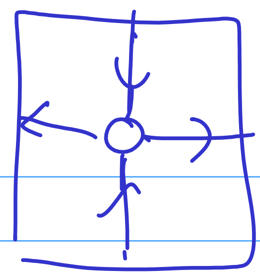
cf Schom. Pr. Thesis



19.1.23

Case $1+\varepsilon \rightsquigarrow 1+1$

$\uparrow$ sym Frob : $Z(pt^+) = A\text{-mod}$

$Z(pt^-) = \bar{A}\text{-mod}$ $\quad (\bar{A} := A^{op})$

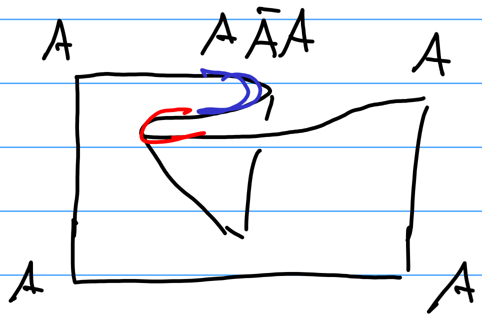
$Z(\supset) : Vec \xrightarrow{{}_{A\bar{A}}A} A\otimes\bar{A}\text{ mod}$

$\longleftarrow$

$Z(\subset) : A\bar{A}\text{-mod} \longrightarrow Vec$

$A_{A\bar{A}} \otimes_{A\bar{A}} -$

If we take $Z(pt^-) = B\text{-mod}$, then equiv $\bar{A}\text{-mod} \to B\text{-mod}$

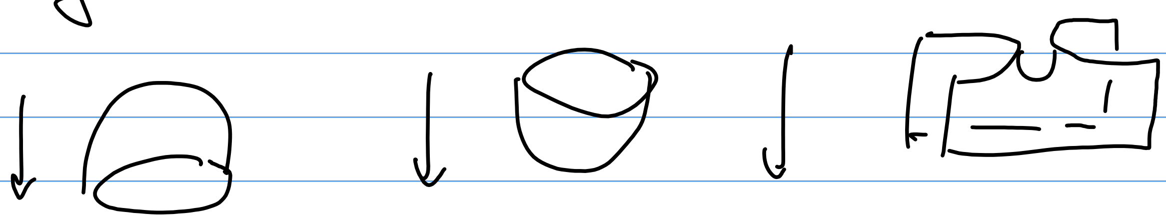


is data : nat iso f_1

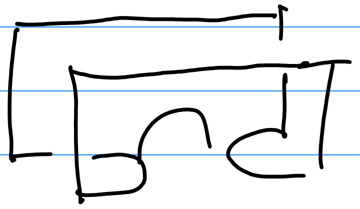
$$A_{A\bar{A}} \otimes_{\bar{A}} {}_{A\bar{A}}A \otimes_{A} - \;\Downarrow\; id$$

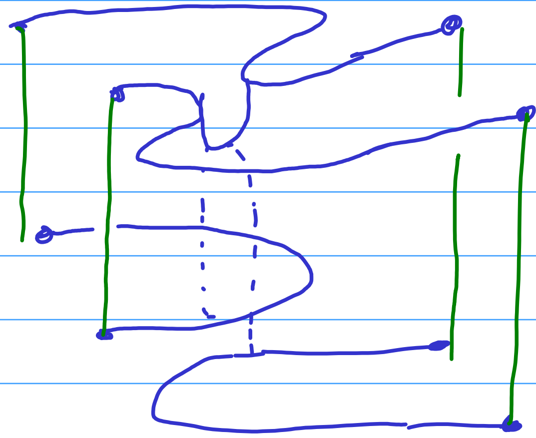
ie $\bar{A}$, B Morita equiv.

To go to 1+1 need :

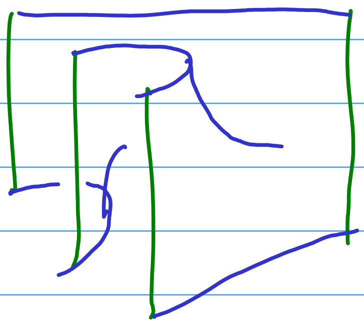


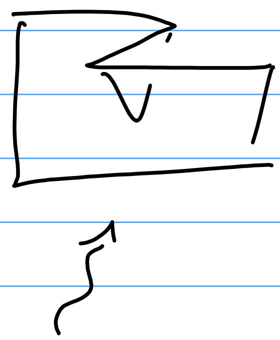
After saddle automatic





Relations from looking at





etc

Restrict to invertible 2-morph : $1+\varepsilon$ TFT

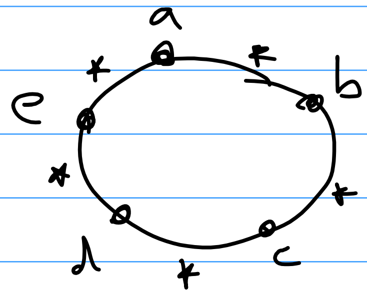
- restr. on these, then data on saddles

CS: Question: Do skein construction for $1+\varepsilon$ and see what $1+1$ we get and how cond. come about.

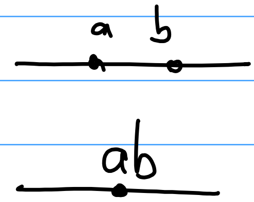
$1+\varepsilon$ skein: 1 mf $\mapsto$ VS

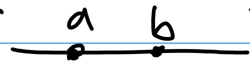
0 mf $\mapsto$ cat ("cylinder cat")

Input datum: lin cat $*/\!\!/A$



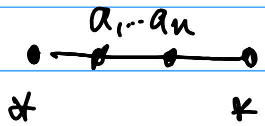
local rel.



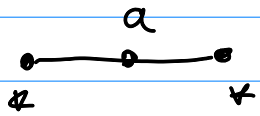
as eval() $= ab \in A$

So $Z(S^1) = A/[A,A]$ cocentre

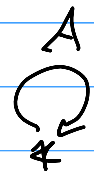
For cylinder:



enough:

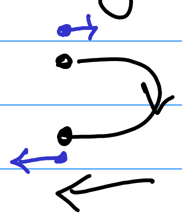


so get 1 obj cat with

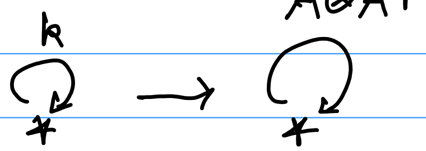


If A is ssi, then Karoub($\circlearrowleft_*^A$) $\cong A\text{-mod}$

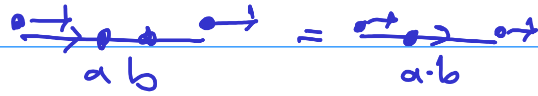
Investigate



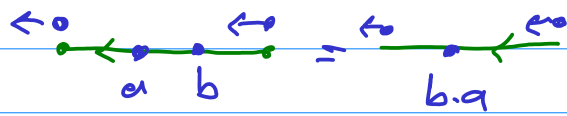
should be



nb:



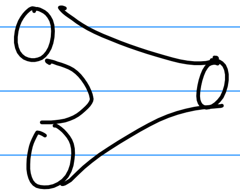
framing of pt induced by interval



so

local rel. are oriented!
ie embedding of oriented discs

In 2+ε d: $\bigcirc \rightsquigarrow$ circle cat $\overset{Kar}{\rightsquigarrow} Z(A)$

 should give $\otimes$ on $Z(A)$

CS: Is this written down somewhere?

Try profunctors: $\mathcal{A} \overset{}{\underset{F}{\not\to}} \mathcal{B}$ k-lin cats

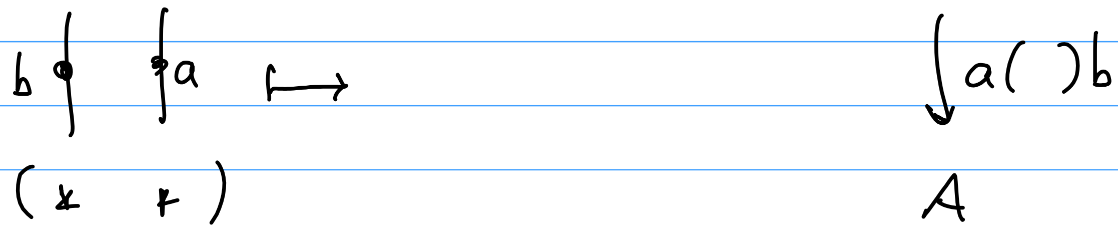
is functor $\mathcal{A}^{op} \boxtimes \mathcal{B} \longrightarrow vec$

Given F get profun via $(a,b) \mapsto Hom(Fa, b)$

In 1+ε d: Want $+ \longrightarrow +$

$F: {}^{*}/\!/_{A^{op}} \boxtimes {}^{*}/\!/_{A} \longrightarrow vect$

$(*, *) \longmapsto skein(\underset{+}{\bullet}\!\!\longrightarrow\!\!\underset{+}{\bullet}) \cong A$



This is indeed the profunc. for the id. fun.

It also gives C° $\longleftarrow$

$$\underbrace{{}^{*}/\!\!/_{A} \boxtimes {}^{*}/\!\!/_{A^{op}}}_{\cong\ {}^{*}/\!\!/_{A^{op}}} \boxtimes \left({}^{*}/\!\!/_{k}\right)^{op} \to \mathrm{vect}$$, again Hom

$$\mathrm{Hom}(\underbrace{F(*)}_{* \boxtimes *}, * \boxtimes *) \simeq \overbrace{A \otimes A^{op}}^{A}$$

$$F\left(1 \Big|_{*}^{*}\right) = \Big|_{*}^{*} 1 \otimes 1$$

Aside: $F : \mathcal{A} \to \mathcal{B}$ abelian

$\mathrm{Rex}(\mathcal{A}, \mathcal{B}) = \mathcal{B} \boxtimes \overline{\mathcal{A}}$ (essentially Eilenberg–Watts)

$= \mathcal{B} \boxtimes \overline{\mathcal{A}} \boxtimes \mathrm{vect}$

$= \mathrm{Rex}(\mathcal{A} \boxtimes \overline{\mathcal{B}}, \mathrm{vect})$

<u>TODO</u>:
- When can we turn profunctors into functors?
- How does above give $1+\varepsilon$ TFT

YHT: Before we go to 1+1, there is the fact that

$$\emptyset \underset{C}{\overset{\supset}{\rightleftarrows}} S^0 \quad (1\ \mathrm{T})$$ two sided adjoint

In A-mod setting:

$$\begin{tikzcd}
\mathrm{Vec} \arrow[r, bend left, "{}_{A\bar A}A \otimes_k -"] & A\bar A\text{-mod} \arrow[l, bend left, "A_{A\bar A} \otimes_{A\bar A} -"]
\end{tikzcd}$$

← always exact

← not always exact

right adj

s|| need

$\mathrm{Hom}_{A\bar A}({}_{\bar A A}A, -)$

left adj

$A^*_{A\bar A} \otimes_{A\bar A} -$