Section 1 – Basic Probability Concepts

Learning Objectives: General Probability with a focus on: Set functions including set notation and basic elements of probability, Mutually exclusive events, Addition and multiplication rules.

Basic Terms:

- The **union** of two events *A* and *B*: the set of outcomes that belong either to *A*, to *B*, or to both.

  Notation: ____________.

- The **intersection** of two events *A* and *B*: the set of outcomes that belong both to *A* and to *B*.

  Notation: ___________.

- The **complement** of event *A*: the set of outcomes that do not belong to *A.*

  Notation: ____________.

DeMorgan's Laws:

- $(A \cup B)' =$
- $(A \cap B)' =$

Some rules concerning operations on events:

I. $A \cap (B_1 \cup B_2 \cup \cdots \cup B_n) =$

   $A \cup (B_1 \cap B_2 \cap \cdots \cap B_n) =$

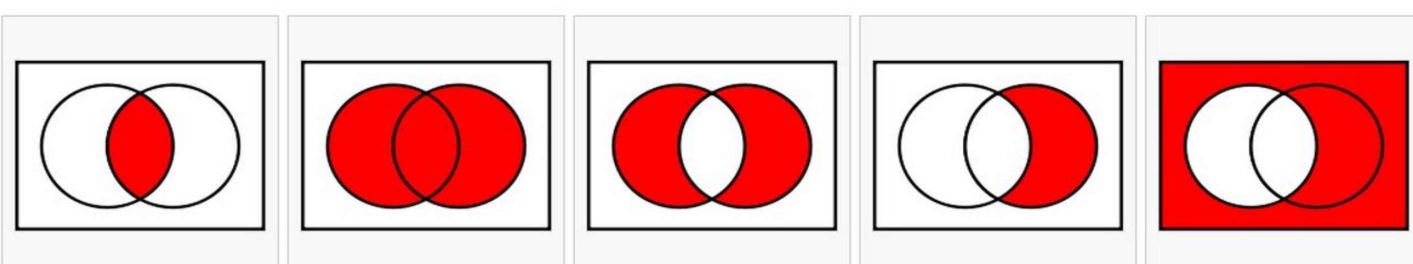
II. If $B_1, B_2, \dots, B_n$ are exhaustive events ($\cup_{i=1}^{n} B_i = S$, the entire probability space), then for any event A,

III. For any event A, $A \cup A' =$ ______________ and $A \cap A' =$______________.

IV. $A \cap B'$ is sometimes denoted $A - B$.

V. If $A \subset B$, then $A \cup B =$ ______________ and $A \cap B =$ ______________.

Venn Diagram (from Wikipedia)



Intersection of two sets $A \cap B$ | Union of two sets $A \cup B$ | Symmetric difference of two sets $A \,\Delta\, B$ | Relative complement of A (left) in B (right) $A^c \cap B = B \setminus A$ | Absolute complement of A in U $A^c = U \setminus A$

Some rules concerning probability:

I. $P(S) =$ ______________ where S is a sample space.

II. Let ϕ denote the empty set. Then $P(\phi) =$ ______________.

III. If *A* and *B* are mutually exclusive events, $P(A \cup B) =$ ______________.
More general, if $A_1, A_2, \ldots, A_n$ are mutually exclusive events,

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) =$$

IV. $0 \leq P(A) \leq 1$ for any event A.

V. If $A \subset B$ then $P(A) \leq P(B)$.

VI. **General Addition Rule:** Let A and B be any two events.

Then $P(A \cup B) =$________________________________.

VII. $P(A') =$ ______________ for any event A.

VIII. For any events A and B, $P(A) =$ ______________________________.

IX. For exhaustive events $B_1, B_2, \ldots, B_n$, $P(\cup_{i=1}^{n} B_i) = 1$. If $B_1, B_2, \ldots, B_n$ are exhaustive and mutually exclusive, we have $P(A) =$

X. If P is a uniform probability function on a probability space with k points, and if event A consists of m of those points, then $P(A) =$

XI. For any events $A_1, A_2, \ldots, A_n$, $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$, with the equality holding iff the events are mutually exclusive.

Exercise 1 (#2) Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a random chosen member of this group visits a physical therapist.

A) 0.26 B) 0.38 C) 0.40 D) 0.48 E) 0.62

Exercise 2 (#3) An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose supplementary coverages A, B, and C are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$, respectively. Determine the probability that a randomly chosen employee will choose no supplementary coverage.

A) 0 B) $\frac{47}{144}$ C) $\frac{1}{2}$ D) $\frac{97}{144}$ E) $\frac{7}{9}$

Exercise 3 (#4) An auto insurance company has 10,000 policyholders. Each policyholder is classified as

i. Young or old;
ii. Male or female; and
iii. Married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

A) 280 B) 423 C) 486 D) 880 E) 896

Exercise 4 (#5) The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office. 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

A) 0.05 B) 0.12 C) 0.18 D) 0.25 E) 0.35

Exercise 5 (#6) You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$.

A) 0.2 B) 0.3 C) 0.4 D) 0.6 E) 0.8

Exercise 6 (#7) A survey of a group's viewing habits over the last year revealed the following information:

i. 28% watched gymnastics
ii. 29% watched baseball
iii. 19% watched soccer
iv. 14% watched gymnastics and baseball
v. 12% watched baseball and soccer
vi. 10% watched gymnastics and soccer
vii. 8% watched all three sports

Calculate the percentage of the group that watched none of the three sports during the last year.

A) 24 B) 36 C) 41 D) 52 E) 60

Exercise 7 (#8) Under an insurance policy, a maximum of five claims may be filed per year by a policyholder. Let p_n be the probability that a policyholder files n claims during a given year, where $n = 0, 1, 2, 3, 4, 5$. An actuary makes the following observations:

i. $p_n \geq p_{n+1}$ for $n = 0, 1, 2, 3, 4$.
ii. The difference between p_n and p_{n+1} is the same for $n = 0, 1, 2, 3, 4$.
iii. Exactly 40% of policyholders file fewer than two claims during a given year.

Calculate the probability that a random policyholder will file more than three claims during a given year.

A) 0.14 B) 0.16 C) 0.27 D) 0.29 E) 0.33