Financial Derivatives FINE 448

1. Introduction

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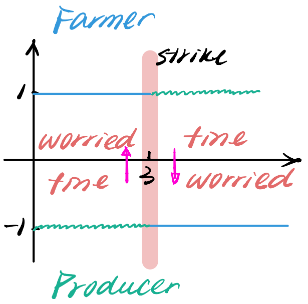
McGill

Winter 2020

What is a Derivative?



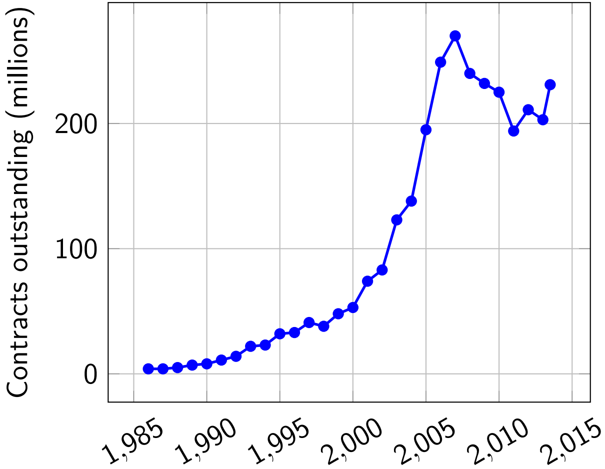
zero-sum game

OTC (over the counter)

- Definition
  - An agreement between two parties which has a value determined by the price of something else:
    - A stock like **Apple**
    - A bond such as a **T-Bond**
    - A currency such as the **EUR/CHF rate**
    - An index such as the **S&P500**
    - A metal like **Gold**
    - A commodity like **Soy beans**
- Types
  - Options, futures, and swaps.
- Uses
  - Risk management
  - Speculation
  - Reduce transaction costs
  - Regulatory arbitrage

① Market risk
② Counterparty risk
(counterparty might bankrupt)
③ Liquidity risk
(might cannot sell or cover position)
④ legal risk

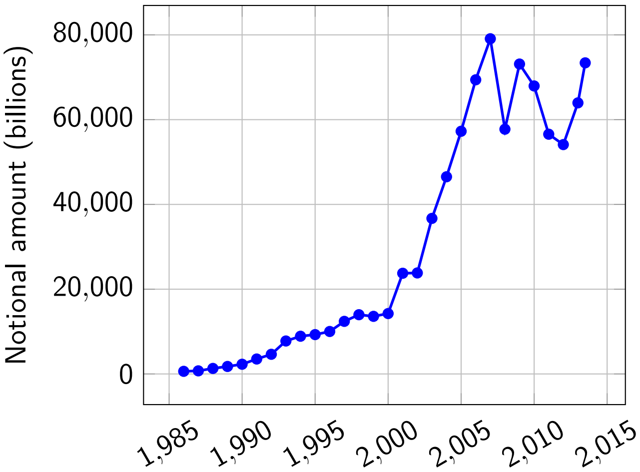
Exchange Traded Derivatives: Contracts Outstanding



source: Bank for International Settlements, Quarterly Review, June 2013.

http://www.bis.org/statistics/extderiv.htm

Exchange Traded Derivatives: Notional Amount



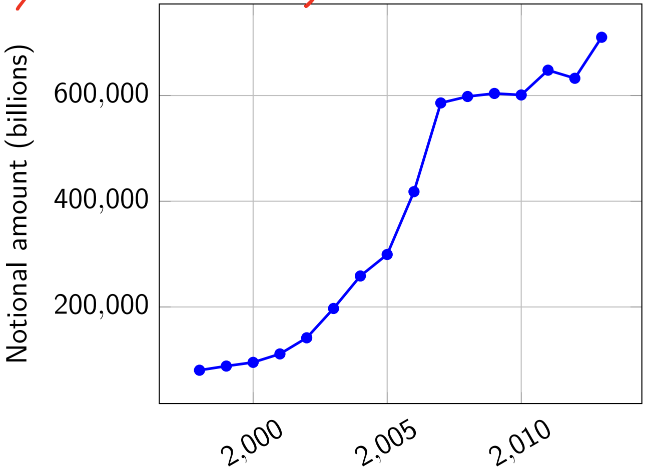
source: Bank for International Settlements, Quarterly Review, June 2012.

http://www.bis.org/statistics/extderiv.htm

Over-the-Counter (OTC) Derivatives

much larger than exchange traded

- There might be double counting
- Many banks are not surveyed.



source: Bank for International Settlements, Quarterly Review, June 2012.

http://www.bis.org/statistics/derstats.htm

Buying and Selling a Financial Asset

- Brokers: commissions
- Market-makers: bid-ask spread **(reflects the perspective of the market-maker)**

The price at which you can buy	**ask (offer)**	**What the market-maker will sell for**
The price at which you can sell	**bid**	**What the market-maker pays**

- Example: Buy and sell 100 shares of XYZ
  - XYZ: bid=\$49.75, ask=\$50, commission=\$15
  - Buy: $(100 \times \$50) + \$15 = \$5,015$
  - Sell: $(100 \times \$49.75) - \$15 = \$4,960$
  - Transaction cost: $\$5,015 - \$4960 = \$55$

$15 \times 2 = 30$ broker
25 market maker
(big financial institution: Goldman)

Problem 1: ABC stock has a bid price of $40.95 and an ask price of $41.05. Assume that the brokerage fee is quoted as 0.3% of the bid or ask price.

a. What amount will you pay to buy 100 shares?

$$(\$41.05 \times 100) + (\$41.05 \times 100) \times 0.003 = \$4,117.32$$

b. What amount will you receive for selling 100 shares?

$$(\$40.95 \times 100) - (\$40.95 \times 100) \times 0.003 = \$4,082.72$$

c. Suppose you buy 100 shares, then immediately sell 100 shares. What is your round-trip transaction cost?

$$\$4,117.32 - \$4,082.72 = \$34.6$$

Short-Selling

- When price of an asset is expected to fall
  - First: borrow and sell the asset (get $$)
  - Then: buy back and return the asset (pay $) risky
  - If price fell in the mean time: Profit $ = $$ - $
  - The lender must be compensated for dividends received
- Example: Cash flows associated with short-selling a share of IBM for 90 days. Note that the short-seller must pay the dividend, D, to the share-lender.

	Day 0	**Dividend Ex-Day**	**Day 90**
Action	Borrow shares	—	Return shares
	Sell shares	—	Purchase shares
Cash	$+S_0$	$-D$	$-S_{90}$

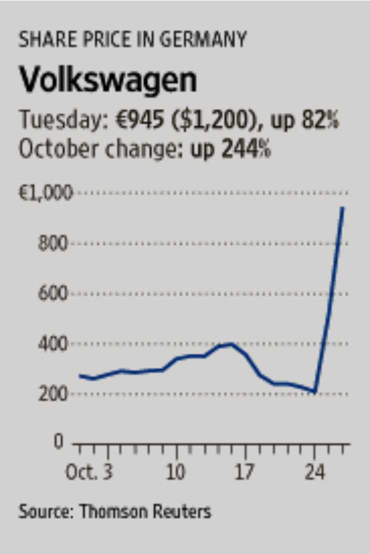
Short selling form the perspective of a broker

- A trader places a short sale order
- The broker searches its own inventory, another trader's margin account, or even another brokerage firm's inventory to locate the shares that the client wants to borrow
- If the stock is located, the short sale order is filled and the trader sells the shares in the market
- Once the transaction is placed, the broker does the lending $\Rightarrow$ any benefit (interest for lending out the shares) belongs to the broker
- The broker is responsible for returning the shares (not a big risk due to margin requirements)

VW's 348% Two-Day Gain Is Pain for Hedge Funds

From the *Wall Street Journal*, 2008:

In short squeezes, investors who borrowed and sold stock expecting its value to fall exit from the trades by buying those shares, or **covering** their positions. That can send a stock upward if shares are hard to come by. When shares are scarce, that can push a company-s market capitalization well beyond a reasonable valuation. [...] Indeed, the recent stock gains left Volkswagen's market value at about $346 billion, just below that of the world's largest publicly traded corporation, Exxon Mobil Corp.



Problem 2: Suppose you short-sell 300 shares of XYZ stock at \$30.19 with a commission charge of 0.5%. Supposing you pay commission charges for purchasing the security to cover the short-sale, how much profit have you made if you close the short-sale at a price of \$29.87?

Initially, we will receive the proceeds form the sale of the asset, less the proportional commission charge:

$$300 \times (\$30.19) - 300 \times (\$30.19) \times 0.005 = \$9,011.72$$

When we close out the position, we will again incur the commission charge, which is added to the purchasing cost:

$$300 \times (\$29.87) + 300 \times (\$29.87) \times 0.005 = \$9,005.81$$

Finally, we receive total profits of: $\$9,011.72 - \$9,005.81 = \$5.91$.

Continuous Compounding

- Terms often used to to refer to interest rates:
  - **Effective annual rate** r: if you invest \$1 today, T years later you will have

$$(1+r)^T$$

  - **Annual rate** r**, compounded** n **times per year**: if you invest \$1 today, T years later you will have

$$\left(1+\frac{r}{n}\right)^{nT}$$

  - **Annualized continuously compounded rate** r: if you invest \$1 today, T years later you will have

$$e^{rT} \equiv \lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{nT}$$

Continuous Compounding: Example

- Suppose you have a zero-coupon bond that matures in 5 years. The price today is \$62.092 for a bond that pays \$100.
  - The effective annual rate of return is

$$\left(\frac{\$100}{\$62.092}\right)^{1/5} - 1 = 0.10$$

  - The continuously compounded rate of return is

$$\frac{\ln(\$100/\$62.092)}{5} = \frac{0.47655}{5} = 0.09531$$

- The continuously compounded rate of return of 9.53% corresponds to the effective annual rate of return of 10%. To verify this, observe that

$$e^{0.09531} = 1.10$$

$$1 + r_d = e^{r_c}$$ continous

discrete

or

$$\ln(1.10) = \ln\left(e^{0.09531}\right) = 0.09531$$

Continous Compounding

- When we multiply exponentials, exponents add. So we have

$$e^x e^y = e^{x+y}$$

  This makes calculations of average rate of return easier.
- When using continuous compounding, increases and decreases are symmetric.
- Moreover, continuously compounded returns can be less than -100% (unbounded)

when $S' = 0$, $r_c = -\infty$

limit of r_d

Problem 3: Suppose that over 1 year a stock price increases from \$100 to \$200. Over the subsequent year it falls back to \$100.

- What is the effective return over the first year? What is the continuously compounded return?

$$\text{effective return} = \frac{\$200 - \$100}{\$100} = 100\%$$

$$\text{continuously compounded return} = \ln\left(\frac{\$200}{\$100}\right) = 69.31\%$$

discrete return: asymmetric

continous return: symmetric

(possibly normal)

- What is the effective return over the second year? The continuously compounded return?

$$\text{effective return} = \frac{\$100 - \$200}{\$200} = -50\%$$

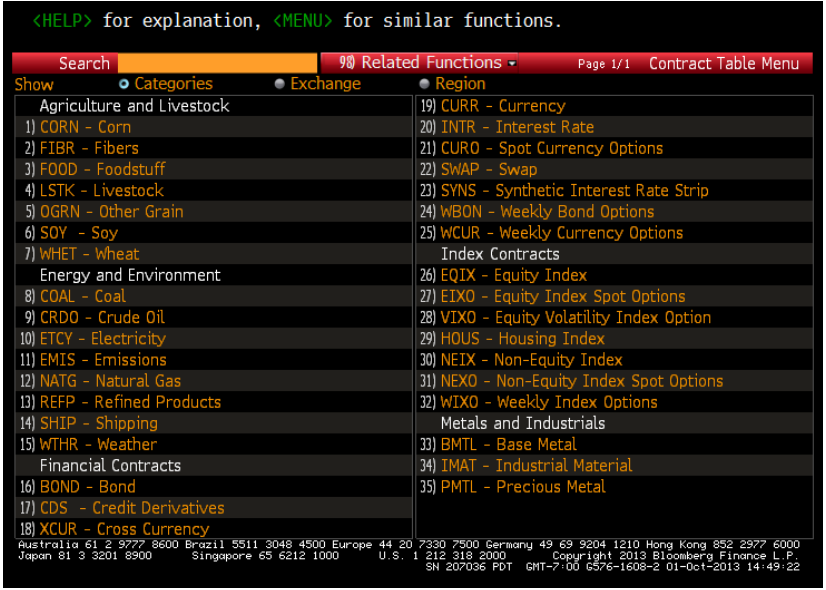
$$\text{continuously compounded return} = \ln\left(\frac{\$100}{\$200}\right) = -69.31\%$$

- What do you notice when you compare the first- and second-year returns computed arithmetically and continuously?

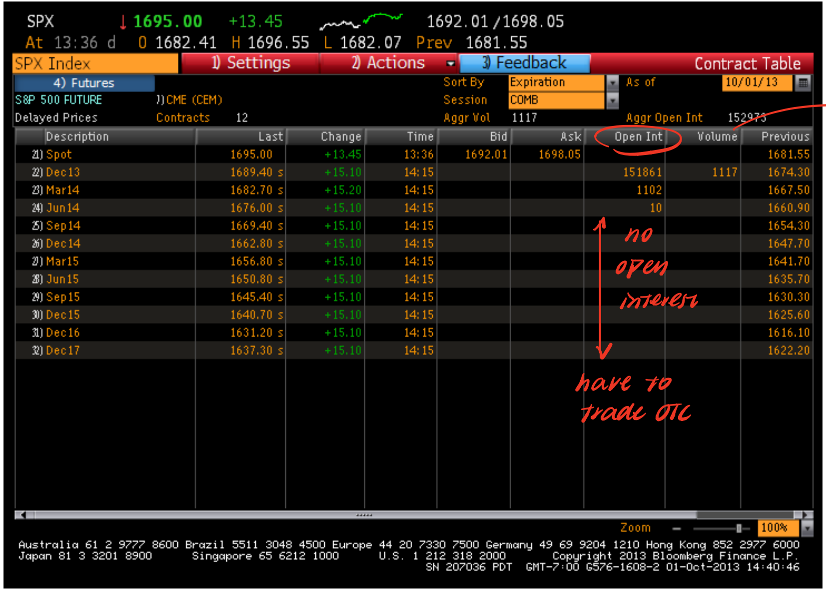
Forward Contracts

- Definition: a binding agreement (obligation) to buy/sell an underlying asset in the future, at a price set today.
- A forward contract specifies:
  1. The features and quantity of the asset to be delivered
  2. The delivery logistics, such as time, date, and place
  3. The price the buyer will pay at the time of delivery

Bloomberg: CTM <GO> contract table menu



Bloomberg: SPX <INDEX> CT <GO>



Description	Last	Change	Time	Bid	Ask	Open Int	Volume	Previous
21) Spot	1695.00	+13.45	13:36	1692.01	1698.05			1681.55
22) Dec13	1689.40 s	+15.10	14:15			151861	1117	1674.30
23) Mar14	1682.70 s	+15.20	14:15			1102		1667.50
24) Jun14	1676.00 s	+15.10	14:15			10		1660.90
25) Sep14	1669.40 s	+15.10	14:15					1654.30
26) Dec14	1662.80 s	+15.10	14:15					1647.70
27) Mar15	1656.80 s	+15.10	14:15					1641.70
28) Jun15	1650.80 s	+15.10	14:15					1635.70
29) Sep15	1645.40 s	+15.10	14:15					1630.30
30) Dec15	1640.70 s	+15.10	14:15					1625.60
31) Dec16	1631.20 s	+15.10	14:15					1616.10
32) Dec17	1637.30 s	+15.10	14:15					1622.20

change in the day

no open interest

have to trade OTC

Bloomberg: SPZ3 <INDEX> DES <GO>

SP500 index (Dec)

always Fri of that month

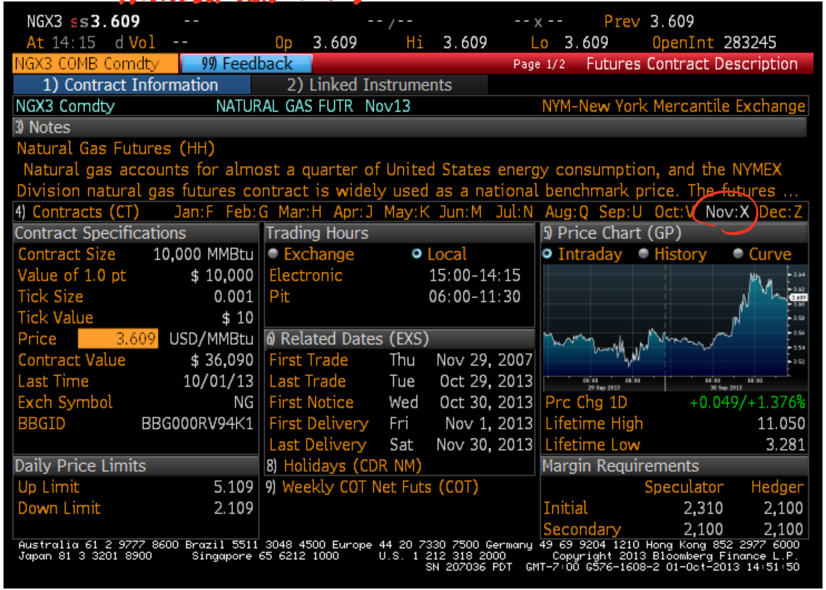
hedge fund

airline

no deliver of index

Bloomberg: NGX3 <CMDTY> DES <GO>

Natural Gas (Nov)



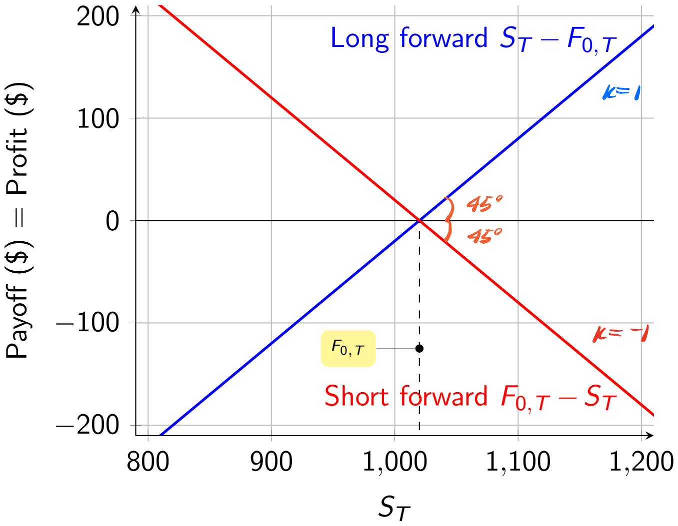
Payoff (Value at Expiration) of a Forward Contract

- Every forward contract has both a buyer and a seller.
- The term **long** is used to describe the buyer and **short** is used to describe the seller.
- Payoff for

  S_T $F_{0,T}$

  - Long forward = Spot price at expiration – Forward price
  - Short forward = Forward price – Spot price at expiration
- Example: S&R index:
  - Today: Spot price = \$1,000. 6-month forward price = \$1,020
  - In 6 months at contract expiration: Spot price = \$1,050
    - Long position payoff = \$1,050 - \$1,020 = \$30
    - Short position payoff = \$1,020 - \$1,050 = -\$30

Payoff Diagram for a Forward



Problem 4: Suppose you enter in a long 6-month forward position at a forward price of \$50. What is the payoff in 6 months for prices of \$40, \$45, \$50, \$55, and \$60?

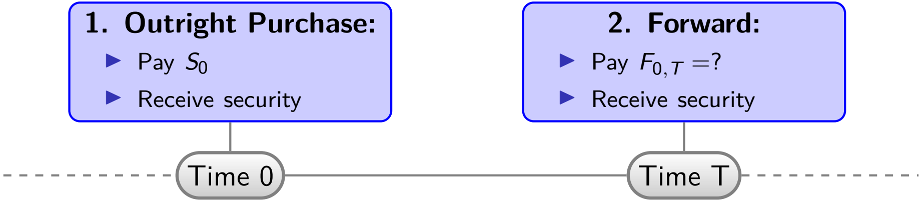
The payoff to a long forward at expiration is equal to:

$$\text{Payoff to long forward} = \text{Spot price at expiration} - \text{Forward price}$$

Therefore, we can construct the following table:

Price of asset in 6 months	Payoff ot the long forward
40	-10
45	-5
50	0
55	5
60	10

Alternative ways to buy a stock



- A forward contract is an arrangement in which you both pay for the stock and receive it at time T, with the time T price specified at time 0.
- **What should you pay for the stock in this case?**
- Arbitrage ensures that there is a very close relationship between prices and forward prices

Pricing a Forward Contract

- Let S_0 be the spot price of an asset at time 0, and r the continuously compounded interest rate. Assume that dividends are continuous and paid at a rate δ.
- Then the forward price at a future time T must satisfy

cannot affect

$$F_{0,T} = S_0 e^{(r-\delta)T} \qquad (1)$$

determine

- Suppose that $F_{0,T} > S_0 e^{(r-\delta)T}$. Then an investor can execute the following trades at time 0 (**buy low and sell high**) and obtain an arbitrage profit:

What does "tailed" mean ?

	Cash Flows	
Transaction	Time 0 div	Time T (expiration)
Buy tailed position in stock ($e^{-\delta T}$ units)	$-S_0 e^{-\delta T}$	S_T
Borrow $S_0 e^{-\delta T}$	$+S_0 e^{-\delta T}$	$-S_0 e^{(r-\delta)T}$
Short forward	0	$F_{0,T} - S_T$
Total	0	$F_{0,T} - S_0 e^{(r-\delta)T} > 0$

Pricing a Forward Contract (cont'd)

- Suppose that $F_{0,T} < S_0 e^{(r-\delta)T}$. Then an investor can execute the following trades at time 0 (**buy low and sell high**) and obtain once again an arbitrage profit:

	Cash Flows	
Transaction	Time 0	Time T (expiration)
Short tailed position in stock ($e^{-\delta T}$ units)	$S_0 e^{-\delta T}$	$-S_T$
Lend $S_0 e^{-\delta T}$	$-S_0 e^{-\delta T}$	$S_0 e^{(r-\delta)T}$
Long forward	0	$S_T - F_{0,T}$
Total	0	$S_0 e^{(r-\delta)T} - F_{0,T} > 0$

- Consequently, and assuming that the non-arbitrage condition holds, we have

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

Forward Contracts vs Futures Contracts

- Forward and futures contracts are essentially the same except for the daily resettlement feature of futures contracts, called **marking-to-market**.
- Because futures are exchange-traded, they are standardized and have specified delivery dates, locations, and procedures.
- Plenty of information is available from: `www.cmegroup.com`

The S&P 500 Futures Contract

Specifications for the S&P 500 index futures contract

- Underlying: S&P 500 index
- Where traded: Chicago Mercantile Exchange
- Size: $\$250 \times$ S&P 500 index
- Months: Mar, Jun, Sep, Dec
- Trading ends: Business day prior to determination of settlement price
- Settlement: Cash-settled, based upon opening price of S&P 500 on third Friday of expiration month

- Suppose the futures price is 1100 and you wish to enter into 8 long futures contracts.

  per S&P point ?
- The notional value of 8 contracts is

$$8 \times \$250 \times 1100 = \underbrace{\$2,000}_{\text{multiplier}} \times 1100 = \$2.2 \text{ million}$$

The S&P 500 Futures Contract (cont'd)

- Suppose that there is 10% margin and weekly settlement (in practice settlement is daily). The margin on futures contracts with a notional value of $2.2 million is $220,000.
- The margin balance today from long position in 8 S&P 500 futures contracts is

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance ($)
0	2000.00	1100.00	—	220,000.00

- Over the first week, the futures price drops 72.01 points to 1027.99. On a mark-to-market basis, we have lost

$$\$2,000 \times (-72.01) = -\$144,020$$

- Thus, if the continuously compounded interest rate is 6%, our margin balance after one week is

weekly

$$\$220,000 \times e^{0.06\times 1/52} - \$144,020 = \$76,233.99$$

The S&P 500 Futures Contract (cont'd)

- Because we have a 10% margin, a 6.5% decline in the futures price results in a 65% decline in margin. The margin balance after the first week is

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance ($)
0	2000.00	1100.00	—	220,000.00
1	2000.00	1027.99	-72.01	76,233.99

- The decline in margin balance means the broker has significantly less protection should we default. For this reason, participants are required to maintain the margin at a minimum level, called the **maintenance margin**. This is often set at 70% to 80% of the initial margin level.
- In this example, the broker would make a **margin call**, requesting additional margin.

  profit = final balance − $\sum$ FV (margin balance)
  remain the same as no margin call if ending at maturity

- We can go on for a period of 10 weeks, assuming weekly marking-to-market and a continuously compounded risk-free rate of 6%.

Note: More loss if close at margin call (market will recover)

The S&P 500 Futures Contract (cont'd)

- The margin balance after a period of 10 weeks is

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance ($)
0	2000.00	1100.00	—	220,000.00
1	2000.00	1027.99	-72.01	76,233.99
2	2000.00	1037.88	9.89	96,102.01
3	2000.00	1073.23	35.35	166,912.96
4	2000.00	1048.78	-24.45	118,205.66
5	2000.00	1090.32	41.54	201,422.13
6	2000.00	1106.94	16.62	234,894.67
7	2000.00	1110.98	4.04	243,245.86
8	2000.00	1024.74	-86.24	71,046.69
9	2000.00	1007.30	-17.44	36,248.72
10	2000.00	1011.65	4.35	44,990.57

- The 10-week profit on the position is obtained by subtracting from the final margin balance the future value of the original margin investment:

$$\$44,990.57 - \underbrace{\$220,000 \times e^{0.06\times 10/52}}_{\text{interest}} = -\$177,562.60$$

The S&P 500 Futures Contract (cont'd)

- What if the position had been forwarded rather than a futures position, but with prices the same? In that case, after 10 weeks our profit would have been

$$(1011.65 - 1100) \times \$2,000 \ = -\$176,700$$

- The futures and forward profits differ because of the interest earned on the mark-to-market proceeds (in the present cases, we have founded losses as they occurred and not at expiration, which explains the loss).

Uses Of Index Futures

- Why buy an index futures contract instead of synthesizing it using the stocks in the index? — Lower transaction costs
- Asset allocation: switching investments among asset classes. Example: invested in the S&P 500 index and wish to temporarily invest in bonds instead of the index. What to do?
  - Alternative #1: sell all 500 stocks and invest in bonds
  - Alternative #2: take a short forward position in S&P 500 index
- General asset allocation: futures overlay, alpha-porting
- Cross-hedging: hedge portfolios that are not exactly the index
- Risk management for stock-pickers
- More in Chapter 5, Section 5.5 of McDonald (2009)

Call Options

- A non-binding agreement (right but not an obligation) to buy an asset into the future, at a price set today
- Preserves the upside potential, while at the same time eliminating the downside
- The seller of a call option is obligated to deliver if asked

Definition and terminology

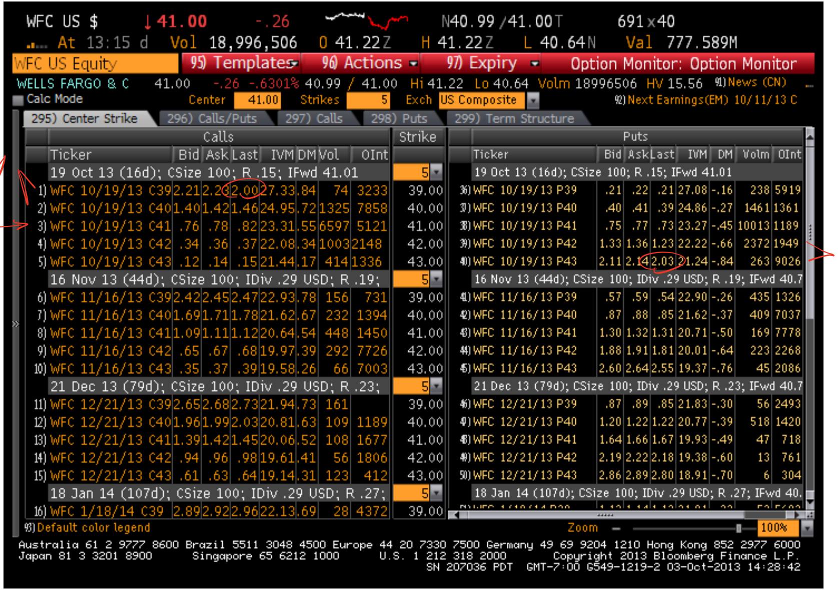
- A call option gives the owner the right but not the obligation to buy the underlying asset at a predetermined price during a predetermined time period
- Strike (or exercise) price: the amount paid by the option buyer for the asset if he/she decides to exercise
- Exercise: the act of paying the strike price to buy the asset
- Expiration: the date by which the option must be exercised or becomes worthless
- Exercise style: specifies when the option can be exercised
  - European-style: can be exercised only at expiration date
  - American-style: can be exercised at any time before expiration
  - Bermudan-style: can be exercised during specified periods

Moneyness

- In-the-money option: positive payoff if exercised immediately
- At-the-money option: zero payoff if exercised immediately
- Out-of-the-money option: negative payoff if exercised immediately

Bloomberg: WFC US <EQUITY> OMON <GO>

verify Call-Put Parity ?



Calls									Strike	Puts								
	Ticker	Bid	Ask	Last	IVM	DM	Vol	OInt			Ticker	Bid	Ask	Last	IVM	DM	Volm	OInt
	19 Oct 13 (16d); CSize 100; R .15; IFwd 41.01								5		19 Oct 13 (16d); CSize 100; R .15; IFwd 41.01							
1)	WFC 10/19/13 C39	2.21	2.23	2.00	27.33	.84	74	3233	39.00	36)	WFC 10/19/13 P39	.21	.22	.21	27.08	-.16	238	5919
2)	WFC 10/19/13 C40	1.40	1.42	1.46	24.95	.72	1325	7858	40.00	37)	WFC 10/19/13 P40	.40	.41	.39	24.86	-.27	1461	1361
3)	WFC 10/19/13 C41	.76	.78	.82	23.31	.55	6597	5121	41.00	38)	WFC 10/19/13 P41	.75	.77	.73	23.27	-.45	10013	1189
4)	WFC 10/19/13 C42	.34	.36	.37	22.08	.34	1003	2148	42.00	39)	WFC 10/19/13 P42	1.33	1.36	1.23	22.22	-.66	2372	1949
5)	WFC 10/19/13 C43	.12	.14	.15	21.44	.17	414	1336	43.00	40)	WFC 10/19/13 P43	2.11	2.14	2.03	21.24	-.84	263	9026
	16 Nov 13 (44d); CSize 100; IDiv .29 USD; R .19;								5		16 Nov 13 (44d); CSize 100; IDiv .29 USD; R .19; IFwd 40.7							
6)	WFC 11/16/13 C39	2.42	2.45	2.47	22.93	.78	156	731	39.00	41)	WFC 11/16/13 P39	.57	.59	.54	22.90	-.26	435	1326
7)	WFC 11/16/13 C40	1.69	1.71	1.78	21.62	.67	232	1394	40.00	42)	WFC 11/16/13 P40	.87	.88	.85	21.62	-.37	409	7037
8)	WFC 11/16/13 C41	1.09	1.11	1.12	20.64	.54	448	1450	41.00	43)	WFC 11/16/13 P41	1.30	1.32	1.31	20.71	-.50	169	7778
9)	WFC 11/16/13 C42	.65	.67	.68	19.97	.39	292	7726	42.00	44)	WFC 11/16/13 P42	1.88	1.91	1.81	20.01	-.64	223	2268
10)	WFC 11/16/13 C43	.35	.37	.39	19.58	.26	66	7003	43.00	45)	WFC 11/16/13 P43	2.60	2.64	2.55	19.37	-.76	45	2086
	21 Dec 13 (79d); CSize 100; IDiv .29 USD; R .23;								5		21 Dec 13 (79d); CSize 100; IDiv .29 USD; R .23; IFwd 40.7							
11)	WFC 12/21/13 C39	2.65	2.68	2.73	21.94	.73	161		39.00	46)	WFC 12/21/13 P39	.87	.89	.85	21.83	-.30	56	2493
12)	WFC 12/21/13 C40	1.96	1.99	2.03	20.81	.63	109	1189	40.00	47)	WFC 12/21/13 P40	1.20	1.22	1.22	20.77	-.39	518	1420
13)	WFC 12/21/13 C41	1.39	1.42	1.45	20.06	.52	108	1677	41.00	48)	WFC 12/21/13 P41	1.64	1.66	1.67	19.93	-.49	47	718
14)	WFC 12/21/13 C42	.94	.96	.98	19.61	.41	56	1806	42.00	49)	WFC 12/21/13 P42	2.19	2.22	2.18	19.38	-.60	13	761
15)	WFC 12/21/13 C43	.61	.63	.64	19.14	.31	123	412	43.00	50)	WFC 12/21/13 P43	2.86	2.89	2.80	18.91	-.70	6	304
	18 Jan 14 (107d); CSize 100; IDiv .29 USD; R .27;								5		18 Jan 14 (107d); CSize 100; IDiv .29 USD; R .27; IFwd 40.							
16)	WFC 1/18/14 C39	2.89	2.92	2.96	22.13	.69	28	4372	39.00									

in the money

at the money

in the money

Call Option Example

- Consider a call option on the S&R index with 6 months to expiration and strike price of \$1,000.
- In six months at contract expiration: if spot price is
  - \$1,100 $\Rightarrow$ call buyer's payoff = \$1,100 – \$1,000 = \$100, call seller's payoff = -\$100
  - \$900 $\Rightarrow$ call buyer's payoff = \$0, call seller's payoff = \$0
- The **payoff** of a call option is then

$$C_T = \max\left[S_T - K, 0\right] \tag{2}$$

where K is the strike price, and S_T is the spot price at expiration.

- The option **profit** is computed as

$$\text{Call profit} = \max\left[S_T - K, 0\right] - \text{future value of premium} \tag{3}$$

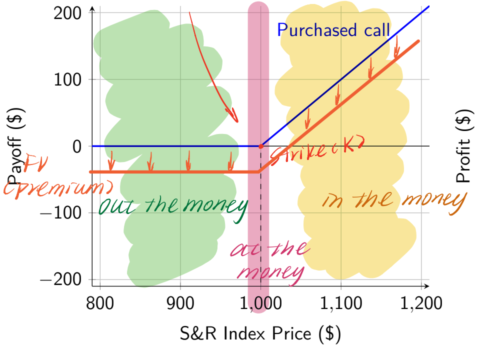
convex

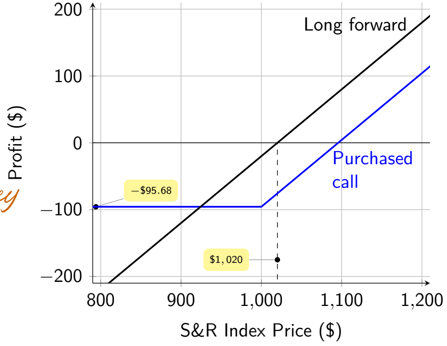
Diagrams for Purchased Call

At maturity

convex (trade asset & asset volatility)

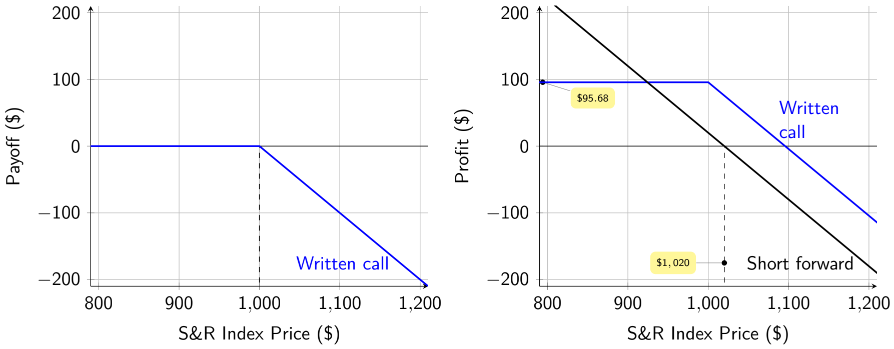
$\max\{0, S_T - K\}$





profit = payoff$_T$ − FV(premium)

Diagrams for Written Call



Problem 5: Consider a call option on the S&R index with 6 months to expiration and strike price of \$1,000. The future value of the option premium is \$95.68. For the figure below, which plots the profit on a purchased call, find the S&R index price at which the call option diagram intersects the x-axis. break even point

The profit of the long call option is:

$$\max[0, S_T - \$1{,}000] - \$95.68$$

To find the S&R index price at which the call option diagram intersects the x-axis, we have to set the above equation equal to zero. We get

$$S_T = \$1{,}095.68$$

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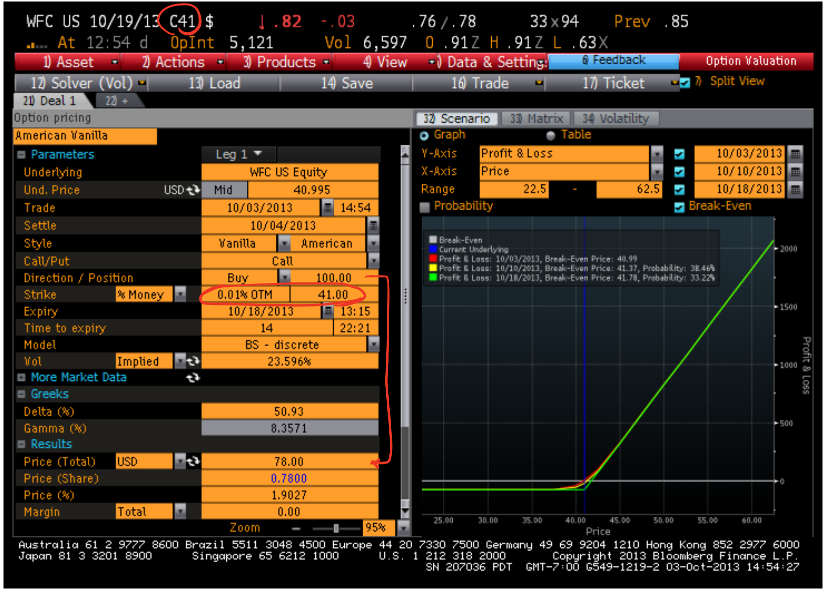


at time T

① 41 → 44 ≈ 10%

0.78 → 3 ≈ 300%

② 41 → 39 ≈ 10%

0.78 → 0 −100%

leverage

Put Options

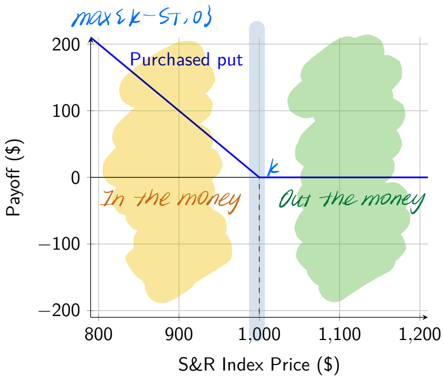
- A put option gives the owner the right but not the obligation to sell the underlying asset at a predetermined price during a predetermined time period.
- The **payoff** of the put option is

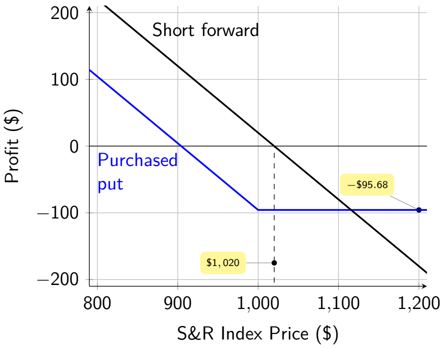
$$P_T = \max\left[K - S_T, 0\right] \tag{4}$$

- The option **profit** is computed as

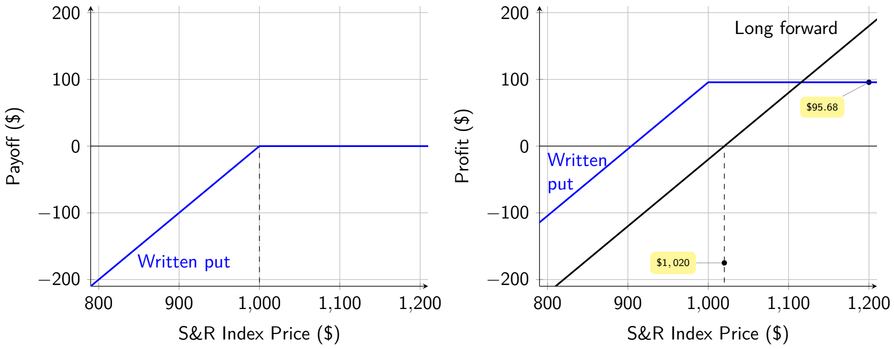
$$\text{Put profit} = \max\left[K - S_T, 0\right] - \text{future value of premium} \tag{5}$$

Diagrams for Purchased Put





Diagrams for Written Put

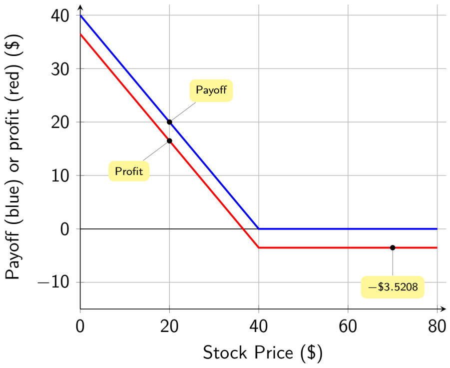


Problem 6: Suppose the stock price is \$40 and the *effective* annual interest rate is 8%. Draw payoff and profit diagrams for a 40-strike put with a premium of \$3.26 and maturity of 1 year.

In order to be able to draw the profit diagram, we need to find the future value of the put premium:

$$FV(\text{premium}) = \$3.26 \times (1 + 0.08) = \$3.5208$$

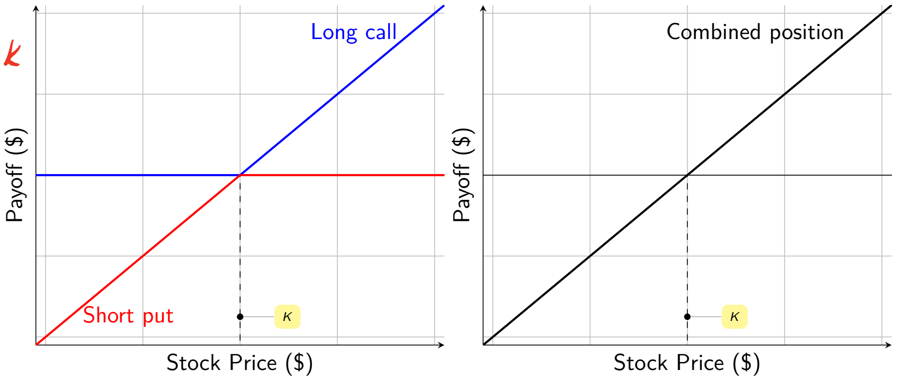
We get the following payoff and profit diagram:



Put-Call Parity

- Suppose you are buying a call option and selling a put option on a non-dividend paying stock. Both options have maturity T and strike price K:

$C_T - P_T = S_T - K$



① European

② Same maturity

③ Same strike

Put-Call Parity (cont'd)

- Your payoff at maturity is

$$
\begin{aligned}
C_T - P_T &= \max\left[S_T - K, 0\right] - \max\left[K - S_T, 0\right] \\
&= \max\left[S_T - K, 0\right] + \min\left[S_T - K, 0\right] \\
&= S_T - K
\end{aligned}
$$

- We have two strategies with the same payoff at maturity:
  - Buy a call and sell a put, thus paying a premium of $C_t - P_t$ today
  - Buy a share of the stock and borrow $PV(K)$, thus paying a premium of $S_t - PV(K)$ today
- Positions that have the same payoff should have the same cost (**Law of one price**):

$$
C_t - P_t = S_t - PV(K) \tag{6}
$$

- Equation (6) is known as **put-call parity**, and one of the most important relations in options.

Put-Call Parity (cont'd)

- Parity provides a cookbook for the synthetic creation of options. It tells us that

$$C_t = P_t + S_t - PV(K) \tag{7}$$

use interest rate of bond with same maturity

and that

$$P_t = C_t - S_t + PV(K) \tag{8}$$

- The first relation says that a call is equivalent to a leveraged position on the underlying asset, which is insured by the purchase of a put. The second relation says that a put is equivalent to a short position on the stock, insured by the purchase of a call
- Parity generally fails for American-style options, which may be exercised prior to maturity.

Why Does the Price of an At-the-Money call Exceed the Price of an At-the-Money put?

- Parity shows that the reason for the call being more expensive is the time value of money:

$$C_t - P_t = \overset{S_t}{\overset{\downarrow}{K}} - PV(K) > 0 \tag{9}$$

- A common erroneous explanation is that the profit on a call is unlimited, while the profit on a put can be no greater than the strike price, which seems to suggest that the call should be more expensive than the put.
- This argument also seems to suggest that every stock is worth more than its price!

Problem 7: The S&R index price is \$1,000 and the *effective* 6-month interest rate is 2%. Suppose the premium on a 6-month S&R call is \$109.20 and the premium on a 6-month put with the same strike price is \$60.18. What is the strike price?

This question is a direct application of the Put-Call Parity:

$$C_t - P_t = S_t - PV(K)$$
$$\$109.20 - \$60.18 = \$1,000 - \frac{K}{1.02}$$
$$K = \$970.00$$

Put-Call Parity for Dividend Paying Stocks

- If the stock is paying dividends over the lifetime of the option, the put-call parity becomes

$$C_t - P_t = [S_t - PV(\text{Div})] - PV(K) \tag{10}$$

where $PV(\text{Div})$ is the present value of the stream of dividends paid on the stock until maturity.

dividend smoothing
⇒ can treat dividend as risk-free

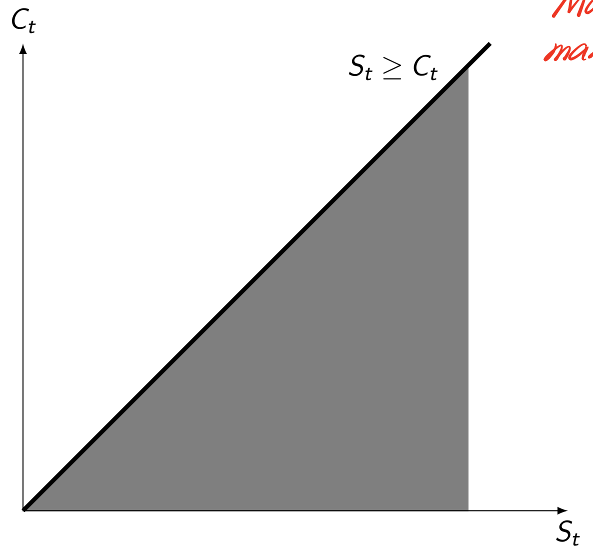
- Hence, we can write

$$C_t = P_t + [S_t - PV(\text{Div})] - PV(K) \tag{11}$$

$$P_t = C_t - [S_t - PV(\text{Div})] + PV(K) \tag{12}$$

- Equations (11)–(12) help us to find maximum and minimum option prices.

Maximum and Minimum Option Prices: Call Price

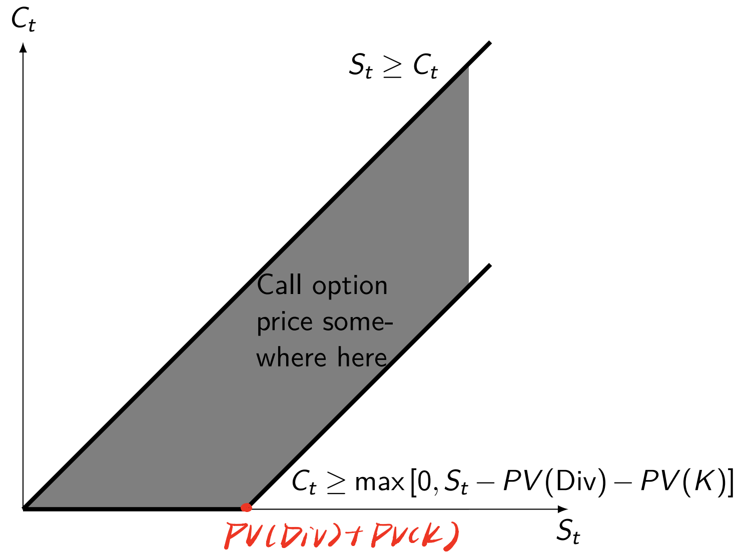


Maximum payoff

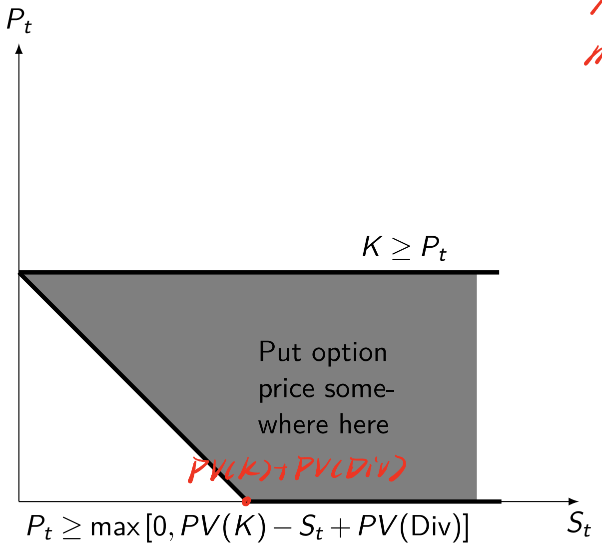
$\max\{S_K - K, 0\} = S_K$

$K \to 0$

Maximum and Minimum Option Prices: Call Price



Maximum and Minimum Option Prices: Put Price



Maximum payoff

$\max \{K - S_T, 0\} = K$, with $S_T \downarrow 0$

Problem 8 (Minimum and Maximum Bounds, Arbitrage)

- A 1M European put option on a non-dividend paying stock is currently selling for 2.50. The option has a strike of 50 and the underlying is currently worth 46. The interest rate is 10%.
- Is there an arbitrage opportunity? If yes, show how you would implement it.

2 Buy Stock −46

3 Borrow 48.5

1 Buy put −2.5

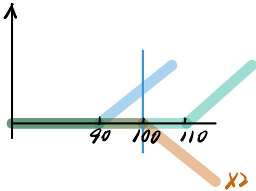
2.5 $P_t \le K$ 50 ✓

2.5 $P_t \ge 0$ ✓

$P_t \ge PV(K) + PV(Div) - S_t$ ✗

$50 \cdot e^{-0.1/12} + 0 - 46 = 3.6$

$S_T < 50$	$S_T \geq 50$
$50 - S_T$	0
S_T	S_T
−48.9	−48.9
$1.1 > 0$	$S_T - 48.9 > 0$



Problem 9 (Building Payoffs)

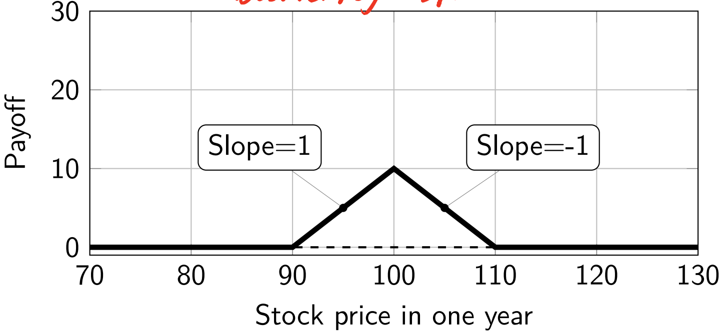
Below is a **payoff** diagram for a position. All options have 1 year to maturity and the stock price today is \$100. The yearly interest rate (**continuously compounded**) is 8%. The underlying asset (the stock) is not paying any dividends.

call (90) +1
call (100) −2
call (110) +1

total price > 0
(no negative payoff)

$P(90) - 2P(100) + P(110)$?

butterfly spread

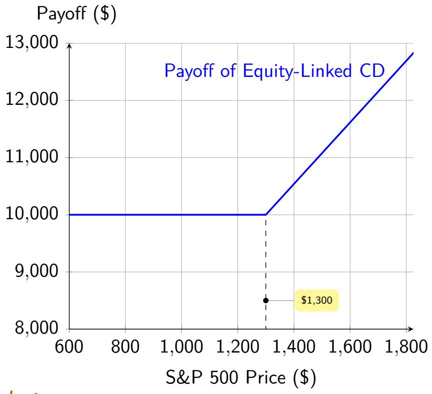


Option	Call(90)	Call(100)	Call(110)
Position			

Example: Equity-Linked CDs

- A 1,999 First Union National Bank CD promises to repay in 5.5 years initial invested amount and 70% of the gain in S&P 500 index (this is a *principal protected equity-linked CD*)

- Assume \$10,000 invested when S&P 500 = 1,300
- Final payoff is



$$\$10,000 \times \left(1 + 0.7 \times \max\left[0, \frac{S_{final}}{1300} - 1\right]\right)$$

$$10000 + \underbrace{7000 \frac{1}{1300}}_{5.38} \underbrace{\max[0, S-1300]}_{call(1300)\ T=5.5}$$

$$PV = 10000 \cdot e^{-0.05 \cdot 5.5} + 5.38 \times 289 < 10000$$

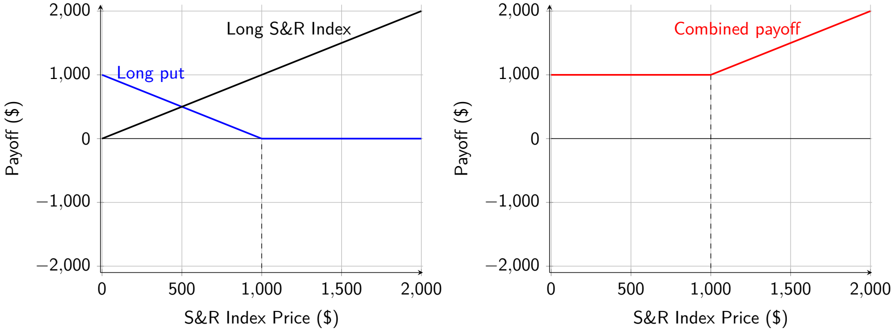
where S_{final} = value of the S&P 500 after 5.5 years.

289

black sholes calculator

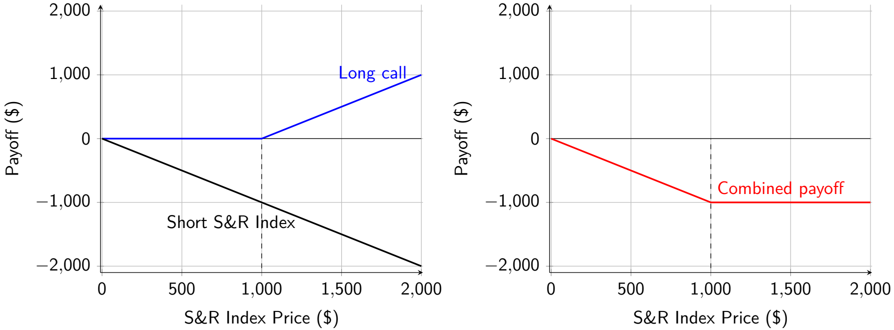
Options are Insurance: Insuring a Long Position (Floors)

- A put option is combined with a position in the underlying asset
- Goal: to insure against a fall in the price of the underlying asset (when one has a long position in that asset)

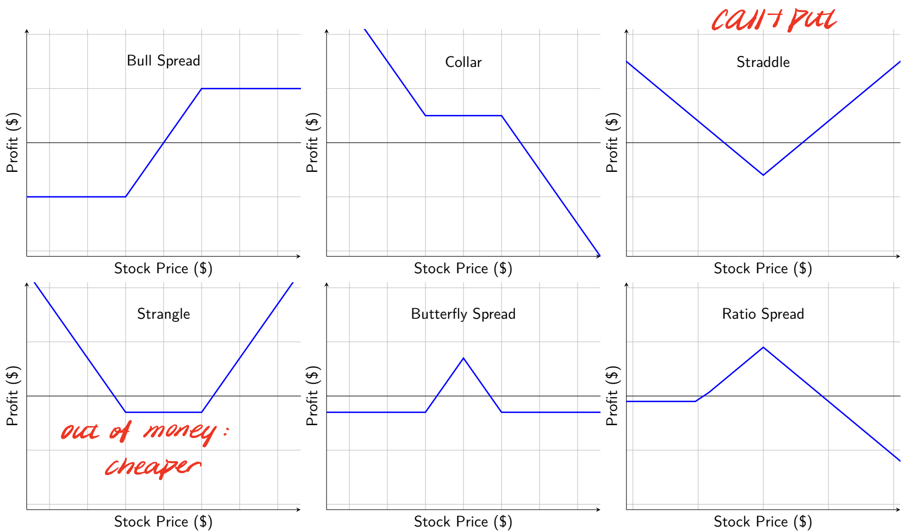


Options are Insurance: Insuring a Short Position (Caps)

- A call option is combined with a position in the underlying asset
- Goal: to insure against an increase in the price of the underlying asset (when one has a short position in that asset)



Various Strategies: Payoffs



Various Strategies: Positions

Bull Spread

	K_{low}	K_{ATM}	K_{high}
Call		Buy	Sell
Put			

Collar

	K_{low}	K_{ATM}	K_{high}
Call			Sell
Put	Buy		

Straddle

	K_{low}	K_{ATM}	K_{high}
Call		Buy	
Put		Buy	

Strangle

	K_{low}	K_{ATM}	K_{high}
Call			Buy
Put	Buy		

Butterfly Spread

	K_{low}	K_{ATM}	K_{high}
Call	Buy	Sell (2)	Buy
Put			

Ratio Spread

	K_{low}	K_{ATM}	K_{high}
Call		Buy	Sell (n)
Put			

Note that you can achieve the same results with different combinations (**but always at the same cost!**)

Various Strategies: Rationales

Bull Spread

- You believe a stock will appreciate ⇒ buy a call option (forward position insured)
- You can lower the cost if you are willing to reduce your profit should the stock appreciate ⇒ sell a call with higher strike
- Surprisingly, you can achieve the same result by buying a low-strike put and selling a high-strike put
- Opposite: **bear spread**

Collar

- A collar is fundamentally a short position (resembling a short forward contract)
- Often used for insurance when we own a stock (**collared stock**)
- The collared stock looks like a bull spread; however, it arises from a different set of transactions
- Opposite: **written collar**

Straddle

expect volatility

- A straddle can profit from stock price moves in both directions
- The disadvantage is that it has a high premium because it requires purchasing two options
- Opposite: **written straddle**

best time: option is cheap

⇒ Option is cheap when volatility is low.

Strangle

- To reduce the premium of a straddle, you can buy out-of-the-money options rather than at-the-money options.
- Opposite: **written strangle**

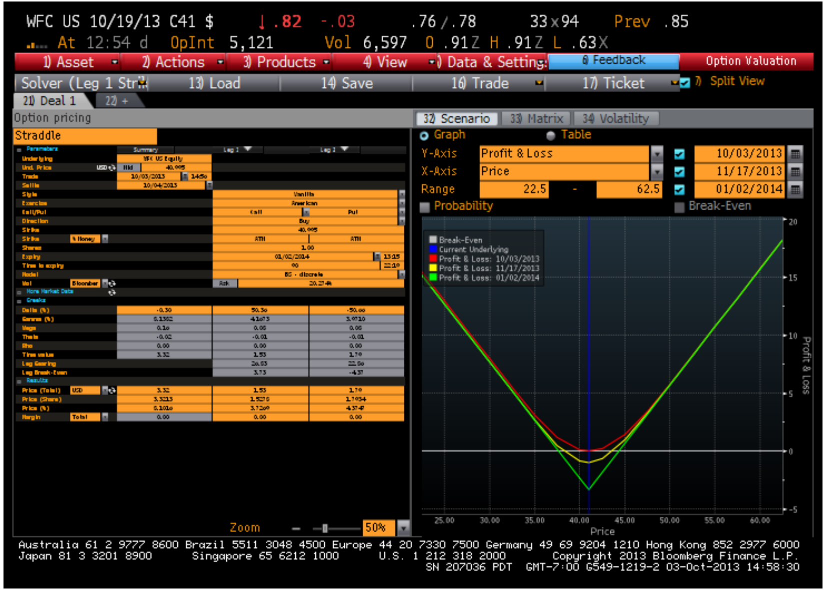
Butterfly Spread

- A butterfly spread is a written straddle to which we add two options to safeguard the position: An out-of-the money put and an out-of-the money call.
- A butterfly spread can be thought of as a written straddle for the timid (or for the prudent!)
- Opposite: **long iron butterfly**

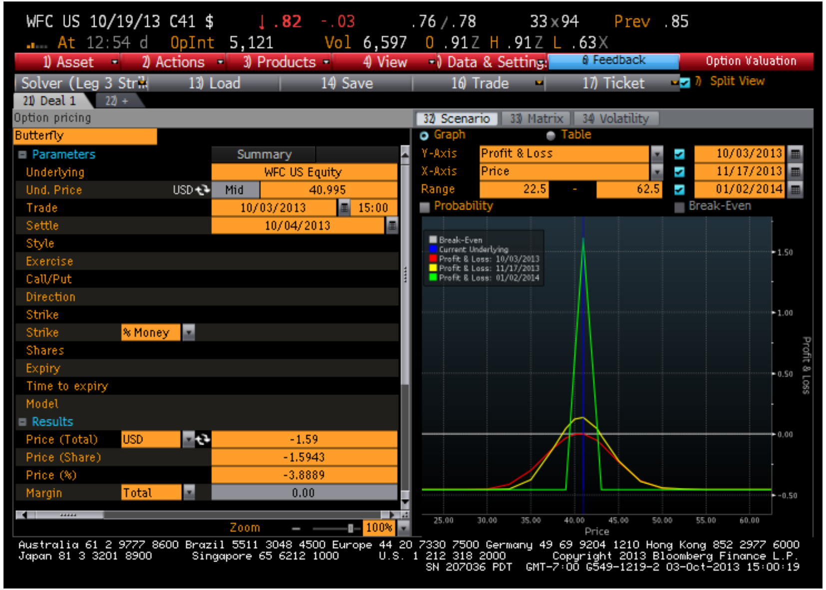
Ratio Spread

- Ratio spreads involve buying one option and selling a greater quantity (n) of an option with a more out-of-the money strike
- The ratio (i.e., "1 by n") is the number of short options divided by the number of long options
- The options are either both calls or both puts
- It is possible to construct ratio spreads with zero premium ⇒ we can construct insurance that costs nothing if it is not needed!

Bloomberg: Products → Option Strategies

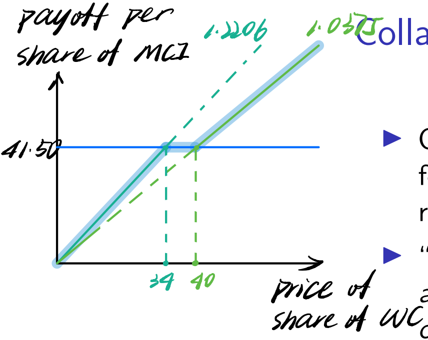


Bloomberg: Products → Option Strategies



Lakonishok, Lee, Pearson, and Poteshman, *Option Market Activity*, The Review of Financial Studies, 2006

- Stylized facts about option trading
  - Written option positions are more common than purchased positions
  - About 4 times more purchased calls than puts
  - Main driver of option market activity is speculating on the direction of underlying stock movements
- Option trading strategies
  - Small fraction of volatility trading strategies (straddles and strangles)
  - Large fraction of covered-call strategies
- Option market activity during the stock market bubble of the late 1990s
  - Call buying and put writing increased dramatically (mostly on growth stocks)
  - Purchased puts less common (little appetite for betting against the bubble)



Collars in Acquisitions: WorldCom/MCI

Aquirer

- On October 1, 1997, WorldCom Inc. CEO (Bernard Ebbers) sent the following note to the CEO of MCI (Bert Roberts), and it was also released through the typical newswires: Target
- *"I am writing to inform you that this morning WorldCom is publicly announcing that it will be commencing an offer to acquire all the outstanding shares of MCI for \$41.50 of WorldCom common stock per MCI share. The actual number of shares of WorldCom common stock to be exchanged for each MCI share in the exchange offer will be determined by dividing \$41.50 by the 20-day average of the high and low sales prices for WorldCom common stock prior to the closing of the exchange offer, but will not be less than 1.0375 shares (if WorldCom's average stock price exceeds \$40) or more than 1.2206 shares (if WorldCom's average stock price is less than \$34)."* avoid manipulation

① Buy WorldCom 1.2206 shares

② Sell C(34) 1.2206 shares

③ Buy C(40) 1.0375 shares

Price of portfolio X → 1 share MCI

- The payoff is contingent upon price of WorldCom's 20-day average stock price prior to the closing exchange offer: