COMP 250
INTRODUCTION TO COMPUTER SCIENCE

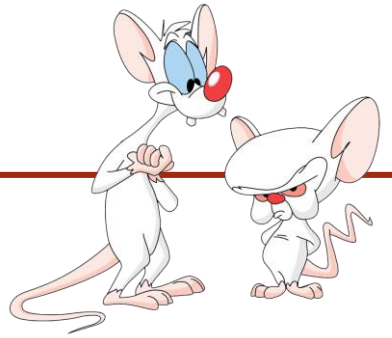
Lecture 21 – Recursion 3 (Mergesort, Quicksort)

Giulia Alberini, Fall 2018

FROM LAST CLASS

- Binary search

WHAT ARE WE GOING TO DO TODAY?



- Merge sort
- Quick sort

TIME COMPLEXITY

$O(log_2 n)$	$O(n)$	$O(n^2)$
• convert to binary • binary search • ……	▪ List operations: findMax, remove ▪ grade school addition or subtraction ▪ …..	▪ insertion/selection/ bubble sort ▪ grade school multiplication ▪ ……

~10^9 OPERATIONS PER SECONDS

$$2^{10} \approx 10^3$$

$$2^{20} \approx 10^6$$

$$2^{30} \approx 10^9$$

~10^9 OPERATIONS PER SECONDS

$log_2\, n$	n	n^2
10	$2^{10} \approx 10^3$	10^6
20	$2^{20} \approx 10^6$	10^{12}
30	$2^{30} \approx 10^9$	10^{18}

minutes…hours

second

centuries

BETTER SORTING ALGORITHM?

$$O(n) \quad < \quad ? \quad < \quad O(n^2)$$

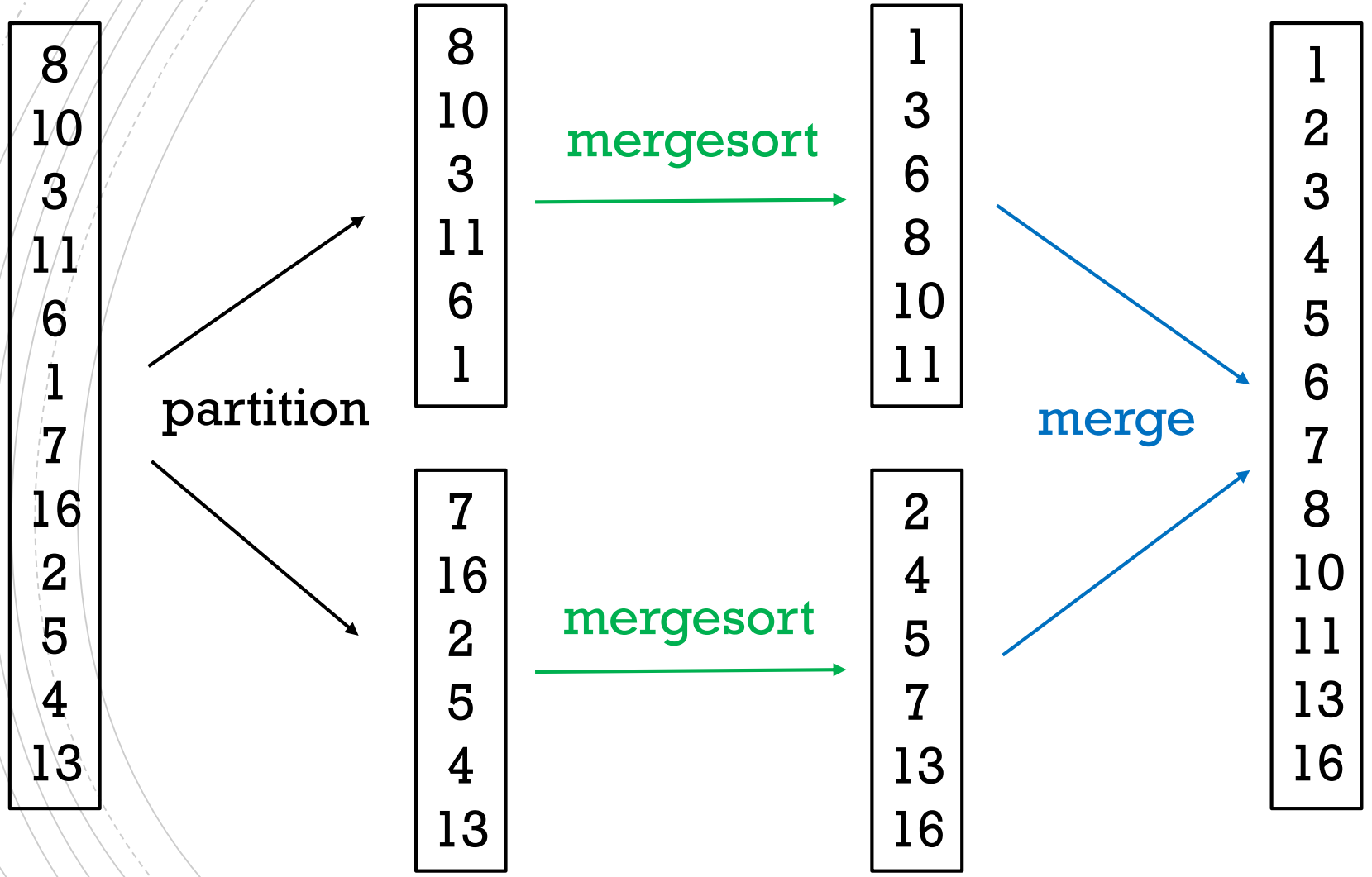
MERGE SORT

MERGE SORT

Merge Sort is a divide and conquer algorithm.

- GOAL: Sort an list.
- IDEA:
  - Partition the list into two halves.
  - Sort each half recursively
  - Merge the sorted half maintaining the order.

IDEA


8
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partition
8
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3
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1
7
16
2
5
4
13
mergesort
mergesort
1
3
6
8
10
11
2
4
5
7
13
16
merge
1
2
3
4
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6
7
8
10
11
13
16

IMPLEMENTATION

```
mergesort(list){
      if (list.size() == 1)
            return list
      else {
          mid = (list.size() - 1) / 2
          list1 = list.getElements(0,mid)
          list2 = list.getElements(mid+1, list.size()-1)
          list1 = mergesort(list1)
          list2 = mergesort(list2)
          return merge(list1, list2)
      }
}
```

IMPLEMENTATION

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            return merge(list1, list2)
        }
}
```

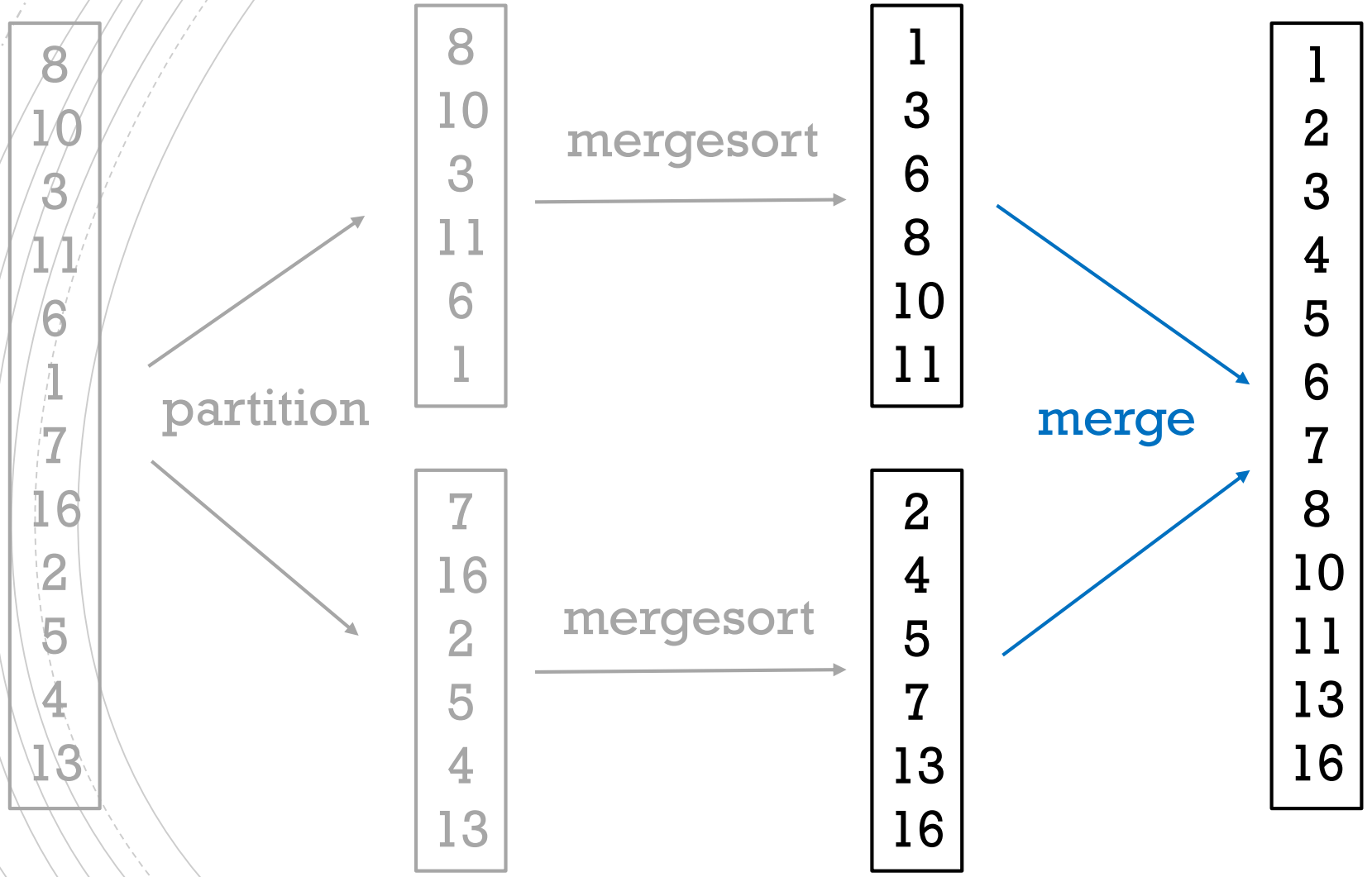
Base case

Partition

Recursive sort

Merge

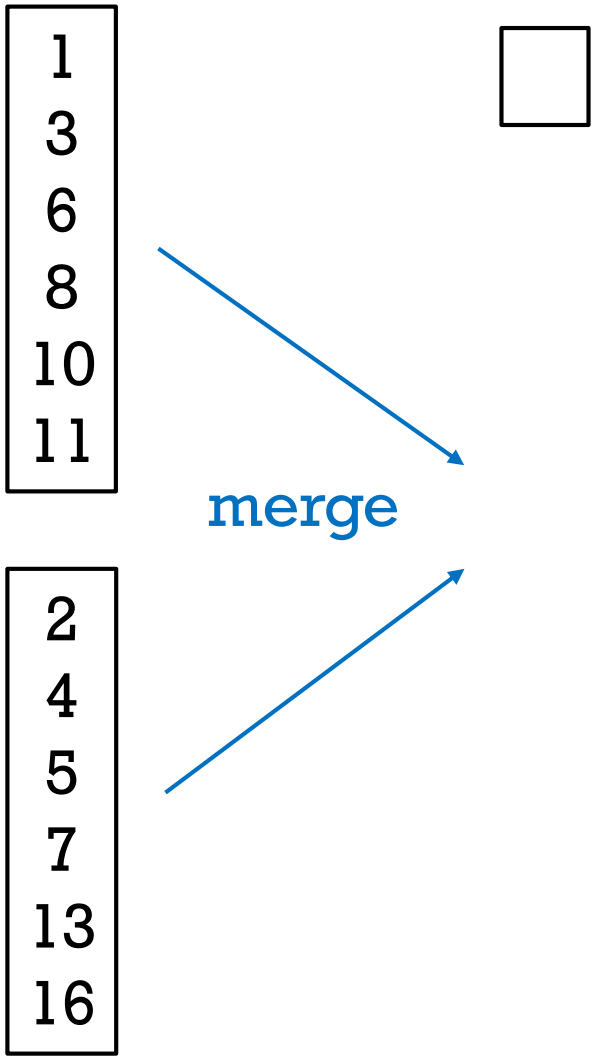
MERGING PRESERVING ORDER



MERGING PRESERVING ORDER

Iterate through the elements of the two sorted list. Depending on how they compare decide which element comes first in the merged list.

Similar to efficient add() in A2!



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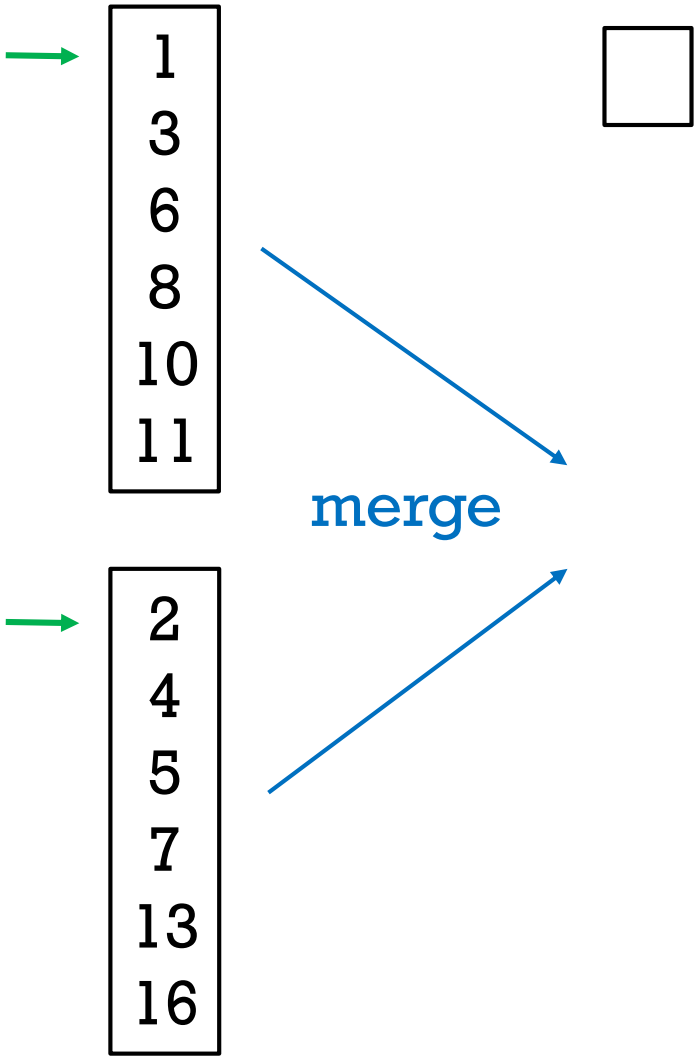

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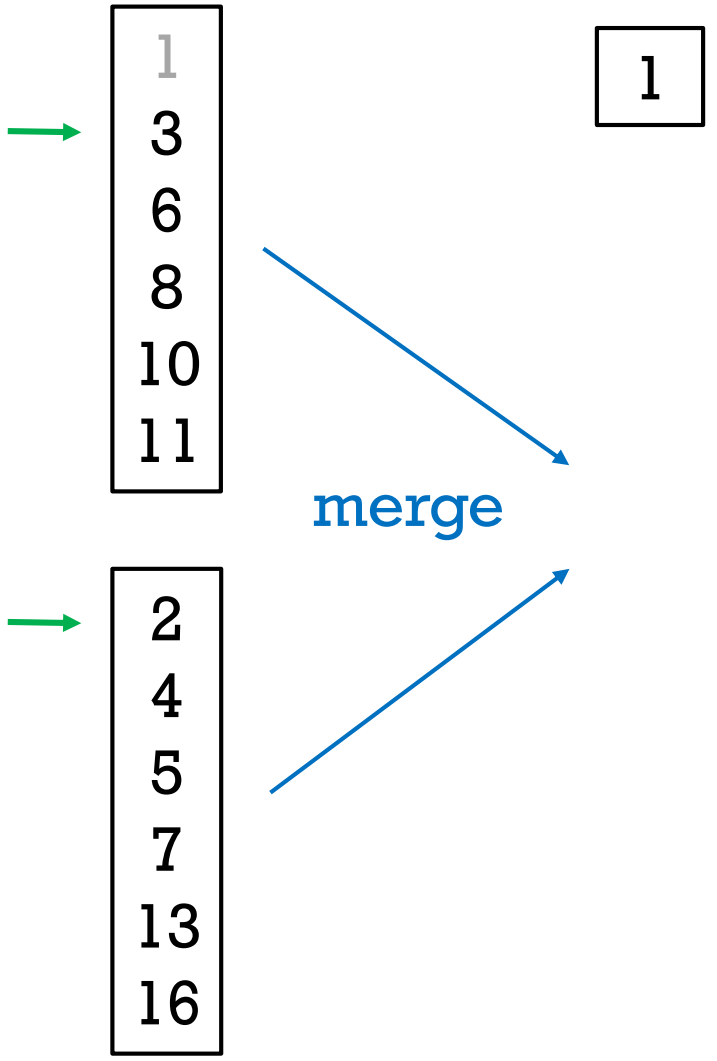

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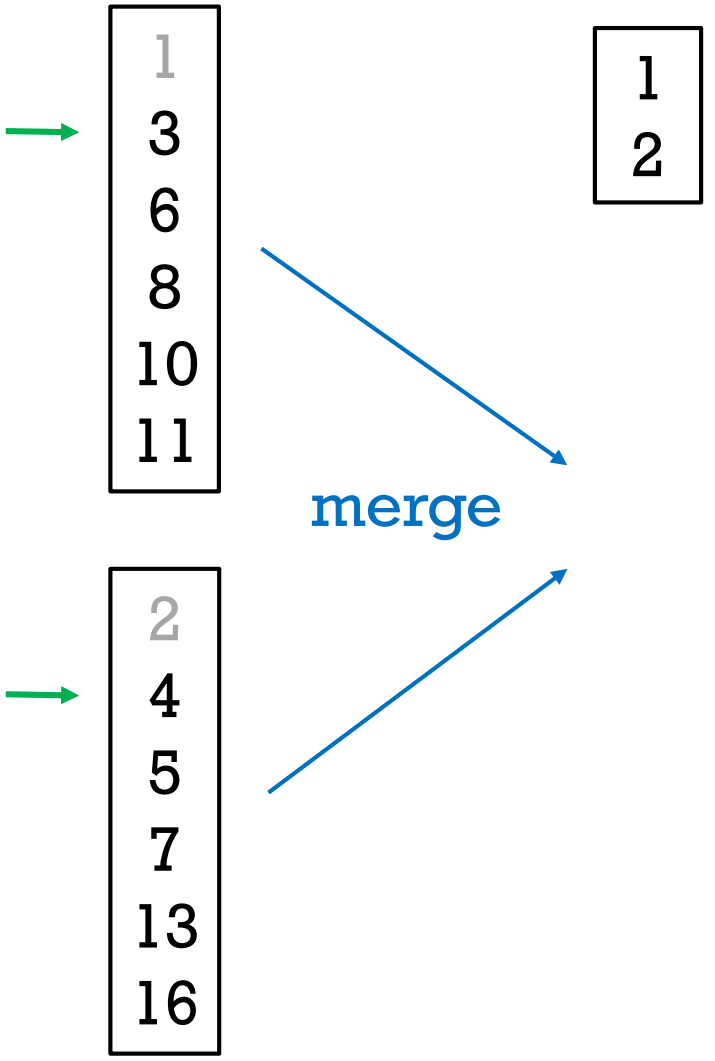

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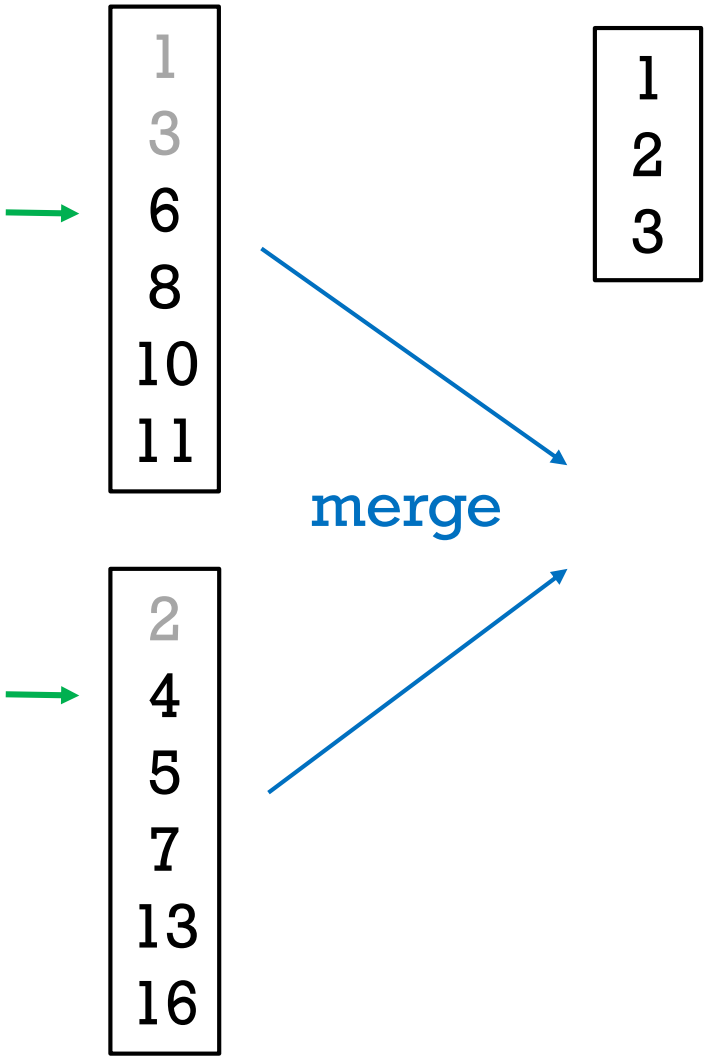

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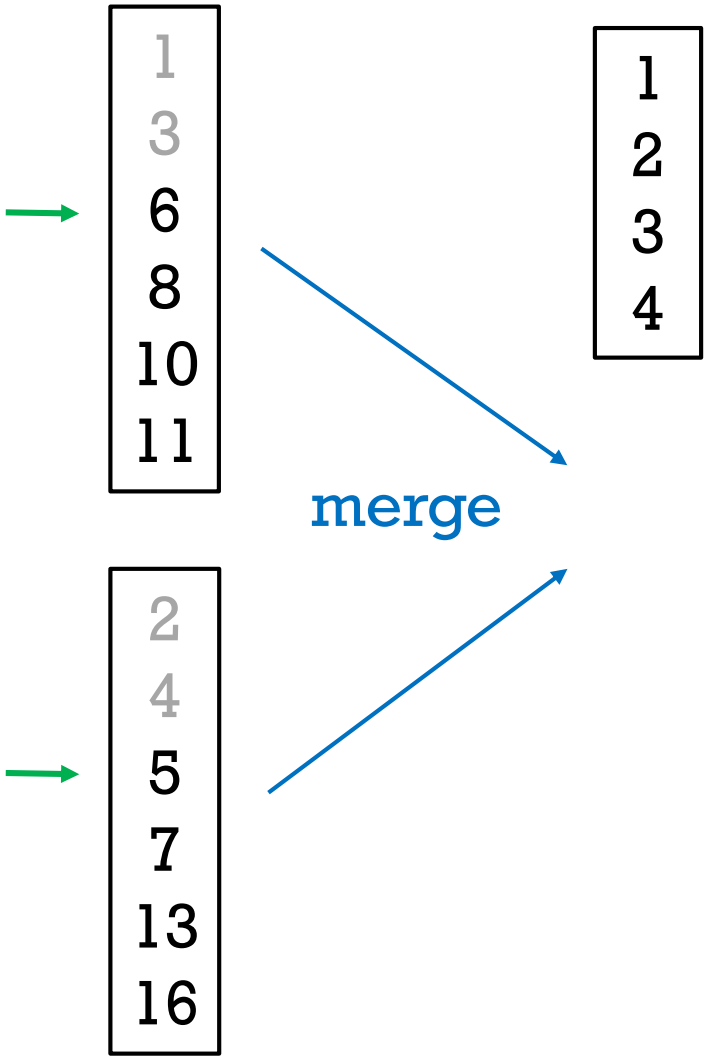

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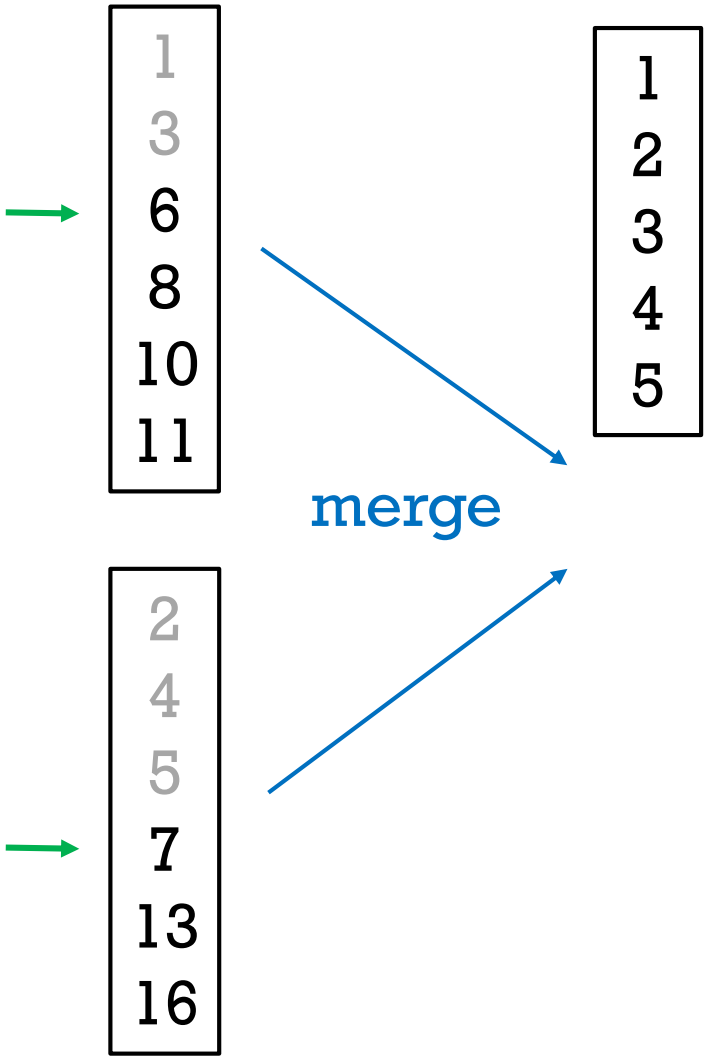

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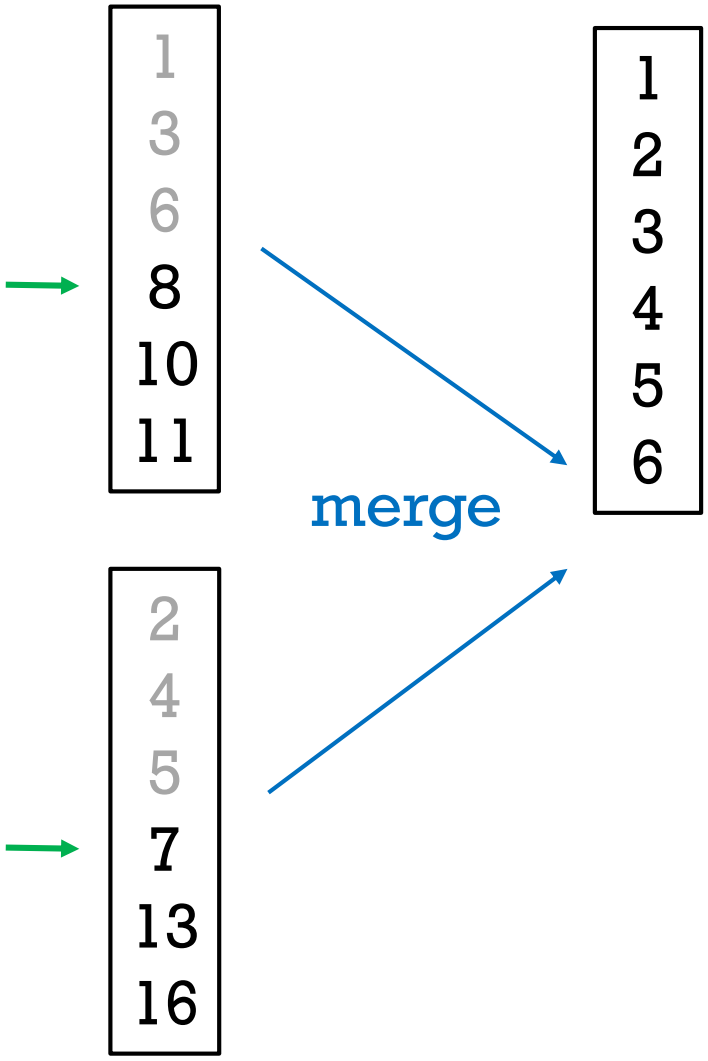

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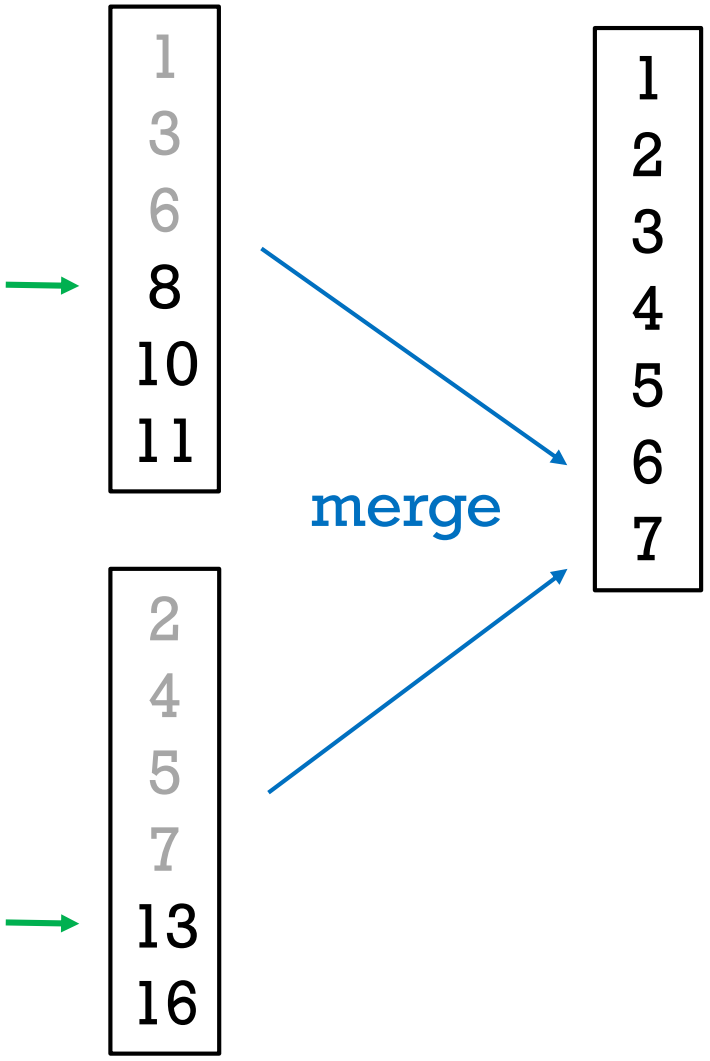
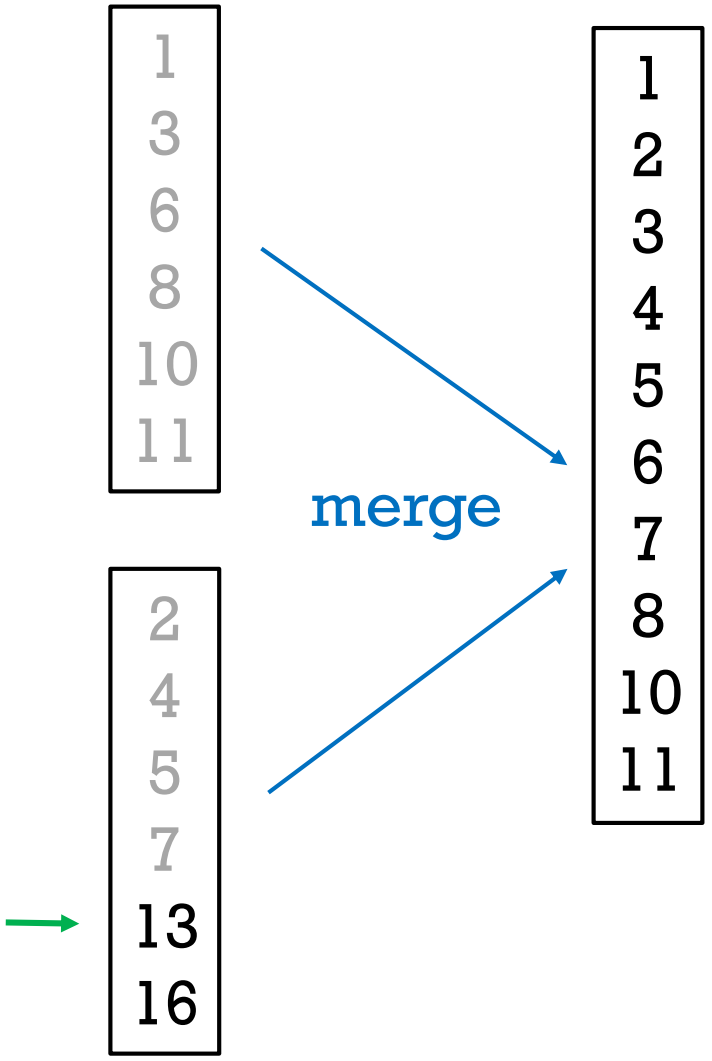


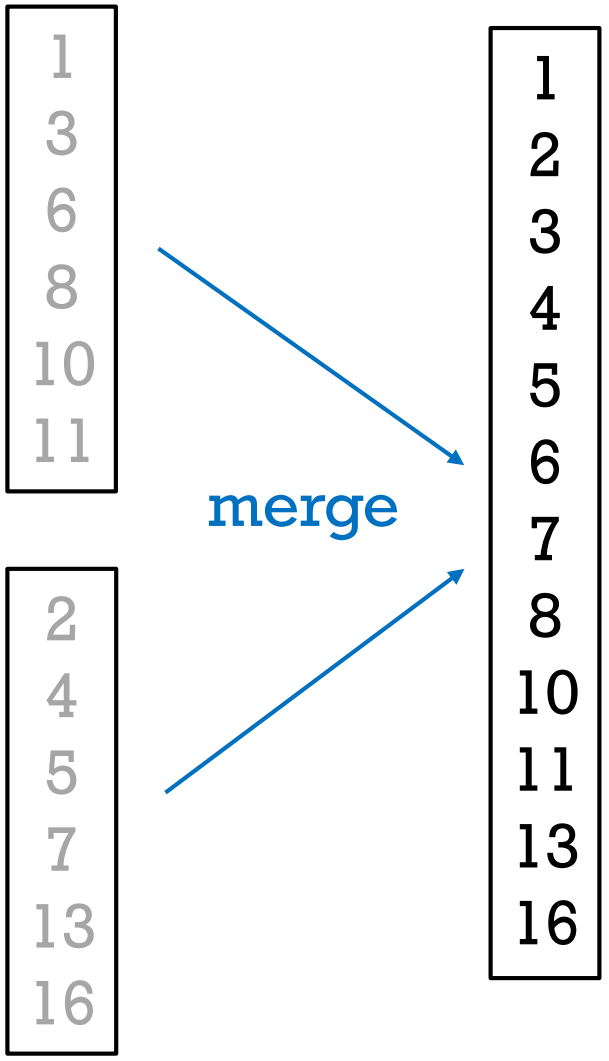
MERGING PRESERVING ORDER

And so on until one list is empty!



MERGING PRESERVING ORDER

Then copy the remaining elements!



IMPLEMENTATION OF MERGE

```
merge(list1, list2){
        list = ...initialize with empty list...
        while (!list1.isEmpty() && !list2.isEmpty()){
                if (list1.get(0) < list2.get(0))
                        list.addlast(list1.removeFirst())
                else
                        list.addlast(list2.removeFirst())
        }
        while (!list1.isEmpty())
                list.addlast(list1.removeFirst())
        while (!list2.isEmpty())
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}
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        }
        while (!list1.isEmpty())
                list.addlast(list1.removeFirst())
        while (!list2.isEmpty())
                list.addlast( list2.removeFirst())
}
```

Pick elements to add until one of the two lists is empty

Then add the remaining elements

1) Partitions into list and call mergesort until base case is reached.

EXAMPLE OF EXECUTION

8
10
3
11
6
1
7
16
2
5
4
13

```
mergesort(list){
        if (list.size() == 1)
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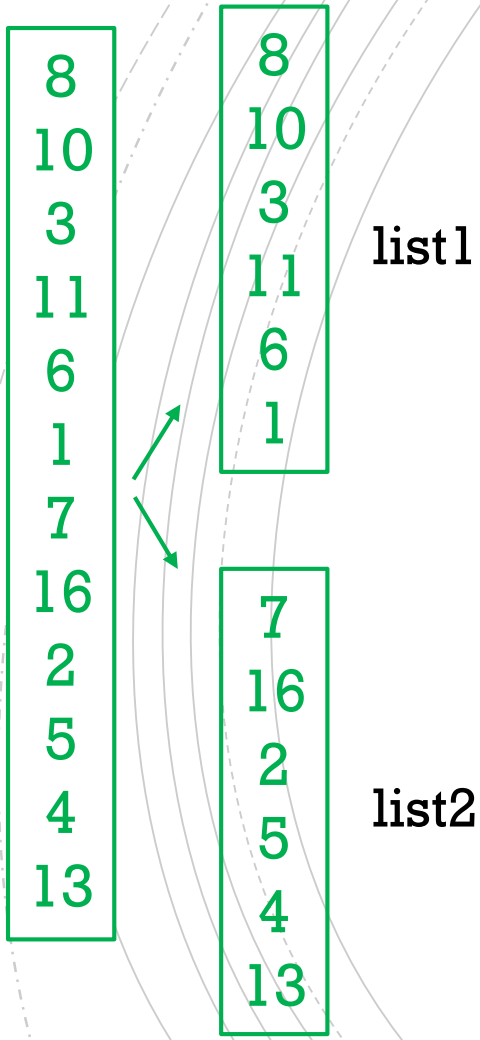

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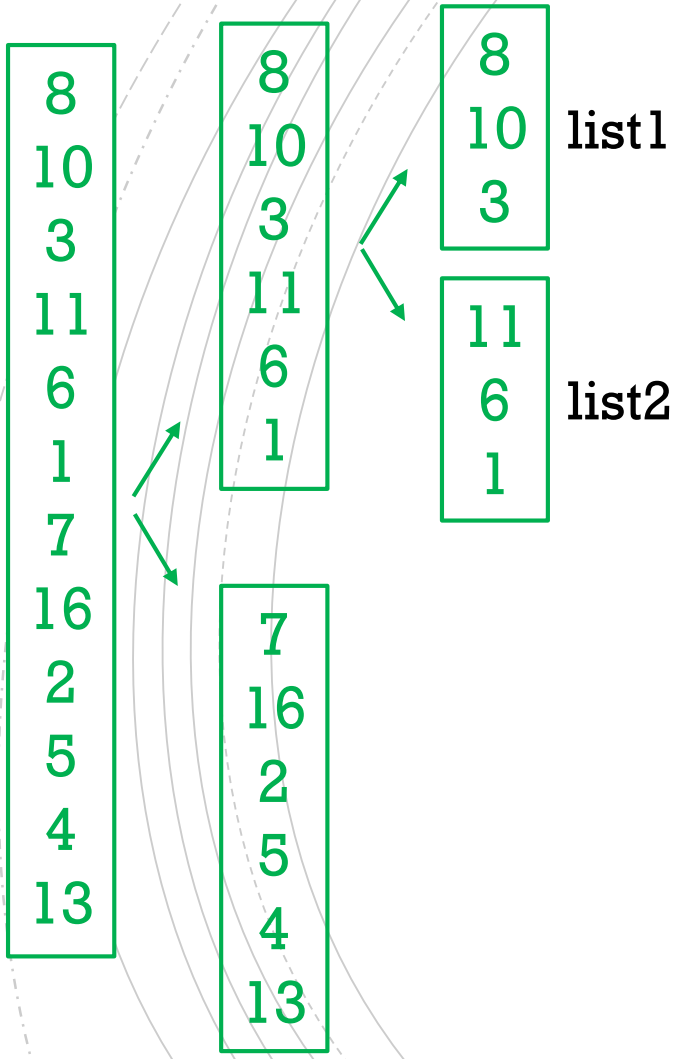

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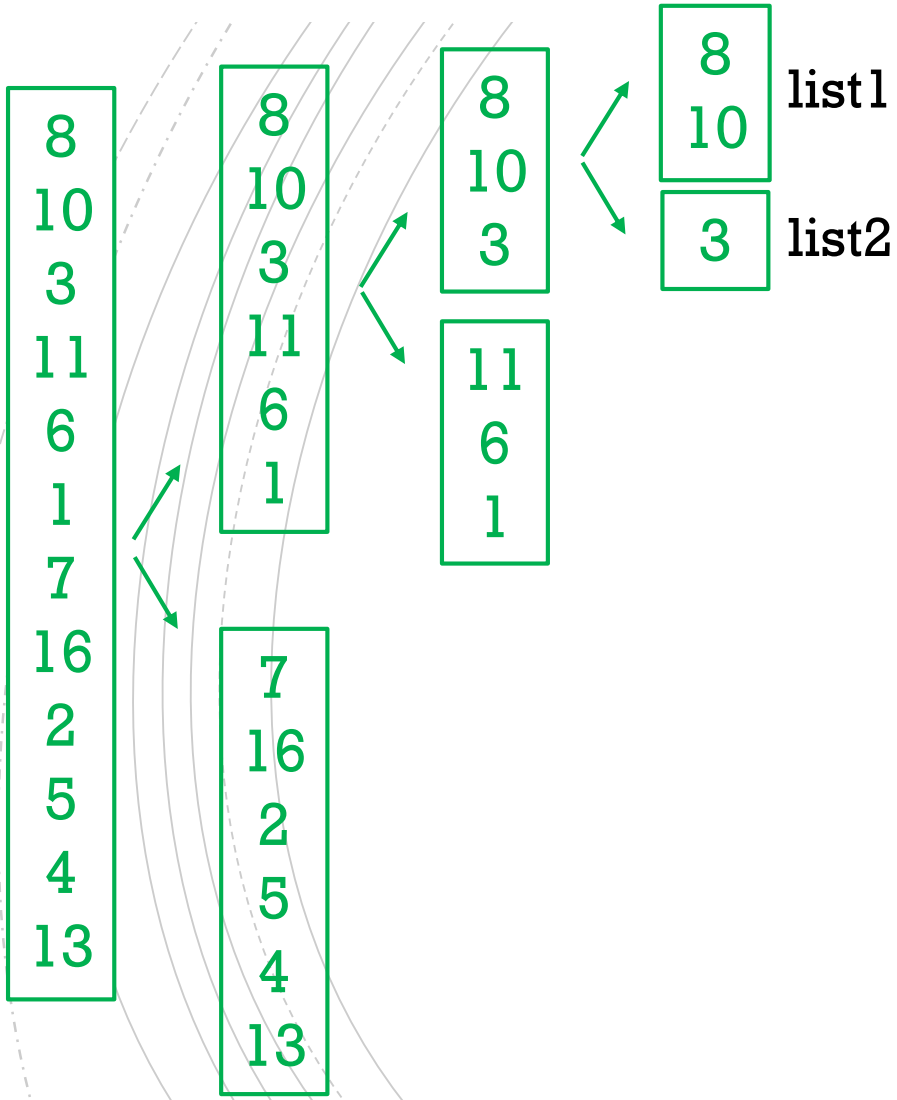

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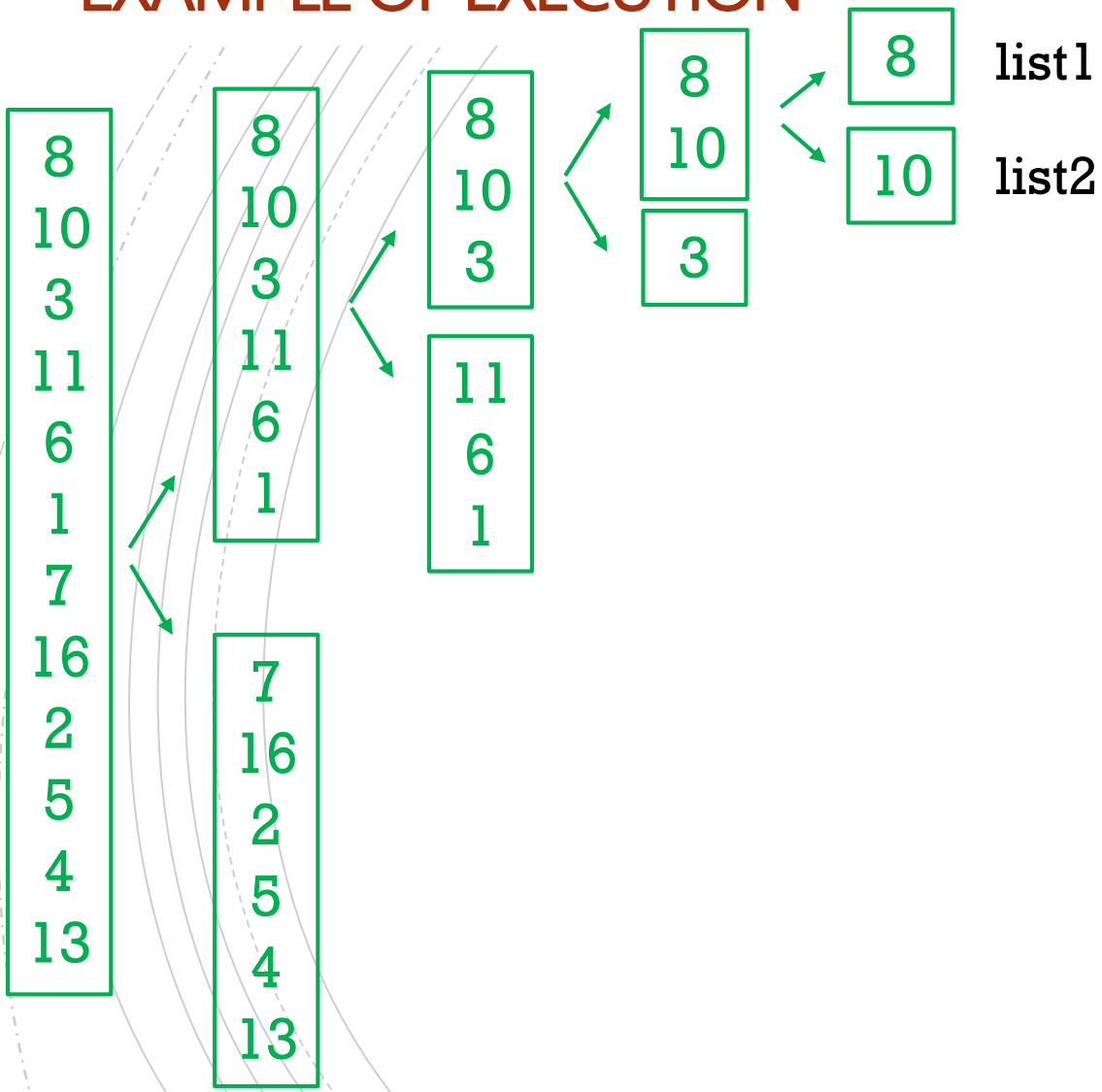
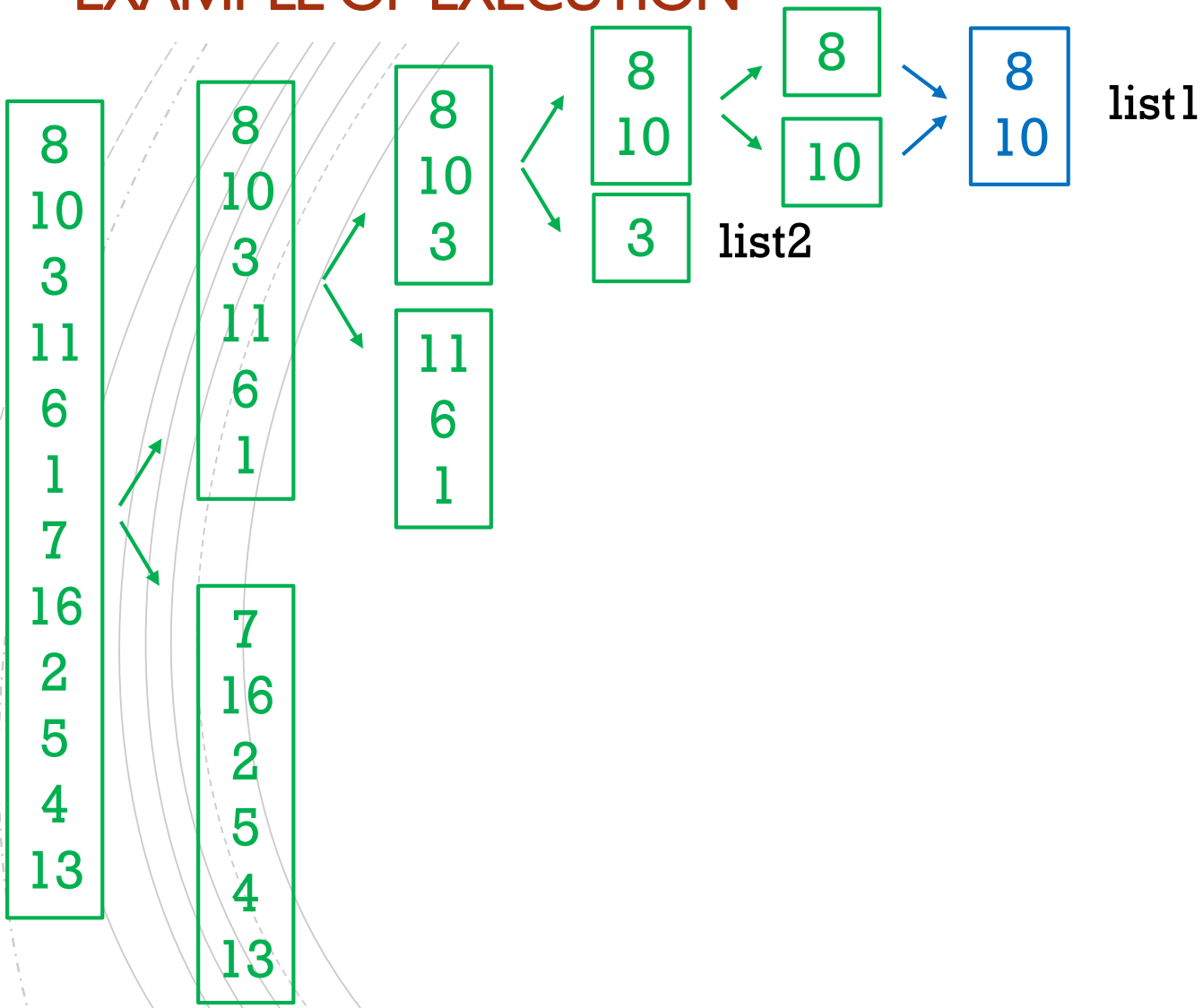


Then start to merge!

EXAMPLE OF EXECUTION



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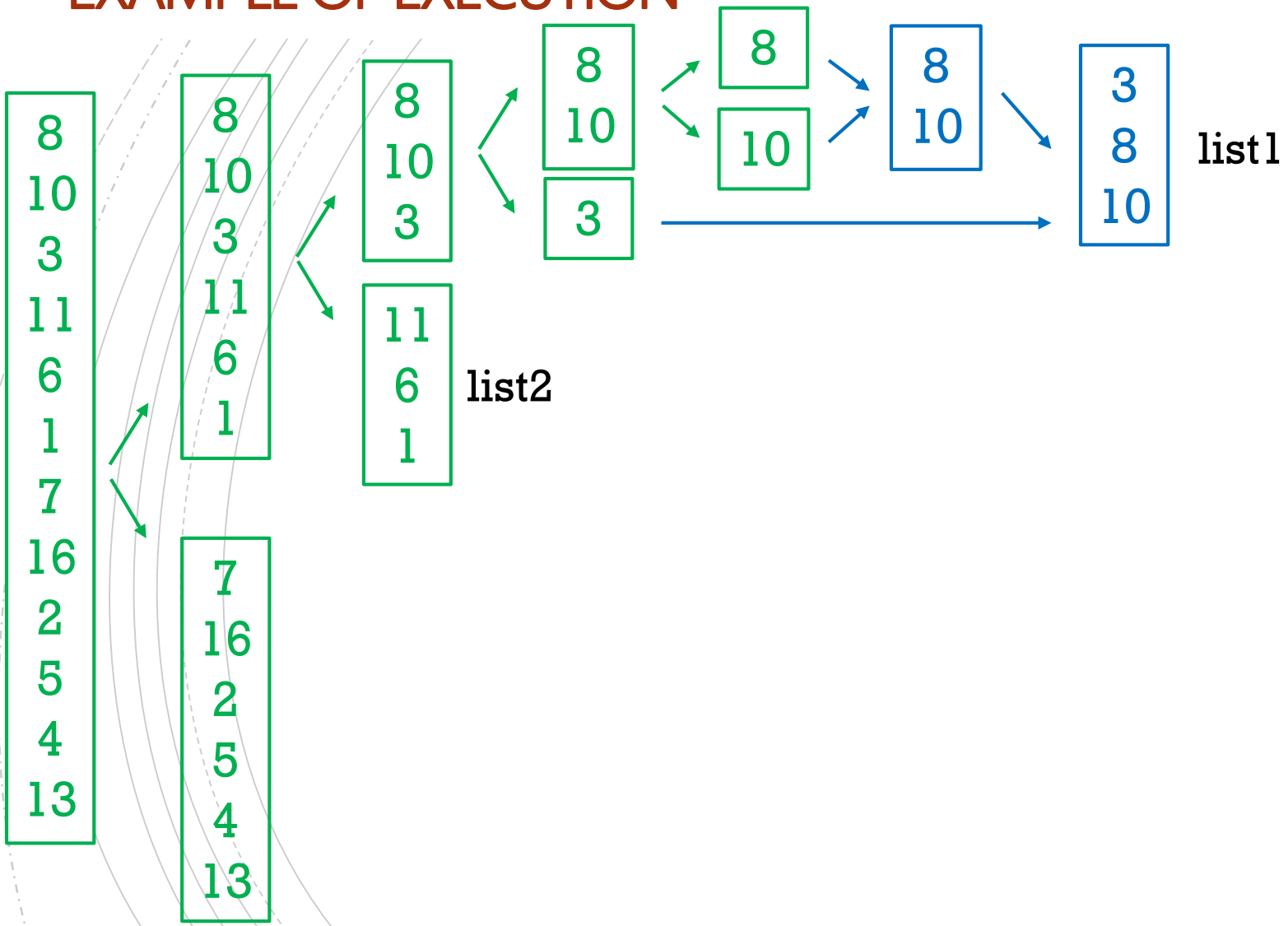

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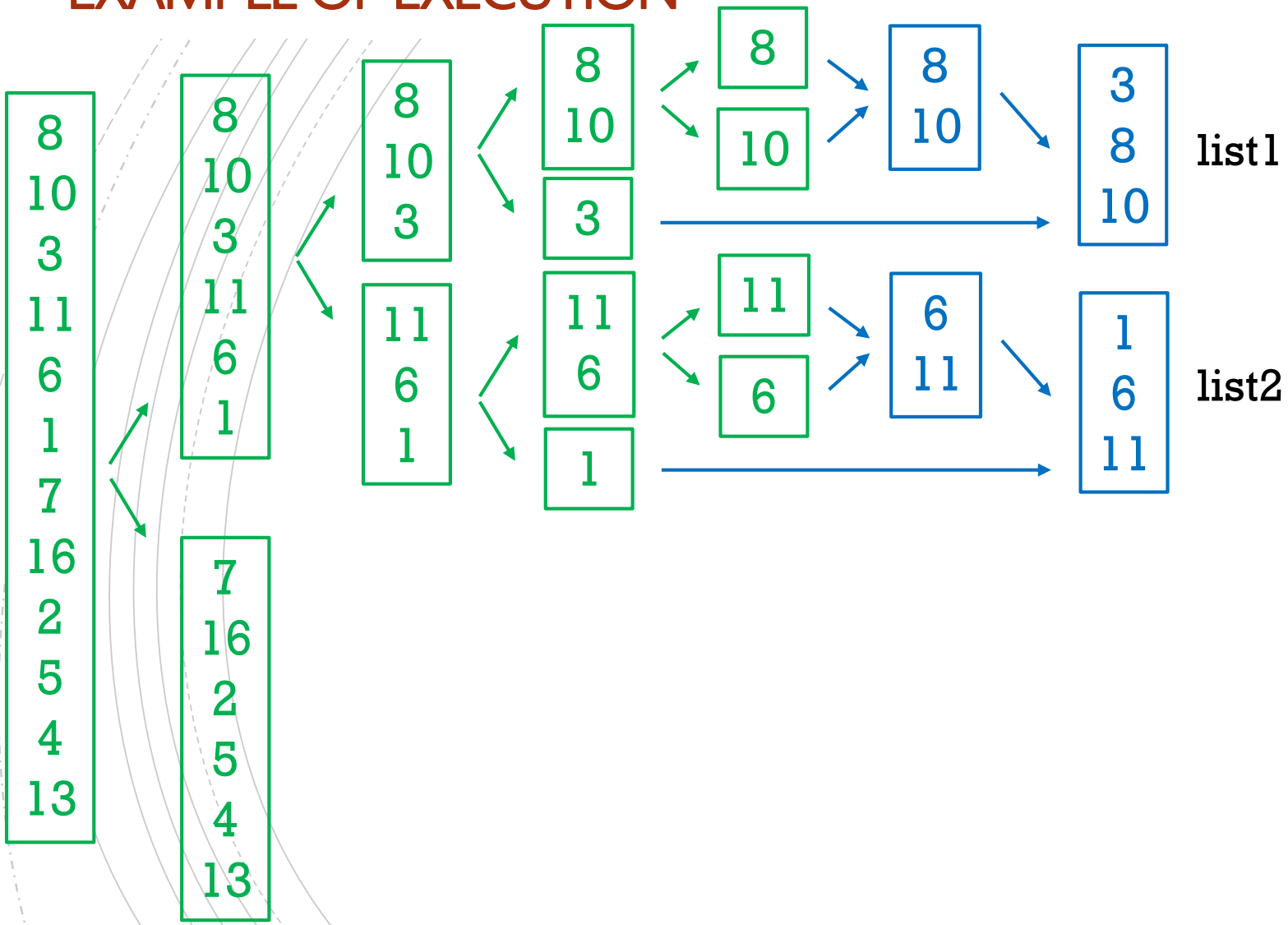

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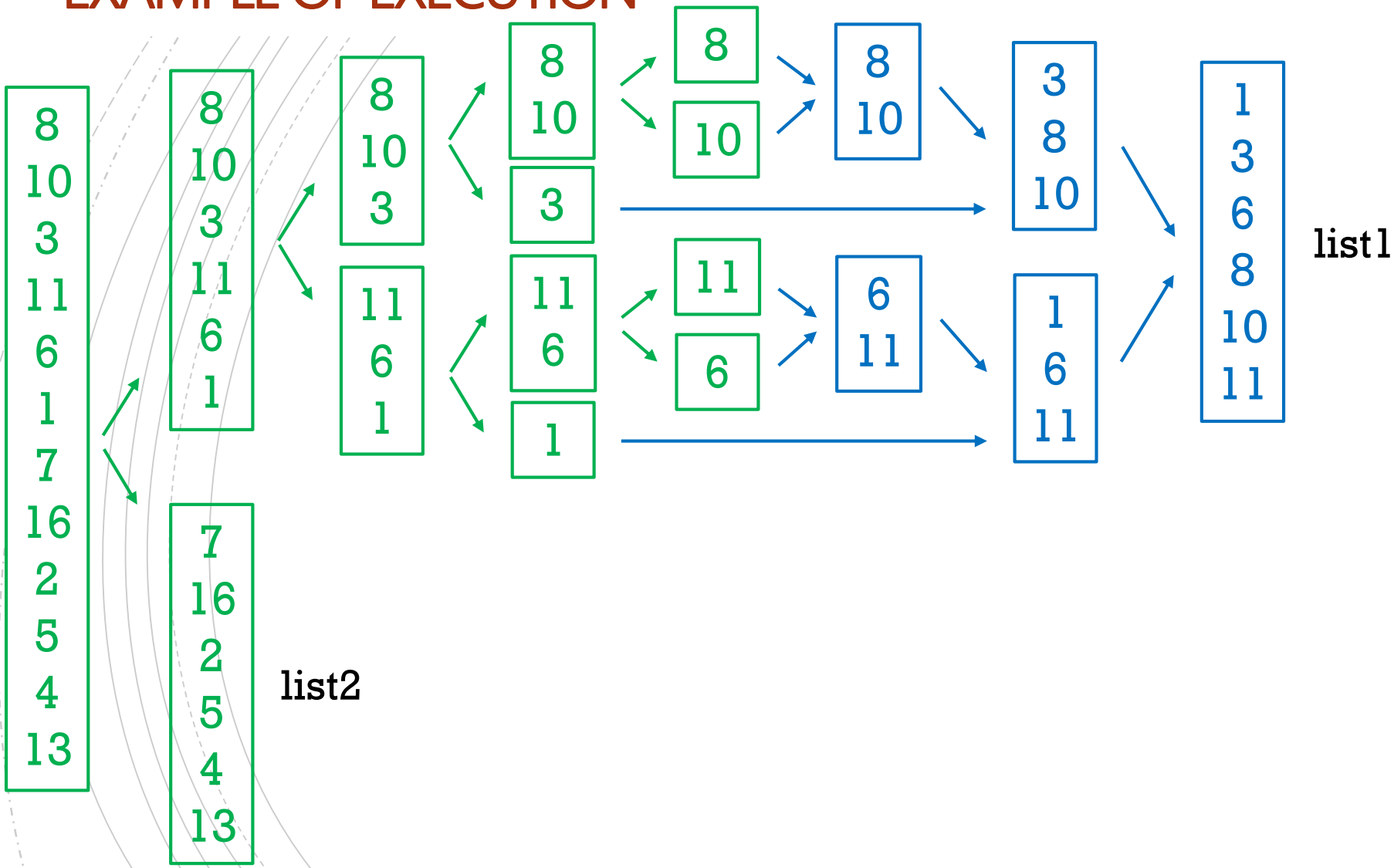

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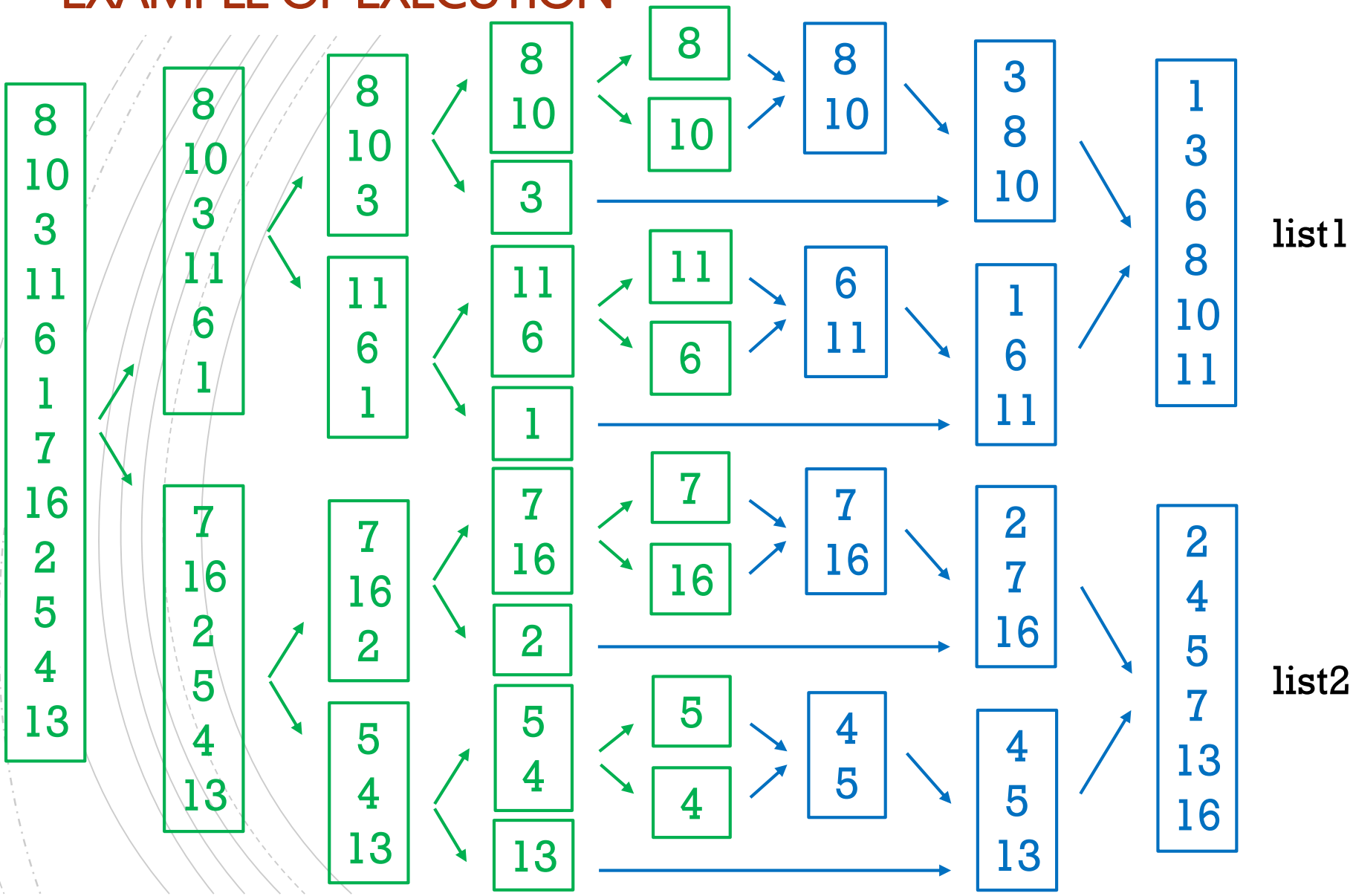

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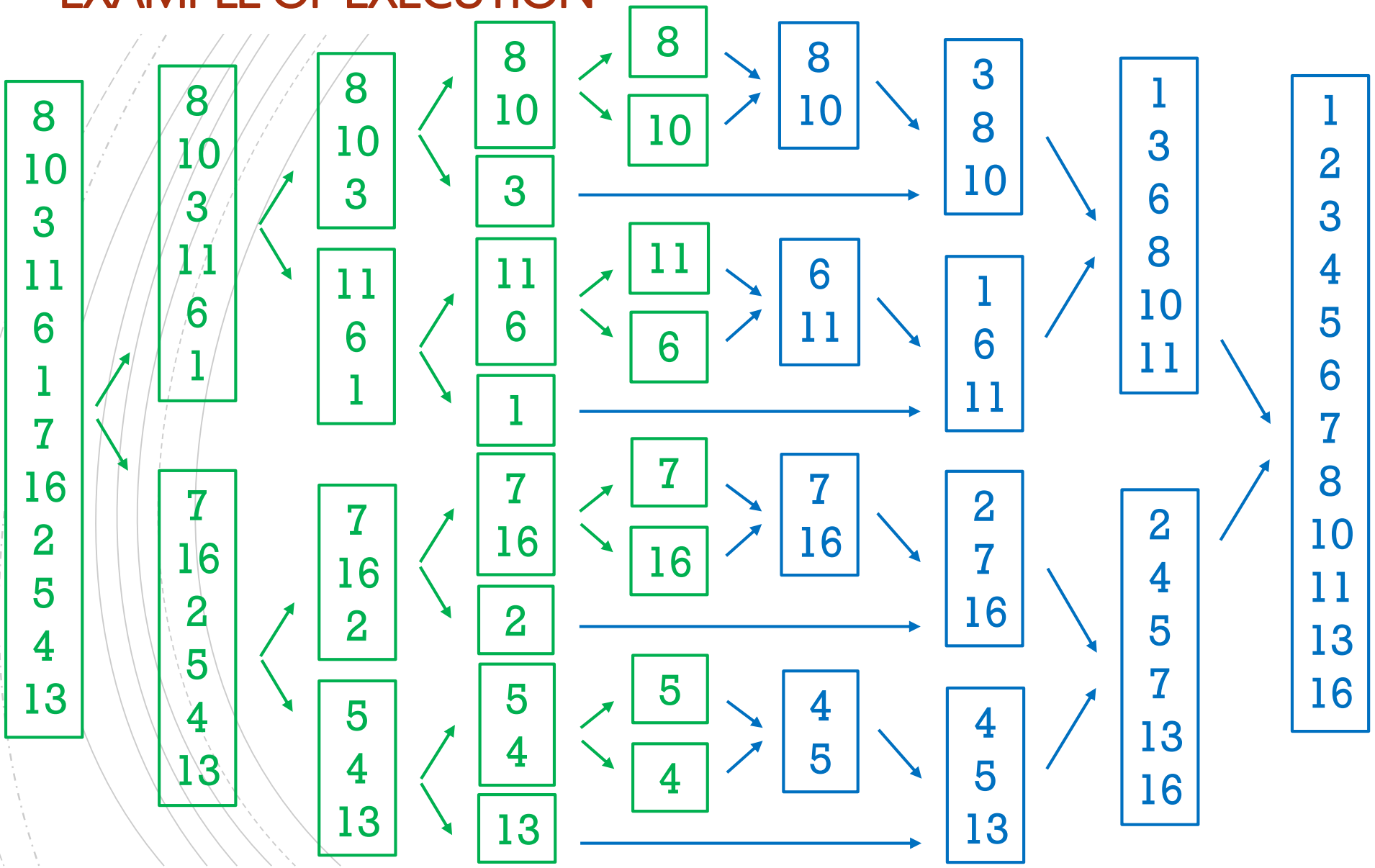
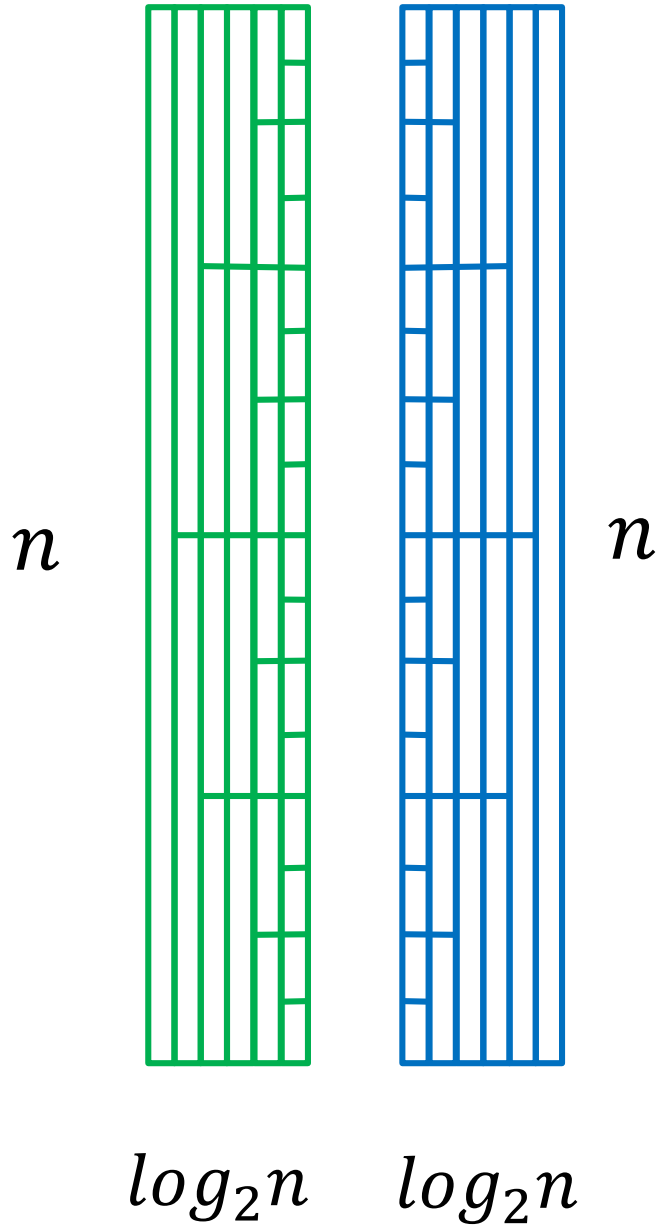


Q: How many operations are required to mergesort a list of size n ?

A: $O(\ n\ \ log_2 n\ )$

This will become more clear at the end of the semester when we discuss recurrences.



COMPLEXITY

$n \log_2 n$ **is much closer to** n **than to** n^2

$\log_2 n$	n	$\boldsymbol{n \log_2 n}$	n^2
10	$2^{10} \approx 10^3$	$\mathbf{10^4}$	10^6
20	$2^{20} \approx 10^6$	$\mathbf{\sim 10^7}$	10^{12}
30	$2^{30} \approx 10^9$	$\mathbf{\sim 10^{10}}$	10^{18}

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milliseconds seconds minutes/hours centuries

QUICK SORT

QUICK SORT

- Quick Sort is a divide and conquer algorithm.
- GOAL: Sort a list.
- IDEA:
  - Pick an element of the array (the pivot).
  - Partition the list moving the pivot to its correct position making sure that all the lower elements are on its left and all the larger elements are on its right.
  - Sort the left part AND the right part of the list recursively.
  - Keep doing it until there's nothing left to sort.

IDEA

IDEA:

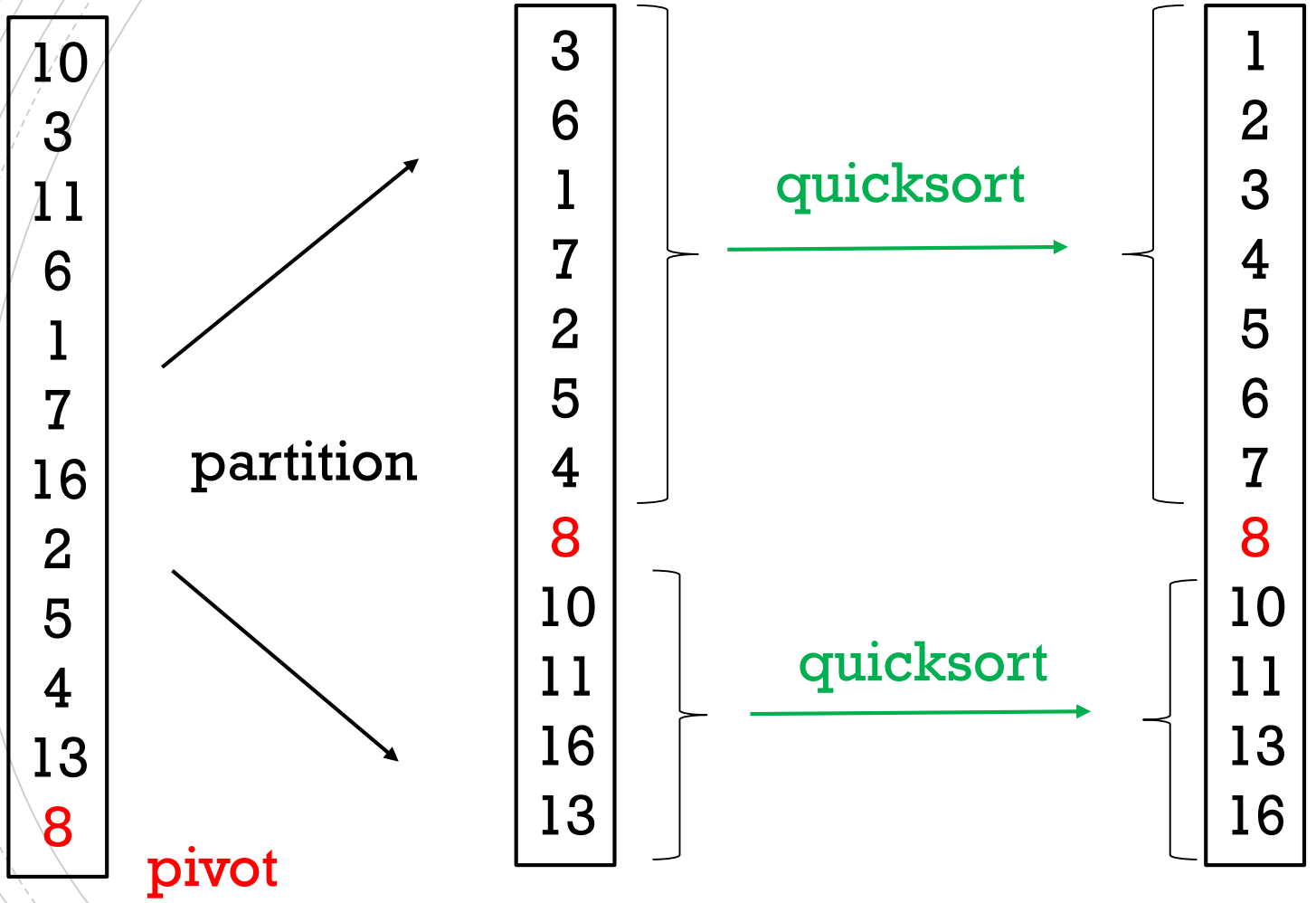
- Pick an element of the array (the pivot).
- Move the pivot to its correct position making sure that all the lower elements are on its left and all the larger elements are on its right.
- Sort the left part AND the right part
- Keep doing it until there's nothing left to sort.

This is the crucial process of the algorithm!

Recursive Step

Base case

IDEA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pivot
partition
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16
13
quicksort
quicksort
1
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16

THE PIVOT

Different versions of Quick Sort pick the pivot in different ways:

- Always pick the first element as the pivot
- Always pick the last element as the pivot
- Pick a random element
- Pick the median as pivot

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QUICK SORT EXAMPLE

QUICK SORT EXAMPLE

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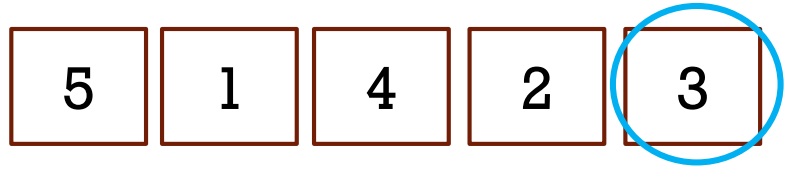
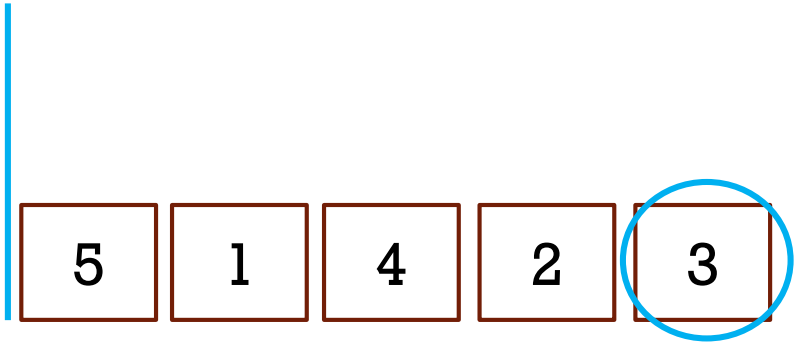


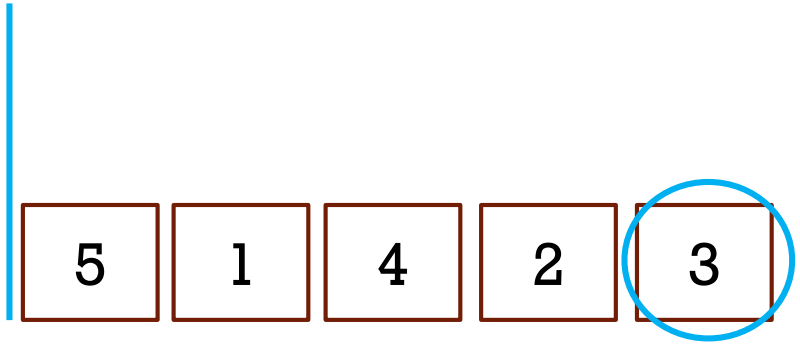
QUICK SORT EXAMPLE

1. Pick the pivot.
2. Set the wall on the left



QUICK SORT EXAMPLE

1. Pick the pivot.
2. Set the wall on the left
3. Go through all the elements of the list that are not the pivot.
   - If the element is smaller than the pivot, move the wall right by 1, and place the element just behind the wall.
   - Otherwise, do nothing.



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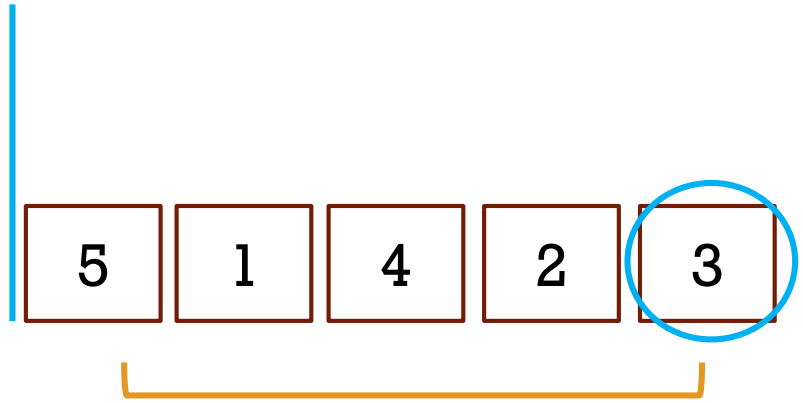

compare:
5<3 false → do nothing!

QUICK SORT EXAMPLE

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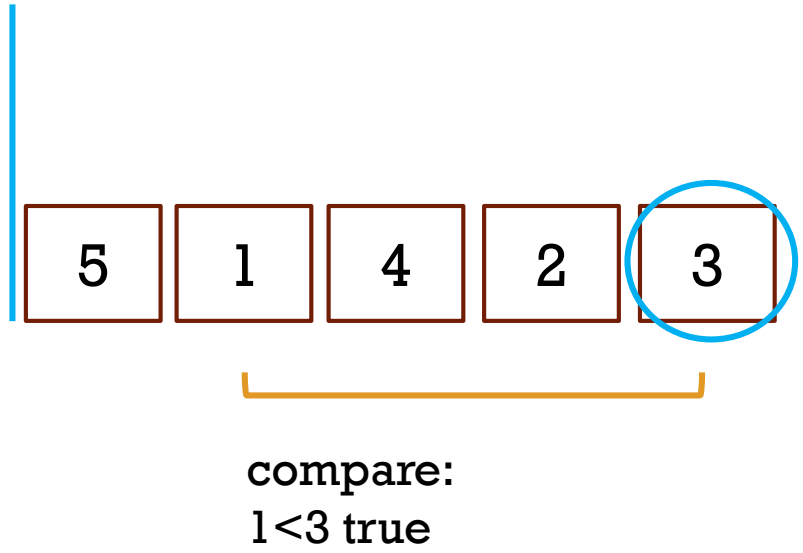

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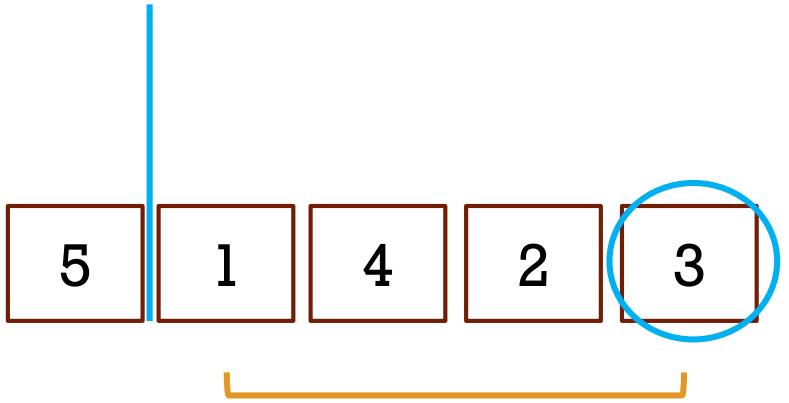

compare:
1<3 true → move the wall

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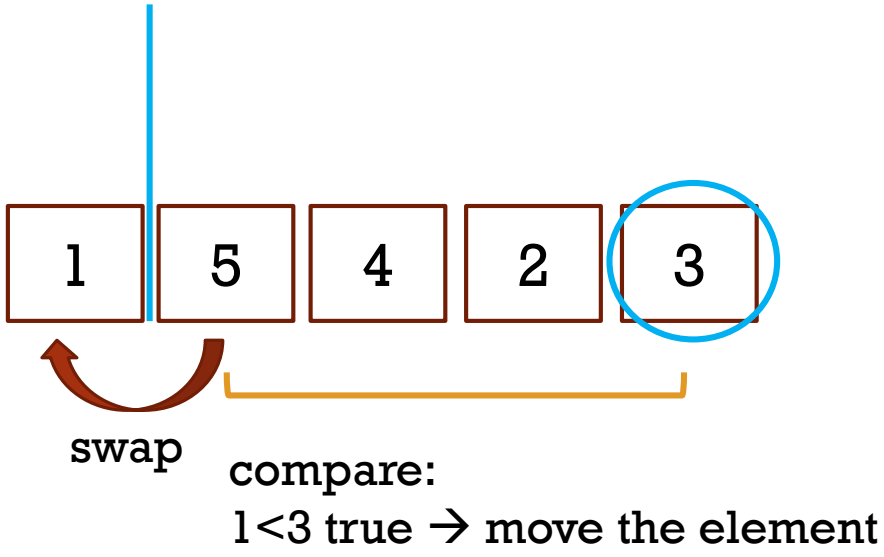

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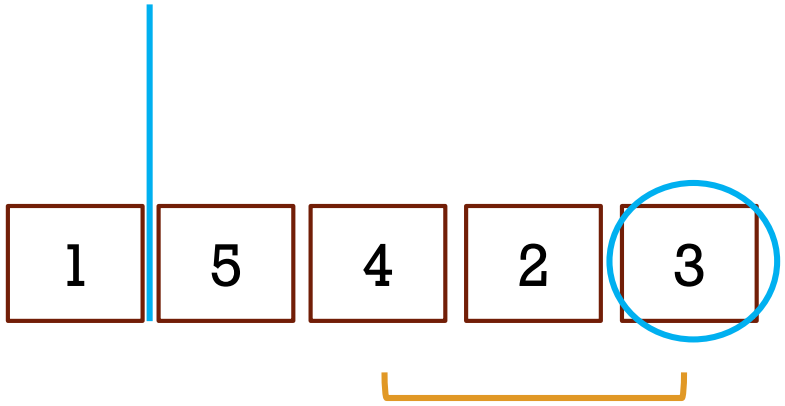

compare:
4<3 false → do nothing

QUICK SORT EXAMPLE

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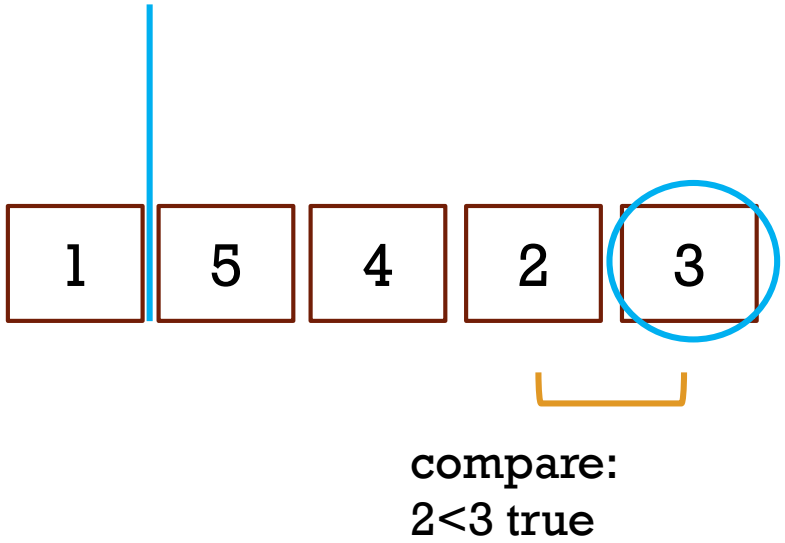

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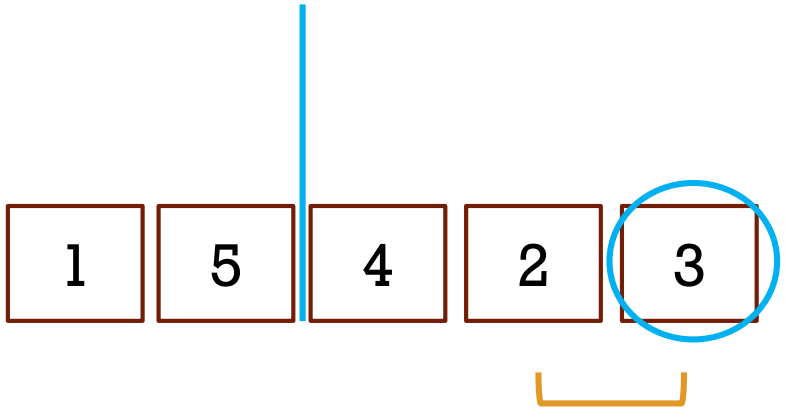

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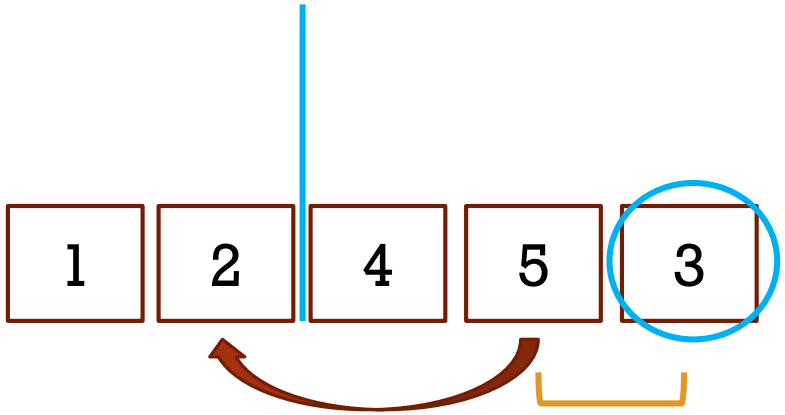

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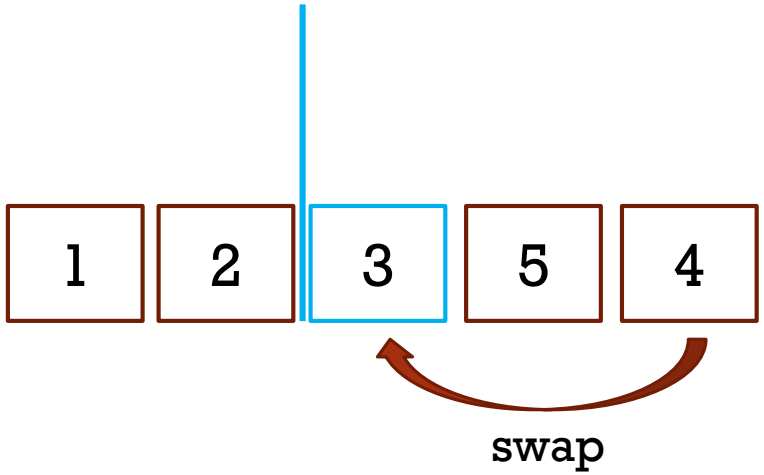
3. Go through all the elements of the list that are not the pivot.
   - If the element is smaller than the pivot, move the wall right by 1, and place the element just behind the wall.
   - Otherwise, do nothing.
4. Move the pivot next to the wall.



QUICK SORT EXAMPLE

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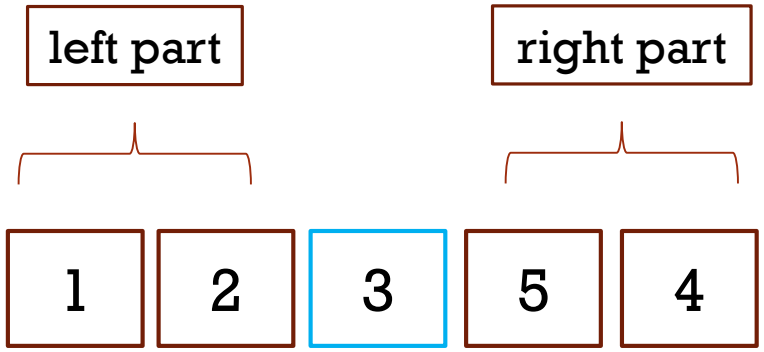
5. Use Quick sort on left part and then on the right part



QUICK SORT EXAMPLE

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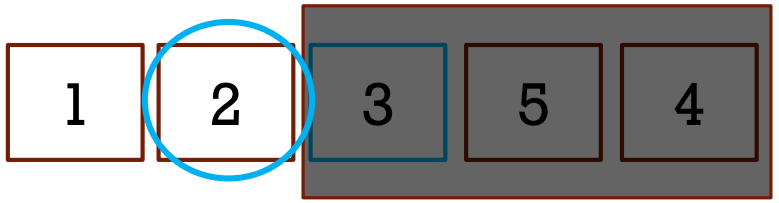

QUICK SORT EXAMPLE

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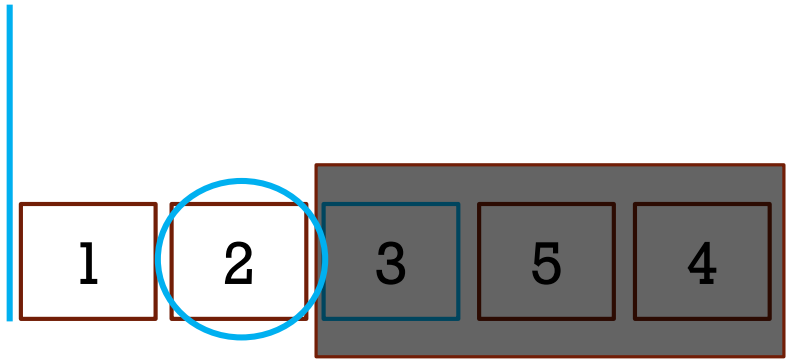

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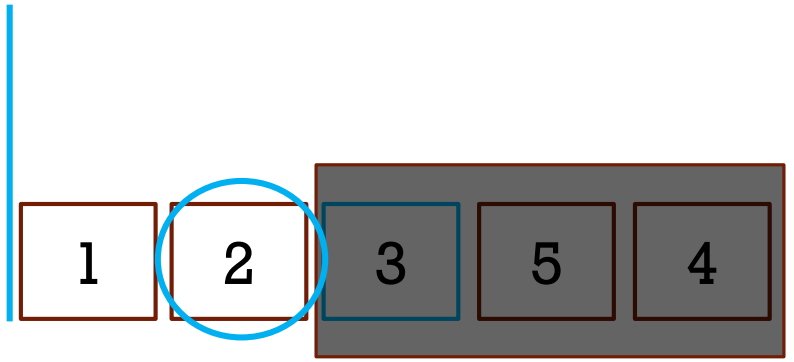

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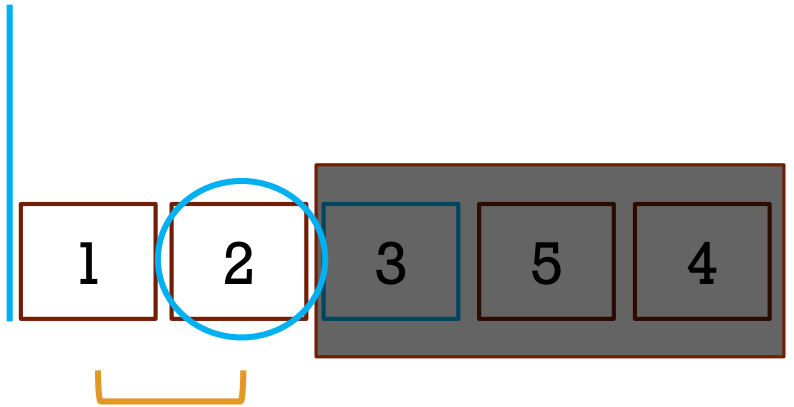

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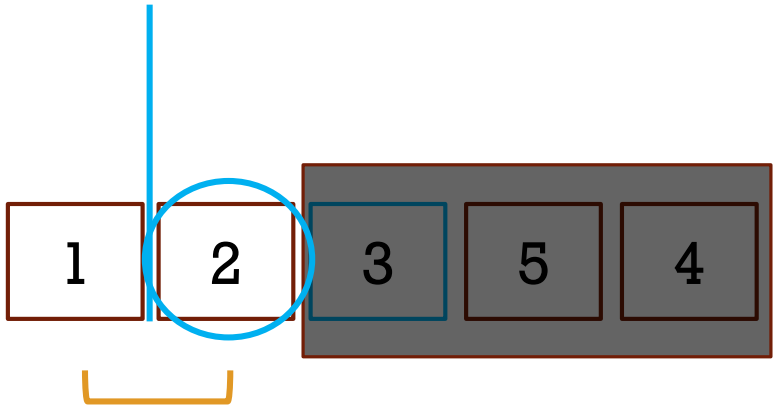

compare:
1<2 true → move wall

QUICK SORT EXAMPLE

1. Pick the pivot.
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   - If the element is smaller than the pivot, move the wall right by 1, and place the element just behind the wall.
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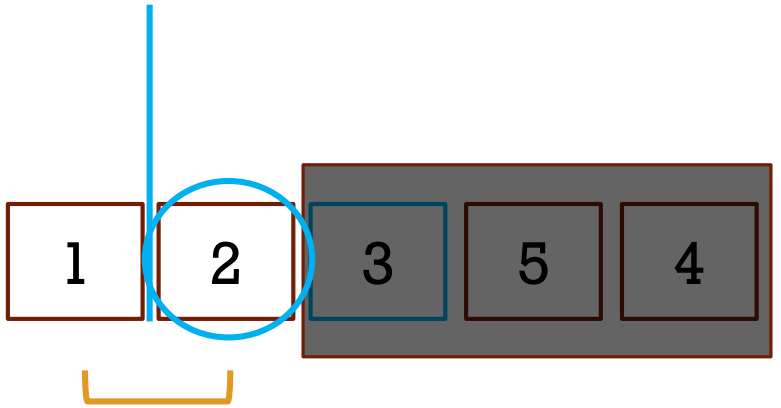

compare:
1<2 true → element already in position.

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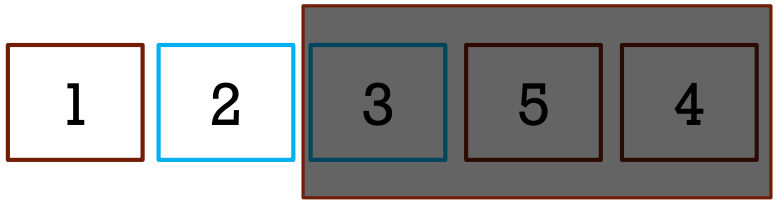
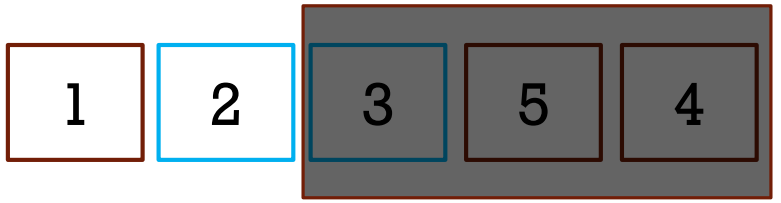
   - Otherwise, do nothing.
4. Move the pivot next to the wall.
5. Use Quick Sort on left part and then on the right part.



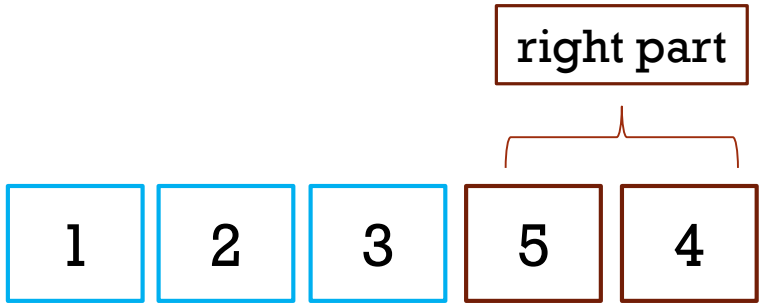
QUICK SORT EXAMPLE

- In this case the left part and the right part are base cases.
- The left part has 1 element → already sorted!
- The right part is empty → sorted!



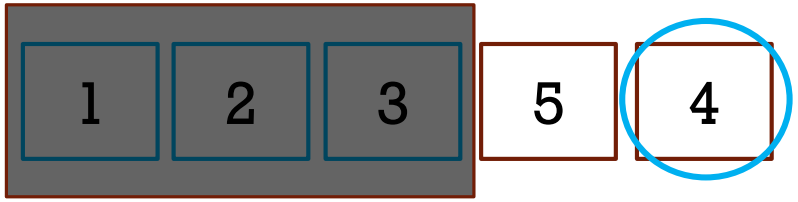
QUICK SORT EXAMPLE

- It is left to sort the part of the list to the right of the first pivot.



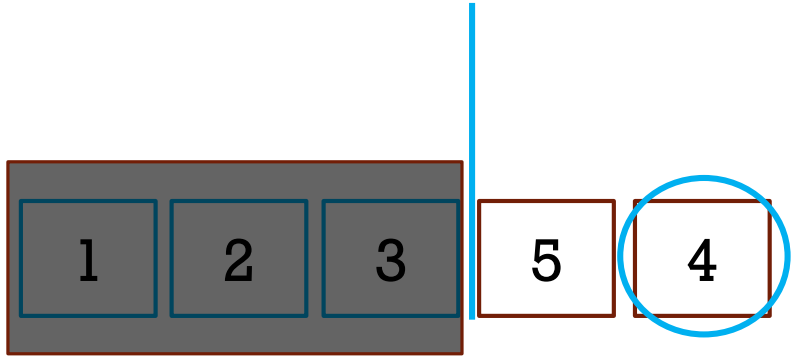
QUICK SORT EXAMPLE

1. Pick the pivot.



QUICK SORT EXAMPLE

1. Pick the pivot.
2. Set the wall on the left



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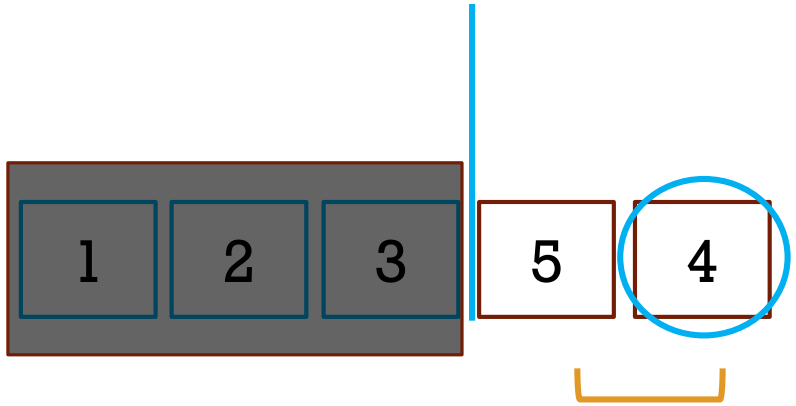

compare:
5<4 false → do nothing!

QUICK SORT EXAMPLE

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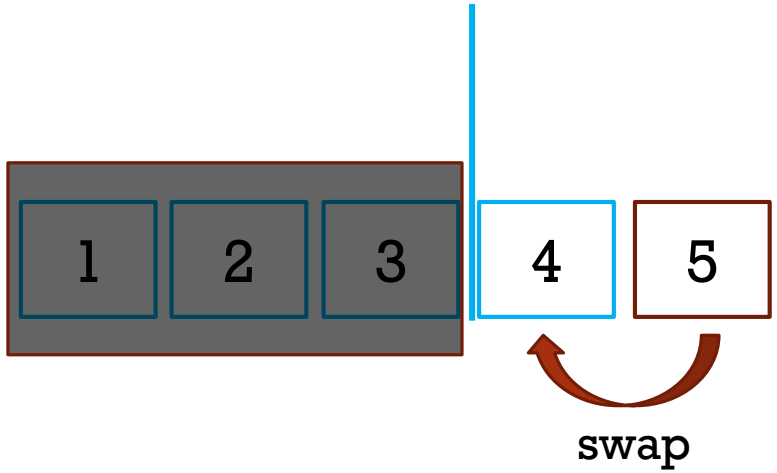

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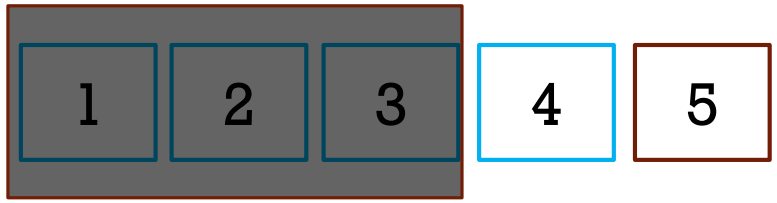
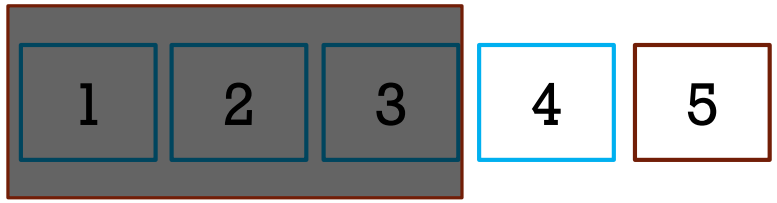
5. Use Quick Sort on left part and then on the right part.



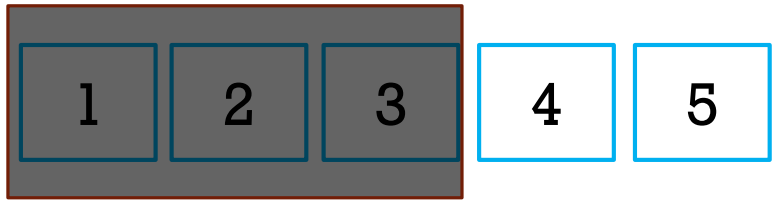
QUICK SORT EXAMPLE

- Once again, both the left part and the right part are base cases.



QUICK SORT EXAMPLE

- Once again, both the left part and the right part are base cases.
- The array is sorted



QUICK SORT EXAMPLE

- Once again, both the left part and the right part are base cases.
- The array is sorted
- The original array is sorted!

QUICK SORT – IMPLEMENTATION

What do we need to implement this algorithm?

- A method that swaps two elements
- A way to refer to parts of the list
- A method that places the pivot in its correct position and moves the elements around so that all the lower elements are on the left, and all the larger elements are on the right. Call it `placeAndDivide`
- A method that implements the Quick Sort, that is:
  - Pick a pivot
  - placeAndDivide
  - quickSort left part
  - quickSort right part

PARTS OF THE LIST

What can we use to denote a part of the list?

- We can use the same idea used from binary search → keep track of the left and right index denoting where the part begins and ends.
- Consider for example the list $\{5,3,6,1,2\}$. Then:
  - The indices 0 and 4 denote the entire list.
  - The indices 0 and 2 denote the part of the list with the 3 left most elements.
  - The indices 1 and 3 denote the part of the list with the 3 middle elements.

QUICK SORT – PSEUDO CODE

```
quickSort(list, leftIndex, rightIndex) {
      // Base case:
      if(leftIndex >= rigthIndex) {
       return; // done!
      } else { // recursive step:
       i ← placeAndDivide(list, leftIndex, rightIndex)
       // i = index where the pivot is placed
       quickSort(list, leftIndex, i-1)
       quickSort(list, i+1, rightIndex)
      }
}
```

PLACEANDDIVIDE – PSEUDO CODE

```
placeAndDivide(list, leftIndex, rightIndex) {
        // pick the right most element
        pivot ← list.get(rigthIndex)
        // place the wall to the left
        wall ← leftIndex -1
        // go through all elements and compare them to the pivot
        for(int i=leftIndex; i< rigthIndex; i++) {
                if(list.get(i)<pivot) {
                        wall++; // move wall
                        swap list.get(i) list.get(wall)// move element behind wall
                }
        }
        swap list.get(rigthIndex) list(wall+1) // move pivot next to wall
        return wall+1;
}
```

MERGESORT VS. QUICKSORT

- Mergesort typically uses an extra list. More space can hurt performance for big lists.
- We will discuss worst case performance of quicksort later in the course.
- See stackoverflow if you want opinions on which is better. The answer is, it depends …