Section 10.3 Multiplication of Matrices

Learning Objective: multiply a matrix by a scalar, multiply a matrix by a matrix, know the basic properties of matrices

Product of a Matrix and a Scalar
If A is a matrix and c is a scalar, the **product** cA is the matrix obtained by multiplying each entry of A by c.

Example 1:
Compute $2A - 3B$ if

$$A = \begin{bmatrix} 0 & 3 & -2 & -1 \\ 2 & 0 & -3 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -6 & 0 & 5 \\ 1 & 1 & -1 & 2 \end{bmatrix}.$$

Solution:

Matrix Multiplication

Let's rethink about how a linear system of equations is related to matrices. For example,

$$\begin{cases} x + 2y + 3z = 15 \\ 2x + y - z = 7 \\ x - y + z = 10 \end{cases} \Rightarrow \begin{cases} 1x + 2y + 3z = 15 \\ 2x + 1y - 1z = 7 \\ 1x - 1y + 1z = 10 \end{cases} \Rightarrow \begin{bmatrix} & & \end{bmatrix} \cdot \begin{bmatrix} \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$

Product of Two Matrices
If A is an $m \times n$ matrix and B is an $n \times p$ matrix, the **product** AB is the $m \times p$ matrix whose entry in row i and column j is obtained by multiplying each entry in row i of A by the corresponding entry in column j of B and then adding the resulting products.

Example 1: Compute the product AB if

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -1 & 2 \\ 2 & -6 & 1 \end{bmatrix}$$

Solution:

Note:

1. The product AB of two matrices A and B can be found only if the number of columns of A is the same as the number of rows of B.
2. If $AB = BA$, we say that A and B are commuting.

Example 2: Multiple the matrices

a) $\begin{bmatrix}1 & 0\\5 & 4\end{bmatrix}\begin{bmatrix}-3 & 1\\2 & 0\end{bmatrix}$

b) $\begin{bmatrix}-3 & 1\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\5 & 4\end{bmatrix}$

c) $\begin{bmatrix}4 & -2 & 3\\1 & 0 & -3\end{bmatrix}\begin{bmatrix}0 & -2\\2 & 1\\-1 & 3\end{bmatrix}$

Example 3:

A publisher with production facilities in New York and San Francisco is preparing for the publication of several new texts and novels. The numbers of books for each type that will be produced in each facility are summarized in matrix A.

$$A = \begin{array}{c} \\ N.Y. \\ S.F. \end{array}\begin{array}{cc} Text & Novel \\ 5 & 6 \\ 1 & 3 \end{array}$$

Matrix B shows the numbers of hours of work by various specialists that go into the production of each type of book.

$$B = \begin{array}{c} \\ Text \\ Novel \end{array}\begin{array}{ccc} Production\ Edition & Designer & Copy\ Editor \\ 50 & 8 & 30 \\ 30 & 2 & 20 \end{array}$$

For each type of specialist, calculate the total number of hours that will be needed in each city to produce the books as planned and summarize your results in matrix form.

Solution: