Image Features

HW4.
CSCI 5722
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Q1. $a=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in R^3$, $\hat{a}=\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$

a) show that the rank of $\hat{a}$ is 2, $\Rightarrow \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}$

Consider. $\hat{a}=\begin{bmatrix} 0 & z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{matrix} r_1 \\ r_2 \\ r_3 \end{matrix} \Big\}$ rows.

Let $r_2+\frac{z}{y}r_3 \to r_2$. $\quad r_2+\frac{x}{y}r_1 \to r_2 \quad r_3\to r_1\ r_1\to r_2\ r_2\to r_3$

$$\begin{bmatrix} 0 & -z & y \\ 0 & \frac{xz}{y} & -x \\ -y & x & 0 \end{bmatrix} \to \begin{bmatrix} 0 & -z & y \\ 0 & 0 & 0 \\ -y & x & 0 \end{bmatrix} \to \begin{bmatrix} -y & x & 0 \\ 0 & -z & y \\ 0 & 0 & 0 \end{bmatrix} \to$$

$-\frac{1}{z}\cdot r_2 \to r_2 \quad r_1 - x\cdot r_2 \to r_1 \quad -\frac{1}{y}\cdot r_1 \to r_1$

$$\begin{bmatrix} -y & x & 0 \\ 0 & 1 & -\frac{y}{z} \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} -y & 0 & \frac{xy}{z} \\ 0 & 1 & -\frac{y}{z} \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -\frac{x}{z} \\ 0 & 1 & -\frac{y}{z} \\ 0 & 0 & 0 \end{bmatrix}$$

So $Rank(\hat{a})=2$.

b) show that $\hat{a}^2 = aa^T - \|a\|^2 I$.

The left side of equation is,

$$\hat{a}^2 = \begin{bmatrix} -z^2-y^2 & xy & xz \\ xy & -z^2-x^2 & yz \\ xz & yz & -y^2-x^2 \end{bmatrix} \qquad aa^T = \begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & xy & z^2 \end{bmatrix}$$

$\|a\|^2 = \cancel{a^2}\, x^2+y^2+z^2. \qquad I = I_3 = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$

The right side of equation is,

$$aa^T - \|a\|^2 I_3 = \begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & xy & z^2 \end{bmatrix} - (x^2+y^2+z^2)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -y^2-z^2 & xy & xz \\ xy & -z^2-x^2 & yz \\ xz & yz & -y^2-x^2 \end{bmatrix} = \hat{a}^2$$

c) show that $\hat{a}^3 = -\|a\|^2 \hat{a}$

The left side of equation is

$$\hat{a}^3 = \begin{bmatrix} 0 & zx^2 - z(-z^2-y^2) & y(-z^2-y^2) - yx^2 \\ z(-z^2-x^2) - zy^2 & 0 & y^2x - x(-z^2-x^2) \\ z^2y - y(-y^2-x^2) & x(-y^2-x^2) - z^2x & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & z(x^2+y^2+z^2) & -y(x^2+y^2+z^2) \\ -z(x^2+y^2+z^2) & 0 & x(x^2+y^2+z^2) \\ y(x^2+y^2+z^2) & -x(x^2+y^2+z^2) & 0 \end{bmatrix}$$

$$= (x^2+y^2+z^2) \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix}$$

~~because~~. The right side is.

$$\|a\|^2 = x^2+y^2+z^2, \quad -\hat{a} = \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix}$$

So $\hat{a}^3 = -\|a\|^2 \cdot \hat{a}$

Q2

a) Descriptors ~~is~~ are the way to compare the keypoints. The are in vector format to save some features of key points. If I make a patch descriptor, I will choose the keypoints intensity in the direction of their most pronounced orientation. firstly, the ~~p~~ keypoints position must be independent. Secondly, they should be against image transformation. The last one ~~is~~, they also should be scale independent.

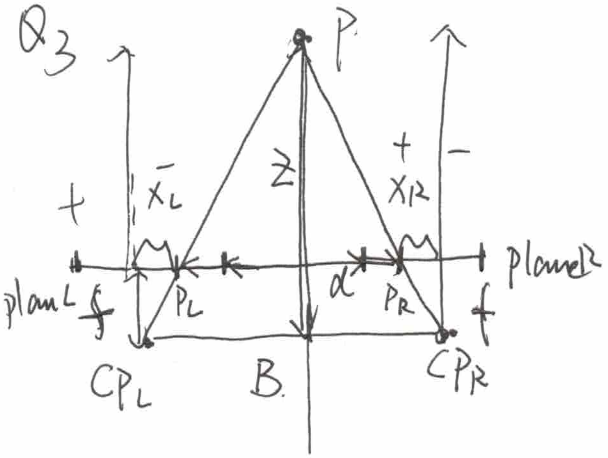
b) 1. Constructing a scale space to create internal representations of the original image to ensure scale invariance.

2. Using Gaussian to find interesting points or ~~key points~~. or using approximation to find key points.

3. Assigning an orientation to the key points, to make sure it rotation invariance.

4. with scale and rotation invariance inplace to generate SIFT features.

Q3



$$\frac{B - x_L + x_r}{z - f} = \frac{B}{z} \Rightarrow z = f\frac{B}{x_L - x_R}$$

$$\cancel{\frac{B-d}{z-f} = \frac{B}{z}}$$

$$z = -f\frac{B}{d}$$

$d = x_R - x_L$

Consider a f is constant

$d = -(x_L - x_R)$ So $z = -B/d$