Problem Set 2

Due date: May 18 26, 2015, before lecture

Instructions: Please hand in a single PDF-file containing all your answers and results. Show the names of the group members on top. Make use of figures and tables and always provide a short interpretation of your results. Include the source code in the appendix. Additionally, hand in the source code as a separate zip-file. The code should be well documented and readable.

Problem 1: Getting Started with Github

1. Revisit the solution for Problem Set 1, Problem 3 (Schelling's segregation model). Run it for `racism = 0.5`. What do you get?
2. Download Github from https://github.com/
3. Store the current version of the code on Github.
4. Now work on the code to remove the bug and store (committing and syncing) intermediate versions on Github.
5. From now on use Github as a version control device to manage your codes.

Problem 2: Univariate Problems

1. Code the Bisection algorithm for any function $f(x)$
2. Use bisection to compute the zeros of the functions
   - $f(x) = x^3 + 4 - \frac{1}{x}$
   - $f(x) = -\exp(-x) + \exp(-x^2)$
3. Revisit the demand-supply example of problem set 1:

$$\mathbf{D}: \qquad p = a - b \cdot q \tag{1a}$$

$$\mathbf{S}: \qquad p = c + d \cdot q^{\psi} \tag{1b}$$

   where p is the price, q is the quantity, a, b, c, d, ψ are some parameters.

   - Write it as a univariate problem:

$$b \cdot q + d \cdot q^{\psi} - (a - c) = 0.$$

   - Parameterize the model with $a = 3$, $b = 0.5$, $c = d = 1$, $\psi = 0.5$. Compute the solution analytically.
   - Compute the solution with your bisection algorithm.
   - Compute the solution with the Matlab internal function `fzero`.
   - Implement a Gauss-Seidel fixed-point iteration for solving the system of equations. Initialize the iteration with $(q, p) = (0.1, 0.1)$. Does it converge? When you reorder the system of equations? How about dampening?

Problem 3: A Contribution to the Empirics of Economic Growth

Consider the empirical analysis conducted in Mankiw, Romer, and Weil (1992): A Contribution to the Empirics of Economic Growth, in: *The Quarterly Journal of Economics*, Vol. 107(2), pp. 407-437.

> "This paper examines whether the Solow growth model is consistent with the international variation in the standard of living. It shows that an augmented Solow model that includes accumulation of human as well as physical capital provides an excellent description of the cross-country data. [...]"

1. Load the data set of Mankiw, Romer and Weil 1992 (file: `MRW92QJE-data.xls`). The data set contains the country number, a non-oil country dummy, an intermediate country dummy, an OECD country dummy, GDP per adult in 1960, GDP per adult in 1985, the annualized GDP growth rate, the annualized population growth rate, the average investment to GDP ratio, average secondary school enrollment rate. Delete countries with missing values (hint: after having loaded the data, check with `isnan`).

2. Generate sub-samples for non-oil countries, intermediate countries, and OECD countries (hint: test for equality ==).

3. For each sub-sample, compute the regression coefficients and respective standard errors for the following regression model

$$\begin{aligned}\log(gdp1985_j) - \log(gdp1960_j) = \beta_0 + \beta_1 \cdot log(gdp1960_j) + \beta_2 \cdot log(investment/gdp_j) + \\ \beta_3 \cdot log(popgrowth_j + g + \delta) + \beta_4 \cdot log(schoolenrol_j) + \epsilon_j\end{aligned}$$

   where $g + \delta = 0.05$.

4. Provide a table with your results that is similar to Table V (p. 426) in the article. No need to replicate it exactly.

Problem 4: Solving The Augmented Solow Growth Model

Consider the human capital augmented Solow growth model that we introduced in lecture 4. Let the production function be Cobb-Douglas.

Table 1: Parameters

s_K	s_H	n	g	δ_k	δ_h	z	α_K	α_H	k_0	h_0
0.200	0.200	0.010	0.015	0.10	0.06	1	0.33	0.33	3	3

Program the different algorithms enumerated below. The numerical root-finding algorithms should automatically adjust to the dimension of the problem, i.e., accept inputs that can be scalars or matrices.[1] Use each in turn to solve the augmented Solow model, i.e., find a root of the appropriate nonlinear equation system. The parameterization is given in table 1, with k_0 and h_0 denoting the starting values.[2] For each algorithm, report the number of iterations and the running time, and compare them.

1. Derive the analytic solution $x^* = (k^*, h^*)$, and compute its value for the given parameterization.

[1] By contrast, the economic model, i.e., the augmented Solow model, has a fixed dimension.

[2] Other starting values may yield results that are not real, i.e., have a complex part. You may want to check with `isreal`.

2. Now assume an analytic solution was not available, so that the nonlinear system of equations has to be solved numerically. Write a Matlab function that returns the function value and the Jacobian of the steady state condition.

3. Program and use Newton's method for root-finding. Letting $\hat{x}^* = \left(\hat{k}^*, \hat{h}^*\right)$ denote the approximate solution, report $\|\hat{x}^* - x^*\|_\infty$. Increase the accuracy of your algorithm by setting stricter convergence criteria, again report $\|\hat{x}^* - x^*\|_\infty$, and compare. (Alternatively, you may also decrease the accuracy and compare.)

4. Program and use the Broyden method for root-finding. Use the identity matrix as the initial guess for the Jacobian.

5. Program and use the Inverse Broyden method for root-finding . Use the identity matrix as the initial guess for the Jacobian.

6. Program and use a fixed-point iteration (hint: the supplied function has a slightly different output than in the previous methods). Save all results from the iteration steps. Display a plot showing the results for k over the iterations along with a horizontal line showing at the value of k^*. Do the same for h. Provide an economic interpretation of the plots.

Problem 5: Cournot Oligopoly

Consider the following Cournot oligopoly model with n firms. The inverse demand function is

$$P(Q) = q^{-1/\lambda} = \Big(\sum_{i=1}^{n} q_i\Big)^{-1/\lambda}$$

Production costs are

$$c_i(q_i) = \frac{1}{2}\zeta_i q_i^2, \quad \forall i = 1, \dots, n$$

Firm i's profits are given by

$$\pi_i(q_1, \dots, q_n) = \Big(\sum_{i=1}^{n} q_i\Big)^{-1/\lambda} q_i - \frac{1}{2}\zeta_i q_i^2$$

For its optimization problem, firm i takes the output of all other firms as given. Thus, equilibrium output levels $(q_1, \dots, q_i)$ are the solution to

$$\frac{\partial \pi_i(q_1, \dots, q_n)}{\partial q_i} = \Big(\sum_{i=1}^{n} q_i\Big)^{-1/\lambda} - \frac{1}{\lambda}\Big(\sum_{i=1}^{n} q_i\Big)^{-1/\lambda - 1} q_i - \zeta_i q_i = 0, \quad \forall i = 1, \dots, n$$

Let $\lambda = 1.6$. The firm-specific costs, ζ_i, take their values on an equally spaced grid between 0.6 and 0.8, i.e., $\{\zeta_i = 0.6 + (i-1)\frac{0.8-0.6}{n-1},\ i = 1, \dots, n\}$.

1. Compute the equilibrium allocations for $n = 2$ firms.

2. Compute the equilibrium allocations for $n = 5$ firms.

3. Compute the equilibrium allocations for $n = 10$ firms.