Problem Set 1

Due date: April 04, 2015, before lecture

Instructions: Please hand in a single PDF-file containing all your answers and results. Show the names of the group members on top. Make use of figures and tables. Include the source code in the appendix. Additionally, hand in the source code as a separate zip-file. The code should be well documented and readable.

Problem 1: Simple linear equation example

Consider a simple market equilibrium model where the demand and supply functions are respectively given by

$$\mathbf{D}: \qquad p = a - b \cdot q \tag{1a}$$

$$\mathbf{S}: \qquad p = c + d \cdot q \tag{1b}$$

where p is the price, q is the quantity, a, b, c, d are some parameters.

1. Show that market equilibrium is characterized by the relationship

$$b \cdot q + d \cdot q - (a - c) = 0.$$

2. Analytically compute the equilibrium allocation and corresponding price, $(q^\star, p^\star)$.

3. Next, transform the system of equations in (1) into a standard linear equation system of the form $Ax = y$ for coefficient matrix A, variable vector $x = [p, q]'$ and data vector y. Analytically solve this system of equations by an LU decomposition applying the steps from the slides of the lecture (see the appendix).

4. Now parameterize the model with $a = 3$, $b = 0.5$, $c = d = 1$. Compute $(q^\star, p^\star)$.

5. Implement a Gauss-Seidel fixed-point iteration for solving the system of equations. Initialize the iteration with $(q, p) = (0.1, 0.1)$. For which order of the equation system does the system converge? Illustrate convergence and non-convergence graphically.

6. Revisit the non-convergent case. Apply a dampening factor (or overrelaxation parameter) λ. Consider a grid for $\lambda \in [0.1, 0.2, \ldots, 0.9]$. For which value of ω do you find the fastest convergence?

Problem 2: Determine the output gap

In this problem, you will use OECD data on quarterly GDP for Germany and Greece to determine the output gap of the two countries. The output gap is a measure of how much an economy is running below its capacity (it can also temporarily run above). Formally, the output gap $G_{j,t}$ of country j at time t is defined as the percentage deviation of GDP, $Y_{j,t}$, from its trend, $\hat{Y}_{j,t}$:

$$G_{j,t} = \frac{Y_{j,t} - \hat{Y}_{j,t}}{\hat{Y}_{j,t}}$$

A crucial question is how to determine the trend. We will compare two approaches that were discussed in the lecture: OLS and the Hodrick-Prescott (HP) Filter.

1. Load the quarterly GDP data from `OECD-Germany_Greece_GDP.xls`. For both countries, calculate log GDP, denoted by $\log Y_{j,t}$.

2. Determine the trend of $\log Y_{j,t}$ using the HP-filter with $\lambda = 1600$, which is a common value for quarterly data.

3. Determine the linear trend of $\log Y_{j,t}$ by OLS regression, i.e. $\log Y_{j,t} = \beta_{0,j} + \beta_{1,j} \cdot t + \epsilon_{j,t}$.

   (a) Calculate the OLS estimator $\hat{\beta}_j$.

   (b) Calculate the linear trend of log GDP, $\widehat{\log Y_{j,t}} = \hat{\beta}_{0,j} + \hat{\beta}_{1,j} \cdot t$

4. Calculate the output gap using both the HP-trend and the linear trend. Don't forget to transform the log variables back to levels, i.e., $\hat{Y}_{j,t} = \exp\left(\widehat{\log Y_{j,t}}\right)$.

5. For both Germany and Greece separately, provide the following two plots

   (a) $\log Y_{j,t}$ together with its HP-trend and its linear trend.

   (b) $G_{j,t}$ for each of the two trends. Show the zero line.

Problem 3: Schelling's Segregation

This exercise uses a modified version Schelling's famous segregation result to deepen your understanding of logical indexing of matrices and control flow statements.

Suppose your residential area consists of 225 houses, spatially organized as a 15×15 check board. There are 220 occupied houses, 110 by black and 110 by white people. In each period, the people take a look around their closest neighbors, i.e., the occupied houses in the eight surrounding squares. They decide to move when more than 35 percent of their neighbors are of a different color. For simplicity assume that the residential area is surrounded by additional non-occupied houses, that remain non-occupied. Movers all move simultaneously and are randomly allocated to non-occupied houses (previously unoccupied or just getting unoccupied). Simulate the dynamics of the residential area over 45 periods. Visualize the composition of the residential area after 0, 15, 30, and 45 periods with plots that look like check boards (`imagesc` and `colormap(flipud(gray))` may help).