COMP 250
INTRODUCTION TO COMPUTER SCIENCE

Lecture 2 – Logarithms and Mod Function

Giulia Alberini, Fall 2018

CSUS

Computer Science Undergraduate Society

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Work to improve student academics and life in the CS department

What are we looking for?

- U0 Rep
- U1 Rep
- Equity Comissioner

Where can you apply?

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Sunday Sep 13th

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HOURS: 10am - 5pm (mon-fri)
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WHO ARE WE? WHAT DO WE DO?

- U2 and U3 students who have taken this course and want to help you!
- We are a **FREE** drop-in tutoring service, perfect for study help, and guidance on assignments.
- We provide review sessions for midterms and finals for intro courses!

ABOUT ME

Giulia Alberini

- Faculty Lecturer in SOCS
- PhD in Computer Science from McGill (Cryptography)
- MSc and BSc in Mathematics from University of Padova
- Love to play Ultimate

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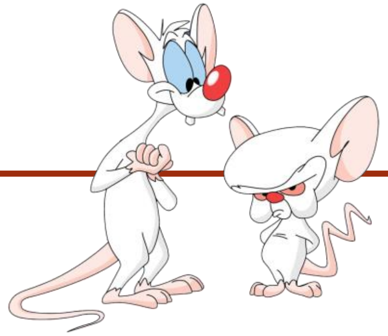
(or by appointment)

WHAT ARE WE GOING TO DO TODAY?



- Logarithms (quick review)
- Modulo function

LOGARITHMS

LOGARITHMS – DEFINITION

- The logarithm is the inverse function of the exponential.
- Let $b > 0$, then the exponential function for the base b is as follows

$$y = b^x$$

- Then the inverse function

$$\mathrm{y} = \log_b(x)$$

is called the *logarithm* of that base b.

LOGARITHMS – DEFINITION

From the definition it directly follows

1. $\log_b(b^x) \equiv x$

2. $b^{\log_b x} \equiv x$

QUICK EXAMPLES

- $\log_2 8 \quad = 3$

  $\text{i.}e., \quad 2^3 = 8$

- $\log_4 16 \quad = 2$

  $\text{i.}e., \quad 4^2 = 16$

- $\log_5 1 \quad = 0$

  $\text{i.}e., \quad 5^0 = 1$

- $\log_8 2 \quad = \frac{1}{3}$

  $\text{i.}e., \quad 8^{\frac{1}{3}} = \left(2^3\right)^{\frac{1}{3}} = 2^{3\cdot\frac{1}{3}} = 2$

- $\log_2 \frac{1}{8} \quad = -3$

  $\text{i.}e., \quad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

- $\log_8 \frac{1}{2} \quad = -\frac{1}{3}$

  $\text{i.}e., \quad 8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$

USEFUL PROPERTIES YOU SHOULD KNOW

Let $b, x, y > 0$

a. $b^{n+m} = b^n \cdot b^m$

b. $(b^n)^m = b^{nm}$

c. $\log_b(xy) = \log_b x + \log_b y$

d. $\log_b(x^n) = n \cdot \log_b x$

LESS OBVIOUS, BUT STILL USEFUL PROPERTIES

Let $a, b, c > 0$

- $\log_b a = (\log_b c) \cdot (\log_c a)$

  Proof:

$$\begin{aligned} \log_b a &= \log_b\left(c^{\log_c a}\right) && \text{from the definition (slide 8, property 2)} \\ &= (\log_c a) \cdot (\log_b c) && \text{slide 10, property d} \end{aligned}$$

LESS OBVIOUS, BUT STILL USEFUL PROPERTIES

Let $a, b, c > 0$

- $a^{\log_b c} = c^{\log_b a}$

Proof. idea: let's prove $\log_b\left(a^{\log_b c}\right) = \log_b\left(c^{\log_b a}\right)$

$$\begin{aligned}\log_b\left(a^{\log_b c}\right) &= (\log_b c) \cdot (\log_b a) \\ &= (\log_b a) \cdot (\log_b c) \\ &= \log_b\left(c^{\log_b a}\right)\end{aligned}$$

MODULO

THE QUOTIENT REMAINDER THEOREM

Theorem (Quotient Remainder)

Given any integer a, and **a positive** integer b, there exist **unique** integers q and r such that

$$a = b \cdot q + r$$

where $0 \leq r < b$.

We call:

a the **dividend** q the **quotient**

b the **divisor** r the **remainder**

MOD FUNCTION

We might be interested only in the remainder. For this cases, in mathematics, there is an operator called the **modulo operator**, abbreviated as **mod**. We write:

$$\boldsymbol{a} \bmod \boldsymbol{b} = \boldsymbol{r}$$

Which reads as "a modulo b is equal to r". b is called the **modulus**.

- **In mathematics, the convention is that $\mathbf{0} \leq \boldsymbol{r} < \boldsymbol{b}$**

QUICK EXAMPLES

- What is $11 \; mod \; 3$?
  - 11 is equal to 3*3 + 2, so when we divide 11 by 3 we get a quotient equal to 3 and a remainder of 2.

$$11 \; mod \; 3 \; = 2$$

- What is $10 \; mod \; 5$?
  - 10 is multiple of 5, thus when we divide 10 by 5 we get a remainder equal to 0.

$$10 \; mod \; 5 \; = \; 0$$

- What is $-3 \; mod \; 5$
  - By convention, we need to find a positive remainder. -3 is equal to 5*(-1) + 2. So, the remainder of the division by 5 is 2.

$$-3 \; mod \; 5 \; = \; 2$$

MODULO – OBSERVATIONS

- If a is a multiple of b, then

$$a \; mod \; b = 0$$

- For all integers k,

$$a \; mod \; b = (a + k * b) \; mod \; b$$

- We use modulo all the time in our daily life
  - Suppose Lauryn Hill is coming to Montreal and I want to get tickets for her concert. It's now 5 pm and the tickets will start to sell in 17 hours. When should I check again to buy the ticket?
  - Suppose my birthday is on December 5^{th}. Last year it fell on a Tuesday. Considering that this year is not a leap year, on which day of the week will my birthday be?

USEFUL PROPERTIES YOU SHOULD KNOW

- **Addition** property of modular arithmetic

$$(a+b) \bmod c = ((a \bmod c) + (b \bmod c)) \bmod c$$

- **Multiplication** property of modular arithmetic

$$(a \cdot b) \bmod c = ((a \bmod c) \cdot (b \bmod c)) \bmod c$$

MODULO IN JAVA

- The *remainder operator*: %
  It is defined to produce a result value such that `(a/b)*b+(a%b)` is equal to `a`

- When both operands have type `int`, the remainder operator evaluates to `int`.

- By definition, the sign of the result depends on the sign of the left operand.

  That is,

  - `(5%2)` is a positive `int` because 5 is positive, while
  - `(-8%3)` is a negative `int`.

https://docs.oracle.com/javase/specs/jls/se7/html/jls-15.html#jls-15.17.3

% - WHAT IS IT USEFUL FOR?

- Check if a number is even or odd

```
if(x%2 == 0) {…
```

```
if(x%2 == 1) {…
```

- Check if a number is a multiple of another number

```
if(x%7 == 0) {…
```

- And more…

EXAMPLE

```
int max = 12;
for(int i = 0; i < max; i++) {
   int x = i%5;
   System.out.print(x + " ");
}
System.out.println();
```

- **What prints?**
  - 0 1 2 3 4 0 1 2 3 4 0 1

EXAMPLE

- What if we still want to print 12 numbers, but we want to start counting from 2 instead of 0?

```
int max = 12;
for(int i = 0; i < max; i++) {
   int x = (i+2)%5;
   System.out.print(x + " ");
}
System.out.println();
```

Then the program will print:

➢ 2 3 4 0 1 2 3 4 0 1 2 3

EXAMPLE

```
int[] arr = {8, 6, 9, 5, 3, 1, 7};
for(int i = 0; i < arr.length; i++) {
   System.out.print(arr[i] + " ");
}
System.out.println();
```

- **What prints?**
  - 8 6 9 5 3 1 7

EXAMPLE

- Let's now create a new array containing the same elements as the original array, but shifted 2 positions to the left
- Idea:

```
int[] arr = {8, 6, 9, 5, 3, 1, 7};
int[] copyArr = new int[arr.length];
for(int i = 0; i < arr.length; i++) {
   copyArr[i] = arr[i+2];
}
```

Problem: in the last 2 iterations the index will be out of bounds!

EXAMPLE

- Solution 2.

```
int[] arr = {8, 6, 9, 5, 3, 1, 7};
int[] copyArr = new int[arr.length];
for(int i = 0; i < arr.length; i++) {
   int index = (i+2)%arr.length;
   copyArr[i] = arr[index];
}
```

i	index
0	2
1	3
2	4
3	5
4	6
5	0
6	1

Much better! If we want to change the shift we just need to change the 2. Actually, instead of 2 we can use a variable where we store the value of the shift.

EXAMPLE

- What if we want to shift the elements toward the right instead?
- Idea:

```
int[] arr = {8, 6, 9, 5, 3, 1, 7};
int[] copyArr = new int[arr.length];
for(int i = 0; i < arr.length; i++) {
   int index = (i-2)%arr.length;
   copyArr[i] = arr[index];
}
```

i	index
0	-2
1	-1
2	0
3	1
4	2
5	3
6	4

Problem: in the first 2 iterations the index will be out of bounds!

EXAMPLE

- When using subtraction be careful!
- To make sure you get a positive integer you can do the following:

```
int[] arr = {8, 6, 9, 5, 3, 1, 7};
int[] copyArr = new int[arr.length];
for(int i = 0; i < arr.length; i++) {
   int n = (i - 2 + arr.length);
   int index = n%arr.length;
   copyArr[i] = arr[index];
}
```

i	n	index
0	5	5
1	6	6
2	7	0
3	8	1
4	9	2
5	10	3
6	11	4

TRY IT! - `shiftElements`

Write a method `shiftElements` that takes an integer array and an `int n` as input and shifts all its elements by `n` positions to the left without creating a new array.

- This week:
  - Number bases and binary numbers
  - Char and Unicode, review of type conversions
- Next week:
  - Java!