa-common)
   rotated(tn*90)} .. unitvector(endofb-common))
     scaled r shifted common;
   endgroup
enddef;

def mark_right_angle(expr endofa, common, endofb) =
   begingroup
   save tn; tn :=
   turningnumber(common--endofa--endofb--cycle);
   draw ((1,0)--(1,1)--(0,1)) zscaled(mark_size*
     unitvector((1+tn)*endofa+(1-tn)*endofb-2*common))
     shifted common;
   endgroup
enddef;

def tick(expr p, n) =
   begingroup
   save midpnt;
   midpnt = 0.5*arclength(p);
   % find the time when half-way the path
   for i=-(n-1)/2 upto (n-1)/2:
     draw_mark(p, midpnt+mark_size*i/2);
     % place n tick marks
   endfor;
   endgroup
enddef;

def draw_mark(expr p, m) =
   begingroup
   save t, dm; pair dm;
   t = arctime m of p;
   % find a vector orthogonal to p at time t
   dm = mark_size*unitvector(direction t of p rotated 90);
   draw(-1/2dm..1/2dm) shifted (point t of p);
   % draw tick mark
   endgroup
enddef;

beginfig(1);
pair A, B, C, D;
A := (0,0);  B := (3cm,0);
C := (1.5cm,4cm);  D := (1.5cm,0);
draw A--B--C--cycle;  draw C--D;
% draw triangle and altitude
```

```
label.bot("A", A);  label.bot("B", B);
label.top("C", C); label.bot("D", D);
tick(A--D,1);  tick(D--B,1);  tick(A--C,2);  tick(B--C,2);
mark_angle(C,A,B,2);  mark_angle(A,B,C,2);
mark_angle(B,C,A,1); mark_right_angle(C,D,B);
endfig;

end;
```

Vectorfields

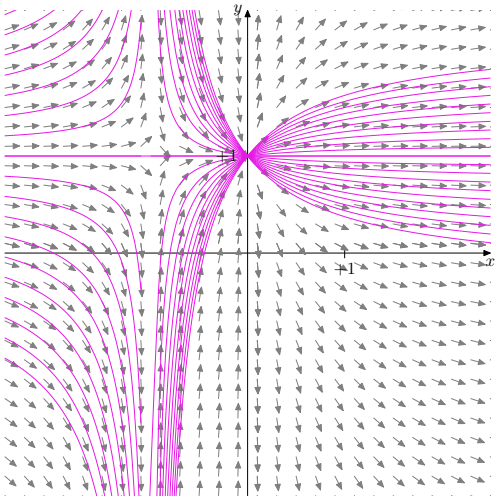
In the first example below we show the directional field corresponding with the ordinary differential equation $y' = y$. For clarity, we show a vectorfield instead of a directional field with small line segments.




```
beginfig(1);
% some constants
numeric xmin, xmax, ymin, ymax, xinc, u;
xmin := -1.5; xmax := 1.5; ymin := 0; ymax := 4.5;
xinc := 0.05; u := 1cm;

% draw axes
draw (xmin-0.5,0)*u -- (xmax+0.5,0)*u;
draw (0,ymin-0.5)*u -- (0,ymax+0.5)*u;

% define f making up the ODE y' = f(x,y).
% Here we take y' = y with the exponential curve
% as solution curve
vardef f(expr x,y) = y enddef;

% define routine to compute function values
def compute_curve(suffix g)(expr xmin, xmax, xinc) =
  ( (xmin,g(xmin))
  for x=xmin+xinc step xinc until xmax: .. (x,g(x)) endfor )
enddef;

% compute and draw exponential curve
vardef exp(expr x) =  (mexp 256)**x enddef;
path p;  p := compute_curve(exp, xmin, xmax, xinc) scaled u;
draw p;

% draw direction field
pair vec; path v;
for x=xmin step 0.5 until xmax:
  for y=ymin+0.5 step 0.5 until ymax-0.5:
    vec := unitvector( (1,f(x,y)) ) scaled 1/2u;
    v := ((0,0)--vec) shifted -1/2vec;
    drawarrow v shifted (x*u,y*u) withcolor blue;
  endfor;
endfor;
% draw directions along the exponential curve
for x=-0.5 step 0.5 until xmax:
  vec := unitvector( (1,f(x,exp(x))) ) scaled 1/2u;
  v := ((0,0)--vec) shifted -1/2vec;
  drawarrow v shifted (x*u,exp(x)*u) withcolor red;
endfor;
% draw ticks and labels
for x=round(xmin) upto xmax:
  draw (x,-0.05)*u--(x,0.05)*u;
endfor;
for y=round(ymin) upto ymax:
  draw (-0.05,y)*u--(0.05,y)*u;
endfor;
label.bot(btex $x$ etex, (xmax+0.5,0)*u);
label.lft(btex $y$ etex, (0,ymax+0.5)*u);
```

```
label(btex $y=e^x$ etex, (xmax, exp(xmax)+0.5)*u);
endfig;
end;
```

Now we shall show some other examples of directional fields corresponding with ODE's and solution curves using the macro package `courbe` from Jean-Michel Sarlat, which we downloaded from `http://melusine.eu.org/syracuse/metapost/courbes/`.

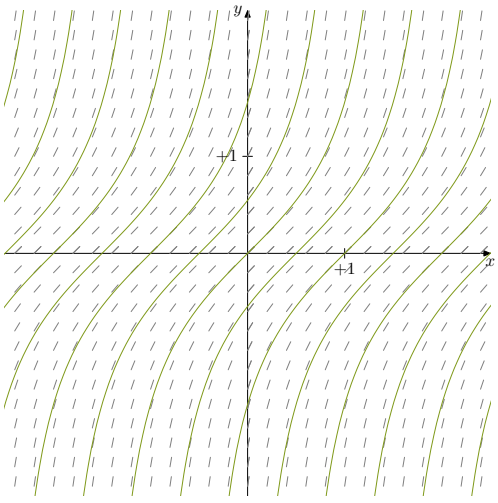


$(x + x^2)y' - y = -1$

```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

input courbes;
vardef fx(expr t) = t enddef;
vardef fy(expr t) = 1+a*t/(1+t) enddef;

beginfig(1);
repere(10cm,10cm,5cm,5cm,2cm,2cm);
trace.axes(0.5pt);
marque.unites(1mm);
%% Champs de vecteurs
vardef F(expr x,y) = (y-1)/(x+x**2) enddef;
champ.vecteurs(0.1,0.1,0.2,0.15,0.5white);
%% Courbes intégrales
color la_couleur;
la_couleur = (0.9,0.1,0.9);
for n = 0 upto 20:
  a := (n/8) - 1.25;
  draw ftrace(-0.995,2.5,50) en_place withcolor la_couleur;
  draw ftrace(-2.5,-1.1,50) en_place withcolor la_couleur;
endfor;
%
draw rpoint(r_xmin,1)--rpoint(r_xmax,1)
  withcolor la_couleur;
decoupe.repere;
etiquette.axes;
etiquette.unites;
label(btex $(x+x^2)y'-y=-1$ etex scaled 2.5,rpoint(0,-3));
endfig;
end;
```

In the next example, we added a macro to the `courbes.mp` package for drawing a directional field with line segments instead of arrows.

```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

input courbes;
% ======================================== fonctions
vardef fx(expr t) = t enddef;
vardef fy(expr t) = tan(t+a) enddef;
% ======================================== figure
```

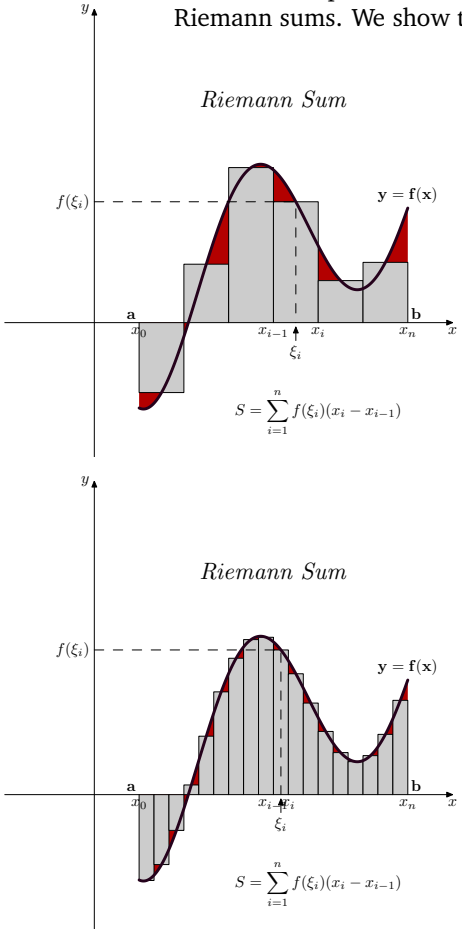


$y' = 1 + y^2$

```
beginfig(1);
repere(10cm,10cm,5cm,5cm,2cm,2cm);
trace.axes(0.5pt);
marque.unites(1mm);

%% Champs de directions
vardef F(expr x,y) = 1+y**2 enddef;
champ.segments(0,0,0.2,0.1,0.5white);

%% Courbes intégrales
for n = 0 upto 16:
  a := (n/2) - 4;
  draw ftrace(-1.5-a,1.5-a,50) en_place
    withcolor (0.5,0.6,0.1);
endfor;

decoupe.repere;
etiquette.axes;
etiquette.unites;
label(btex $y'=1+y^2$ etex scaled 2.5,rpoint(0,-3));
endfig;

end;
```

Riemann Sums

Another example of the `courbes.mp` macro package is the following illustration of Riemann sums. We show two pictures for different number of segments.




```
verbatimtex
%&latex
\documentclass{article}
\everymath{\displaystyle}
\begin{document}
etex

input courbes;
vardef fx(expr t) = t enddef;
vardef fy(expr t) = (t-5)*sin(t)-cos(t)+2 enddef;

beginfig(1);

numeric a,b,n,h;
a = 1;  b = 7; n = 6; h = (b-a)/n;
color aubergine;  aubergine = (37/256,2/256,29/256);
repere(10cm,10cm,2cm,3cm,1cm,1cm);
fill ((a,0)--ftrace(a,b,200)--(b,0)--cycle) en_place
  withcolor 0.7red;

for i=1 upto n:
  path cc;
  aa := a + (i-1) * h;
  bb := aa + h;
  ff := fy(aa + h/2);
  cc := rpoint(aa,0)--rpoint(aa,ff)--rpoint(bb,ff)--
        rpoint(bb,0);
  fill cc--cycle withcolor 0.8white;
  draw cc;
endfor;

trace.axes(0.5pt);
trace.courbe(a,b,200,2pt,aubergine);
decoupe.repere;
etiquette.axes;
```

```
label.bot(btex $x_{i-1}$ etex, rpoint(a+n/2*h,0));
label.bot(btex $x_{0}$ etex, rpoint(a,0));
label.bot(btex $x_{n}$ etex, rpoint(b,0));
label.ulft(btex $\mathbf{a}$ etex, rpoint(a,0));
label.urt(btex $\mathbf{b}$ etex, rpoint(b,0));
label.top(btex $\mathbf{y=f(x)}$ etex, f(b) en_place);
label.bot(btex $x_{i}$ etex, rpoint(a+n/2*h+h,0));
projection.axes(f(a+(n+1)/2*h),0.5pt,2);

label.lft(btex $f(\xi_i)$ etex, rpoint(0,fy(a+(n+1)/2*h)));
label.bot(btex $\xi_i$ etex,rpoint(a+(n+1)/2*h,-0.4));
drawarrow rpoint(a+(n+1)/2*h,-0.4)--
  rpoint(a+(n+1)/2*h,-0.1);
label(btex \textit{Riemann Sum} etex scaled 2,rpoint(4,5));
label(btex $S = \sum_{i=1}^n f(\xi_i)(x_i - x_{i-1})$ etex
  scaled 1.5,rpoint(5,-2));
endfig;

end;
```

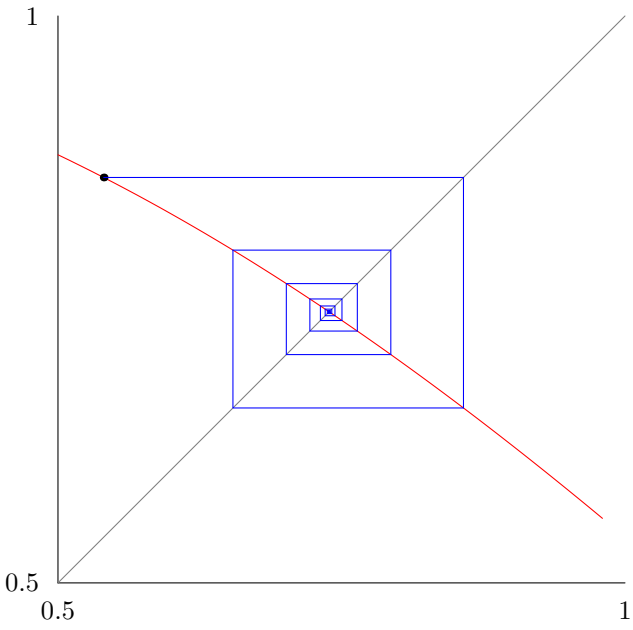
Iterated Functions

The following diagrams are 'standard' in the theory of iterative processes:

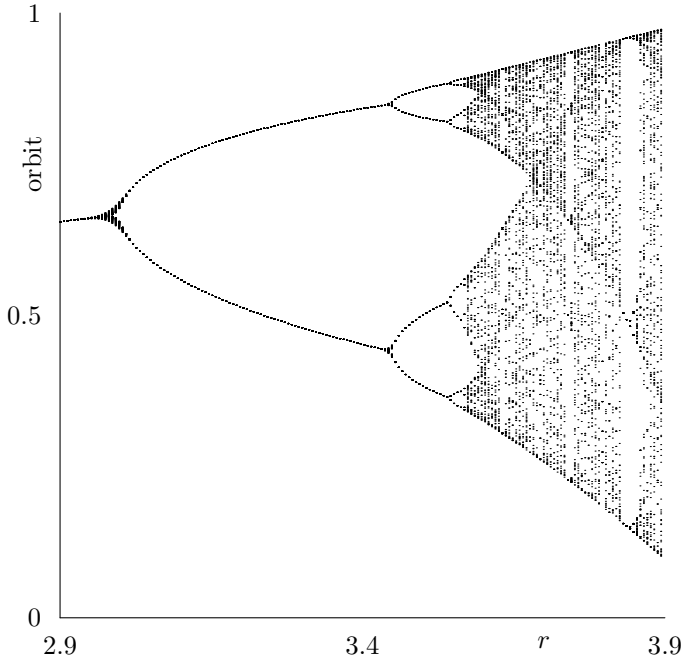
□ The cobweb-graph of applying the cosine function iteratively.

□ The bifurcation diagram of the logistic function, $f(x) = rx(1-x)$ for $0 < r < 4$.

The code that produced these diagrams is shown below.




```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

beginfig(1)
% some constants
u := 10cm;
numeric xmin, xmax, ymin, ymax, xinc;
xmin := 0.5; xmax := 1.0;
ymin := xmin; ymax := xmax;
xinc := 0.02;

% draw axes
draw (xmin,ymin)*u -- (xmax,ymin)*u;
draw (xmin,ymin)*u -- (xmin,ymax)*u;

% define routine to compute function values
def compute_curve(suffix g)(expr xmin, xmax, xinc) =
    ( (xmin,g(xmin))
    for x=xmin+xinc step xinc until xmax:
      .. (x,g(x))
    endfor )
enddef;

% compute and draw cosine curve
numeric pi; pi := 3.1415926;
numeric radian;  radian := 180/pi; % 2pi*radian = 360 ;
vardef cos primary x = (cosd(x*radian)) enddef;
path p;
p := compute_curve(cos, xmin, xmax, xinc) scaled u;
draw p;
% draw identity graph
draw (xmin,ymin)*u -- (xmax,xmax)*u withcolor 0.5white;
```

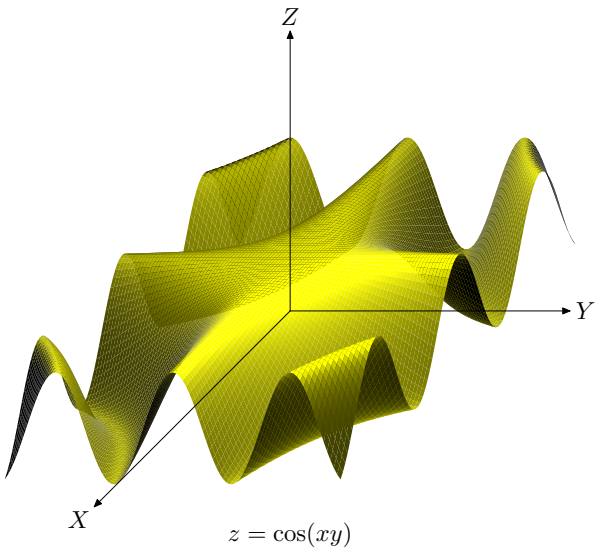



```
% compute the orbit starting from some point
numeric x, initial, orbitlength;
x := 1.0; % the starting point
initial := 1; % some initial iterations
orbitlength := 15; % number of iterations
for i=1 upto initial: % do initial iterations
  x := cos(x);
endfor;
dotlabel("", (x,cos(x))*u); % mark starting point
for i=1 upto orbitlength:  % draw hooks
  draw (x,cos(x))*u -- (cos(x),cos(x))*u --
    (cos(x),cos(cos(x)))*u withcolor blue;
  x := cos(x); % next value
endfor;

% draw axis labels
labeloffset := 0.25cm;
label.bot(decimal(xmin), (xmin,ymin)*u);
label.bot(decimal(xmax), (xmax,ymin)*u);
label.lft(decimal(ymin), (xmin,ymin)*u);
label.lft(decimal(ymax), (xmin,ymax)*u);
endfig;
```

```
beginfig(2)
numeric rmin, rmax, r, dr, n, ux, uy;
rmin := 2.9; rmax := 3.9;
r := rmin; n := 175;
dr := (rmax - rmin)/n;
ux := 8cm; uy := 8cm;
for i = 1 upto n:
  x := 0.5; % our starting point
  for j=1 upto 75: % initial iterations
    x := r*x*(1-x);
  endfor
    for j=1 upto 150: % the next 100 iterations
        x := r*x*(1-x);
        draw (r*ux,x*uy) withpen pencircle scaled .5pt;
    endfor
    r := r+dr;
endfor;

% draw axes and labels
draw (rmin*ux,0) -- (rmax*ux,0);
draw (rmin*ux,0) -- (rmin*ux,uy);
labeloffset := 0.25cm;
label.bot(decimal(rmin), (rmin*ux,0));
label.bot(decimal((rmin+rmax)/2), ((rmin+rmax)/2*ux,0));
label.bot(decimal(rmax), (rmax*ux,0));
label.lft(decimal(0), (rmin*ux,0));
label.lft(decimal(0.5), (rmin*ux,0.5*uy));
label.lft(decimal(1), (rmin*ux,uy));
label.bot(btex $r$ etex, ((rmax-0.2)*ux,0));
label.lft(btex orbit etex rotated 90, (rmin*ux,0.75*uy));
endfig;

end;
```

A Surface Plot

You can draw surface plots from basic principles. We give one example.



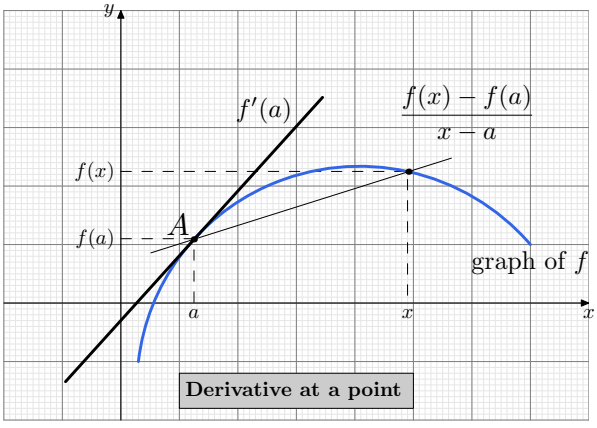
$z = \cos(xy)$

```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

% u: dimensional unit
% xp, yp, zp: coordinates of light source
% bf : brightness factor
% base_color : base color
numeric u,xp,yp,zp,bf;
color base_color;
u = 1cm;
xp := 3;  yp := 3; zp := 5;
bf := 30;
base_color := red+green;

% O, Xr, Yr, Zr : reference frame
pair O,Xr,Yr,Zr;
O = (0,0);
Xr = (-.7,-.7) scaled u;
Yr = (1,0) scaled u;
Zr = (0,1) scaled u;

% for drawing the reference frame
vardef frameXYZ(expr s) =
  drawarrow O--Xr scaled s;
  drawarrow O--Yr scaled s;
  drawarrow O--Zr scaled s;
  label.llft(btex $X$ etex scaled 1.25, (Xr scaled s));
  label.rt(btex $Y$ etex scaled 1.25, (Yr scaled s));
  label.top(btex $Z$ etex scaled 1.25, (Zr scaled s));
enddef;

% from 3D to 2D coordinates
vardef project(expr x,y,z) = x*Xr + y*Yr + z*Zr enddef;

% numerical derivatives by central differences
vardef diffx(suffix f)(expr x,y) =
  numeric h; h := 0.01;
  (f(x+h,y)-f(x-h,y))/(2*h)
enddef;
vardef diffy(suffix f)(expr x,y) =
 numeric h; h := 0.01;
 (f(x,y+h)-f(x,y-h))/(2*h)
enddef;

% Compute brightness factor at a point
vardef brightnessfactor(suffix f)(expr x,y,z) =
 numeric dfx,dfy,ca,cb,cc;
 dfx := diffx(f,x,y);
 dfy := diffy(f,x,y);
 ca := (zp-z)-dfy*(yp-y)-dfx*(xp-x);
 cb := sqrt(1+dfx*dfx+dfy*dfy);
 cc := sqrt((z-zp)*(z-zp)+(y-yp)*(y-yp)+(x-xp)*(x-xp));
 bf*ca/(cb*cc*cc*cc)
enddef;

% compute the colors and draw the patches
vardef
 z_surface(suffix f)(expr xmin,xmax,ymin,ymax,nx,ny) =
 numeric dx,dy,xt,yt,zt,factor[][];
 pair Z[][];
```

```
dx := (xmax-xmin)/nx;
dy := (ymax-ymin)/ny;
for i=0 upto nx:
  xt := xmin+i*dx;
  for j=0 upto ny:
    yt := ymin+j*dy;
    zt := f(xt,yt);
    Z[i][j] = project(xt,yt,zt);
    factor[i][j] := brightnessfactor(f,xt,yt,zt);
  endfor
endfor
for i = 0 upto nx-1:
  for j= 0 upto ny-1:
   fill Z[i][j]--Z[i][j+1]--Z[i+1][j+1]--Z[i+1][j]--cycle
        withcolor factor[i][j]*base_color;
  endfor
endfor
enddef;

beginfig(1);
xp := 3;
yp := 3;
zp := 10;
bf := 100;

numeric pi; pi := 3.14159;
vardef cos primary x = cosd(x/pi*180) enddef;
vardef f(expr x,y) = cos(x*y) enddef;
z_surface(f,-3,3,-3,3,100,100);
frameXYZ(5);
label(btex $z = \cos(xy)$ etex scaled 1.25,(0,-4cm));
endfig;
end;
```

Miscellaneous

We adopt another example from Jean-Michel Sarlat that uses his macro package `courbe` and another one called `grille`. The example has been downloaded and slightly adapted from `http://melusine.eu.org/syracuse/metapost/cours/sarlat/derivation/`.




```
verbatimtex
%&latex
\documentclass{article}
\begin{document}
etex

input courbes;
input grille;

vardef droite(expr a,b,t) =
 (t[a,b])--(t[b,a])
enddef;
path c,cartouche;
c=(2.3cm,1cm)..(4.5cm,4cm)..(9cm,3cm);
cartouche = (3cm,2mm)--(7cm,2mm)--(7cm,8mm)--(3cm,8mm)--
  cycle;
pair A,M;
A = point 0.6 of c;
M = point 1.5 of c;

vardef tangente(expr t,x) =
 pair X,Y;
 X := point (t-.05) of c;
 Y := point (t+.05) of c;
```

```
 droite(X,Y,x)
enddef;
beginfig(1);
grille(1cm,0,10cm,0,7cm);
repere(10cm,7cm,2cm,2cm,1cm,1cm);
trace.axes(.5pt);
marque.unites(0.1);

%% lectures sur la grille
numeric xa,ya,xm,ym;
xa = 1.25; ya = 1.1 ; xm = 4.9 ; ym = 2.25;
pair AA,MM;
AA = (xa,ya) ; MM = (xm,ym) ;
projection.axes(AA,0.5,1.7);
projection.axes(MM,0.5,1.7);
label.bot(btex $a$ etex,rpoint(xa,0));
label.bot(btex $x$ etex,rpoint(xm,0));
label.lft(btex $f(a)$ etex, rpoint(0,ya));
label.lft(btex $f(x)$ etex, rpoint(0,ym));
%% fin des lectures

draw c withpen pencircle scaled 1.5pt withcolor (.2,.4,.9);

draw droite(A,M,1.2);

draw tangente(0.6,9) withpen pencircle scaled 1.5pt;

dotlabel.ulft(btex $A$ etex scaled 1.5,A);
dotlabel("",M);
label(btex $\displaystyle\frac{f(x)-f(a)}{x-a}$ etex
  scaled 1.25,
M shifted (1cm,1cm));
label.ulft(btex $f'(a)$ etex scaled 1.25, (5cm,5cm));
label.bot(btex graph of $f$ etex scaled 1.25, point 2 of c);

%% Cartouche
fill cartouche withcolor .8white;
draw cartouche;
label.rt(btex \textbf{Derivative at a point}
  etex,(3cm,5mm));
%% fin du cartouche

decoupe.repere;
etiquette.axes;
endfig;
end;
```

References

[GRS94] Michel Goossens, Sebastian Rahtz, Frank Mittelbach. *The LaTeX Graphics Companion*, Addison-Wesley (1994), ISBN 0-201-85469-4.

[Hag02] Hans Hagen. *The Metafun Manual*, 2002. downloadable as `www.pragma-ade.com/general/manuals/metafun-p.pdf`

[Hob92a] John D. Hobby: *A User's manual for MetaPost*, AT&T Bell Laboratories Computing Science Technical Report 162, 1992.

[Hob92b] John D. Hobby: *Drawing Graphs with MetaPost*, AT&T Bell Laboratories Computing Science Technical Report 164, 1992.