Review of " "

In this paper, the authors prove that the complement of an augmented cellular alternating link L in a thickened torus can be decomposed into two torihedra, where a torihedron is T x [0,1) with a 4-valent graph on the T x {0} boundary and all vertices removed to make it ideal. They further prove that if L is a weakly prime cellular alternating link in a solid torus with no bigons, then any augmentation will be hyperbolic. Finally, they prove that if there are bigons but every bigon sequence with at least one bigon is augmented in the "bigon direction", then the result is hyperbolic.

This generalizes results of Adams on augmented alternating links in the 3-sphere and Kwon on fully alternating links in the thickened torus. A stronger result on hyperbolicity is known from a paper of Adams et al, which does not need the conditions on bigons, and holds for thickened surfaces of any genus, but the proof techniques applied there are completely different. For that reason, I do think this paper is worth publishing. However, there are a variety of questions.

First, why must the augmented components be in the direction for the bigon sequences. Why couldn't you use the direction along the axis of the twist sequence. You could make sure each bigon in the sequence is punctured by an augmenting component if that is a problem. It seems like your theorems could be extended to these cases as well. (If the chain is two bigons, one augmented component would puncture both bigons for instance.) If not, why not?

Second, could you explain what the problem is with bigons? In the case of Menasco type arguments, bigons are collapsed, which gives a sequence of edges on the ideal polyhedra that are all identified to one another. Is the problem that then the total number of edges that are identified is no longer 4, in which case the choice of $\pi/2$ for angles no longer works? Please explain to the reader the problem.

Further, what if you start with a projection that is not weakly prime. If you augment appropriately, then the resulting augmented link is no longer weakly prime. Can you prove hyperbolicity then?

Here are some specific errata/suggestions:

Page Line

1 10 Explain what twist-reduced is.
1 -11 "We point out that [7]". Not sure what is intended here.

Definition 2.4. "direction of a twist region is a pair of faces that are across from the bigons at the end." Don't know what that means. Please clarify.

Definition 2.5. As mentioned above, are you allowing an augmentation that punctures a bigon? If not, why not?

p. 5 You should reference Figure 4 in the paper.

6 6 “we also three horizontal edges” Missing word “add”?

6 6, 10 Figure 2.1 should be Figure 5.

8 12 You say that for an ideal prism, the sum of the base angles must be π. I believe it as I worked it out for myself, but it would be nice if you explained why rather than expecting the reader to figure out why.

12 21 “ short,long” should be “short, long”

12 -1 Need period after non-zero