COMP 250
INTRODUCTION TO COMPUTER SCIENCE

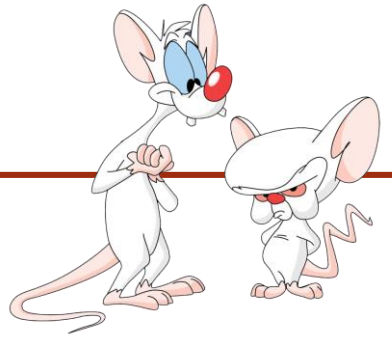
Lecture 12 – Quadratic Sorting a List

Giulia Alberini, Fall 2018

FROM LAST WEEK

- ArrayList
- Linked Lists

WHAT ARE WE GOING TO DO TODAY?



- How to sort a list
  - Bubble sort
  - Selection sort
  - Insertion sort

SORTING

- The process of arranging items in a ordered list following a given criterion.
- For example, sorting a list of integers in ascending order (from smallest to largest):

BEFORE	AFTER
3	-5
17	-2
-5	3
-2	4
23	17
4	23

SORTING ALGORITHMS

There are many techniques for sorting a list

- Selection Sort
- Bubble Sort
- Insertion Sort
- Random Sort :P
- Heap Sort
- Merge Sort
- Quick Sort

SORTING ALGORITHMS

There are many techniques for sorting a list

- Selection Sort
- Bubble Sort
- Insertion Sort

Today $O(N^2)$

- Heap Sort
- Merge Sort
- Quick Sort

Later $O(N \cdot \log N)$

Check out how different algorithms compare: https://www.youtube.com/watch?v=ZZuD6iUe3Pc

OBAMA KNOWS ABOUT SORTING!

https://www.youtube.com/watch?v=k4RRi_ntQc8

OBSERVATION

Today we are concerned with algorithms, not data structures.

The following algorithms are independent of whether we use an array list or a linked list.

BUBBLE SORT

BUBBLE SORT

- Bubble sort is the simplest sorting algorithm.
- Goal: order a list of integers in ascending order
- IDEA: repeatedly iterate through the list and swap adjacent elements if they are in the wrong order.

BUBBLE SORT – PSEUDOCODE

```
for i from 0 to list.length-1 {
   for j from 0 to list.length -2 {
      if(list[j] > list[j+1]) {
         swap(list[j], list[j+1])
      }
   }
}
```

EXAMPLE – ONE ITERATION

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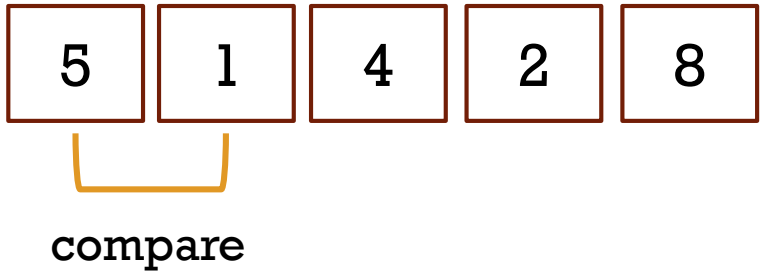
Iteration #1

- Compare all adjacent elements.
- If needed, swap!



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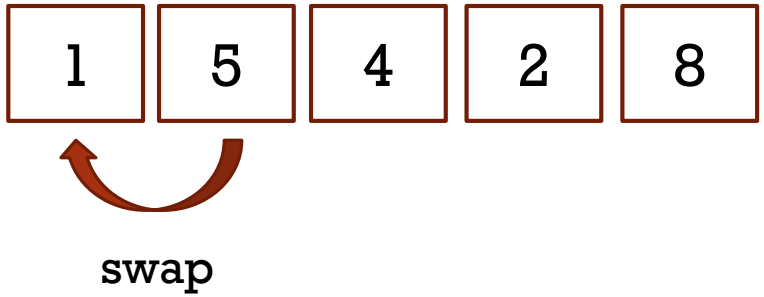

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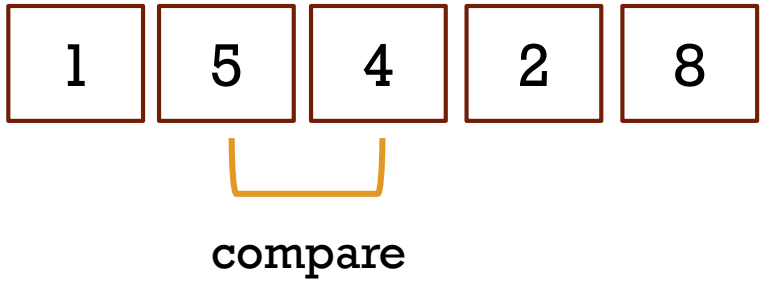

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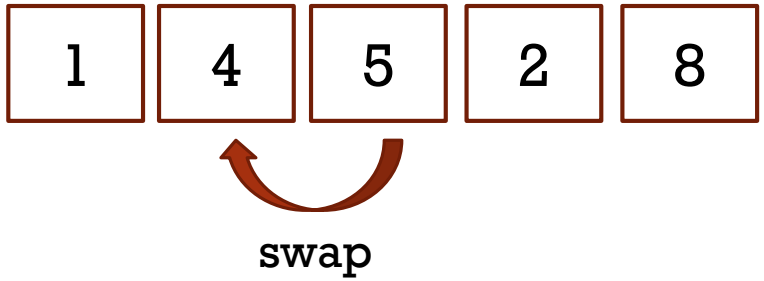

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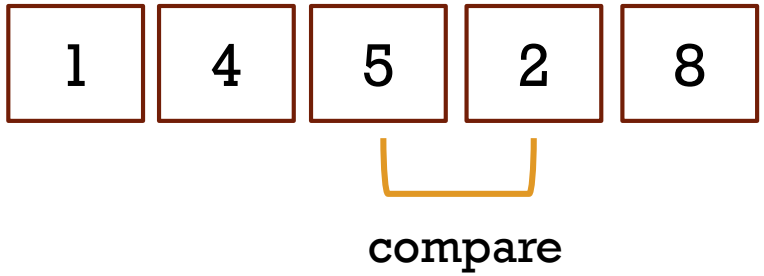

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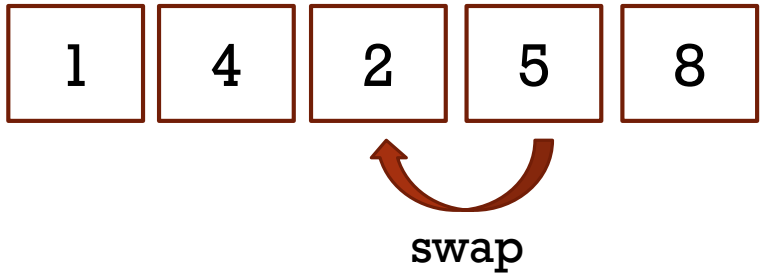

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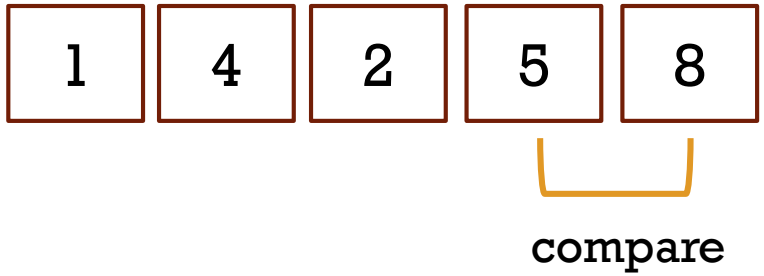


WHAT CAN WE SAY AFTER THE FIRST ITERATION?

Q: Where is the largest element ?

A:

Q: Where is the smallest element?

A:

WHAT CAN WE SAY AFTER THE FIRST ITERATION?

Q: Where is the largest element ?

A: It must be at the end of the list (position N-1)

Q: Where is the smallest element?

A: Anywhere (except position N-1)

WHAT CAN WE SAY AFTER THE FIRST ITERATION?

Q: Where is the largest element ?

A: It must be at the end of the list (position N-1)

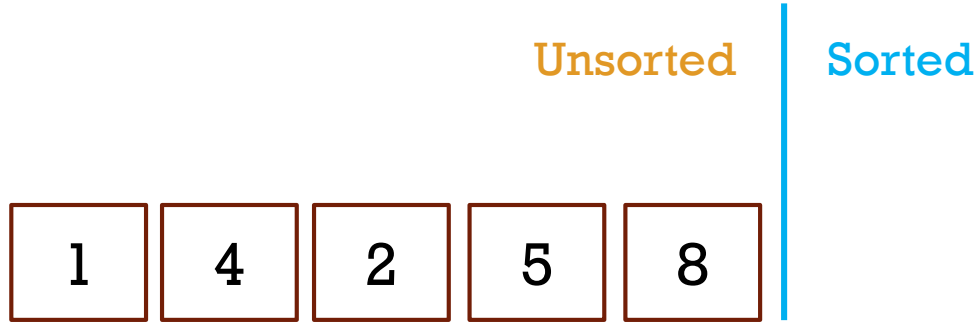
- Since each time we iterate through the list we ensure that the largest element is in the correct position. → at each iteration we can stop comparing adjacent elements one step earlier.

BUBBLE SORT – PSEUDOCODE

```
for i from 0 to list.length-1 {
   for j from 0 to list.length - i -2 {
      if(list[j] > list[j+1]) {
         swap(list[j], list[j+1])
      }
   }
}
```

EXAMPLE

We left off at the end of Iteration #1



EXAMPLE

Iteration #2

- Compare all adjacent elements up to index **3**.
- If needed, swap!

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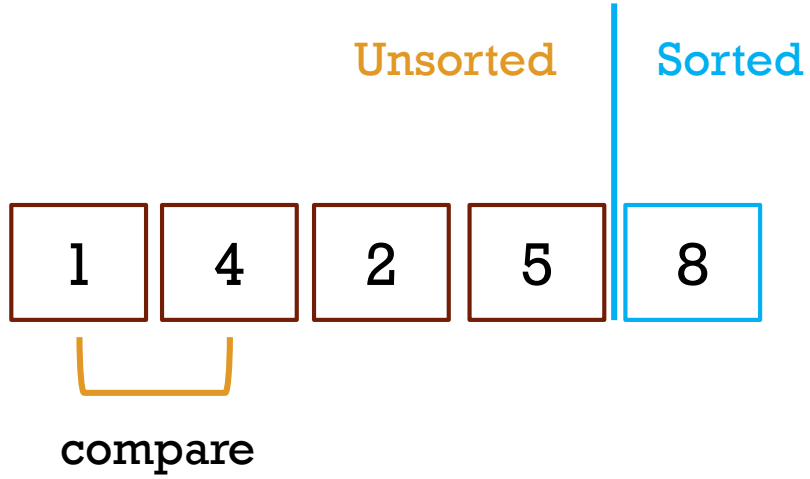

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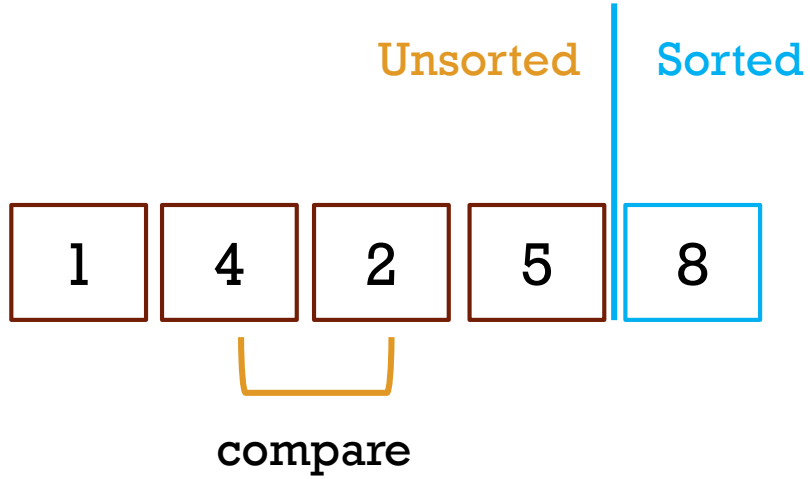

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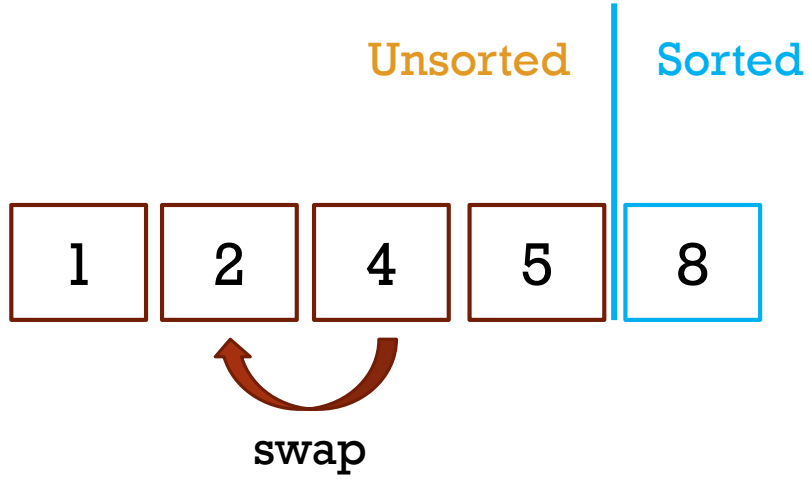

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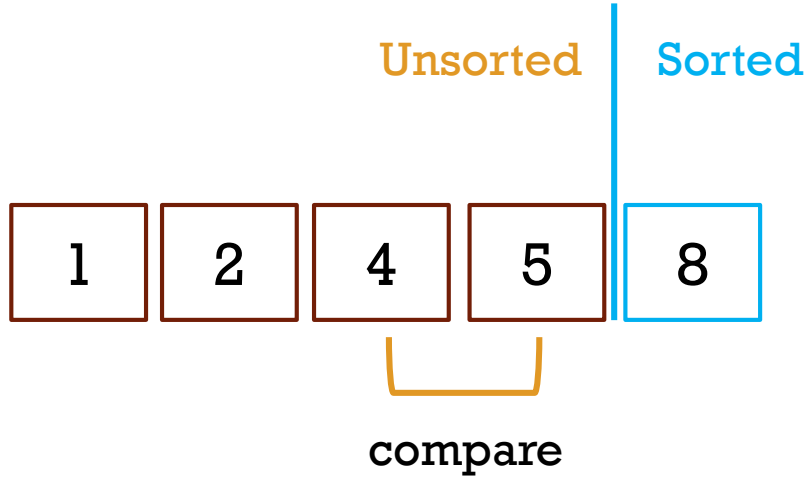
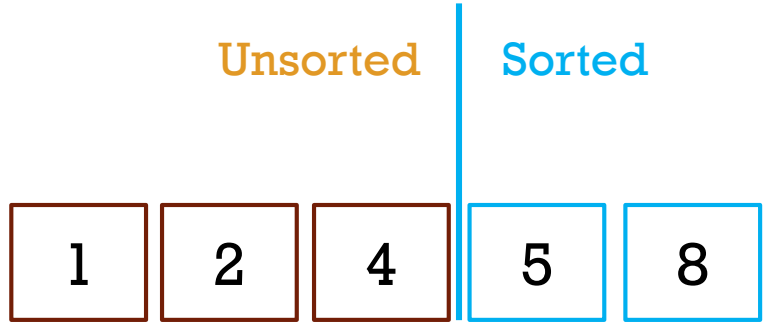


EXAMPLE

Iteration #3

- Compare all adjacent elements up to index **2**.
- If needed, swap!

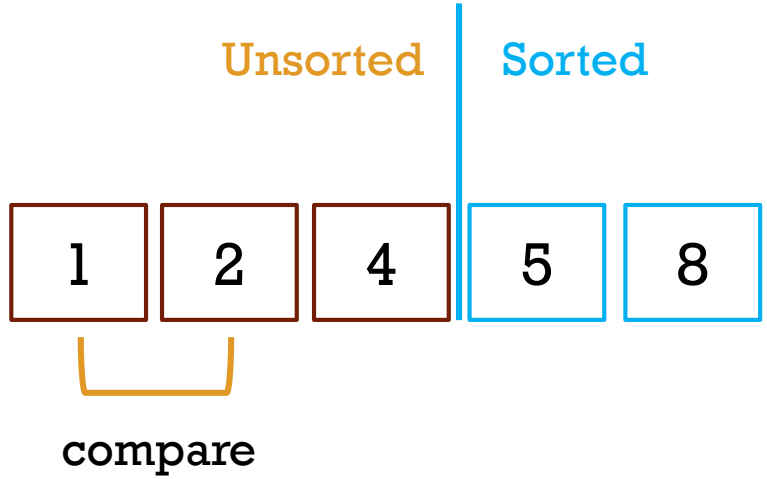


Note: now the list is sorted, but the algorithm does not know that.
When can the algorithm infer that the list is sorted?

EXAMPLE

Iteration #3

- Compare all adjacent elements up to index **2**.
- If needed, swap!



EXAMPLE

Iteration #3

- Compare all adjacent elements up to index **2**.
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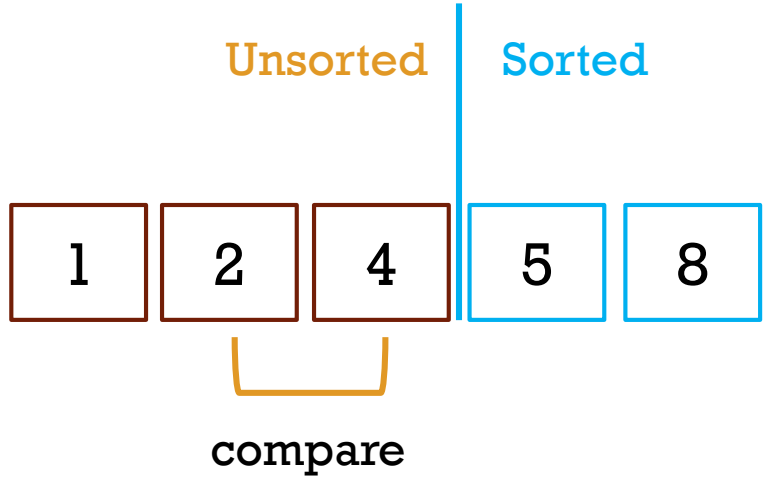


No swap was needed in this iteration → the list is sorted!

EXAMPLE

No swap was needed in the last iteration. We can stop comparing. The list is sorted!

BUBBLE SORT – PSEUDOCODE

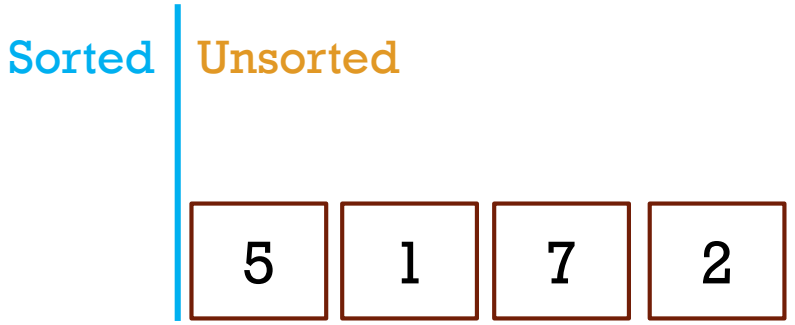
```
sorted = false
i = 0
while (!sorted) {
   sorted = true
   for j from 0 to list.length - i -2 {
      if(list[j] > list[j+1]) {
         swap(list[j], list[j+1])
         sorted = false
      }
   }
   i++
}
```

SELECTION SORT

SELECTION SORT

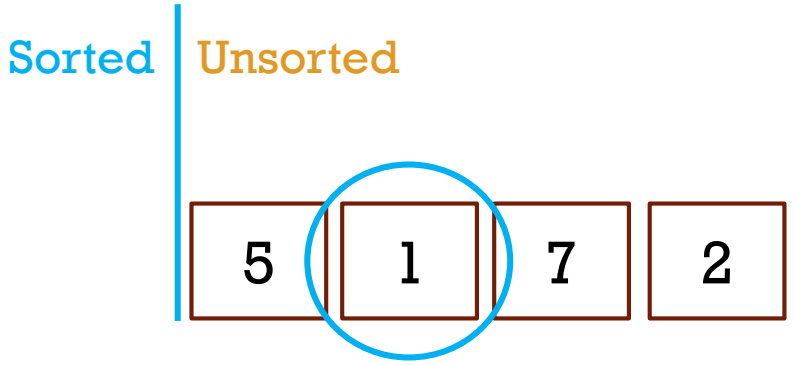
- Goal: order a list of integers in ascending order
- Idea: consider the list as if it was divided into two parts, one sorted and the other unsorted. (note: at the beginning the sorted part is empty)
- Procedure:
  - Select the smallest element in the unsorted part of the list
  - Swap that element with the element in the initial position of the unsorted array
  - Change where you divide the array from the sorted part to the unsorted part.

EXAMPLE



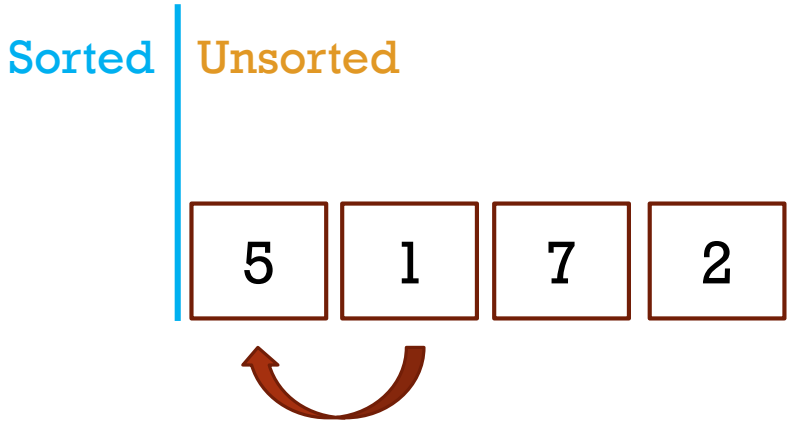
EXAMPLE

- Select



EXAMPLE

- Select
- Swap



EXAMPLE

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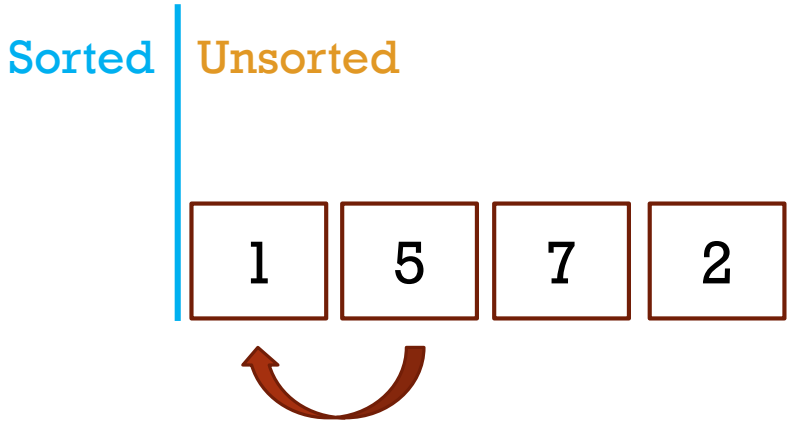
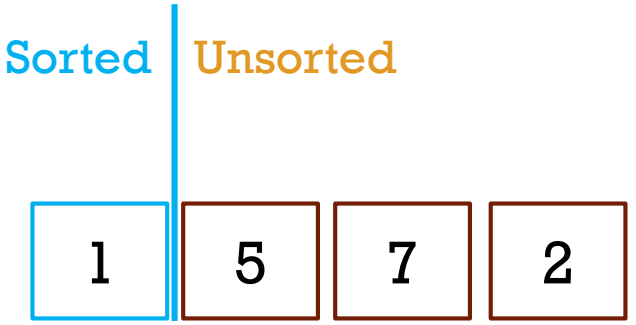


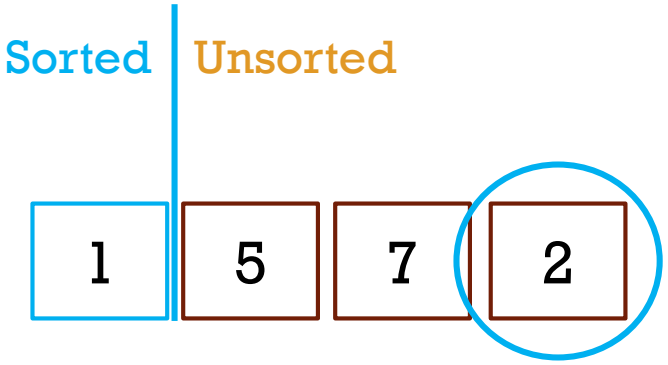
EXAMPLE

- Select
- Swap
- Update delimiter



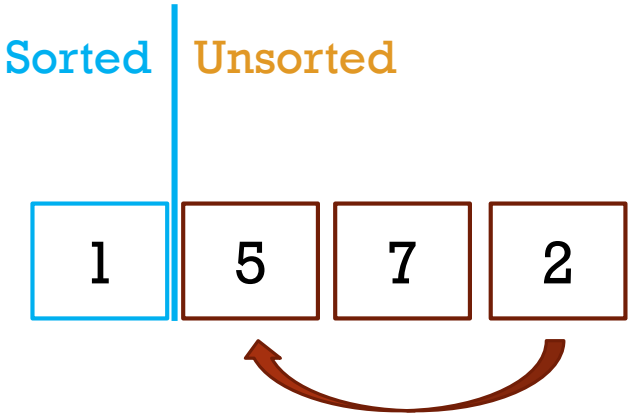
EXAMPLE

- Select



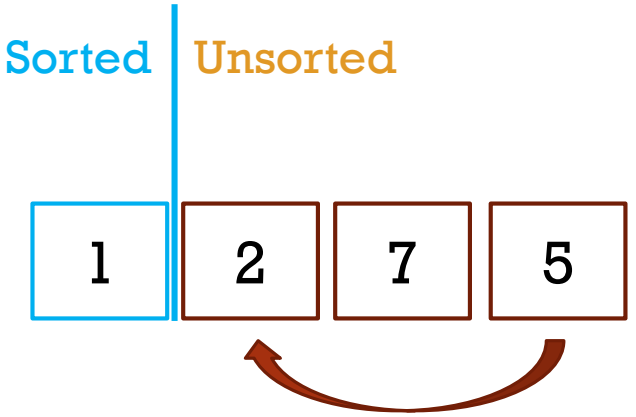
EXAMPLE

- Select
- Swap



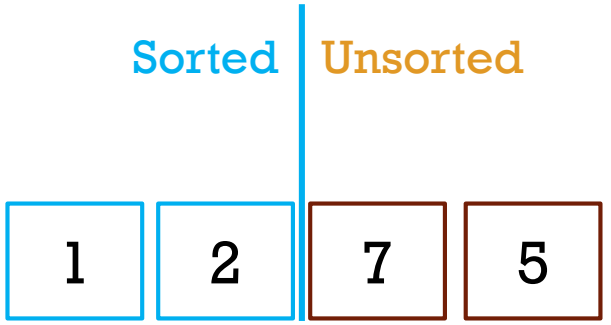
EXAMPLE

- Select
- Swap



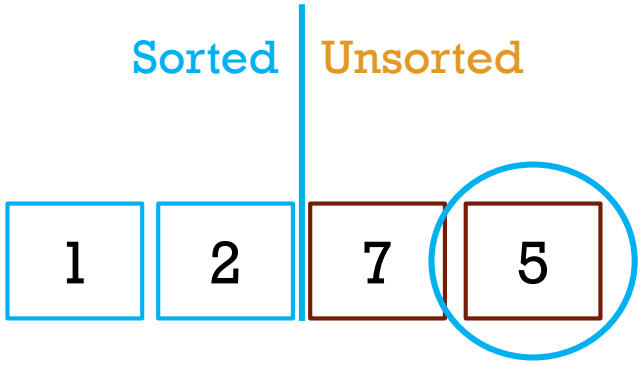
EXAMPLE

- Select
- Swap
- Update



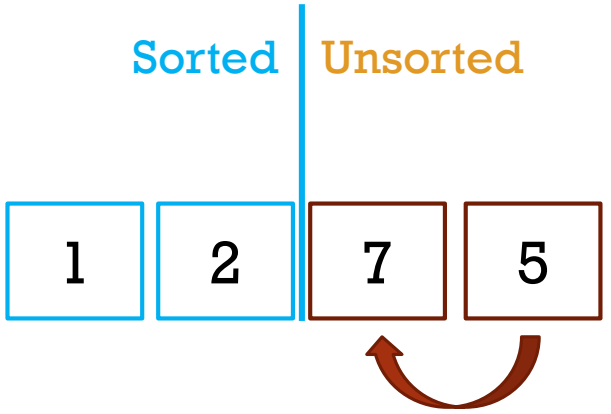
EXAMPLE

- Select



EXAMPLE

- Select
- Swap



EXAMPLE

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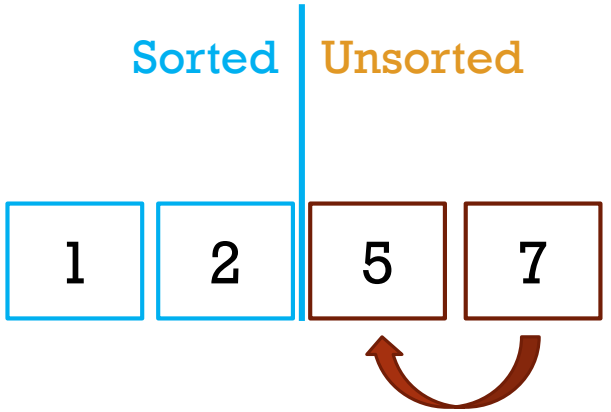

EXAMPLE

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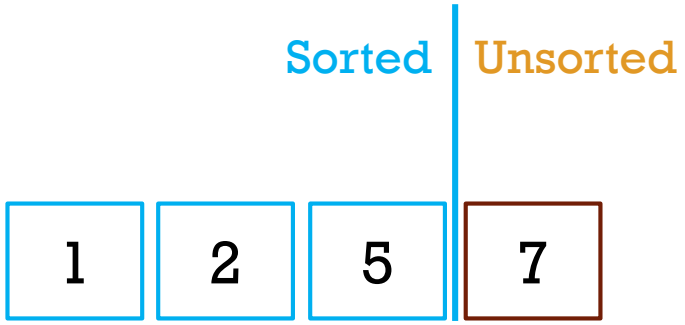
- Swap
- Update



EXAMPLE

- Done!

SELECTION SORT – PSEUDOCODE

```
for delim from 0 to N-2 {
   min = delim
   for i from delim+1 to N-1 {
      if(list[i] < list[min]) {
         min = i
      }
   }
   if(min != delim) {
      swap(list[min], list[delim])
   }
}
```

Repeat until list is all sorted (~N times)

Find the index of the min element in the unsorted part of the list

Swap the min element in the first position of the unsorted part of the list.

SELECTION SORT

```
for delim from 0 to N-2
        for i from delim+1 to N-1
                ...
```

- How many times does the inner loop iterate?

SELECTION SORT

```
for delim from 0 to N-2
        for i from delim+1 to N-1
                ...
```

- How many times does the inner loop iterate?
  - N-1 + N-2 + N-3 +…+ 2 + 1

SELECTION SORT

```
for delim from 0 to N-2
        for i from delim+1 to N-1
                ...
```

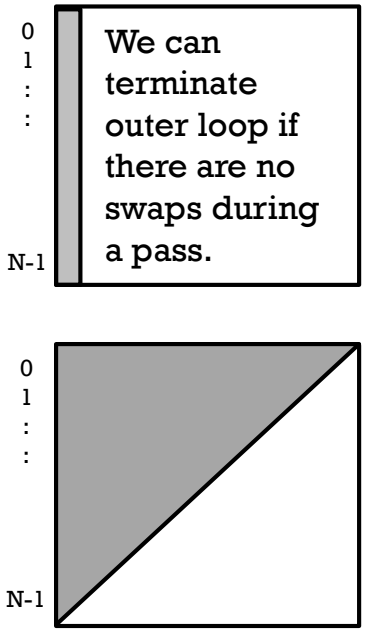
- How many times does the inner loop iterate?
  - N-1 + N-2 + N-3 +…+ 2 + 1 = **N*(N-1)/2**

COMPARISON

Dark area denotes which elements of the list need to be examined at each iteration of the outer loop.

Bubblesort

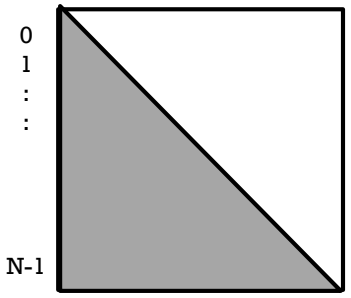
```
while(!sorted)
    for j from 0 to N – 2 – i
```



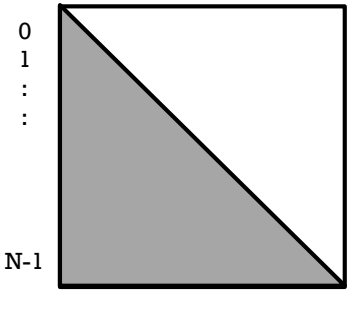
Selection sort

```
for  delim from 0  to N-2
     for i from delim+1  to  N-1
```

Best case



Worst case

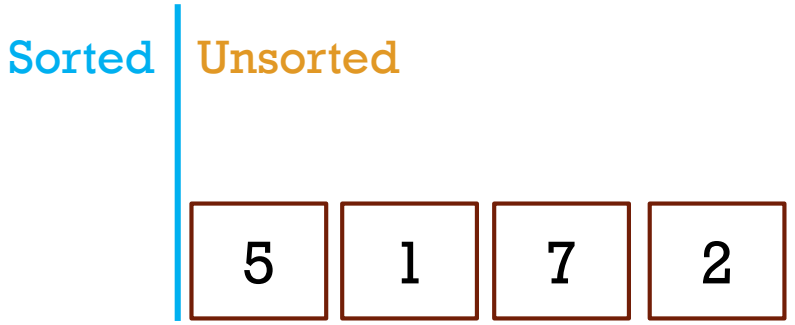


INSERTION SORT

INSERTION SORT

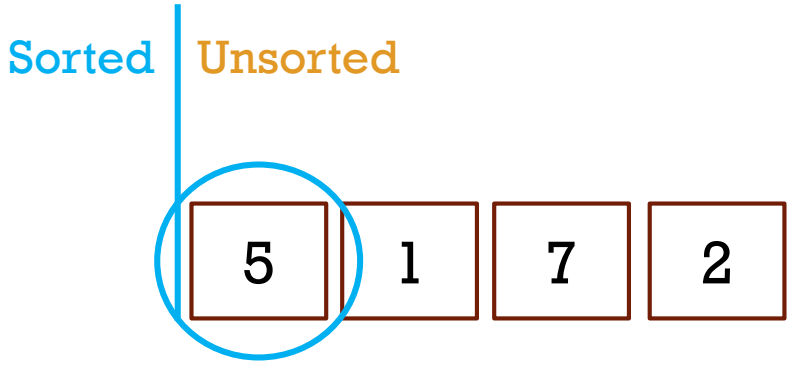
- Goal: order a list of integers in ascending order
- Idea: consider the list as if it was divided into two parts, one sorted and the other unsorted. (note: at the beginning the sorted part is empty)
- Procedure:
  - Select the first element of the unsorted part of the list
  - Insert such element into its correct position in the sorted part of the list.
  - Change where you divide the array from the sorted part to the unsorted part.

EXAMPLE



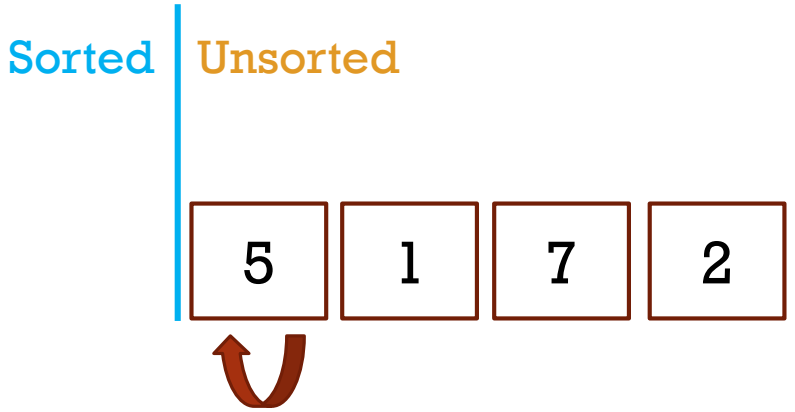
EXAMPLE

- Select



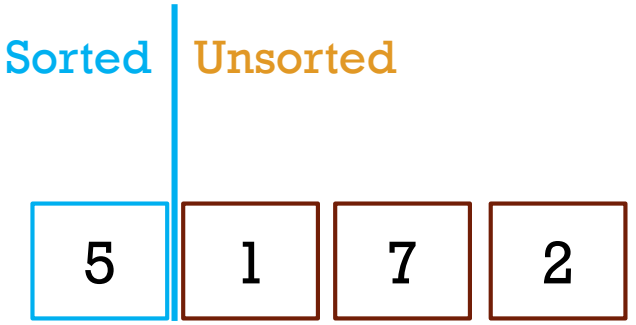
EXAMPLE

- Select
- Insert



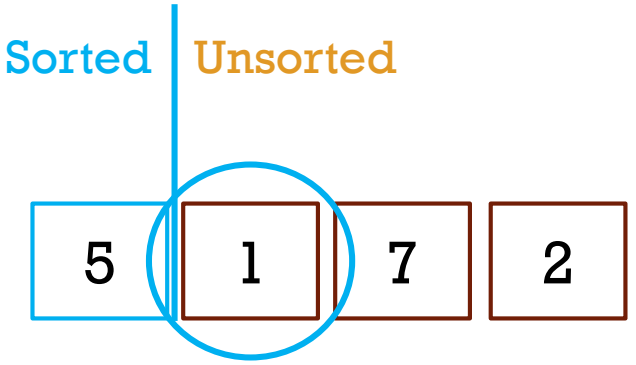
EXAMPLE

- Select
- Insert
- Update



EXAMPLE

- Select



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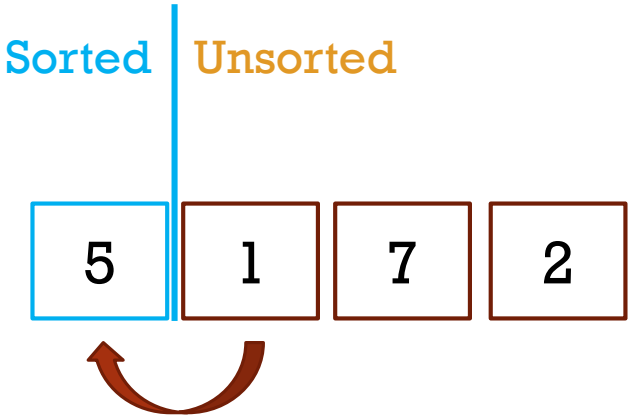
- Insert



EXAMPLE

- Select
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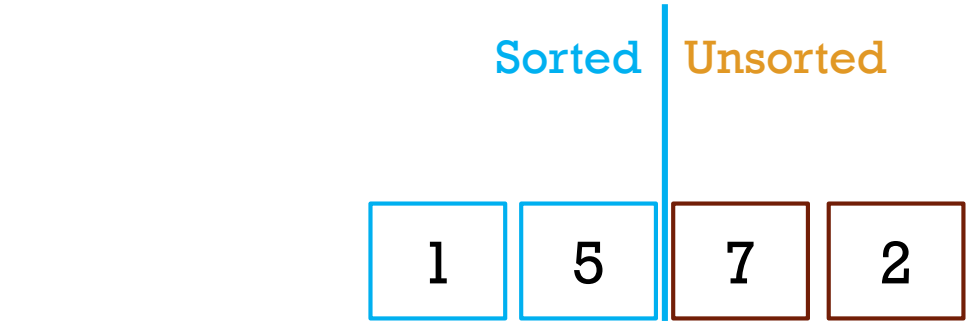
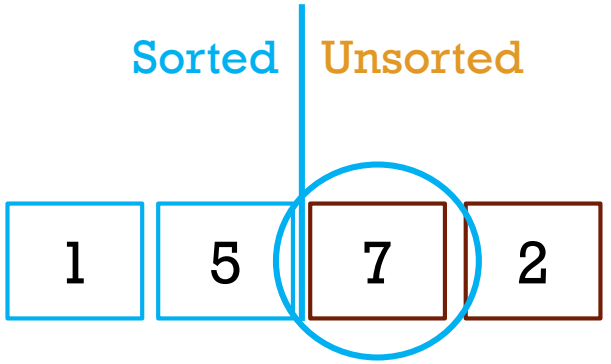


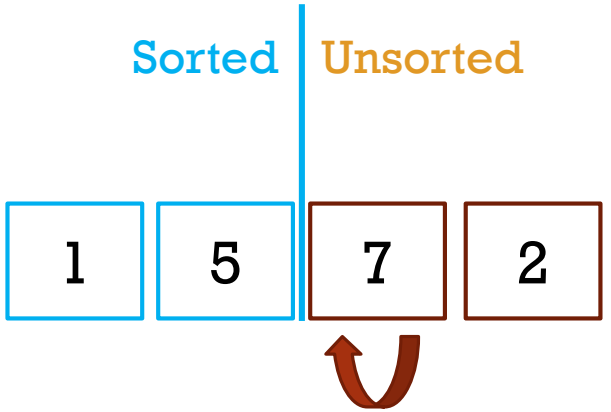
EXAMPLE

- Select



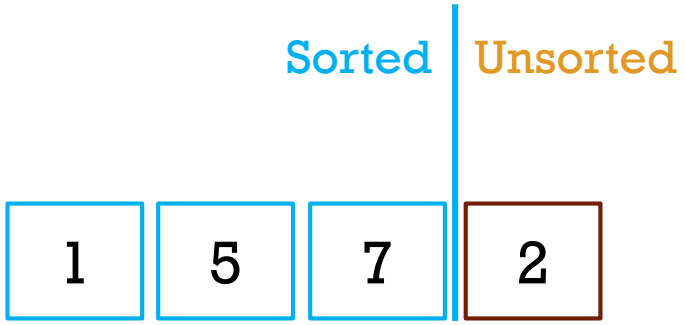
EXAMPLE

- Select
- Insert



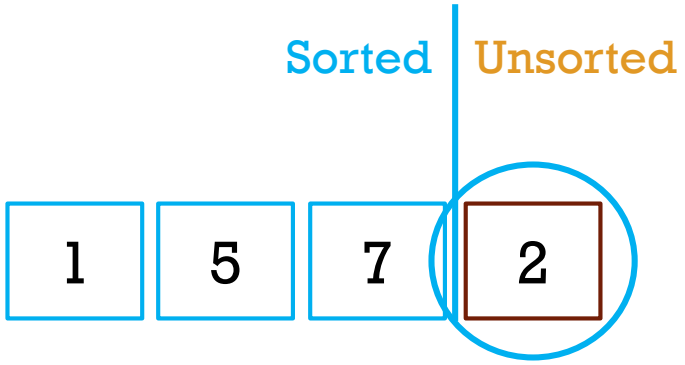
EXAMPLE

- Select
- Insert
- Update



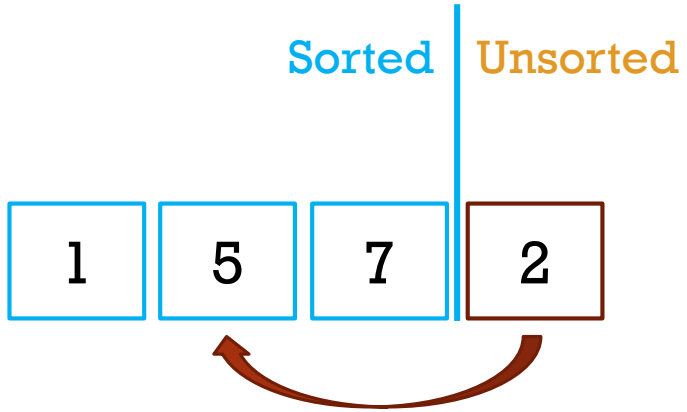
EXAMPLE

- Select



EXAMPLE

- Select
- Insert



EXAMPLE

- Done!

INSERTING

Mechanism is similar to inserting (adding) an element to an array list:

Shift all elements ahead by one position to make a hole, and then fill the hole.

INSERTION SORT – PSEUDOCODE

```
for i from 0 to N-1 {
    element = list[i]
    k = i
    while(k>0 && element<list[k-1]){
        list[k] = list[k-1]
        k--
    }
    list[k] = element
}
```

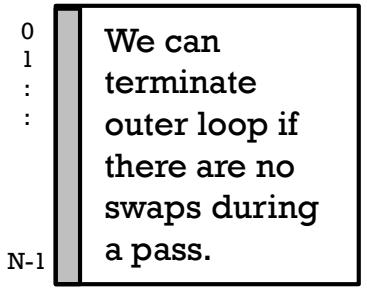
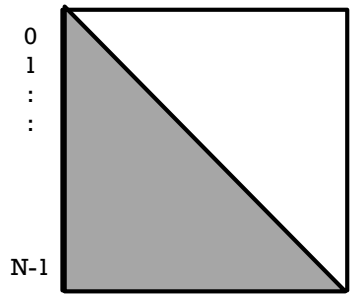
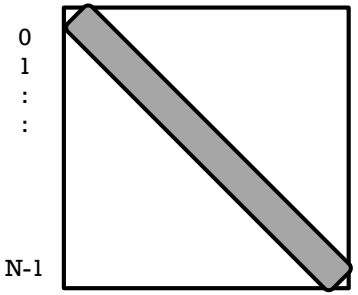
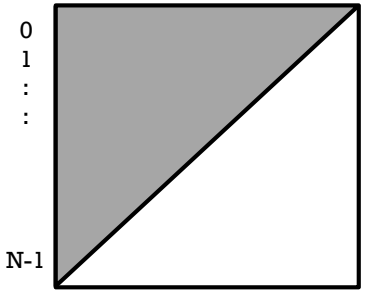
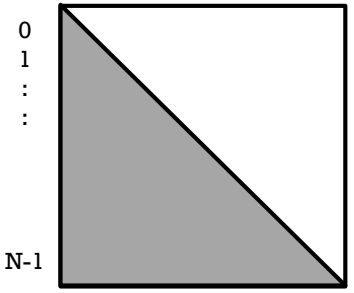
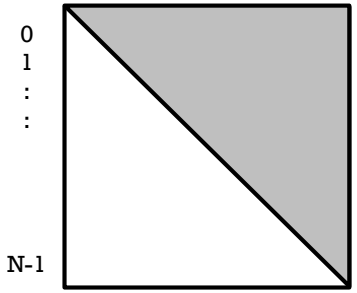
Repeat until list is all sorted (~N times)

Find where the element should be inserted in the sorted part of the list + make space for it (shift all the larger elements to the right)

Insert the element in the sorted part of the list.

COMPARISON OF THE THREE ALGORITHMS

Performance depends highly on initial data. Also, it depends on implementation (array vs. linked list), e.g. what is cost of swap and 'shift'.

	Bubblesort	Selection sort	Insertion sort
	while(!sorted) for j from 0 to N – 2 – i	for delim from 0 to N-2 for i from delim+1 to N-1	for i from 0 to N-1 while ….
Best case			
Worst case			
	Outer loop	Outer loop	Outer loop