BRAC UNIVERSITY
Inspiring Excellence

Lecture 4
SOP & POS

	A	B	C	F
0	0	0	0	**1**
1	0	0	1	**1**
2	0	1	0	**0**
3	0	1	1	**0**
4	1	0	0	**1**
5	1	0	1	**1**
6	1	1	0	**0**
7	1	1	1	**0**

Min Terms : 0,1,4,5
[000,001,100,101]
$F=\Sigma(0,1,4,5)$

Max Terms : 2,3,6,7
[010,011,110,111]
$F=\Pi(2,3,6,7)$

CSE260 : Digital Logic Design

SOP & POS

Sum-of-Products (SOP) Expression:

A product term or a logical sum (OR) of several product terms.

Example: x+yz′, xy′+x′yz, AB+A′B’

Product-of-Sums (POS) Expression:

A sum term or a logical product (AND) of several sum terms.

Example: x(y+z′), (x+y′)(x′+y+z), (A+B)(A′+B’)

SOP & POS [Cont.]

Every boolean expression can either be expressed as sum-of-products or product-of-sums expression.

Example:

SOP: x’y + xy’ + xyz

POS: (x + y’)(x’ + y)(x’ + z’)

MIN & MAX TERM

MIN & MAX TERM

Minterms are sum terms.

For Boolean functions, the minterms of a function are the terms for which the result is 1.

Boolean functions can be expressed as sum- of-Minterms.

Maxterms are Product terms.

For Boolean functions, the maxterms of a function are the terms for which the result is 0.

Boolean functions can be expressed as Products-of-Maxterms.

MIN & MAX TERM

	A	B	C	F
0	0	0	0	**1**
1	0	0	1	**1**
2	0	1	0	**0**
3	0	1	1	**0**
4	1	0	0	**1**
5	1	0	1	**1**
6	1	1	0	**0**
7	1	1	1	**0**

Min Terms : 0,1,4,5

[000,001,100,101] $F=\Sigma(0,1,4,5)$

Max Terms : 2,3,6,7

[010,011,110,111] $F=\Pi(2,3,6,7)$

MIN-SOP and MAX-POS

	A	B	C	F
0	0	0	0	**1**
1	0	0	1	**1**
2	0	1	0	**0**
3	0	1	1	**0**
4	1	0	0	**1**
5	1	0	1	**1**
6	1	1	0	**0**
7	1	1	1	**0**

Min Terms : 0,1,4,5 [000,001,100,101]

F=Σ(0,1,4,5)

F=A'B'C'+A'B'C+AB'C'+AB'C

Max Terms : 2,3,6,7 [010,011,110,111] F=Π(2,3,6,7)

F=(A+B'+C)(A+B'+C')(A'+B'+C) (A'+B'+C')

Min/SOP

0-Prime, 1-No prime

AND among literals

OR among terms

Max/POS

1-Prime, 0-No prime

OR among literals

AND among terms

Conversion of Function to SOP & POS

How to Convert into SOP

Steps

- Check if each term contains all variable,
  - if not then AND (x+x') if x is the missing term
- Remove the duplicates

How to Convert into SOP

Example:

F(A,B,C)=A+B’C

=A (B+B’) (C+C’) + B’C(A+A’)

=(AB+AB’) (C+C’) + B’C(A+A’)

=AB(C+C’) + AB’(C+C’) + B’C(A+A’)

=ABC + ABC’ + *AB’C* + AB’C’ + *B’CA* + B’CA’

Now, removing duplicates and writing the literals in order,

=ABC + ABC’ + AB’C + AB’C’ + A’B’C

= 111, 110, 101, 100, 001

=Σ(1,4,5,6,7)

How to Convert into POS

Steps

1. Apply distributive law **(x+yz)=(x+y)(x+z)** if possible at every stage
2. If then terms, like x, are missing, append the term by OR **x.x'**
3. Check if any sum term is missing any variable, if so then OR missing variable. Again apply distributive law

How to Convert into POS:

F(A,B,C) =A+B'C

= (A+B')(A+C)

= (A+B'+CC')(A+C+BB')

= (A+B'+C)(A+B'+C')(A+C+B)(A+C+B')

Now, removing duplicates and writing the literals in order,

= (A+B'+C)(A+B'+C')(A+B+C)

= 010, 011, 000

= π (2,3,0)

= π (0,2,3)

Example

Find SOP for F(w,x,y,z) =wy+x'z

Solution:

F(w,x,y,z)

= wy + x'z

= wy[(x+x')(z+z')] + x'z[(w+w')(y+y')]

= wy(xz+xz'+x'z+x'z') + x'z(wy+wy'+w'y+w'y')

= wyxz + wyxz' + wyx'z + wyx'z' + x'zwy + x'zwy' + x'zw'y + x'zw'y'

= wxyz + wxyz' + wx'yz + wx'yz' + wx'y'z + w'x'yz + w'x'y'z

= 1111,1110,1011,1010,1001,0011,0001

= Σ(15,14,11,10,9,3,1)

= Σ(1,3,9,10,11,14,15)

Example

Find POS for F(w,x,y,z) =wy+x'z

Solution:

F(w,x,y,z)

= wy+x'z

= (wy+x')(wy+z)

= (x'+w)(x'+y)(w+z)(y+z)

= (x'+w+yy')(x'+y+zz')(w+z+xx')(y+z+xx')

= (x'+w+y)(x'+w+y')(x'+y+z)(x'+y+z')(w+z+x)(w+z+x')(y+z+x)(y+z+x')

= (x'+w+y+zz')(x'+w+y'+zz')(x'+y+z+ww')(x'+y+z'+ww')
(w+z+x+yy')(w+z+x'+yy')(y+z+x+ww')(y+z+x'+ww')

= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')
(w+x'+y+z)(w'+x'+y+z)(w+x'+y+z')(w'+x'+y+z')
(w+x+y+z)(w+x+y'+z)(w+x'+y+z)(w+x'+y'+z)
(w+x+y+z)(w'+x+y+z)(w+x'+y+z)(w'+x'+y+z)

= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x'+y+z)
(w'+x'+y+z')(w+x+y+z)(w+x+y'+z)(w'+x+y+z)

= 0100,0101,0110,0111,1100,1101,0000,0010,1000

= Π(4,5,6,7,12,13,0,2,8) = Π(0,2,4,5,6,7,8,12,13)

Example

Find POS for F(w,x,y,z) =wy+x'z

Alternate Solution: Same thing can be done by taking all missing terms together

F(w,x,y,z)
= wy+x'z
= (wy+x')(wy+z)
= (x'+w)(x'+y)(w+z)(y+z)
= (x'+w+yy'+zz')(x'+y+zz'+ww')(w+z+xx'+yy')(y+z+xx'+ww')
= (x'+w+y+zz')(x'+w+y'+zz')(x'+y+z+ww')(x'+y+z'+ww')
(w+z+x+yy')(w+z+x'+yy')(y+z+x+ww')(y+z+x'+ww')
= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')
(w+x'+y+z)(w'+x'+y+z)(w+x'+y+z')(w'+x'+y+z')
(w+x+y+z)(w+x+y'+z)(w+x'+y+z)(w+x'+y'+z)
(w+x+y+z)(w'+x+y+z)(w+x'+y+z)(w'+x'+y+z)
= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x'+y+z)
(w'+x'+y+z')(w+x+y+z)(w+x+y'+z)(w'+x+y+z)
= 0100,0101,0110,0111,1100,1101,0000,0010,1000
= Π(4,5,6,7,12,13,0,2,8) = Π(0,2,4,5,6,7,8,12,13)