BONDS

Required Reading

- **Chapter 6,** "*Bonds*" from J. Berk et al., Fundamentals of Corporate Finance, Second Canadian Edition.

Bond: A security sold by a government or a corporation to raise money from investors in exchange for a promised future payment.

Examples. T-Bills, T-Notes, T-Bonds, Commercial Paper, Corporate Bonds, Municipal Bonds, Mortgage-backed securities, Asset-backed Securities, etc.

Bonds: Definitions

Maturity (*n*): The final repayment date of a bond.

Coupons Payments (*CPN*): The promised interest payments of a bond, paid periodically until maturity.

Face Value (*FV*): The notional amount of the bond used to compute the interest. Typically the faced value is repaid at maturity. [Also called *par value* or *principal value*.]

Bonds: Definitions

Yield to Maturity (YTM):

- The **YTM** is the *single* discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.

[**Note***:* Yields are typically quoted like APR so to discount you need to find the corresponding effective rate.]

Bonds: Definitions

Coupon Rate: The sum of all the coupon payments during a year divided by the face value of the bond.

- The coupon rate determines the amount of each coupon payment of a bond.

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

Example: A "*$1000 bond with a 10% coupon rate and semiannual payments*" will pay the following coupon payments every six months:

$$CPN = \frac{0.1 \times \$1000}{2} = \frac{\$100}{2} = \$50$$

BONDS vs. STOCKS

Bonds vs stocks (2008, end of year)

Bonds: ≈$66,000 billions

Stocks: ≈$32,000 billions

Bonds vs stocks (2012, end of year)

Bonds: ≈$81,000 billions

Stocks: ≈$55,000 billions

Total Debt Securities (billions of US $)

Q4-2013	All	All	Government	Corporations
Canada	$2,188	2%	$1,294	$894
China	$4,094	5%	$1,507	$2,587
Germany	$4,357	5%	$2,255	$2,102
Japan	$12,244	13%	$9,019	$3,225
US	$36,942	41%	$14,819	$21,896
TOTAL	$90,896		$41,424	$49,242

Source: BIS, Q4-2013

- US GDP: $17,311 billions
- Canada GDP: $1,821 billions

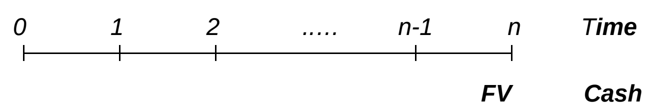
Average Daily Trading Volume in the US Bond Markets ($Billions)

	Municipal	Treasury	Agency MBS	Corporate Debt	Federal Agency Securities	Total
2003	12.6	433.5	206.0	18.0	81.7	751.8
2005	16.9	554.5	251.8	16.7	78.8	918.7
2007	25.0	570.2	320.1	16.4	83.0	1,014.7
2008	19.2	553.1	344.9	11.8	104.5	1,033.4
2009	12.3	407.9	299.9	16.8	77.7	814.5
2010	13.3	528.2	320.6	20.5	11.2	893.7
2012	11.3	518.9	280.4	16.7	9.7	843.1
2013	11.2	545.4	222.8	18.1	6.5	809.4

Sources: Federal Reserve Bank of New York, Municipal Securities Rulemaking Board

Zero-Coupon Bonds

- **Payment Structure of a Zero-Coupon Bond**:



In words: A zero coupon bond just pays its face value (***FV***) at maturity (***n***).

Example: Treasury bills are zero-coupon government bonds with maturity of up to one year.

Zero-Coupon Bonds

- **Price and Yield to Maturity of a Zero-Coupon Bond:**

$$P = \frac{FV}{(1+YTM_n)^n}$$

In the above formula $\boldsymbol{YTM_n}$ is the yield to maturity expressed as an *effective rate* per period for holding the bond from today until maturity $\boldsymbol{n}$ periods in the future.

[**Note**: If $\boldsymbol{n}$ is not in years, $\boldsymbol{YTM_n}$ will not be an effective rate per year. For example, if n is 11 semi-annual periods, YTM_{11} would be an effective rate per semi-annual period for a zero-coupon bond held for 11 semi-annual periods until maturity.]

Example

Consider the following zero-coupon bonds maturing in 1, 2, 3, and 4 years trading at the prices shown below per $100 face value. Find their YTM.

Maturity	**1 Year**	**2 Years**	**3 Years**	**4 Years**
Price	$96.62	$92.45	$87.63	$83.03

$$P = \frac{FV}{(1+YTM_n)^n} \Rightarrow YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1$$

$$YTM_1 = \left(\frac{100}{96.62}\right) - 1 = 3.50\% \qquad YTM_2 = \left(\frac{100}{92.45}\right)^{1/2} - 1 = 4.00\%$$

$$YTM_3 = \left(\frac{100}{87.63}\right)^{1/3} - 1 = 4.50\% \qquad YTM_4 = \left(\frac{100}{83.03}\right)^{1/4} - 1 = 4.75\%$$

Example

Consider a risk-free zero-coupon bond with a face value of $100 maturing in two years. The current one-year risk-free interest rate is 2%. It is also known with certainty that the one-year risk-free interest rate will be 4% the following year.

a) Find the current price of the bond and its yield to maturity:

$$P_0 = \frac{100}{(1+0.02)(1+0.04)} = \$94.27$$

$$YTM_2 = \left(\frac{FV}{P_0}\right)^{1/2} - 1 = \left(\frac{100}{94.27}\right)^{1/2} - 1 = 2.9951\%$$

Example (cont.)

b) At what price will the bond be trading in year one :

$$P_1 = \frac{100}{(1+0.04)} = \$96.15$$

c) What is the return of buying the bond today and selling it at time 1 (i.e., in one year):

$$\text{Return} = \frac{P_1 - P_0}{P_0} = \frac{96.15 - 94.27}{94.27} = 2\%$$

d) What is the return of buying the bond at time 1 (i.e., in one year) and getting its FV at time 2:

$$\text{Return} = \frac{FV - P_1}{P_1} = \frac{100 - 96.15}{96.15} = 4\%$$

Example (cont.**)**

Notice that

- the return of holding the bond during the first year is 2%.
- the return of holding the bond during the second year is 4%.
- the *two-year* return of holding the bond is:

$$\text{Return} = \frac{\boldsymbol{FV} - \boldsymbol{P}_0}{\boldsymbol{P}_0} = \frac{100 - 94.27}{94.27} = 6.08\%$$

[*Note*: (1+0.02) x (1+0.04) -1 = 6.08%]

- the YTM_2 is the return per year of holding the bond from time 0 until maturity in 2 years:

$$\boldsymbol{YTM}_2 = (1 + 0.0608)^{1/2} - 1 = 2.9951\%$$

Coupon Bonds

- **Typical Payment Structure of a Coupon Bond**:

0	*1*	*2*	*3 .....*	*n-1*	*n*	***Time***
	CPN	***CPN***	***CPN***	***CPN***	***CPN+FV***	***Cash***

In words: A coupon bond pays regular coupon interest payments (***CPN***) up to maturity (***n***), when it also pays its face value (***FV***).

Example: Government of Canada bonds are sold with maturities of 2, 5, 10, and 30 years.

Coupon Bonds

Return on a coupon bond comes from:

- The difference between the purchase price and the principal value.
- Periodic coupon payments.

Coupon Bonds

- **Price of a Coupon Bond:**

$$P = \frac{CPN}{YTM_n}\left(1 - \frac{1}{(1+YTM_n)^n}\right) + \frac{FV}{(1+YTM_n)^n}$$

where YTM_n is the yield to maturity expressed in the form of an effective rate per coupon interval.

Note: *YTM* are typically quoted as APRs. For example, bonds with semi-annual payments normally have their yields to maturity quoted as an APR with semi-annual compounding. In the formula, we need to use the YTM expressed as an effective rate per coupon interval.

Example

	Bond A
Maturity (years)	15
Par Value	$1,000
Coupon Rate	3%

Calculate the price of Bond A assuming that its YTM is 8% (APR compounded annually) and that it pays one coupon per year.

$$\boldsymbol{CPN} = 0.03 \times 1000 = \$30 \text{ every year}$$

$$\boldsymbol{YTM} = 8\% \text{ APR} \Rightarrow \frac{8\%}{1} = 8\% \text{ Effective rate per year}$$

$$\boldsymbol{P_A} = \frac{30}{0.08}\left(1 - \frac{1}{1.08^{15}}\right) + \frac{1{,}000}{1.08^{15}} = \$572.03$$

Example

Assume that it is May 15, 2012 and the Government of Canada has just issued securities with May 2017 maturity, $1000 par value, and a 5% coupon rate with semiannual coupons. The first coupon payment will be paid on November 15, 2012. Assuming that the YTM is 6% (APR compounded semi-annually) , calculate the price of the bond.

Solution

$\boldsymbol{FV} = \$1000$

$$\boldsymbol{CPN} = \frac{0.05 \times \$1000}{2} = \$25 \text{ every six months}$$

$$\boldsymbol{YTM} = \frac{6\%}{2} = 3\% \text{ (effective semi-annual rate)}$$

[**Note**: The YTM is quoted as an APR compounded semi-annually, and hence, we divide by 2 to find the YTM expressed as an effective semi-annual rate.]

$\boldsymbol{n}$ = 5 years x 2 = 10 six-month periods

[**Note**: The bond matures in five years and the coupons are paid semiannually, therefore, there are 10 six-month periods until maturity.]

Solution (cont)

Based on the information from the previous slide we can now calculate the price of the bond:

$$\boldsymbol{P} = \frac{25}{0.03}\left(1 - \frac{1}{(1+0.03)^{10}}\right) + \frac{1000}{(1+0.03)^{10}} = \$957.35$$

Example

Buy&Sell.com has semiannual coupon bonds with a 10% coupon rate, a FV of $1,000, and 5 years to maturity. The current price of the bond is $859.52. What is the YTM (expressed as an APR with semiannual compounding)? What is the effective annual yield?

Solution

- *YTM:*

$$859.52 = \frac{1{,}000 \times 0.1 \div 2}{\boldsymbol{YTM}}\left(1 - \frac{1}{(1+\boldsymbol{YTM})^{10}}\right) + \frac{1{,}000}{(1+\boldsymbol{YTM})^{10}}$$

Solving: *YTM is* 7% (This is the yield to maturity expressed as an effective semi-annual rate for holding the bond until maturity.)

Hence 2 x 7% = 14%, is the *YTM* quoted as an APR with semi-annual compounding.

- Effective Annual Yield:

$$\boldsymbol{EAR} = \left(1 + \frac{0.14}{2}\right)^2 - 1 = 14.49\%$$

Example

Consider a risk-free bond with a face value of \$1000, that matures in two years, pays one coupon of \$10 each year, and that has just made a coupon payment. The current one-year risk-free interest rate is 1%. It is also known with certainty that the one-year risk-free interest rate will be 3% the following year.

Find the (a) current price of the bond and (b) its yield to maturity.

a) Bond Price:

0	1	2 **Year**
	\$10	\$10+\$1000

$$P_0 = \frac{10}{1+0.01} + \frac{1010}{(1+0.01)(1+0.03)} = \$980.78$$

Example (cont.)

b) The YTM is the *single* discount rate that sets the present value of the promised bond payments equal to the current market price of the bond:

$$P_0 = \frac{10}{(1+YTM)} + \frac{1010}{(1+YTM)^2} = \$980.78$$

Solving the above equation: ***YTM*** = 1.99%.

Interpretation: The return of holding the bond from now until maturity in two years if there is no default is 1.99% per year.

Bond Prices

- A bond **trades at par** when its price is **equal** to the face value.
- A bond **trades at a premium** when its price is **greater than** its face value.
- A bond **trades at a discount** when its price **is lower** than its face value.

→ Zero-coupon bonds always trade for a discount.

→ Coupon bonds may trade at par, at a discount or at a premium.

Bond Prices *Just After* a Coupon Payment

1. If the *Coupon Rate* is equal to the *YTM* (expressed as an APR) then the **bond trades** at par just after a coupon payment.
2. If the *Coupon Rate* is larger than the *YTM* (expressed as an APR) the **bond trades at a premium** just after a coupon payment.
3. If the *Coupon Rate* is smaller than the *YTM* (expressed as an APR) the **bond trades at a discount** just after a coupon payment.

Note: For example, if the coupon is paid semi-annually, you would need to compare the coupon rate to a the *YTM expressed as an APR with semi-annual compounding.*

Example

Consider three 10-year bonds with *semiannual* coupon payments. One bond has a 4% coupon rate, one bond has a 6% coupon rate, and one bond has a 8% coupon rate. If the yield to maturity of these bonds is 6% (APR with semiannual compounding), what is the price of each bond per $100 face value when they are issued?

Example (cont)

•When the coupon rate (4%) is smaller that the YTM (APR with semiannual compounding) (6%) the bond trades at a discount:

$$\boldsymbol{P}(4\%\text{ Coupon}) = \frac{2}{0.03}\left(1 - \frac{1}{1.03^{20}}\right) + \frac{100}{1.03^{20}} = \$85.12\text{ (at discount)}$$

•When the coupon rate (6%) is equal to the YTM (APR with semiannual compounding) (6%) the bond trades at par:

$$\boldsymbol{P}(6\%\text{ Coupon}) = \frac{3}{0.03}\left(1 - \frac{1}{1.03^{20}}\right) + \frac{100}{1.03^{20}} = \$100\text{ (at par)}$$

•When the coupon rate (8%) is greater that the YTM (APR with semiannual compounding) (6%) the bond trades at a premium:

$$\boldsymbol{P}(8\%\text{ Coupon}) = \frac{4}{0.03}\left(1 - \frac{1}{1.03^{20}}\right) + \frac{100}{1.03^{20}} = \$114.88\text{ (at premium)}$$

Example

Consider three 30-year bonds with *annual* coupon payments. One bond has a 3% coupon rate, one bond has a 5% coupon rate, and one bond has a 10% coupon rate. If the yield to maturity of these bonds is 5%, what is the price of each bond per $100 face value when they are issued?

Example (cont)

$$P(3\%\ \text{Coupon}) = \frac{3}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$69.26\ (\text{at discount})$$

$$P(5\%\ \text{Coupon}) = \frac{5}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$100.00\ (\text{at par})$$

$$P(10\%\ \text{Coupon}) = \frac{10}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86\ (\text{at premium})$$

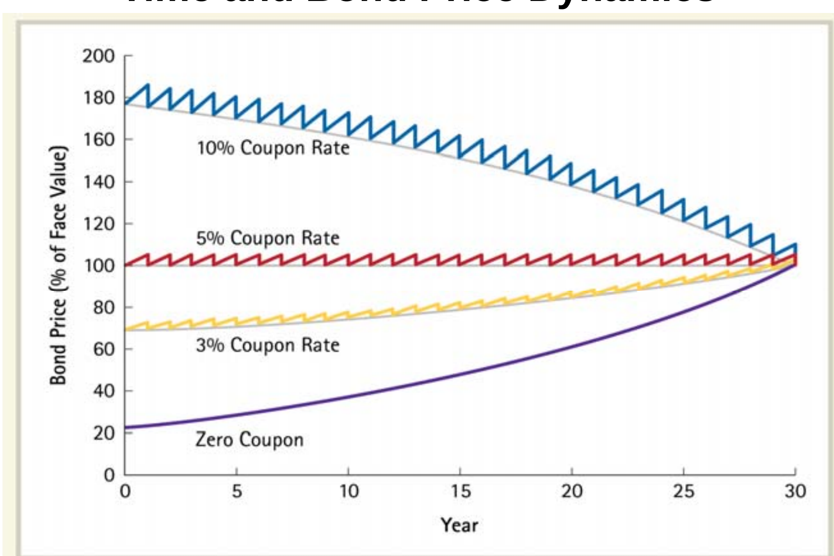
Time and Bond Price Dynamics

Example

Consider the three 30-years bonds of the previous example and calculate the prices 10 years, 5 years, and 1 year before maturity, and at maturity assuming that the YTM remains constant at 5%. (**Note**: **The bond prices below are calculated just after the coupon is paid for the corresponding year**.)

Coupon Rate / *Years to Maturity*	3% Coupon	5% Coupon	10% Coupon
30	$69.26	$100.00	$176.86
29	$69.72	$100.00	$175.71
10	$84.56	$100.00	$138.61
5	$91.34	$100.00	$121.65
1	$98.09	$100.00	$104.76
0	$100.00	$100.00	$100.00

Time and Bond Price Dynamics



Effect of time on bond prices when the *yield remains constant.*

Return From Holding the Bonds for One Year

(from 30 years to maturity to 29 years to maturity)

Coupon Rate / *Years to Maturity*	3% Coupon	5% Coupon	10% Coupon
30	$69.26	$100.00	$176.86
29	$69.72	$100.00	$175.71

$$\text{Return (3\% Coup. Bond)} = \frac{(3 + 69.72) - 69.26}{69.26} = 5\%$$

$$\text{Return (5\% Coup. Bond)} = \frac{(5 + 100) - 100}{100} = 5\%$$

$$\text{Return (10\% Coup. Bond)} = \frac{(10 + 175.71) - 176.86}{176.86} = 5\%$$

Time and Bond Price Dynamics

If a bond's yield to maturity is known to remain constant, the per period return of an investment in the bond equals its yield to maturity even if you sell the bond early.

→ *In the previous example the YTM of the three bonds is known to remain constant at 5%. Let's go back to this example****....***

Return From Holding the Bonds for One Year
(from 30 years to maturity to 29 years to maturity)

For the bond trading at par:

$$\text{Return (5\% Coup. Bond)} = \frac{(5+100)-100}{100} = 5\%$$

The return of holding the bond can be decomposed in two parts:

$$\text{(1) Return from coupon} = \frac{5}{100} = 5\%$$

$$\text{(2) Return from price change} = \frac{100-100}{100} = 0\%$$

$$(1) + (2) = \text{Return (5\% Coup. Bond)} = 5\% + 0\% = 5\%$$

Hence for the bond trading at par the entire return comes from the coupon payment (i.e., there is no price change).

Return From Holding the Bonds for One Year
(from 30 years to maturity to 29 years to maturity)

For the bond trading at a discount:

$$\text{Return (3\% Coup. Bond)} = \frac{(3+69.72)-69.26}{69.26} = 5\%$$

The return of holding the bond can be decomposed in two parts:

$$\text{(1) Return from coupon} = \frac{3}{69.26} = 4.3\%$$

$$\text{(2) Return from price change} = \frac{69.72-69.26}{69.26} = 0.7\%$$

$$(1) + (2) = \text{Return (3\% Coup. Bond)} = 4.3\% + 0.7\% = 5\%$$

Hence for the bond trading at a discount, the return comes from both, the coupon payment and the increase in price.

Return From Holding the Bonds for One Year
(from 30 years to maturity to 29 years to maturity)

For the bond trading at a premium:

$$\text{Return (10\% Coup. Bond)} = \frac{(10 + 175.71) - 176.86}{176.86} = 5\%$$

The return of holding the bond can be decomposed in two parts:

$$\text{(1) Return from coupon} = \frac{10}{176.86} = 5.7\%$$

$$\text{(2) Return from price change} = \frac{175.71 - 176.86}{176.86} = -0.7\%$$

$$(1) + (2) = \text{Return (10\% Coup. Bond)} = 5.7\% - 0.7\% = 5\%$$

Hence for the bond trading at a premium, the return from the coupon is greater than 5% but there is a decrease in price that makes the return of holding the bond for a year equal to 5%.

Main Risk Factors in Bond Valuation

- **Interest Rate Risk**

  The price of the bond changes due to changes in the interest rate.

- **Credit Risk**

  Bonds typically have some probability of default.

Interest Rate Risk

- The price of the bond decreases (resp. *increases*) when the interest rate rises (resp. *falls).* That is, *bond prices and interest rates move in the opposite direction.*

- **Interest Rate Risk**: The risk that arises for bond owners from fluctuating interest rates.

  → All other things equal, the prices of bonds with longer maturities are more sensitive to changes in interest rates.

  → All other things equal, the prices of bonds with higher coupons rates are less sensitive to changes in interest rates

Example 1

	Bond A	**Bond B**
Maturity	15 years	30 years
Par Value	\$1,000	\$1,000
Coupon rate	8%	8%

(Assume Annual Coupon Payments)

- Calculate the bond price if the yield is 8%:

$$P_A = P_B = \$1{,}000$$

- Calculate the bond price if the yield increases to 8.1%:

$$P_A = \frac{1{,}000 \cdot 0.08}{0.081}\left(1 - \frac{1}{1.081^{15}}\right) + \frac{1{,}000}{1.081^{15}} = \$991.49$$

$$P_B = \frac{1{,}000 \cdot 0.08}{0.081}\left(1 - \frac{1}{1.081^{30}}\right) + \frac{1{,}000}{1.081^{30}} = \$988.85$$

Hence, the price of bond A drops by 0.851% while bond B drops by 1.115%

Example 2

	Bond A	Bond B
Maturity	15 years	15 years
Par Value	\$1,000	\$1,000
Coupon rate	8%	3%

(Assume Annual Coupon Payments)

- Calculate the bond price if the yield rate is 8%:

  P_A=\$1,000 ; P_B =\$572

- Calculate the bond price if the yield increases to 8.1%:

  P_A=\$991.49 ; P_B =\$566.12

Hence, the price of bond A drops by 0.851% while bond B drops by 1.03%.

Credit Risk

- **Credit Risk** is the risk of default by the issuer of the bond, so that the bond's cash flows are not known with certainty.

- To compensate for the default risk, investors demand a higher promised payment.

Credit Risk

Interest on Five-Year Bonds (Dec-2010)

Borrower	Interest Rate
Government of Canada	2.39%
Bank of Montreal	3.58%
TD Bank	3.58%
CIBC	3.60
Rogers Communications	3.76%
GE Capital	3.88%
Shaw Communications	4.14%

Bond Ratings

- Several companies rate the creditworthiness of bonds: Standard & Poor's and Moody's
- These ratings help investors assess creditworthiness
- **From Standard & Poor's**: "Credit ratings are forward-looking opinions about credit risk. Standard & Poor's credit ratings express the agency's opinion about the ability and willingness of an issuer, such as a corporation or state or city government, to meet its financial obligations in full and on time."

Bond Ratings

Moody's	Standard & Poor's	Description (Moody's)
Investment Grade Debt		
Aaa	AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa	AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A	A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa	BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.

Speculative Bonds		
Ba	BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B	B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa	CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca	CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C	C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

Source: www.moodys.com.

AAA Firm's Balance Sheet

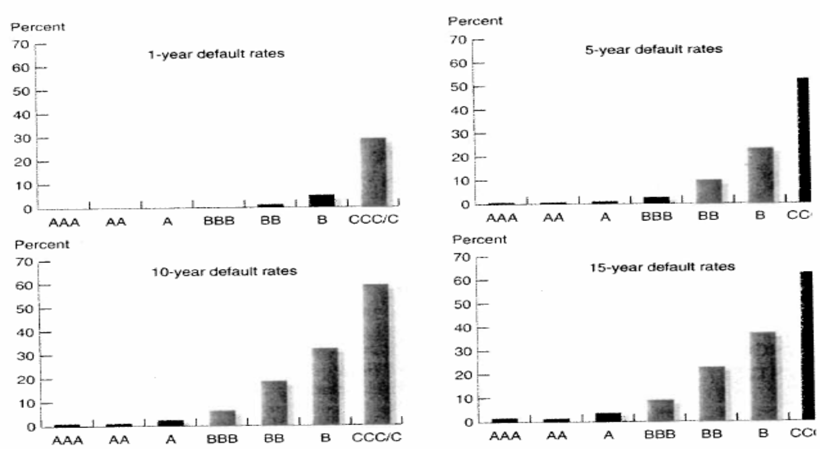
	MSFT		JNJ		ADP		XOM	
Short Term Debt	$	2,000	$	5,435	$	730	$	2,157
Long Term Debt	$	3,746	$	8,179	$	43	$	7,117
Total Debt	$	5,746	$	13,614	$	773	$	9,274
Market Cap	$	237,310	$	166,200	$	20,280	$	352,310
D/E		2.4%		8.2%		3.8%		2.6%
D/E (April 2009)		0%		5.7%		0.3%		2.1%

Note:

- Data in $Millions
- Automatic Data Processing (NYSE:ADP) ; Johnson & Johnson (NYSE:JNJ) ; Microsoft (NASDAQ:MSFT) ; ExxonMobil (NYSE:XOM)

Bond Ratings and Default Probabilities

EXHIBIT 8.7
Cumulative Default Rates for U.S. Corporate Bonds Rated by Standard & Poor's, 1981–2009 (%)



Average Recovery Rates (Moody's)

EXHIBIT 7

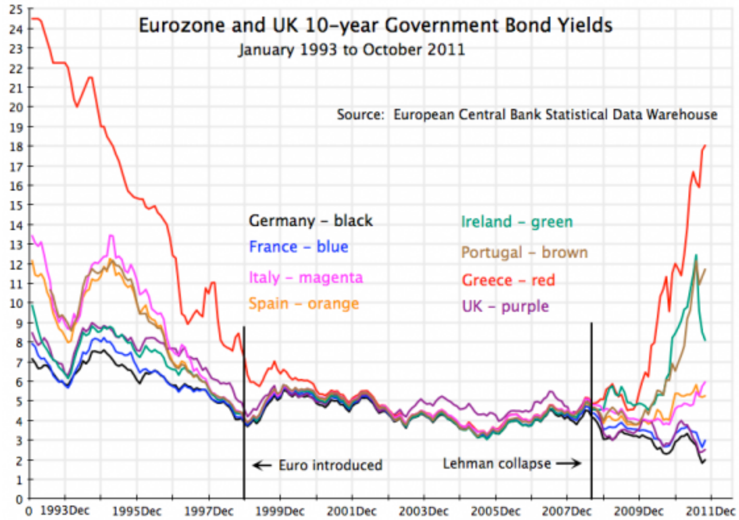
Average Corporate Debt Recovery Rates Measured by Post-Default Trading Prices

	Issuer-weighted			Value-weighted		
Lien Position	2010	2009	1982-2010	2010	2009	1982-2010
1st Lien Bank Loan	70.9%	53.6%	65.8%	72.3%	56.3%	59.6%
2nd Lien Bank Loan	18.2%	16.0%	29.1%	18.4%	20.8%	27.9%
Sr. Unsecured Bank Loan	n.a.	32.6%	47.8%	n.a.	37.9%	39.9%
Sr. Secured Bond	62.5%	37.5%	50.8%	54.7%	29.6%	49.1%
Sr. Unsecured Bond	49.5%	37.1%	36.7%	63.8%	35.5%	37.4%
Sr. Subordinated Bond	33.5%	22.4%	30.7%	39.4%	18.0%	25.3%
Subordinated Bond*	33.7%	45.3%	31.3%	32.2%	25.1%	24.2%
Jr. Subordinated Bond	n.a.	n.a.	24.7%	n.a.	n.a.	17.1%

Credit Risk and Yields

- The yield to maturity of as bond is calculated using the *promised* cash flows instead of the *expected* cash flows.
- Hence, the yield to maturity of a bond that has default risk is not equal to the expected return of investing in the bond.
- A higher yield to maturity does not necessarily imply that a bond's expected return is higher.

Credit Risk and Yields



Example *(A Bond that it is Certain to Default)*

Consider a one-year zero-coupon bond with face value of $1,000 that will default with probability one at the end of year one and that will only pay $900. Assume that the risk free rate is 4%.

a) *Calculate the current price of the bond:*

P = 900 / 1.04 = $865.38

Q. *Why am I using 900 instead of $1,000?*

b) *Calculate the YTM of the bond:*

$$YTM = \frac{FV}{P} - 1 = \frac{1{,}000}{865.38} - 1 = 15.56\%$$

Note: Since the YTM is calculated with the promised cash-flow (i.e., $1,000) rather than the expected cash-flow (i.e., $900) the YTM of a bond that has some probability of default is not equal to the expected return of investing in the bond.

c) *Calculate the expected return of the bond*

$$\frac{900 - 865.38}{865.38} = 4\%$$

Q. *Why is the expected return equal to the risk free rate?*

Example *(A Bond with a 50% Chance to Default)*

Consider a one-year zero-coupon bond with face value of $1,000 that will default with probability 0.5 at the end of year one. In the event that the bond defaults, it will pay only $900 instead of the promised $1,000. Assume that the risk free rate is 4% and that investors demand a 1.1% risk premium for the bond (that is a total of 5.1% return).

a) *Calculate the current price of the bond:*

P = 950 / 1.051 = $903.90

Q. *Why am I using 5.1% instead of 4%?*

b) *Calculate the YTM of the bond:*

$$YTM = \frac{FV}{P} - 1 = \frac{1{,}000}{903.90} - 1 = 10.63\%$$

c) *Calculate the expected return of the bond*

$$\frac{950 - 903.90}{903.90} = 5.1\%$$

Q. *Why is the expected return greater than 4%?*

Summing Up the Previous Examples:

1-Year Zero-Coupon Bond	**Bond Price**	**YTM**	**Expected Return**
Default Free	$961.54***	4.00%	4%
50% Chance of to Default	$903.90	10.63%	5.1%
Certain to Default	$865.38	15.56%	4%

*** P=1,000/1.04 = 961.54