

We consider the cases $n = 2$ and $n = 3$. For $n = 2$, let \mathcal{C} be a non-semisimple finite tensor category. For $n = 3$, let \mathcal{C} be a non-semisimple finite ribbon category. In both cases, consider the tensor ideal \mathcal{P} of projective objects in \mathcal{C} .

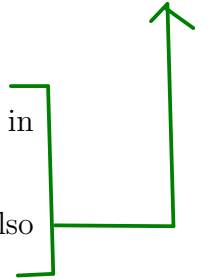
Perform a string-net construction where you admit as labels only objects of \mathcal{P} .

Questions:

1. Compute for $n = 2$ the vector space associated to S^2 and T^2 and a surface of arbitrary genus. ✗ ✓ (compute dim)
2. Compute for $n = 2$ the cylinder category for the circle. Compare to the Drinfeld center. ✓
3. Compute for $n = 3$ the vector space associated to S^3 . Can we make statements about other 3-manifolds? ✗
4. For $n = 3$, compute categories for oriented surfaces. Are they related to modules of known algebras (elliptic Drinfeld center etc.) ✓
5. Investigate whether for $n = 2$ the construction can be extended to 3-manifolds. You are allowed to use the modified trace on \mathcal{P} . ✓
6. Investigate whether for $n = 3$ the construction can be extended to 4-manifolds. You are allowed to use the modified trace on \mathcal{P} . ✓

Possible variations of the problems:

1. Do not work with the monoidal ideal \mathcal{P} , but consider stringnets in which at least one component is colored by a projective object.
2. Replace the monoidal ideal \mathcal{P} by some other monoidal ideal that also admits a modified trace.



Elaboration of variation: For $n = 2 + \epsilon$: Fix a pivotal finite tensor category \mathcal{C} and a tensor ideal $\mathcal{I} \subset \mathcal{C}$. Declare a string net to be \mathcal{I} -admissible if on each connected component of the surface there is an \mathcal{I} -coloured edge.

1. Show that the string net space for \mathcal{I} -admissible string nets on the sphere is the vector space dual of the space of two-sided modified traces on \mathcal{I} . ✗
2. Compute the \mathcal{I} -admissible string net space on the torus. Can this be non-zero even if there are no two-sided traces? ✗

3. Maybe look at the symplectic fermion example. This has intermediate ideals which are not too complicated. (And there is an example of an intermediate ideal which has a left and a right modified trace, but no two-sided trace.) ✓
4. If \mathcal{I} is the projective ideal, are the string net spaces related to state space of the non-ssi TFT for the Drinfeld centre of \mathcal{C} , or, equivalently, to the spaces Lyubasheko's modular functor for $\mathcal{Z}(\mathcal{C})$ assigns to surfaces? ✓

Rinse and repeat for $n = 3 + \varepsilon$.