

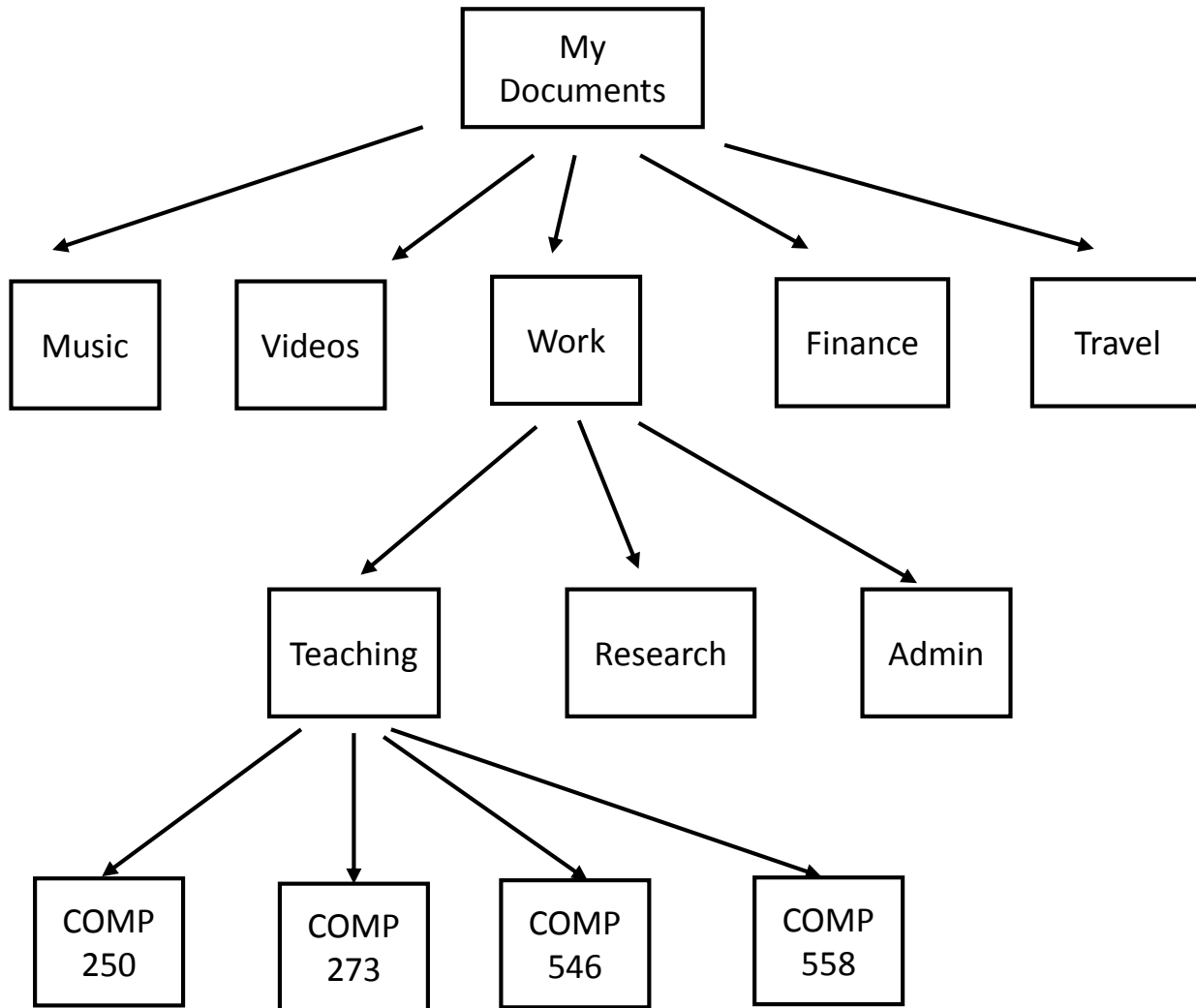
COMP 250

Lecture 23

tree traversal

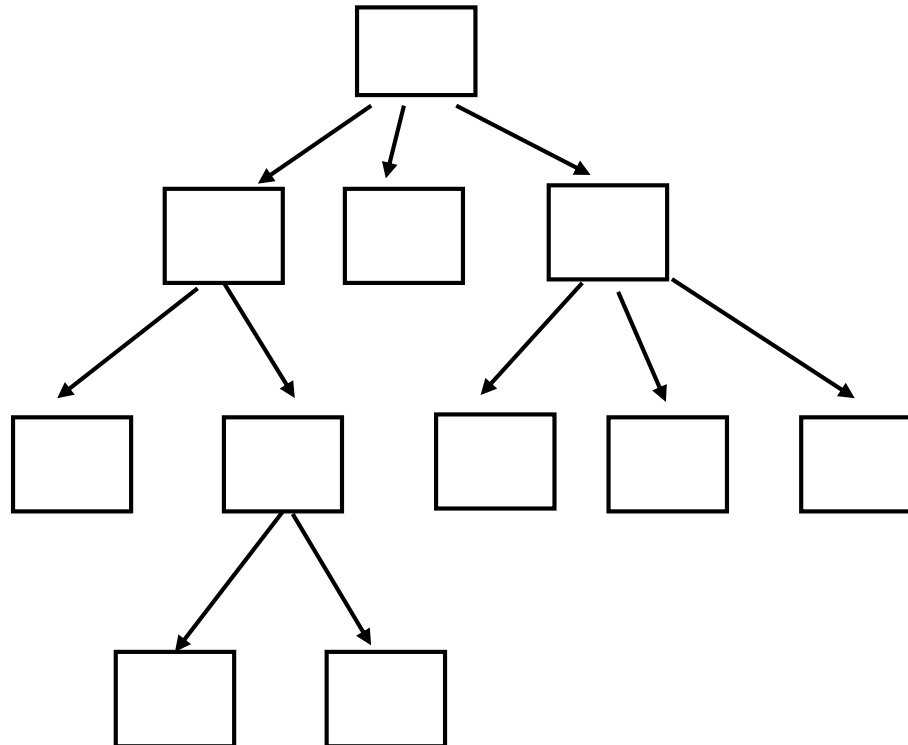
Nov. 2, 2018

Tree: Example

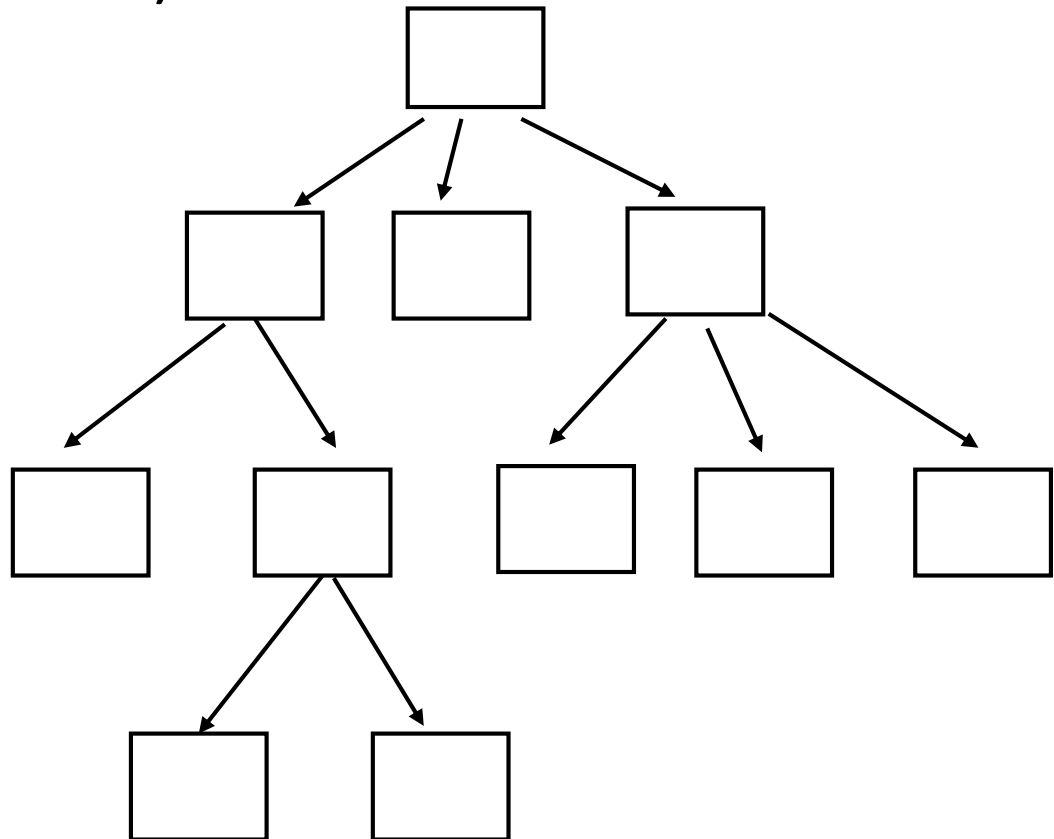


Tree Traversal

How to visit (enumerate, iterate through, traverse...) all the nodes of a tree ?

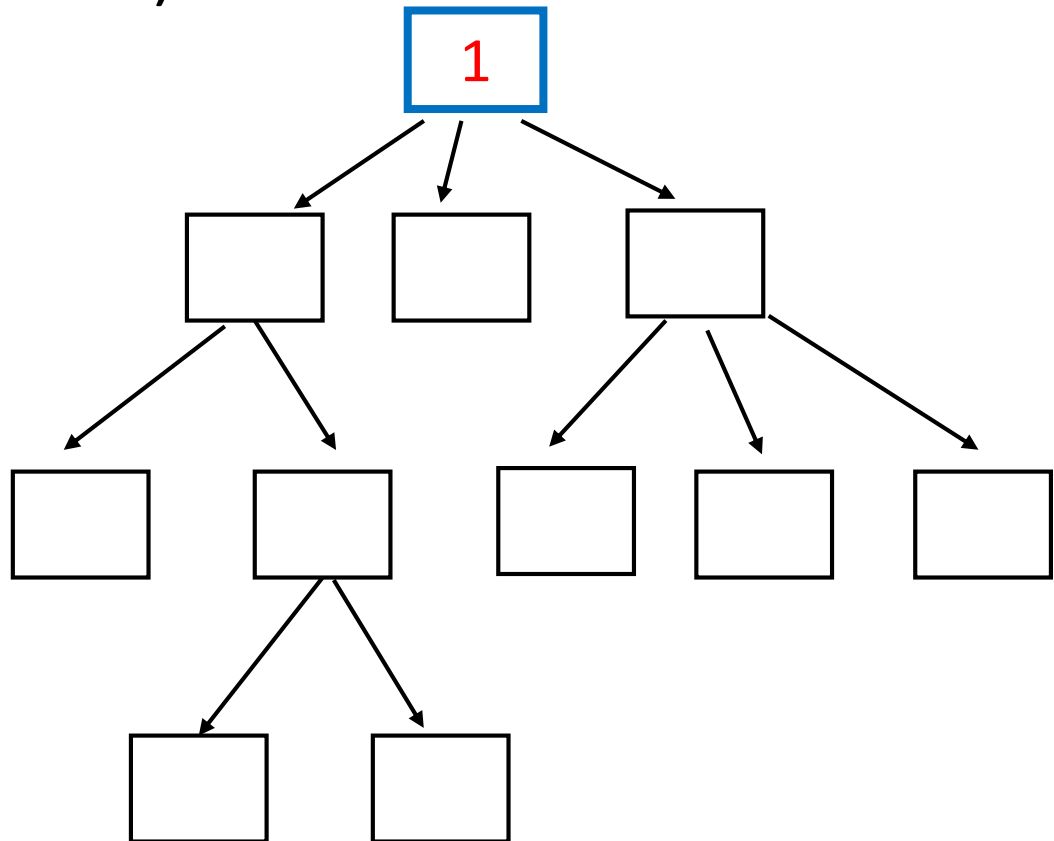


```
depthfirst (root){  
  if (root is not empty){  
    visit root  
    for each child of root  
      depthfirst( child )  
  }  
}
```

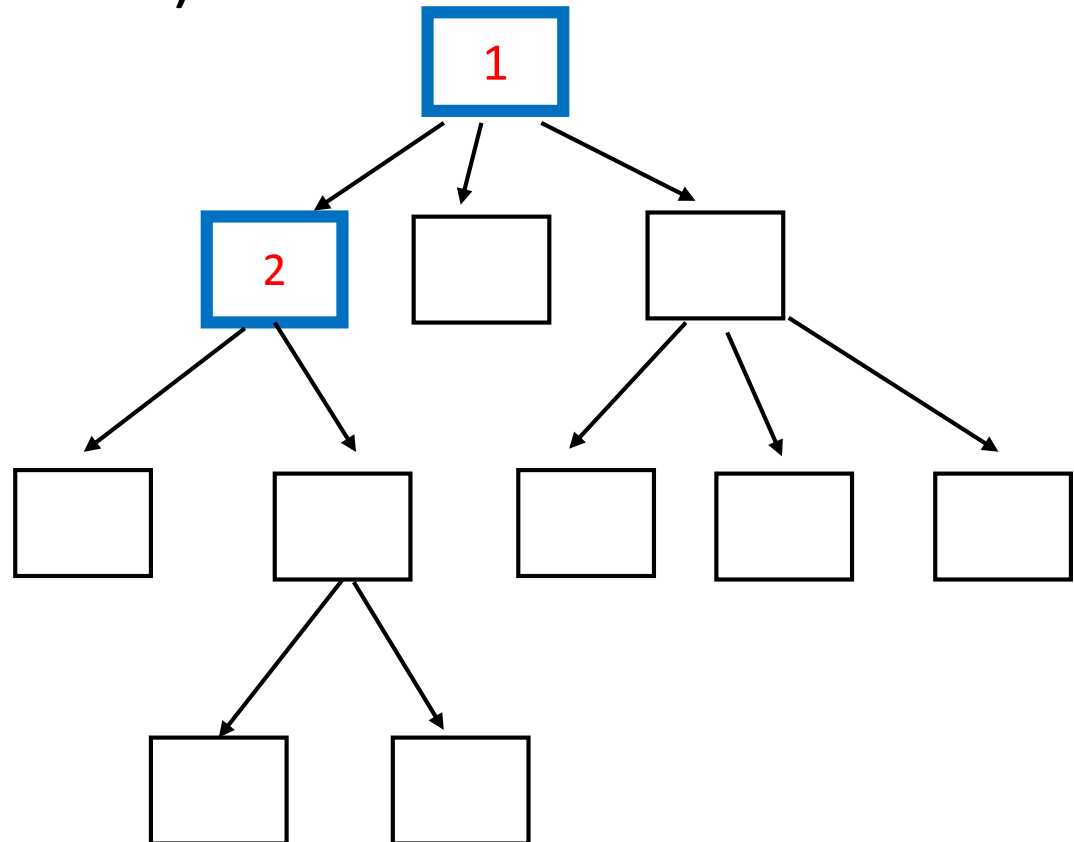


```
depthfirst (root){  
  if (root is not empty){  
    visit root  
    for each child of root  
      depthfirst( child )  
  }  
}
```

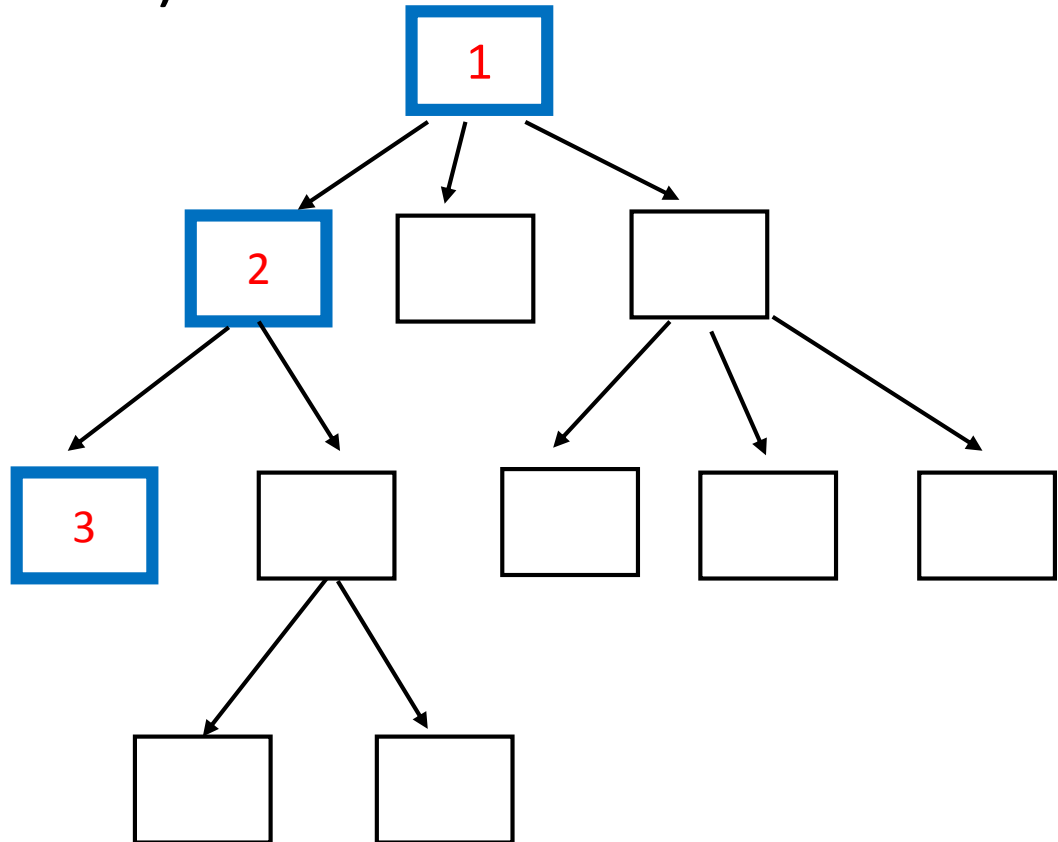
“preorder” traversal:
visit the root before
the children



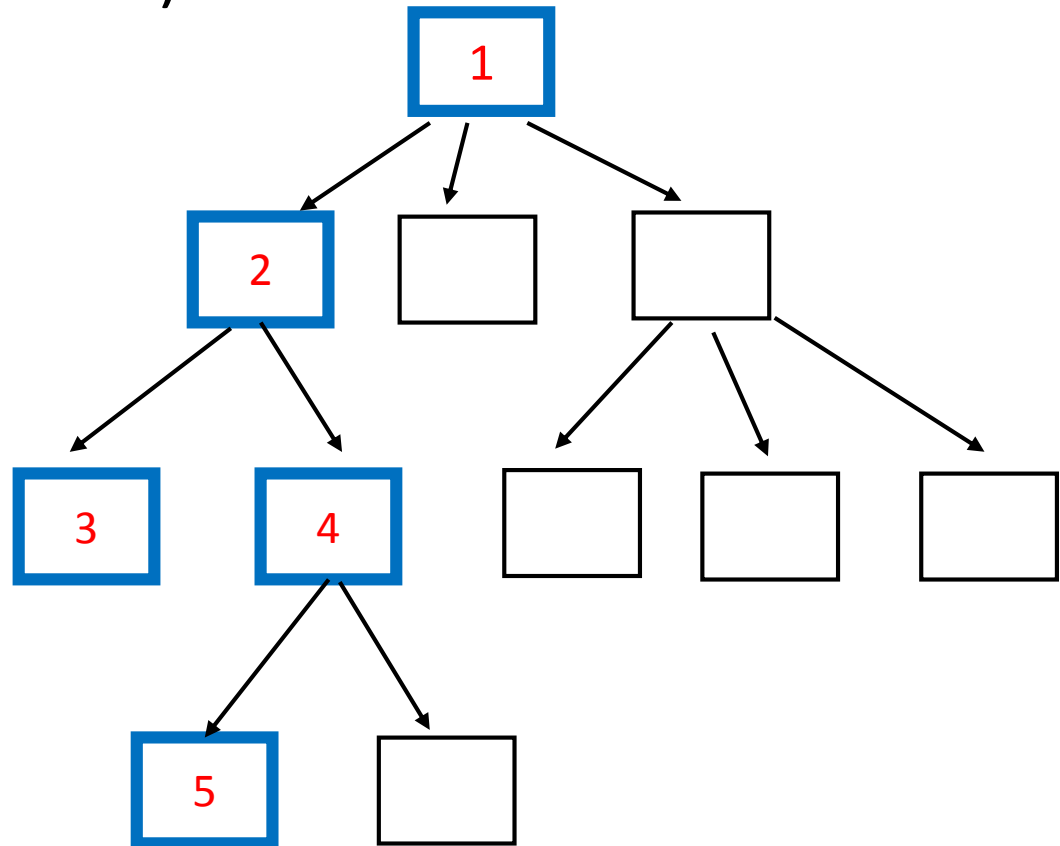
```
depthfirst (root){  
  if (root is not empty){  
    visit root  
    for each child of root  
      depthfirst( child )  
  }  
}
```



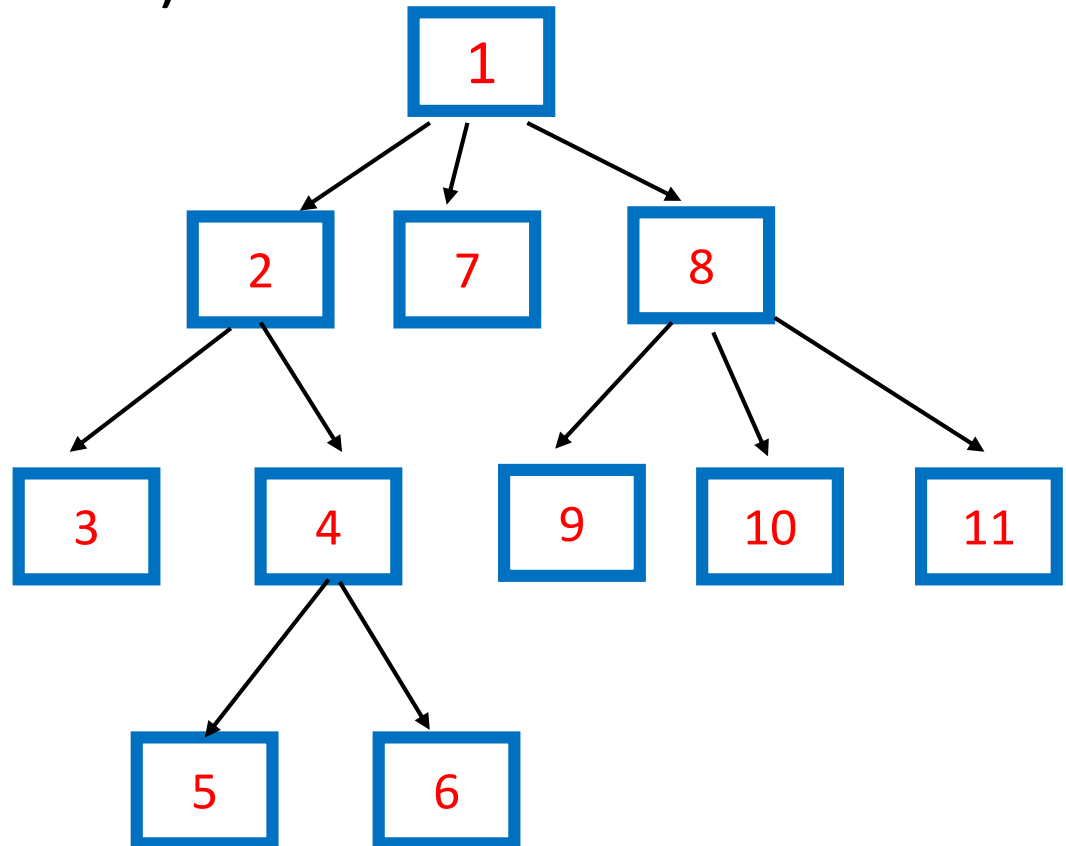
```
depthfirst (root){  
  if (root is not empty){  
    visit root  
    for each child of root  
      depthfirst( child )  
  }  
}
```



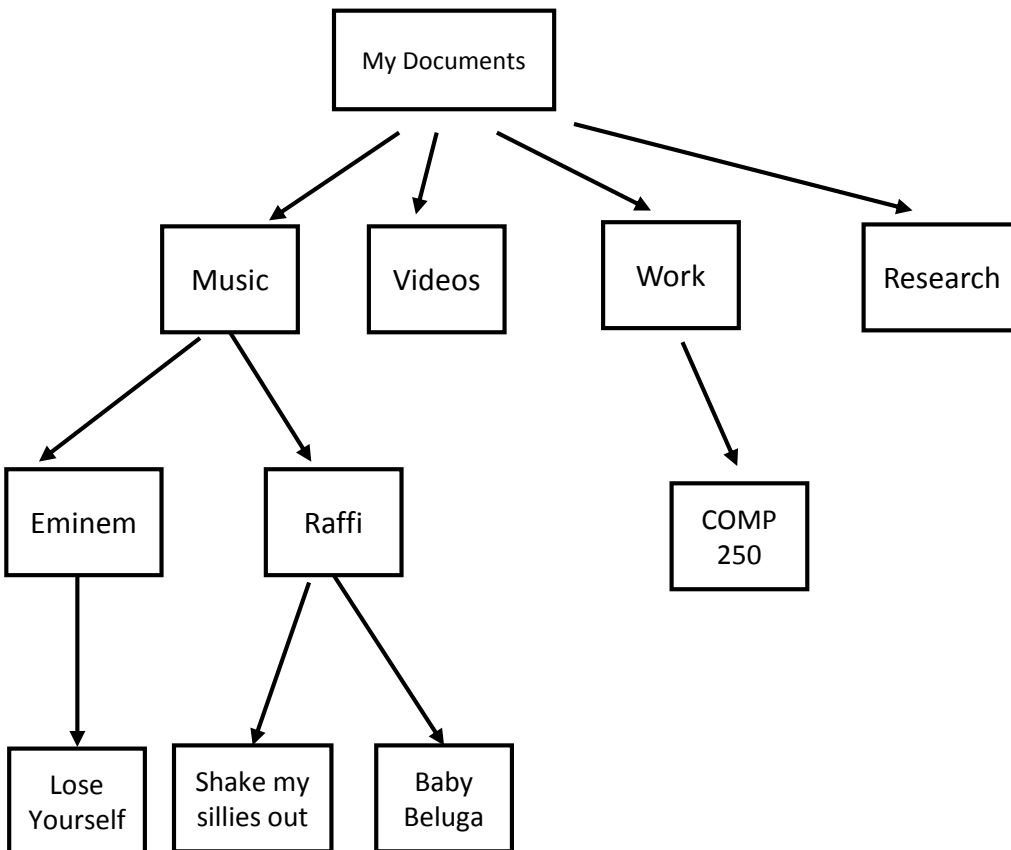
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depthfirst (root){  
  if (root is not empty){  
    visit root  
    for each child of root  
      depthfirst( child )  
  }  
}
```



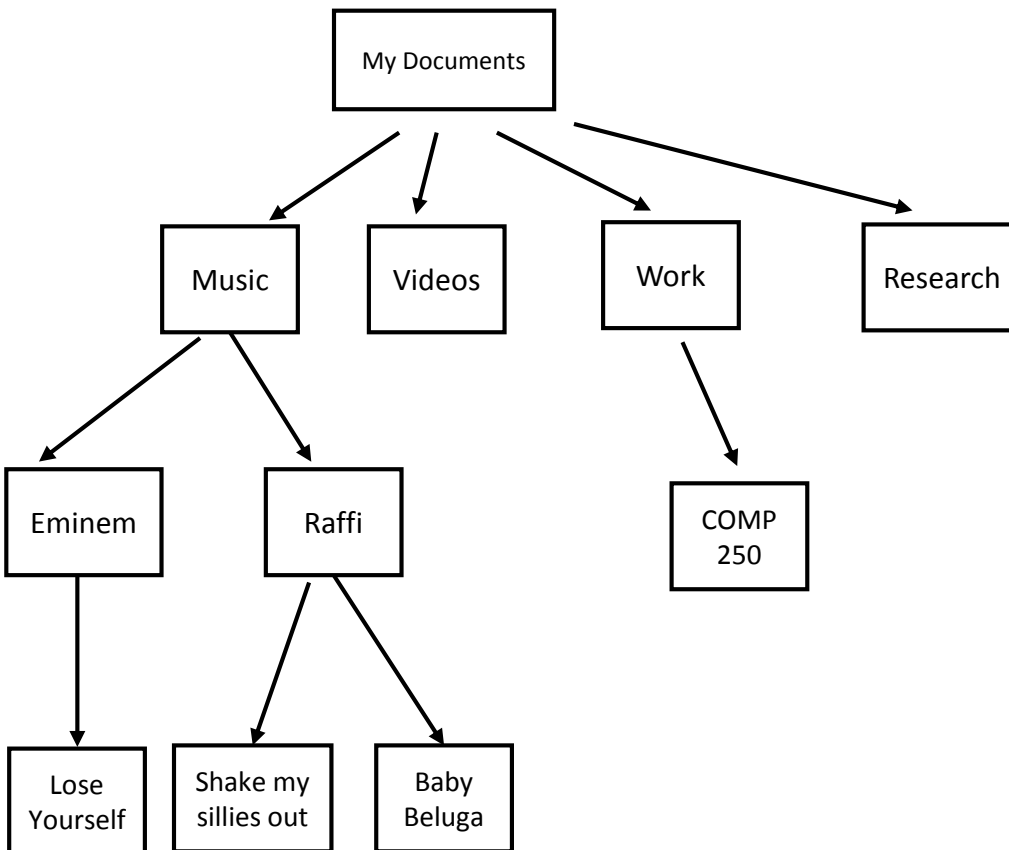

```
depthfirst (root){  
  if (root is not empty){  
    visit root  
    for each child of root  
      depthfirst( child )  
  }  
}
```



Example of Preorder Traversal: printing a hierarchical file system (visit = print directory or file name)



Example of Preorder Traversal: printing a hierarchical file system (visit = print directory or file name)



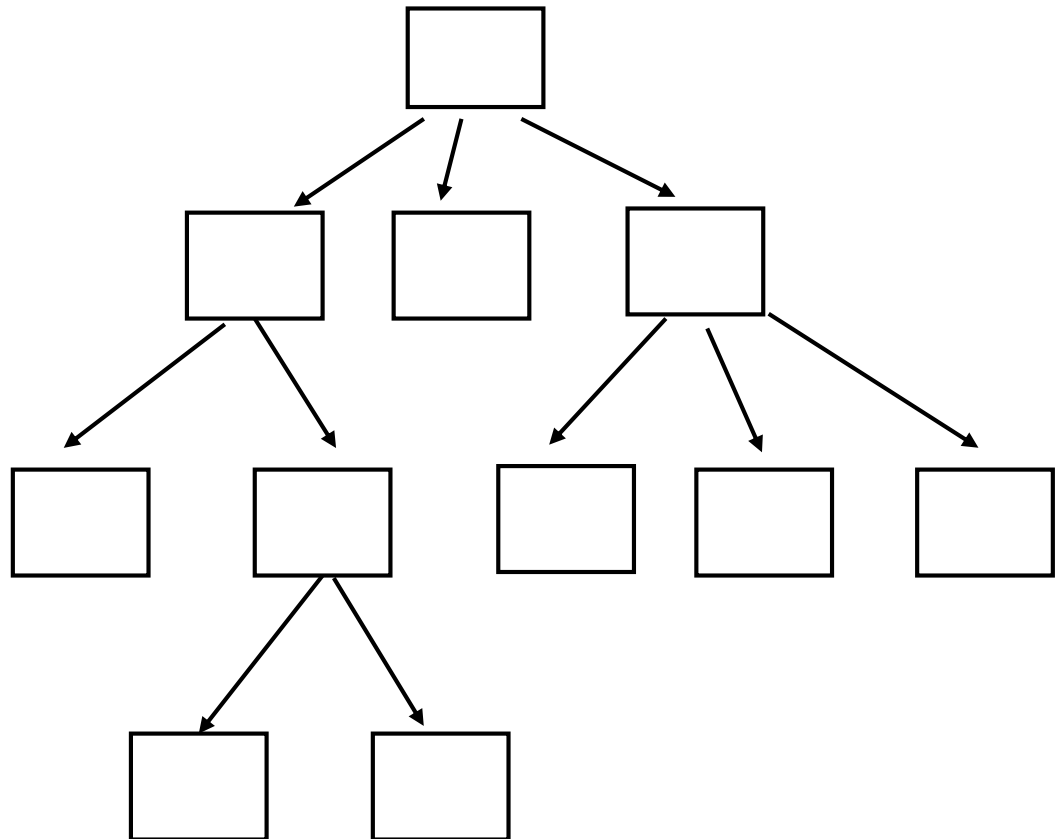
```
Documents      (directory)
Music           (directory)
  Eminem       (directory)
    Lose Yourself (file)
  Raffi        (directory)
    Shake My Sillies Out (file)
    Baby Beluga  (file)
Videos         (directory)
  :            (file)
Work           (directory)
  COMP250      (directory)
  :
Research       (directory)
  :
```

“Visit” implies that you do something at that node.

Analogy: you aren't visiting London UK if you just fly through Heathrow.

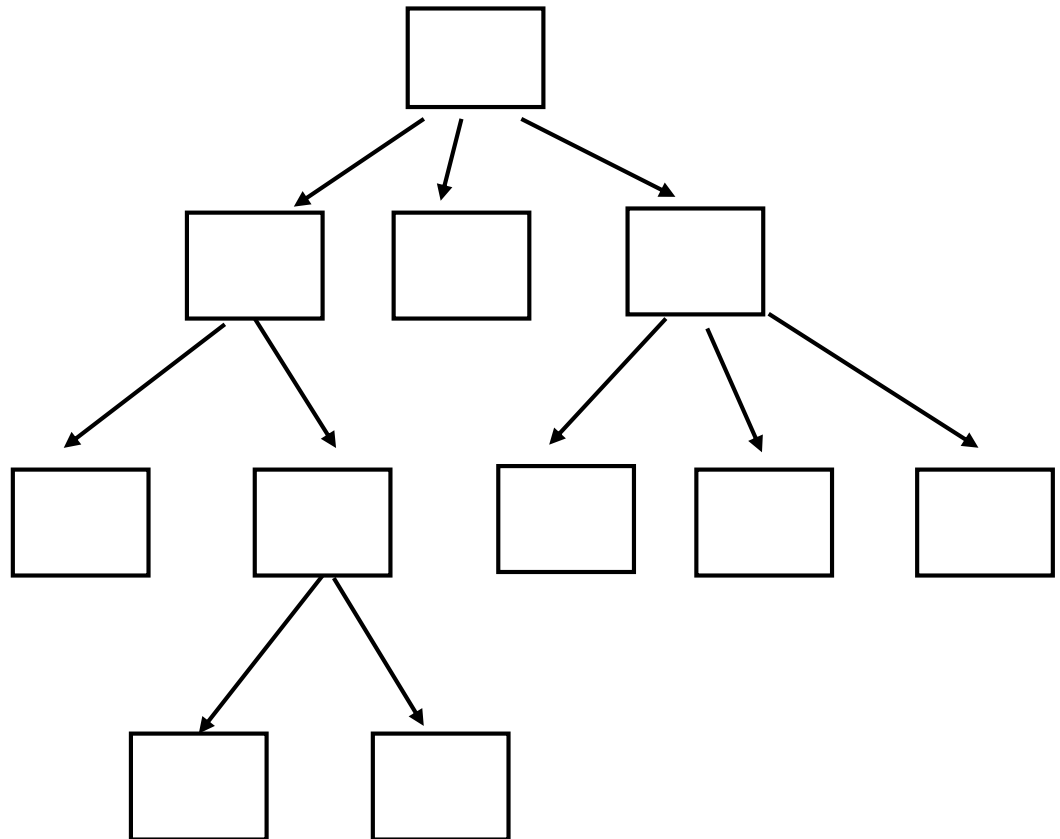
```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
    visit root  
  }  
}
```

“postorder” traversal:
visit the root after the
children



```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
    visit root  
  }  
}
```

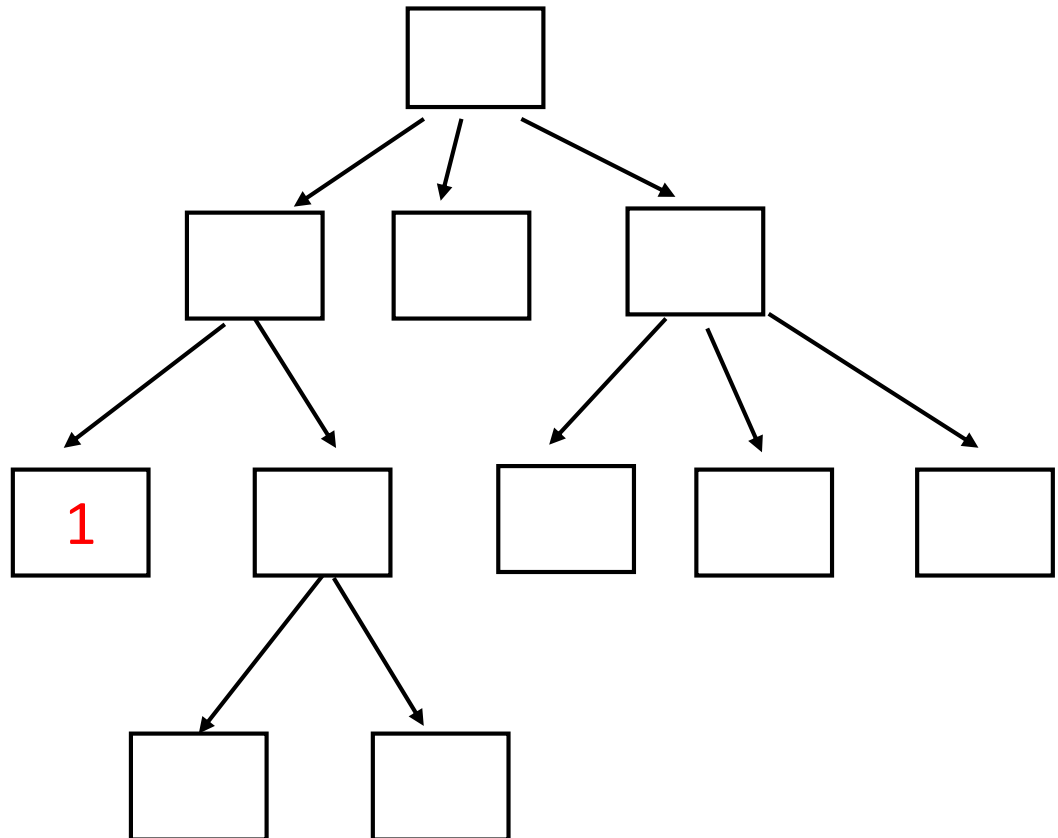
“postorder” traversal:
visit the root after the
children



Q: Which node
is visited first?

```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
    visit root  
  }  
}
```

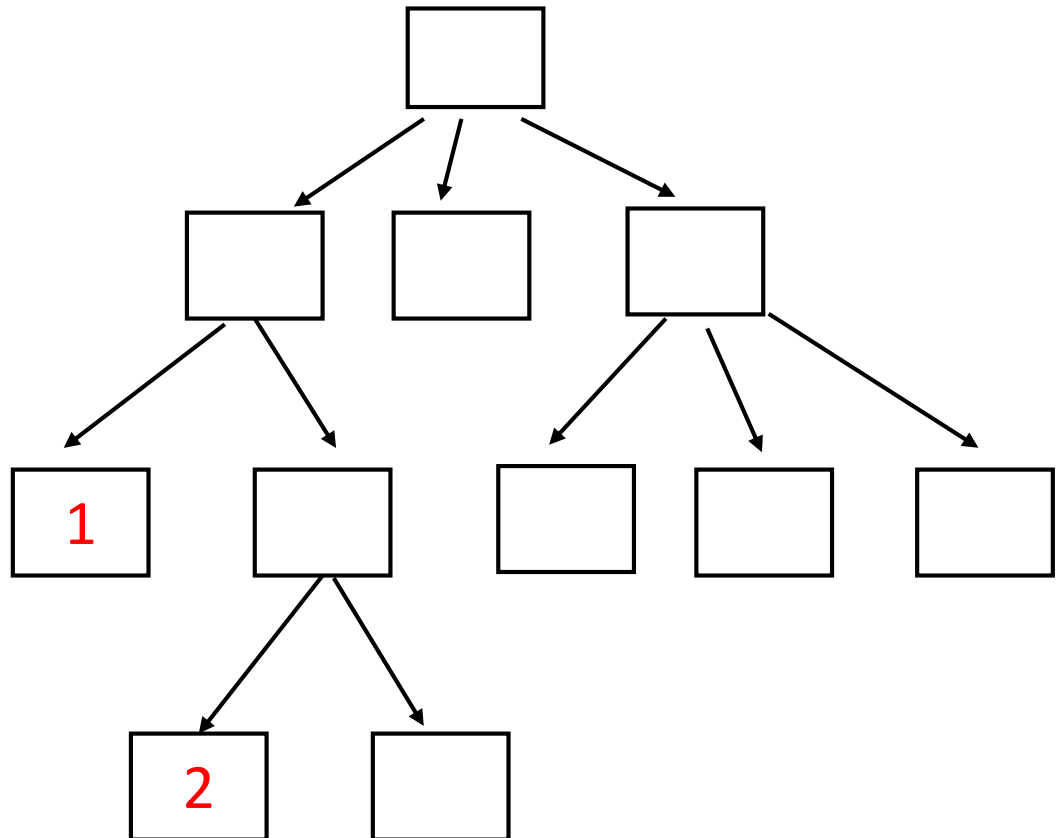
“postorder” traversal:
visit the root after the
children



Q: Which node is
visited second?

```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
    visit root  
  }  
}
```

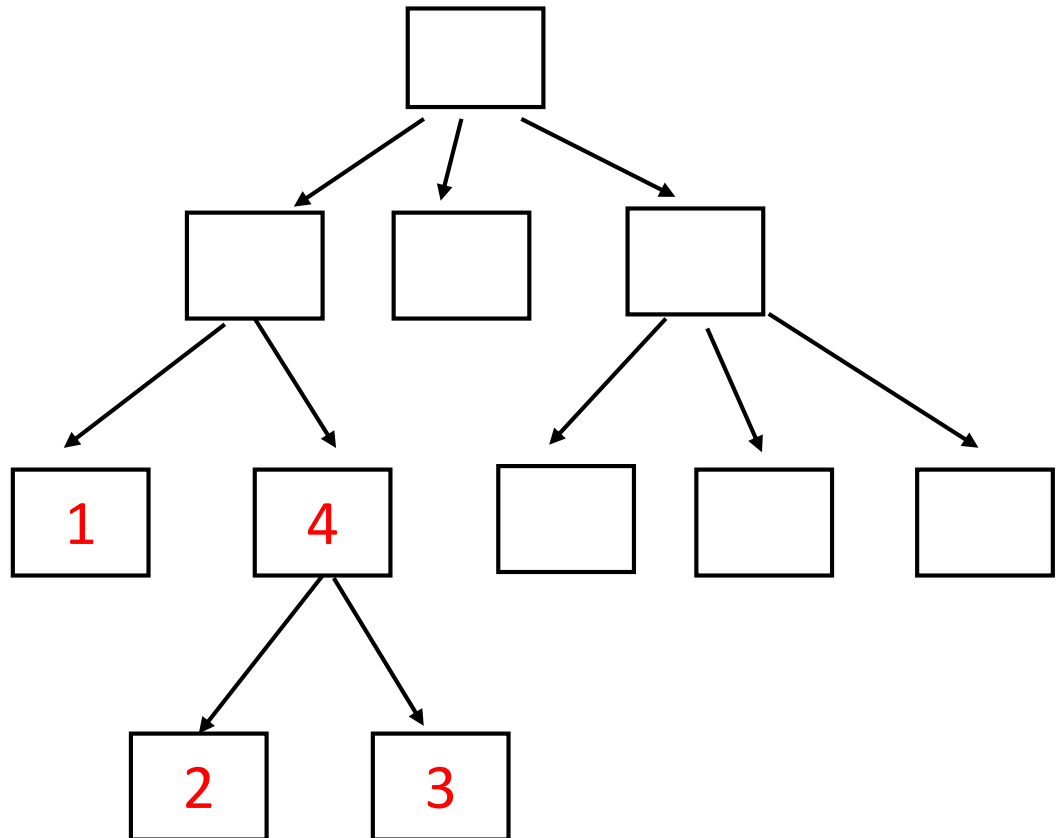
“postorder” traversal:
visit the root after the
children



Q: Which node is
visited 3rd and 4th ?

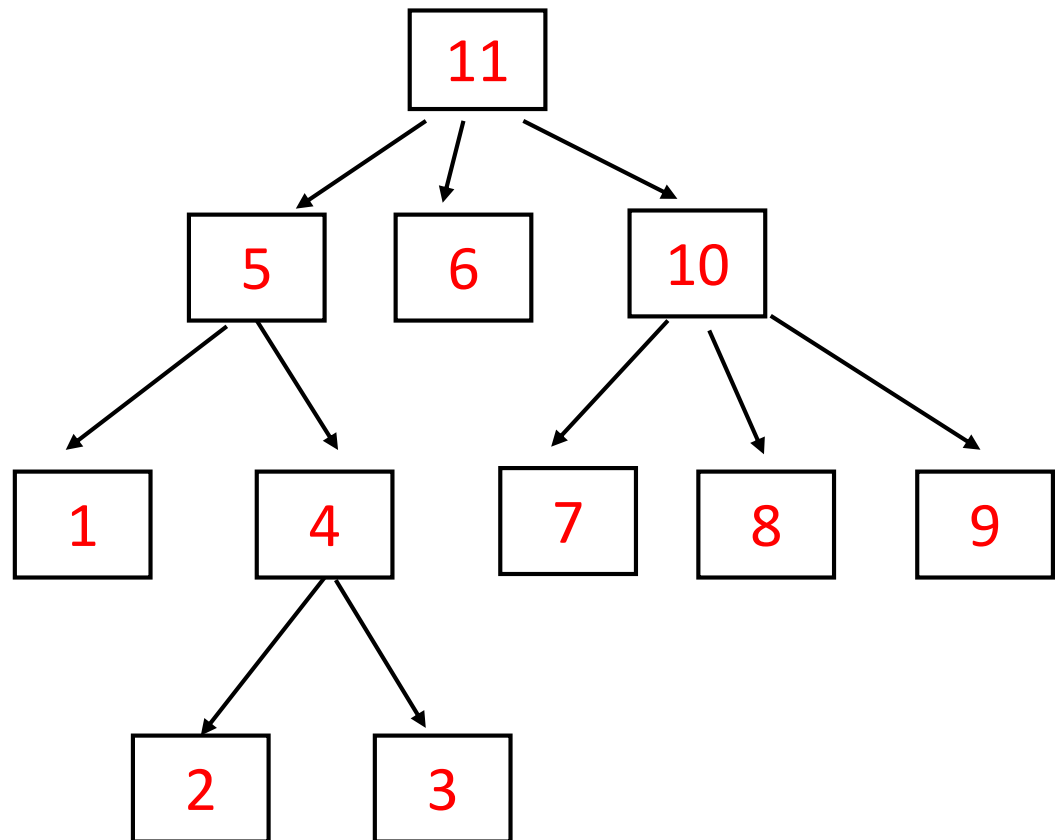

```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
    visit root  
  }  
}
```

“postorder” traversal:
visit the root after the
children

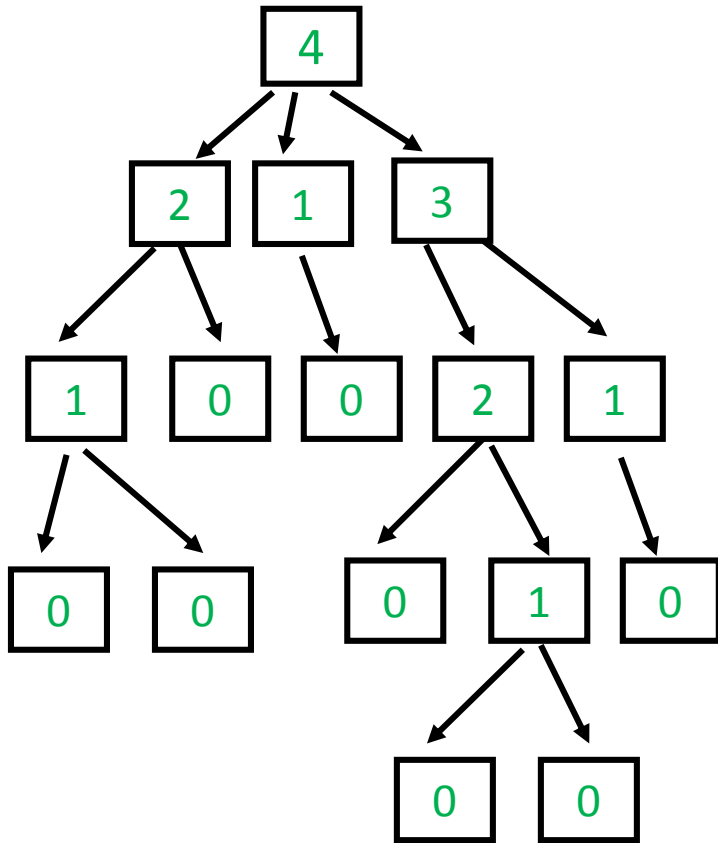


```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
    visit root  
  }  
}
```

“postorder” traversal:
visit the root after the
children



Example 1 postorder

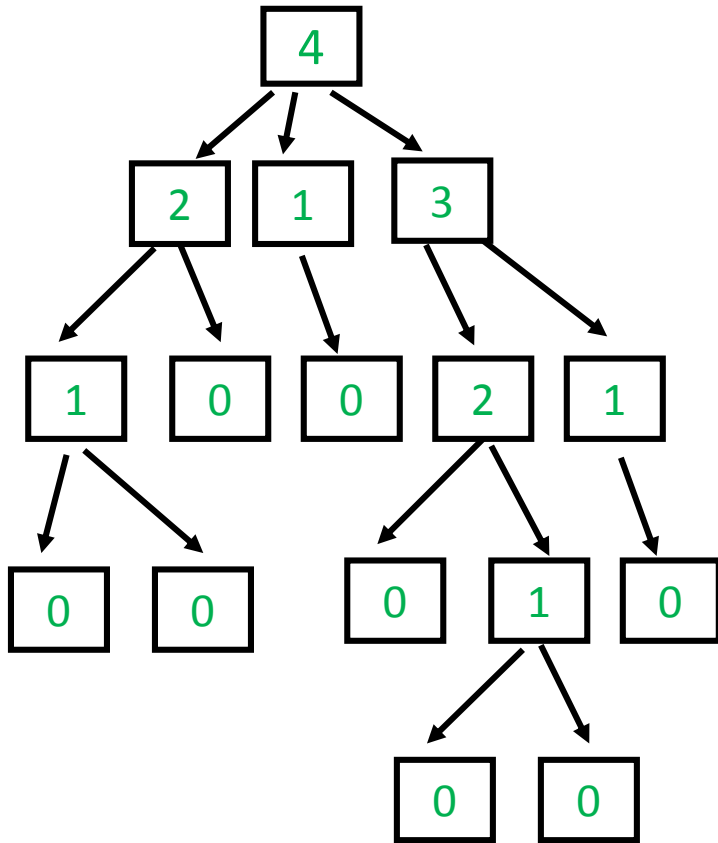


height(v){

?

}

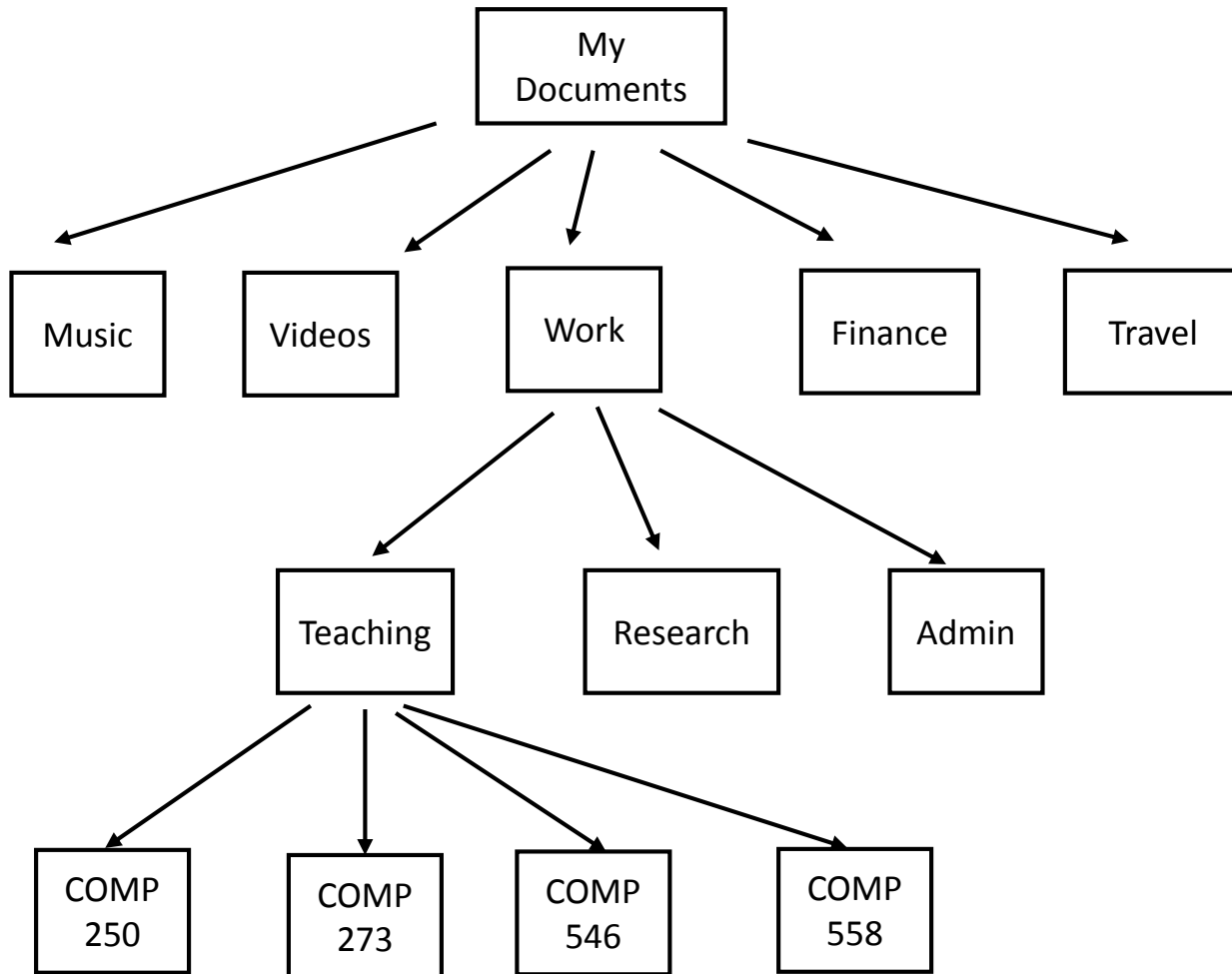
Example 1 postorder



```
height(v){  
    if (v is a leaf)  
        return 0  
    else{  
        h = 0  
        for each child w of v  
            h = max(h, height(w))  
        return 1 + h  
    }  
}
```

visit = return value of height

Example 2 Postorder: What is the total number of bytes in all files in a directory?



```
numBytes(root){  
    if root is a leaf  
        return number of bytes at root  
    else {  
        sum = 0  
        for each child of root{  
            sum += numBytes(child)  
        }  
        return sum  
    }  
}
```

By 'visit' here, we mean determining the number of bytes for a node, e.g. If we were to store 'sum' at the node.

“preorder” traversal

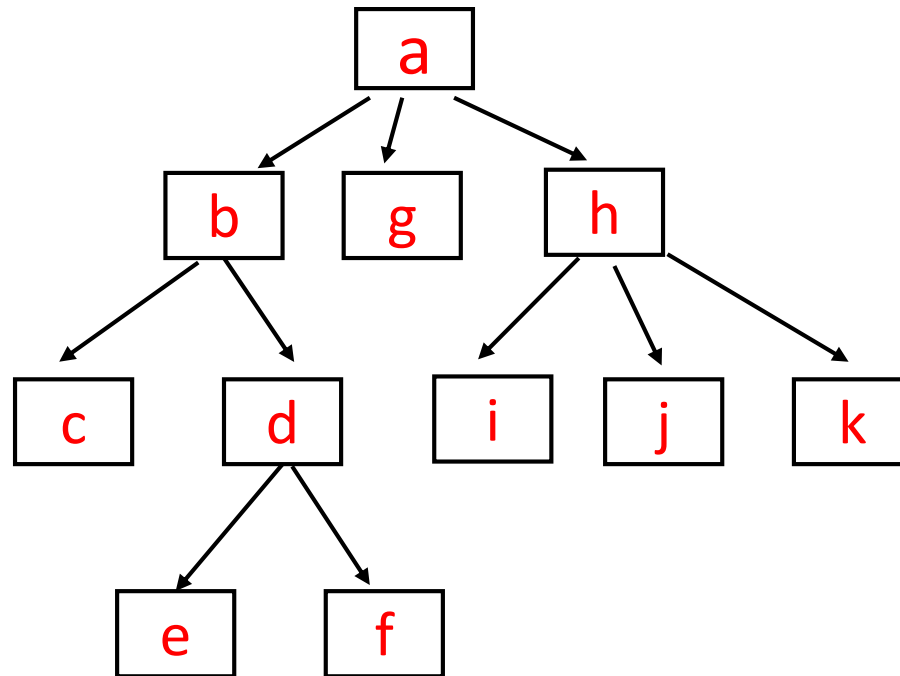
```
depthfirst (root){  
    if (root is not empty){  
        visit root  
        for each child of root  
            depthfirst( child )  
    }  
}
```

“postorder” traversal

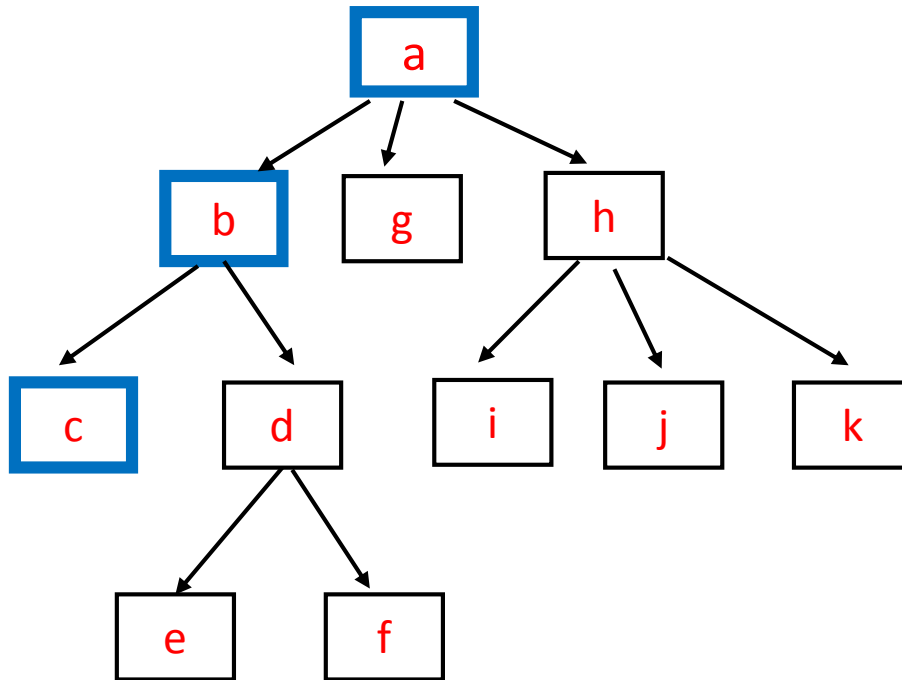
```
depthfirst (root){  
    if (root is not empty){  
        for each child of root  
            depthfirst( child )  
        visit root  
    }  
}
```

Same `depthfirst(root)` call sequence occurs for preorder vs postorder.

In example below, the letter order corresponds to `depthfirst(root)` call order.

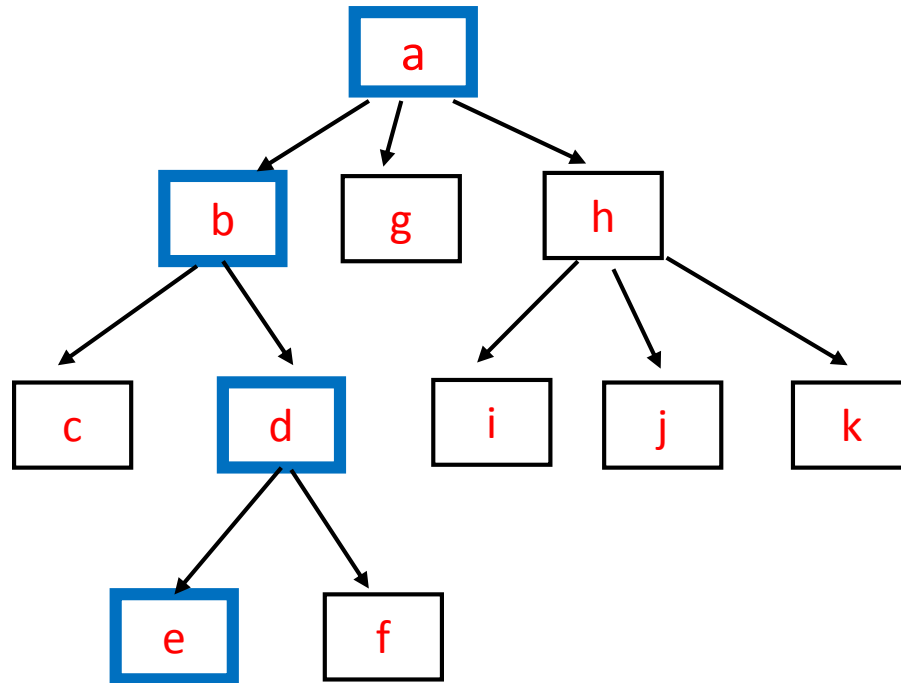


Call stack for `depthfirst(root)`



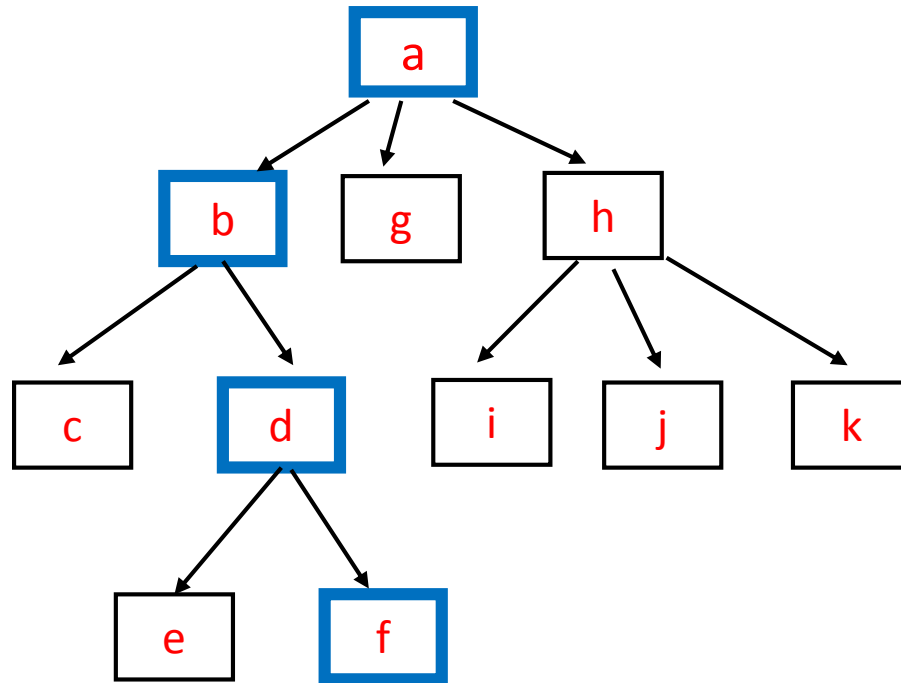
c
b b
a a a

Call stack for `depthfirst(root)`



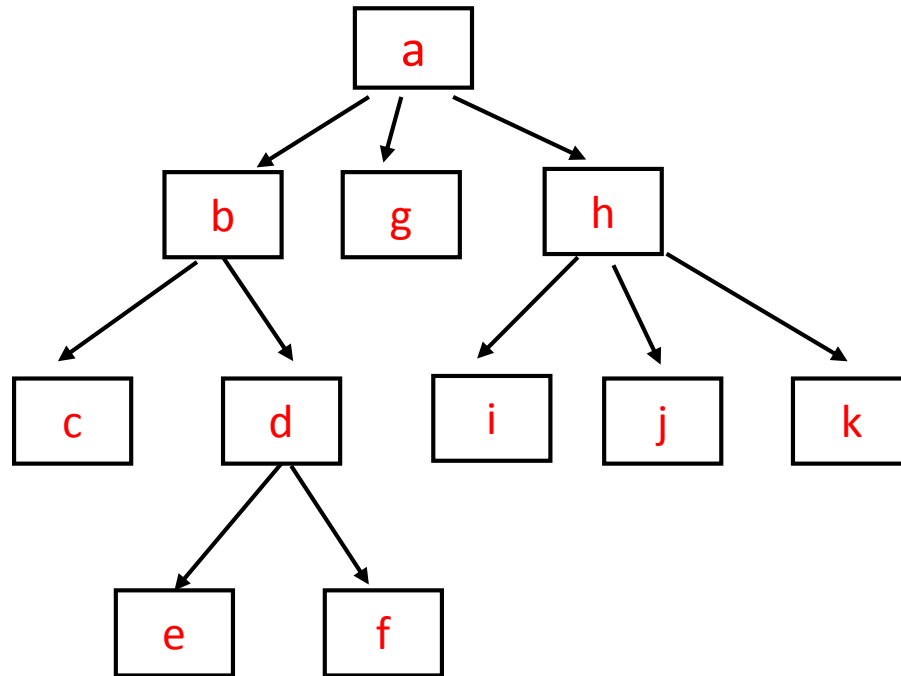
a a a a a a
b b b b b
c d d
e

Call stack for `depthfirst(root)`



a a a a a a a a
b b b b b b b
c d d d d
e f

Call stack for `depthfirst(root)`



Notation: the letters indicate call order of `depthFirst(root)`

 e f
 c d d d d d i j k
 b b b b b b b b g h h h h h h
a a

Tree traversal

Recursive

- depth first (pre- versus post-order)

Non-Recursive

- using a stack
- using a queue


Assume algorithm has access to root of tree.



```
treeTraversalUsingStack(root){  
    initialize empty stack s  
    s.push(root)
```



```
}
```

```
treeTraversalUsingStack(root){  
    initialize empty stack s  
    s.push(root)  
    while s is not empty {  
        cur = s.pop()  
        visit cur  
          
    }  
}
```

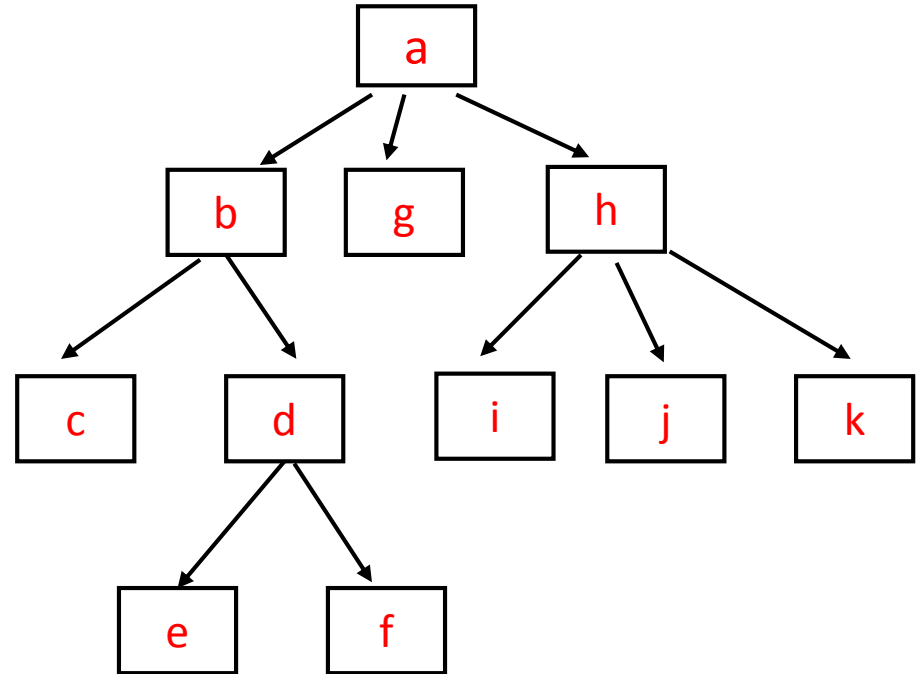
```
treeTraversalUsingStack(root){  
    initialize empty stack s  
    s.push(root)  
    while s is not empty {  
        cur = s.pop()  
        visit cur  
        for each child of cur  
            s.push(child)  
    }  
}
```


What is the order of nodes visited ?

```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

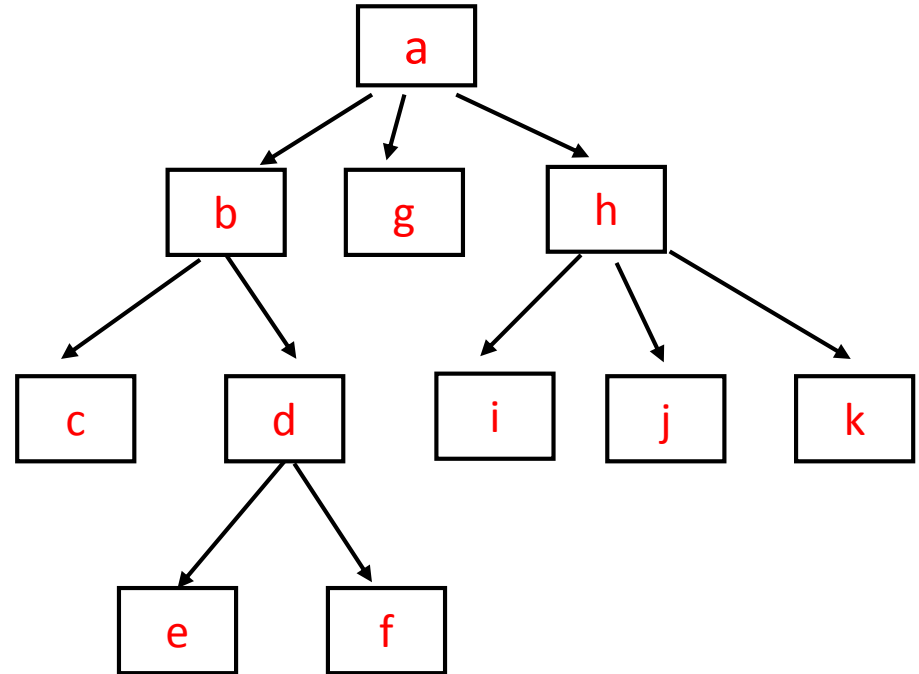
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```

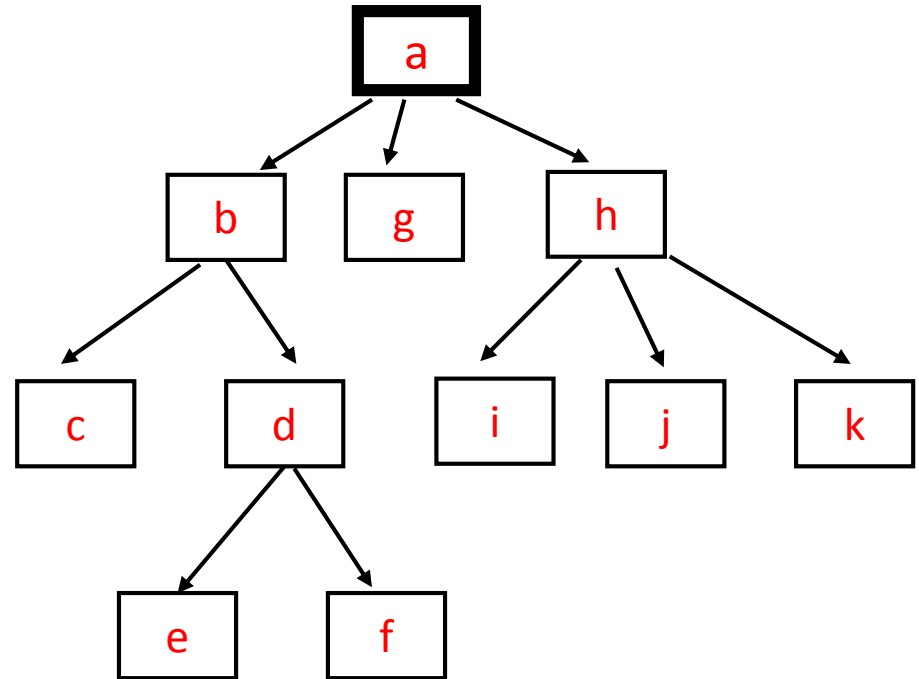


a _

```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```

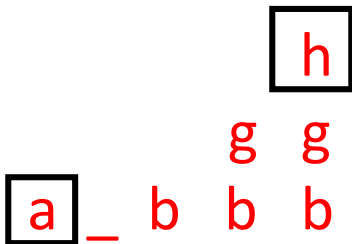
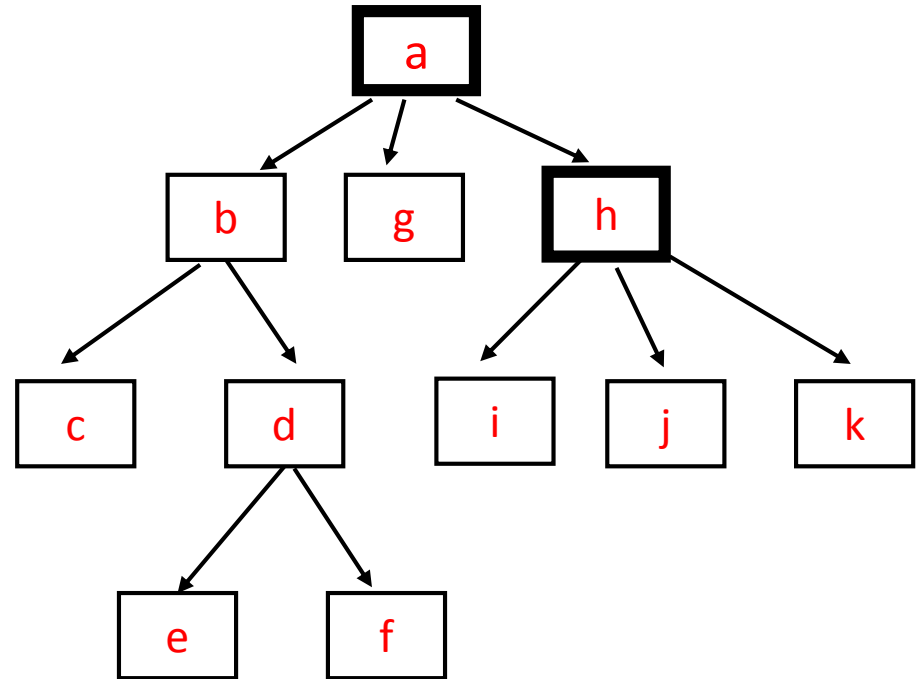


a _ b b b
 g g
 h

```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

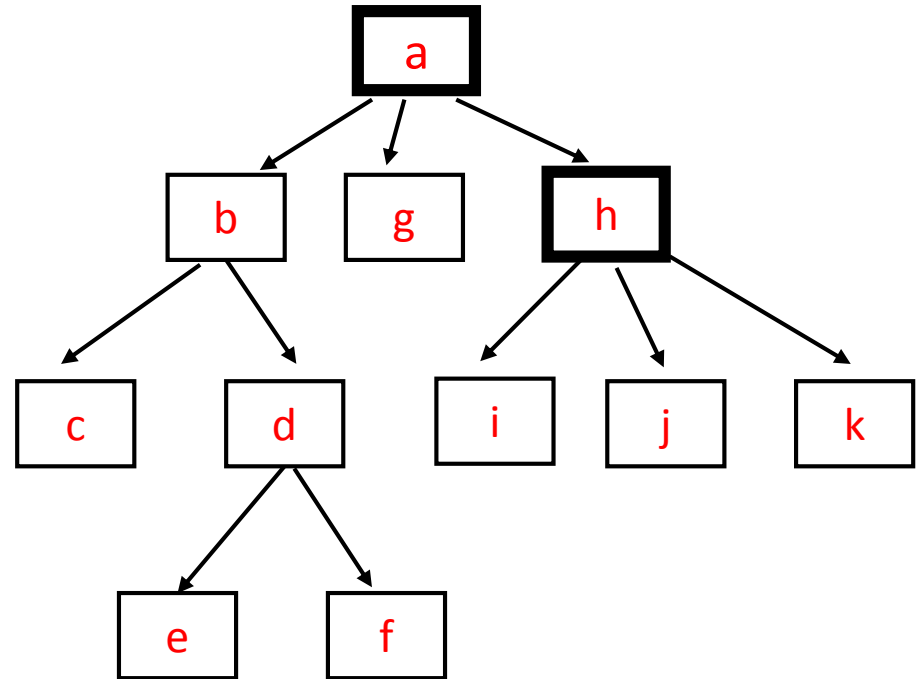
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



a _ b b b b b b
 g g g g g g
 h i i i j j k

```
treeTraversalUsingStack(root){
```

```
  initialize empty stack s
```

```
  s.push(root)
```

```
  while s is not empty {
```

```
    cur = s.pop()
```

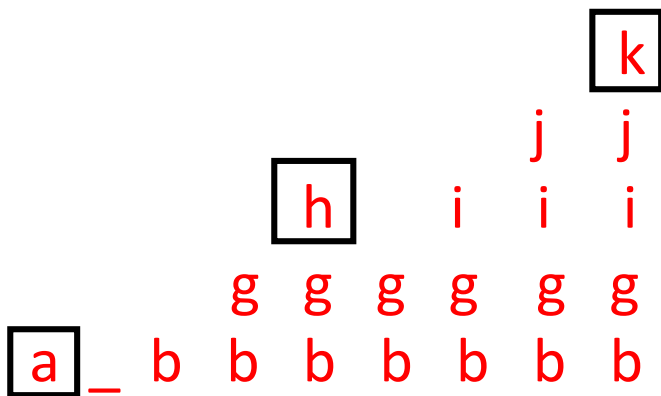
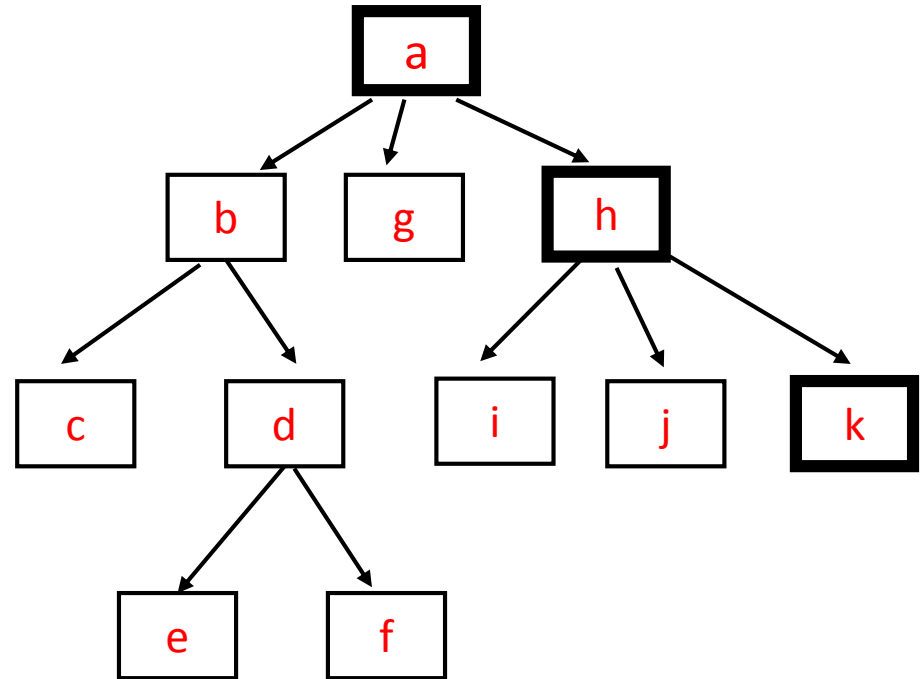
```
    visit cur
```

```
    for each child of cur
```

```
      s.push(child)
```

```
  }
```

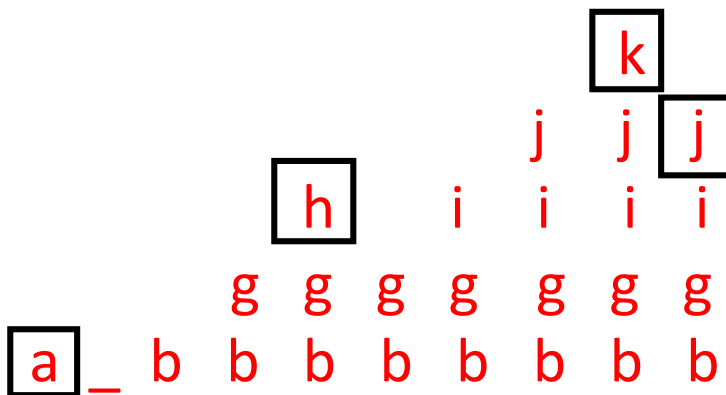
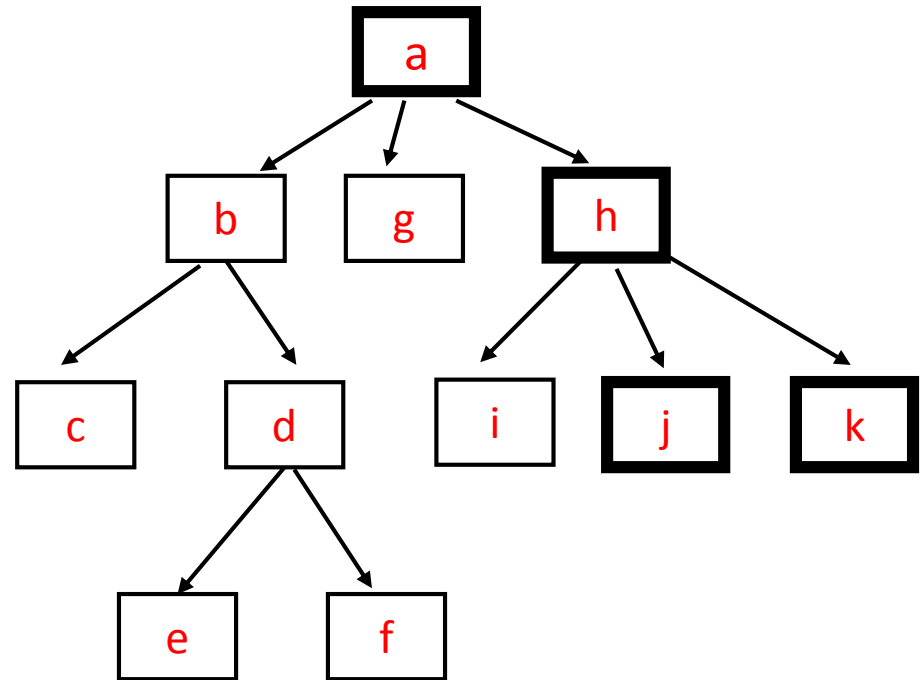
```
}
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

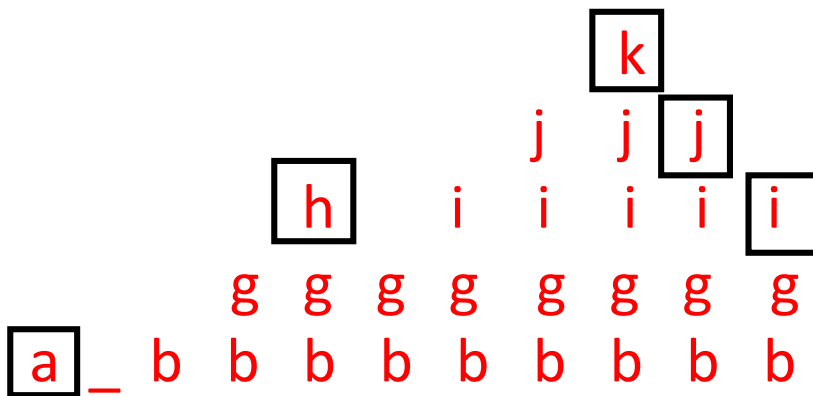
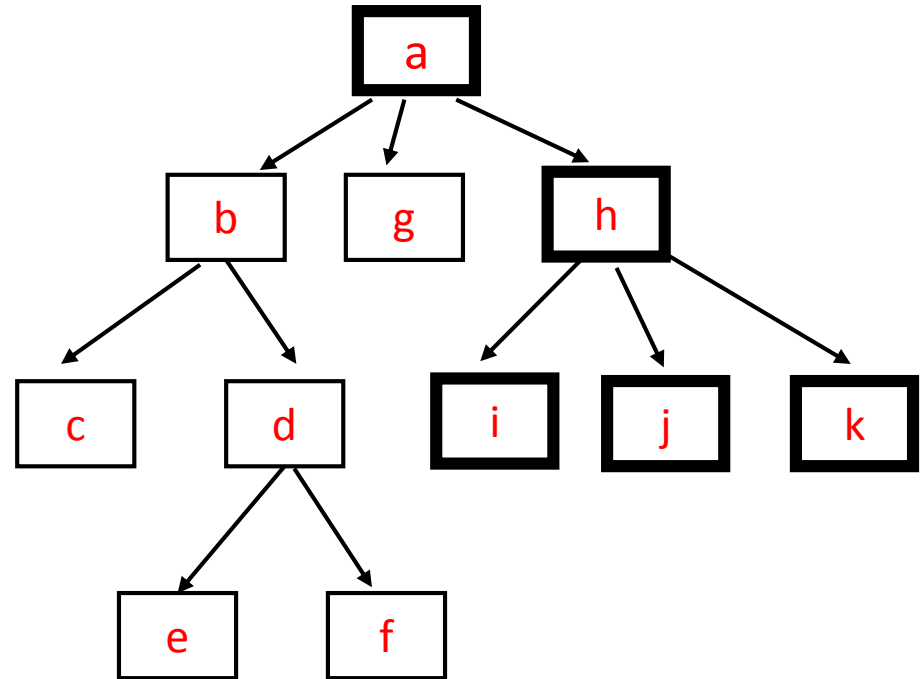
```




```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

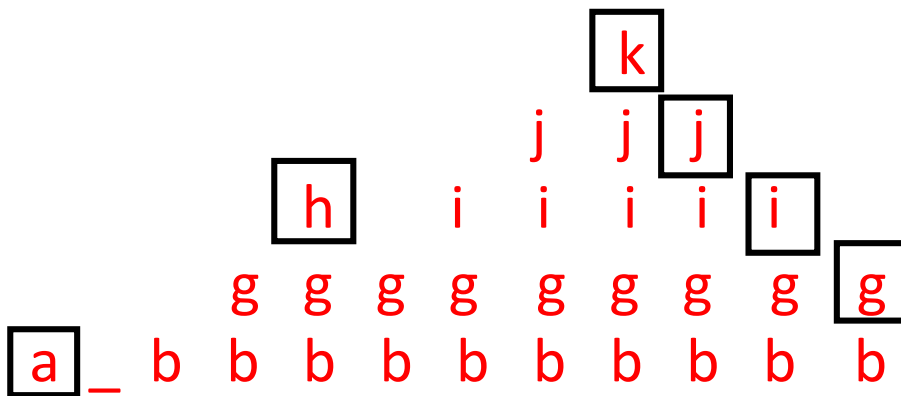
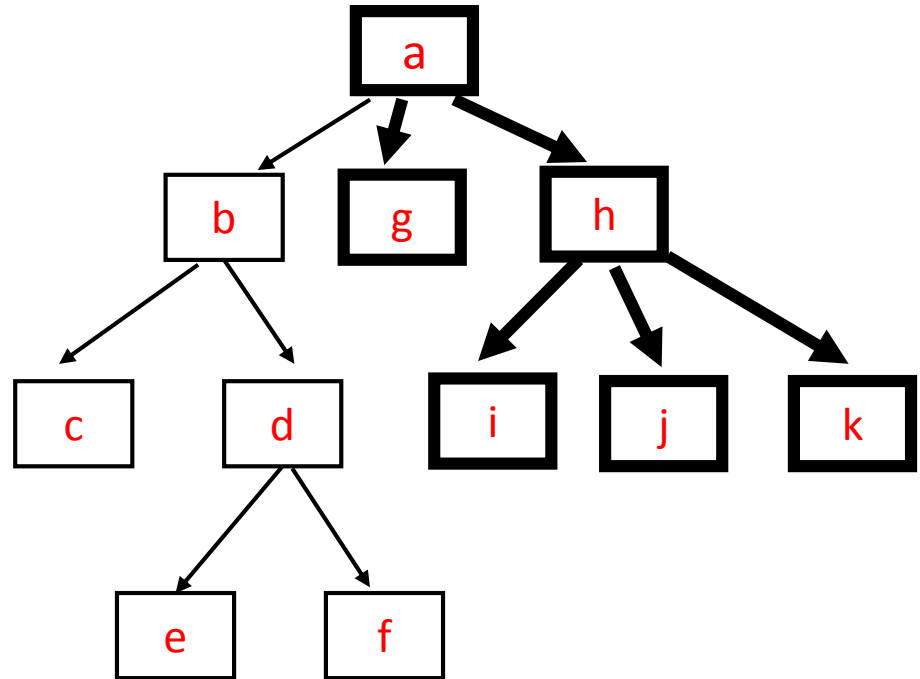
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



```
treeTraversalUsingStack(root){
```

```
  initialize empty stack s
```

```
  s.push(root)
```

```
  while s is not empty {
```

```
    cur = s.pop()
```

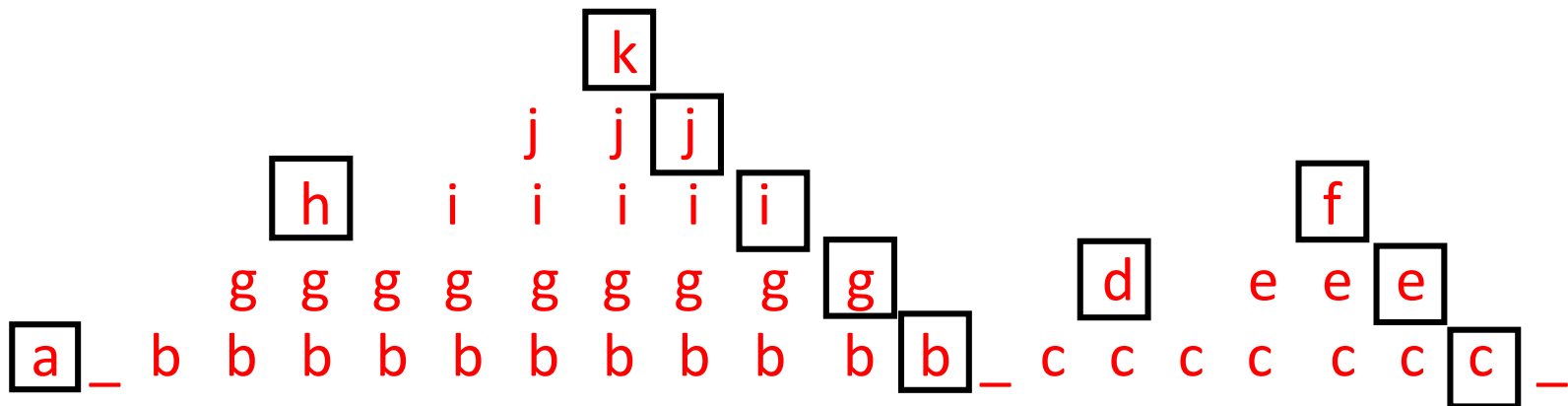
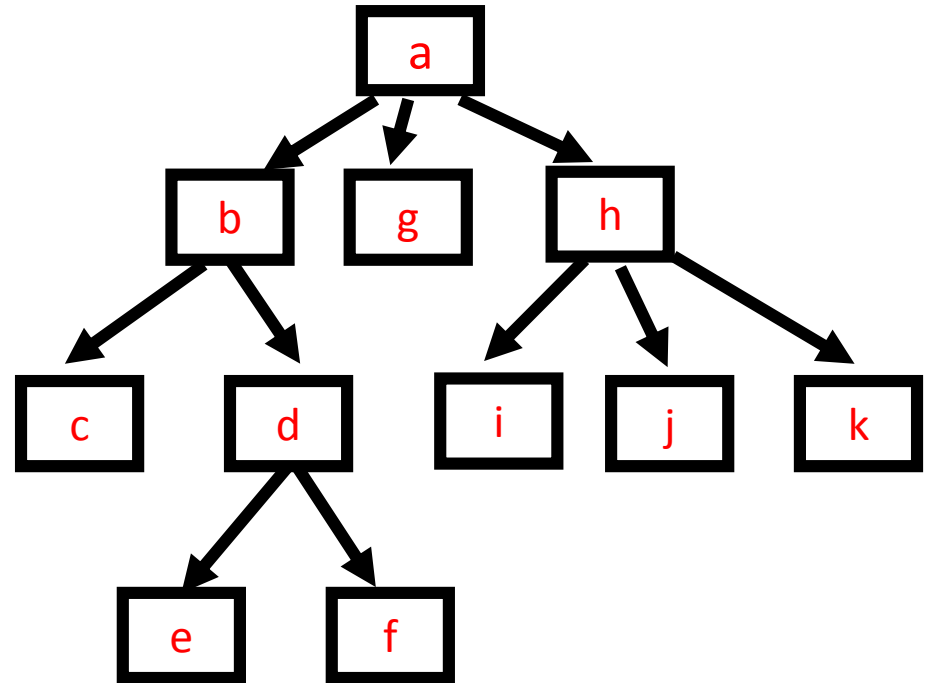
```
    visit cur
```

```
    for each child of cur
```

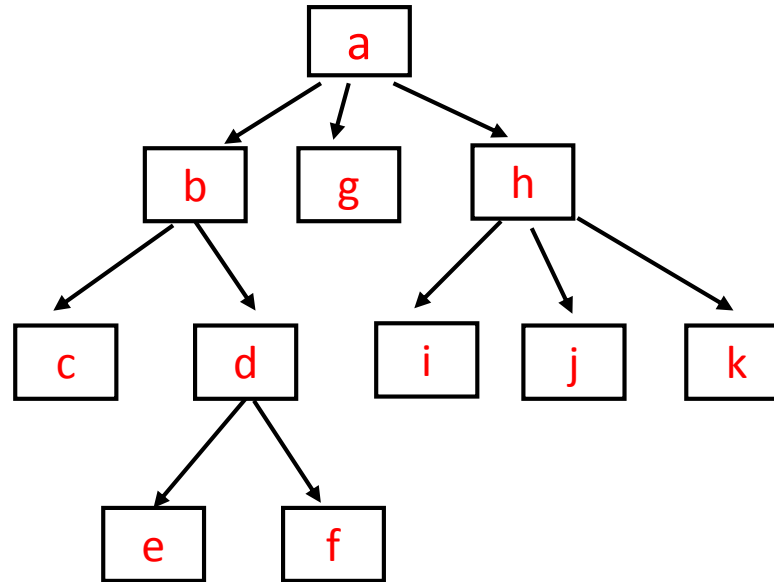
```
      s.push(child)
```

```
  }
```

```
}
```



Stack based method is also depth first,
but visits children from right to left




recursive preorder
recursive postorder

abcdefghijk
cefdbgijkha

non-recursive (stack)

ahkjigbdfec

```
treeTraversalUsingStack(root){  
  initialize empty stack s  
  s.push(root)  
  while s is not empty {  
    cur = s.pop()  
    visit cur  
    for each child of cur  
      s.push(child)  
    visit cur  
  }  
}
```



Moving the visit does
not make it post order.

It is still pre-order.

Why?

What if we use a queue instead?

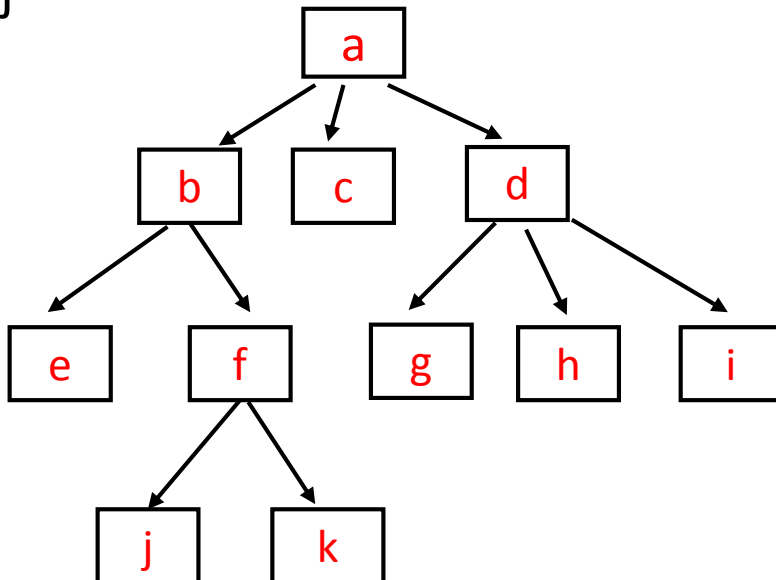
```
treeTraversalUsingStack(root){  
  initialize empty stack s  
  s.push(root)  
  while s is not empty {  
    cur = s.pop()  
    visit cur  
    for each child of cur  
      s.push(child)  
  }  
}
```

```
treeTraversalUsingQueue(root){  
  initialize empty queue q  
  q.enqueue(root)  
  while q is not empty {  
    cur = q.dequeue()  
    visit cur  
    for each child of cur  
      q.enqueue(child)  
  }  
}
```

```
treeTraversalUsingQueue(root){  
  initialize empty queue q  
  q.enqueue(root)  
  while q is not empty {  
    cur = q.dequeue()  
    visit cur  
    for each child of cur  
      q.enqueue(child)  
  }  
}
```

Queue state
at start of the
while loop

a

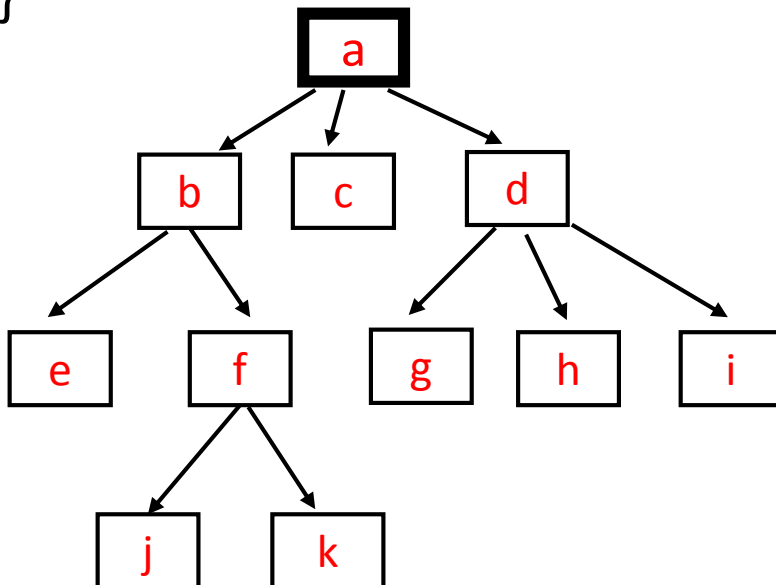


```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

```

Queue state
at start of the
while loop



a
b c d

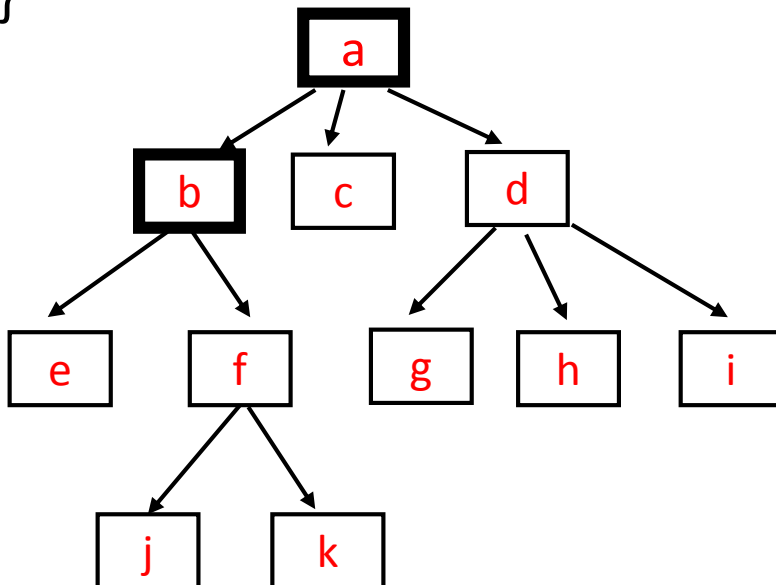



```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

```

Queue state
at start of the
while loop



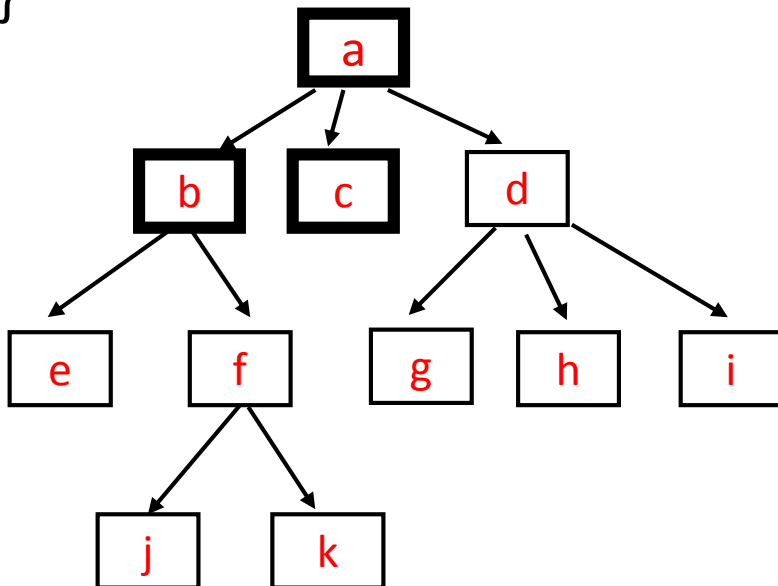
a
b c d
c d e f



```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

```



Queue state
at start of the
while loop

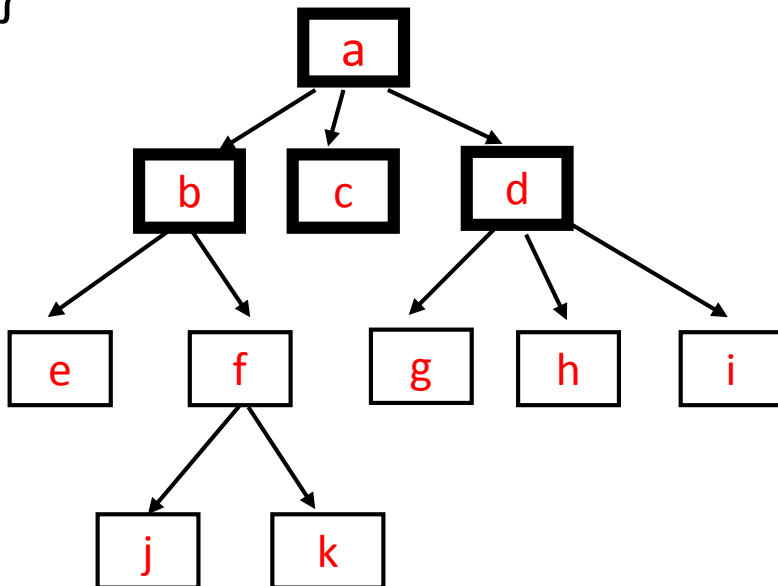
a
b c d
c d e f
d e f



```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

```



Queue state
at start of the
while loop

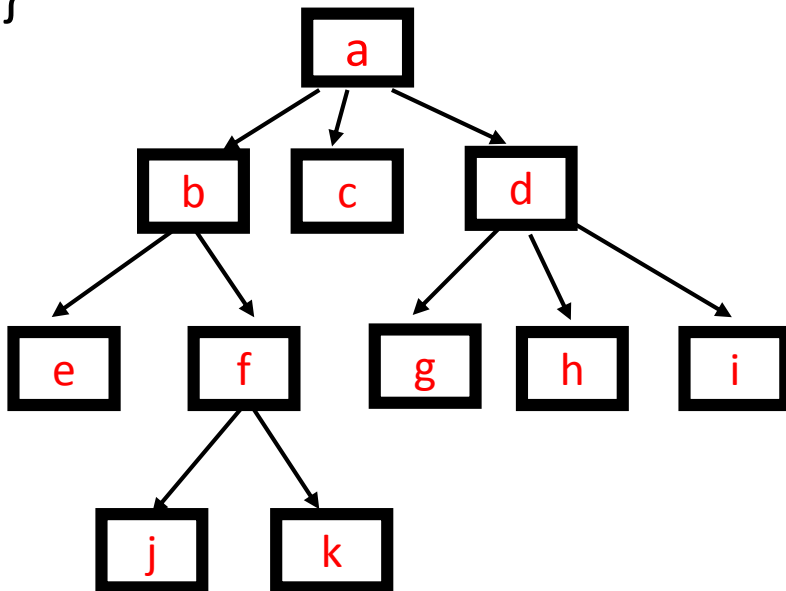
a
b c d
c d e f
d e f
e f g h i



```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

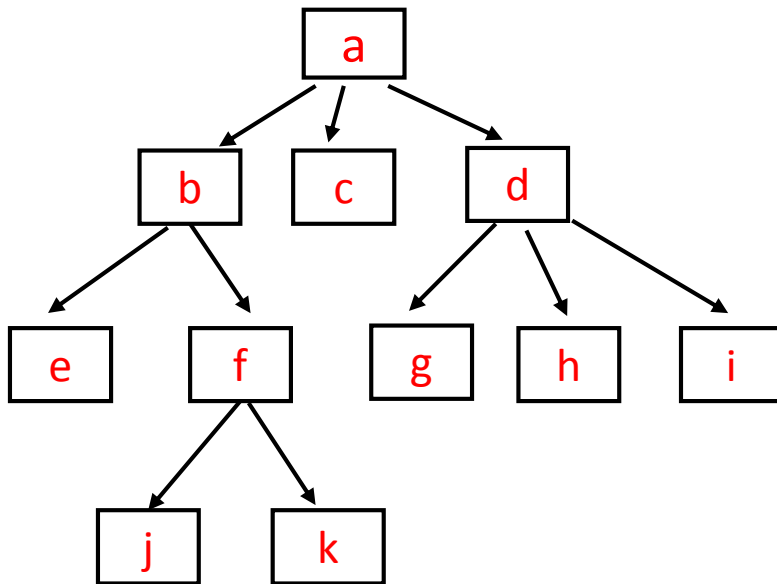
```



a
 b c d
 c d e f
 d e f
 e f g h i
 f g h i
 g h i j k
 h i j k
 i j k
 j k
 k

breadth first traversal

for each level i
visit all nodes at level i



order visited: **abcdefghijkl**