Computational Economics: Problem Set 2

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Problem 1: Getting Started with Github

When we set `racism=0.5`, we get an error message. The error occurs because in line 50 and 51 the command `nrow_mover = length(mover)` and `nrow_freehouse = length(freehouse)` respectively. However, when there is only one or two movers, the `length` commands returns three since there are three columns. Therefore, one should use `size(mover,1)` and `size(freehouse,1)` instead to get the number of rows.

Problem 2: Univariate Problems

Write it as a univariate problem with the corresponding parameters,

$$0.5q + q^{0.5} - 2 = 0$$

Thus the root for the equation can be calculated analytically by letting $x = q^{0.5}$, then

$$x^2 + 2x - 4 = 0$$
$$\Rightarrow x = -1 \pm \sqrt{5}$$

Thus $q = x^2 = 6 \pm 2\sqrt{5}$.

Problem 3: A Contribution to the Empirics of Economic Growth

Dependent varibale: log difference GDP per working-age person 1960-1985			
Sample	Non-oil	Intermediate	OECD
Observation	98	75	22
Constant	1.874	2.498	4.155
	(0.828)	(0.860)	(0.898)
ln(Y60)	-0.288	-0.366	-0.398
	(0.060)	(0.066)	(0.063)
ln(I/GDP)	0.524	0.538	0.332
	(0.085)	(0.099)	(0.156)
ln(n+g+δ)	-0.506	-0.545	-0.863
	(0.283)	(0.281)	(0.304)
ln(School)	0.231	0.270	0.228
	(0.058)	(0.078)	(0.130)
$\bar{R}^2$	0.46	0.43	0.63

Problem 4: Solving The Augmented Solow Growth Model

1. The system of equations reads

$$Y(t) = K(t)^{\alpha_K} H(t)^{\alpha_H} [A(t)L(t)]^{1-\alpha_K-\alpha_H} \tag{1}$$

$$\dot{K}(t) = s_K Y(t) - \delta_k K(t) \tag{2}$$

$$\dot{H}(t) = s_H Y(t) - \delta_h H(t) \tag{3}$$

$$\dot{L}(t) = nL(t) \tag{4}$$

$$\dot{A}(t) = gA(t) \tag{5}$$

where $\dot{X}(t)$ denotes the time derivative of time function $X(t)$. Now transform the system and divide all variables by $A(t)L(t)$. We denote the variables after transformation by $x(t) = X(t)/[A(t)L(t)]$. We obtain after some steps

$$y(t) = k(t)^{\alpha_K} h(t)^{\alpha_H} \tag{6}$$
$$\dot{k}(t) = s_K y(t) - (n + g + \delta_k)k(t) \tag{7}$$
$$\dot{h}(t) = s_H y(t) - (n + g + \delta_h)h(t) \tag{8}$$

At the steady-state, $\dot{k} = \dot{h} = 0$, thus we have

$$s_K y(t) = (n + g + \delta_k)k(t) \tag{9}$$
$$s_H y(t) = (n + g + \delta_h)h(t) \tag{10}$$

Substituting y into the equations and solving for k^* and h^* we obtain

$$k^* = \left(\frac{s_K}{n + g + \delta_k}\right)^{\frac{1-\alpha_H}{1-\alpha_K-\alpha_H}} \left(\frac{s_H}{n + g + \delta_h}\right)^{\frac{\alpha_H}{1-\alpha_K-\alpha_H}} \tag{11}$$
$$h^* = \left(\frac{s_H}{n + g + \delta_h}\right)^{\frac{1-\alpha_K}{1-\alpha_K-\alpha_H}} \left(\frac{s_K}{n + g + \delta_k}\right)^{\frac{\alpha_K}{1-\alpha_K-\alpha_H}} \tag{12}$$

Now substituting in our parameterization, we obtain

$$k^* \approx 5.7932, \qquad h^* \approx 8.5194. \tag{13}$$

2. Substituting Eq.(**??**) into Eq.(**??**) and (**??**), and letting $\dot{k} = \dot{h} = 0$, we obtain at steady state

$$f_1(k, h) := s_K k(t)^{\alpha_K} h(t)^{\alpha_H} - (n + g + \delta_k)k(t) = 0 \tag{14}$$
$$f_2(k, h) := s_H k(t)^{\alpha_K} h(t)^{\alpha_H} - (n + g + \delta_h)h(t) = 0 \tag{15}$$

Dropping all time arguments, the Jacobian is given by

$$\mathcal{J} = \begin{pmatrix} \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial h} \\ \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial h} \end{pmatrix} \tag{16}$$
$$= \begin{pmatrix} \alpha_K s_K k^{\alpha_K - 1} h^{\alpha_H} - (n + g + \delta_k) & \alpha_H s_K k_K^{\alpha} h^{\alpha_H - 1} \\ \alpha_K s_H k^{\alpha_K - 1} h^{\alpha_H} & \alpha_H s_H k_K^{\alpha} h^{\alpha_H - 1} - (n + g + \delta_k) \end{pmatrix} \tag{17}$$

3. For $\epsilon = \delta = 0.001$, convergence is reported after 5 iterations, with infinite norm difference from the true value being 3.133e-11.

   For $\epsilon = \delta = 1e - 20$, convergence is reported after 7 iterations, with infinite norm difference from the true value being 7.105e-15.

4. For $\epsilon = \delta = 0.001$, convergence is reported after 21 iterations, with infinite norm difference from the true value being 0.00019.
   For $\epsilon = \delta = 1e-20$, convergence is reported after 29 iterations, with infinite norm difference from the true value being 1.776e-15.

5. For $\epsilon = \delta = 0.001$, convergence is reported after 21 iterations, with infinite norm difference from the true value being 0.00019.
   For $\epsilon = \delta = 1e-20$, convergence is reported after 29 iterations, with infinite norm difference from the true value being 1.776e-15.

6. Convergence of k and h are shown in the figure below. Agents start to accumulate both types of capital since the beginning, at a decreasing rate when approaching the steady state. They stop capital accumulation after reaching the steady state and continue to produce with the same amount of captial.

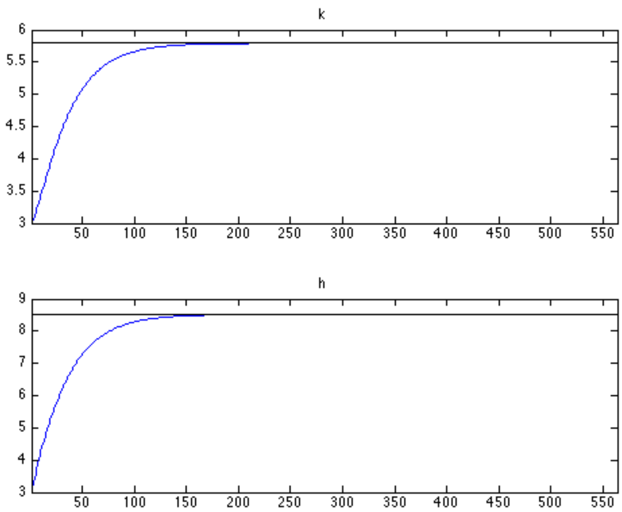

Figure 1: Convergence of k and h