Fixed Income Markets Cont'd

FINE 452: Applied Quantitative Finance

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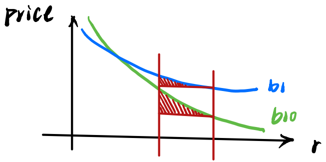
Long-term vs. short-term bonds

Measurement: Duration (like β)

the most important source of risk

- Changes in interest rates have a greater impact on the prices of long-term bonds than on those of short-term bonds.

$$B_0 = \frac{F}{(1+r)^{t}}$$ zero-coupon



Duration

$$B = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$$

$$\frac{dB}{dr} = -\sum_{t=1}^{T} \frac{CF_t}{(1+r)^{t+1}} t = -\frac{1}{1+r} \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} t$$

$$\frac{dB/B}{dr/(1+r)} = -\sum_{t=1}^{T} \left(\frac{\frac{CF_t}{(1+r)^t}}{B} \right) t$$

percentage change in the growth rate.

Definition

- Duration (often called Macaulay duration) is the weighted average of the future times when the bond's cash payments are received, where the weight for each time is the PV of the cash flow received at that time divided by the total PV (i.e. price) of the bond.

$$Duration = \sum_{t=1}^{T} \frac{PV(C_t)}{PV} t \quad \in [1, T]$$

$$\underset{-0.36}{dB_0} = -\underset{3.6}{D} \underset{0.01}{dr}$$

- Duration measures a bond's average maturity.

3 factors

$$Duration = \frac{\frac{CF_1}{1+r}}{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}} \cdot 1 + \frac{\frac{CF_2}{(1+r)^2}}{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}} \cdot 2 + \cdots + \frac{\frac{CF_T}{(1+r)^T}}{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}} \cdot T$$

Properties of Duration

The lower the duration, the less sensitive to fluctuation in interest rates.

- Holding changes in other variables constant:
  1. An increase in the coupon lowers duration
  2. An increase in the interest rate lowers duration
  3. In general, the longer the maturity, the greater the duration.

Later payment are more heavily discounted.

Example

- Bond has
  - Face value \$100.
  - Annual coupon rate 8.5% of face value.
  - Maturity 4 years.
  - Discount rate 3%.
- The PV of the bond is 120.44⇒the duration is

$$
\begin{aligned}
D &= \frac{\frac{8.50}{1.03}}{120.44} \times 1 + \frac{\frac{8.50}{1.03\hat{\ }2}}{120.44} \times 2 + \frac{\frac{8.50}{1.03\hat{\ }3}}{120.44} \times 3 + \frac{\frac{108.50}{1.03\hat{\ }4}}{120.44} \times 4 \\
&= 3.60
\end{aligned}
$$

Duration of Zero-Coupon Bonds

- Zero-coupon bonds repay the principal at maturity, but make no coupon payments along the way.

$\Rightarrow$ the duration of a zero-coupon bond is equal to its maturity.

- Coupon-paying bonds deliver cash payments earlier than zero-coupon bonds with the same maturity and, therefore, have shorter duration (i.e., they are less sensitive to interest rate fluctuations and have higher prices than strips with the same maturity.)

Immunization

Immunization

Immunization is a bond hedging strategy where you match the asset duration with the liability duration.

- Immunization theory tries to eliminate sensitivity to shifts in the term structure by matching the duration of the assets to the duration of the liabilities.

  $D = \frac{-dB/B}{dr/(1+r)}$ small change)

- If duration is truly a measure of sensitivity to interest rate shifts, a shift in the term structure will have the same impact on the present value of both assets and liabilities $\Rightarrow$ will leave unchanged the ability of the program to meet any obligations.

7-year zero-coupon bond

1. Immunization: An insurance company must make a payment of $19, 487 in 7 years. The market interest rate is 10%. The company's portfolio manager wishes to fund the obligation using 3-year zero-coupon bonds and perpetuities paying annual coupons. This problem will take you through the steps that the manager will follow to immunize the obligation.

   ↑FV

   infinite coupon bond

   a. What is the duration of the liability?
   b. What is the duration of the asset portfolio? (Note: the duration of a perpetuity is $\frac{1+y}{y}$ where y is the yield to maturity; the duration of a portfolio of assets is the weighted sum of the durations of the individual assets in the portfolio, where the weight of each asset is the fraction of the portfolio value that is accounted for by that asset .)

      or interest rate (flat term)

      $D = \sum_{i=1}^{n} w_i D_i$
   c. How can the asset mix in the portfolio be chosen so as to make the duration of the portfolio of assets equal to the duration of the liability?
   d. Once a position has been immunized, does the manager need to further rebalance the portfolio in the future? Why or why not?

Answer:

a. The liability is like a 7-year zero-coupon bond. Therefore, it has a duration of 7 years.
b. The asset portfolio is comprised of 3-year zero-coupon bonds and a perpetuity. The duration of the 3-year zero-coupon bond is 3 years and the duration of the perpetuity is 11 years. Therefore, the duration of the asset portfolio is

   $$w \times 3 + (1-w) \times \frac{1+0.1}{0.1}$$

   where w is the fraction of the portfolio invested in the 3-year zero-coupon bond.
c. To find the asset mix, we set the duration of the asset portfolio equal to that of the liability, and solve for w:

   $$w \times 3 + (1-w) \times \frac{1+0.1}{0.1} = 7$$

This implies that w=1/2. The manager should invest half the portfolio in the zero-coupon bond and half in the perpetuity. This will result in an asset duration of 7 years. $\frac{19487}{(1.1)^7} = 10{,}000$

The obligation has a present value of . The asset portfolio will be invested equally in the 3-year zero-coupon bond and the perpetuity . Therefore, to fully fund the obligation, the manager must purchase $5,000 of the zero-coupon bond and $5,000 of the perpetuity.

at date 1:

$3w + 11(1-w) = 6$

- maturity change
- interest rate might also change

d. Even if a position is immunized, there is the need to rebalance the asset portfolio in response to changes in interest rates. Moreover, even if interest rates do not change, the passage of time also will affect duration and require rebalancing.

Immunization is <u>dynamic</u>

Algorithm: $3w + 11(1-w) - T_{new} > \delta$ trigger rebalancing.