Risk Factors in Equity Markets

Anisha Ghosh*

*McGill University

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Multifactor Models

- In the CAPM, bad times are defined as periods of low return of the market portfolio.

Multifactor Models

Recognize that bad times can be defined more broadly than just bad returns on the market $\Rightarrow$ there are multiple factors, with each factor defining its own set of bad times.

- The first multifactor model was the arbitrage pricing theory (APT), developed by Ross (1976).
  - The factors cannot be arbitraged or diversified away (hence the word 'arbitrage').
  - $\Rightarrow$ in equilibrium, investors must be compensated for bearing these multiple sources of factor risk.
  - The APT is silent on the number and identity of the factors.

Outline

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Fama-French Model

- **Size effect:** Stocks with lower market capitalization (small stocks) tend to have higher average returns than large stocks, after adjusting for their betas (Banz (1981), Reinganum (1981)).
- **Value effect:** Value stocks (stocks with high ratios of a fundamental e.g., book value, sales, earnings, dividends, to price) tend to have higher average returns than growth stocks (stocks with low ratios of fundamentals to price) (Basu (1977)).
- Fama and French (1993): propose a three-factor model to capture the patterns in U.S. average returns associated with size and value versus growth:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m}\left(r_{m,t} - r_{f,t}\right) + \beta_{i,SMB} r_{SMB,t} + \beta_{i,HML} r_{HML,t} + \varepsilon_{i,t}$$

  - $r_{SMB,t}$ is a factor mimicking portfolio constructed to capture the size premium.
  - $r_{HML,t}$ is a factor mimicking portfolio constructed to capture the value premium.

Construction of Factors

- The NYSE, AMEX, and NASDAQ stocks are divided into 2 size (market-equity) groups, Small and Big, based on the median of the ranked values of market equity for NYSE stocks.
- The NYSE, AMEX, and NASDAQ stocks are divided into 3 book-to-market-equity groups based on the breakpoints for the bottom 30% (Low), middle 40% (Medium), and top 30% (High) of the ranked values of BE/ME for NYSE stocks.
  - BE/ME is the book common equity for the fiscal year ending in calender year $t-1$, divided by market equity at the end of December of $t-1$.
- The decision to sort firms into 3 groups on BE/ME and only 2 on ME is based on the evidence that the former plays a stronger role in average stock returns than size.

Construction of Factors cont'd

- 6 portfolios $(S/L, S/M, S/H, B/L, B/M, B/H)$ are constructed from the intersection of the two ME and 3 BE/ME groups.
- Monthly value-weighted returns on the 6 portfolios are calculated from July of year t to June of year $t+1$.
- The portfolios are reformed in June of $t+1$.

Size Factor

SMB

SMB is the difference, each month, between the simple average of the returns on the 3 small-stock portfolios and the simple average of the returns on the 3 big-stock portfolios:

$$r_{SMB,t} = \frac{1}{3}\left(r_{S/L,t} + r_{S/M,t} + r_{S/H,t}\right) - \frac{1}{3}\left(r_{B/L,t} + r_{B/M,t} + r_{B/H,t}\right)$$

- SMB is the difference between the returns on small and big-stock portfolios with about the same weighted-average book-to-market equity.
- $\Rightarrow$it is largely free of the influence of BE/ME, focusing instead on the different return behaviors of small and big stocks.

BE/ME Factor

HML

HML is the difference, each month, between the simple average of the returns on the 2 high BE/ME portfolios and the simple average of the returns on the 2 low BE/ME portfolios:

$$r_{HML,t} = \frac{1}{2}\left(r_{S/H,t} + r_{B/H,t}\right) - \frac{1}{2}\left(r_{S/L,t} + r_{B/L,t}\right)$$

- The two components of HML have about the same weighted-average size.
- $\Rightarrow$it is largely free of the size factor in returns, focusing instead on the different return behaviors of high and low BE/ME firms.
- The correlation between monthly $r_{SMB,t}$ and $r_{HML,t}$ factors is -0.08.

Returns to be Explained

- Excess returns on 25 portfolios formed on size and book-to-market equity (to determine whether the factor mimicking portfolios SMB and HML capture common risk factors in stock returns related to size and BE/ME).
- The 25 portfolios also produce a wide spread of average returns to be explained by competing asset-pricing models.
- The 25 size-BE/ME portfolios are formed much like the 6 size-BE/ME portfolios discussed earlier: from the intersection of 5 ME-sorted portfolios and 5 BE/ME-sorted portfolios.

Descriptive Statistics of 25 Portfolios

Table 1

Descriptive statistics for 25 stock portfolios formed on size and book-to-market equity: 1963–1991, 29 years.*

Size quintile	Book-to-market equity (BE/ME) quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Average of annual averages of firm size					Average of annual B/E ratios for portfolio				
Small	20.6	20.8	20.2	19.4	15.1	0.30	0.62	0.84	1.09	1.80
2	89.7	89.3	89.3	89.9	88.5	0.31	0.60	0.83	1.09	1.71
3	209.3	211.9	210.8	214.8	210.7	0.31	0.60	0.84	1.08	1.66
4	535.1	537.4	545.4	551.6	538.7	0.31	0.61	0.84	1.09	1.67
Big	3583.7	2885.8	2819.5	2700.5	2337.9	0.29	0.59	0.83	1.08	1.56
	Average of annual percent of market value in portfolio					Average of annual number of firms in portfolio				
Small	0.69	0.49	0.46	0.48	0.64	428.0	276.6	263.8	291.5	512.7
2	0.92	0.71	0.65	0.61	0.55	121.6	94.0	86.7	79.8	71.3
3	1.78	1.36	1.26	1.14	0.82	102.7	78.3	73.0	64.5	45.9
4	3.95	3.01	2.71	2.41	1.50	90.1	68.9	60.7	53.1	33.4
Big	30.13	15.87	12.85	10.44	4.61	93.6	63.7	52.7	44.0	23.6
	Average of annual E/P ratios (in percent) for portfolio					Average of annual D/P ratios (in percent) for portfolio				
Small	2.42	7.24	8.26	9.06	2.66	1.00	1.94	2.60	3.13	2.82
2	5.20	8.61	10.16	10.95	9.28	1.59	2.45	3.45	4.25	4.53
3	5.91	8.72	10.43	11.62	10.78	1.56	3.03	4.04	4.68	4.64
4	5.85	8.94	10.45	11.64	11.39	1.80	3.09	4.22	5.01	4.94
Big	6.00	9.07	10.30	12.15	13.92	2.56	3.69	4.68	5.49	5.90

Summary Statistics of Dependent Returns

Dependent variables: Excess returns on 25 stock portfolios formed on *ME* and *BE/ME*

Size quintile	Book-to-market equity (*BE/ME*) quintiles: Low	2	3	4	High	Low	2	3	4	High
	Means					Standard deviations				
Small	0.39	0.70	0.79	0.88	1.01	7.76	6.84	6.29	5.99	6.27
2	0.44	0.71	0.85	0.84	1.02	7.28	6.42	5.85	5.33	6.06
3	0.43	0.66	0.68	0.81	0.97	6.71	5.71	5.27	4.92	5.69
4	0.48	0.35	0.57	0.77	1.05	5.97	5.44	5.03	4.95	5.75
Big	0.40	0.36	0.32	0.56	0.59	4.95	4.70	4.38	4.27	4.85
	t-statistics for means									
Small	0.93	1.88	2.33	2.73	2.97					
2	1.11	2.05	2.69	2.91	3.11					
3	1.18	2.12	2.39	3.04	3.15					
4	1.49	1.19	2.08	2.88	3.36					
Big	1.50	1.42	1.34	2.43	2.26					

- Produce a wide range of average excess returns to be explained (.32% to 1.05% per month).
- Confirm negative relation between size and average return.
- Confirm stronger positive relation between average return and book-to-market equity.

Summary Statistics of Explanatory Returns

Table 2

Summary statistics for the monthly dependent and explanatory returns (in percent) in the regressions of tables 3 to 8: July 1963 to December 1991, 342 observations.[a]

				Autocorr. for lag			Correlations				
Name	Mean	Std.	$t(mn)$	1	2	12					
				Explanatory returns							
RM	0.97	4.52	3.97	0.05	−0.05	0.03					
TB	0.54	0.22	45.97	0.94	0.90	0.65					
LTG	0.60	3.03	3.66	0.05	−0.00	0.00					
CB	0.62	2.24	5.10	0.20	−0.04	0.04					
RM-RF	0.43	4.54	1.76	0.05	−0.04	0.03	*RM-RF*	*RMO*	*SMB*	*HML*	*TERM*
RMO	0.50	3.55	2.61	−0.10	−0.05	0.02	0.78	1.00			
SMB	0.27	2.89	1.73	0.19	0.07	0.23	0.32	−0.00	1.00		
HML	0.40	2.54	2.91	0.18	0.06	0.07	−0.38	−0.00	−0.08	1.00	
TERM	0.06	3.02	0.38	0.05	−0.00	−0.00	0.34	0.00	−0.07	−0.05	1.00
DEF	0.02	1.60	0.21	0.20	−0.04	−0.00	−0.07	−0.00	0.17	0.08	−0.69

Note: The average risk premiums for the common factors in returns are just the average values of the explanatory variables.

Common Variation in Returns

- Fama and French (1993) adopt a time-series regression approach to examine whether the risk factors ($r_{m,t} - r_{f,t}$, $r_{SMB,t}$, $r_{HML,t}$) capture common (shared and, thus, undiversifiable) variation in stock and bond returns.
- In time series regressions, the slopes and R^2 provide direct evidence on this.

Regression on the Market Factor

Table 4

Regressions of excess stock and bond returns (in percent) on the excess stock-market return, $RM-RF$: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity

Size quintile	Book-to-market equity (BE/ME) quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	b					$t(b)$				
Small	1.40	1.26	1.14	1.06	1.08	26.33	28.12	27.01	25.03	23.01
2	1.42	1.25	1.12	1.02	1.13	35.76	35.56	33.12	33.14	29.04
3	1.36	1.15	1.04	0.96	1.08	42.98	42.52	37.50	35.81	31.16
4	1.24	1.14	1.03	0.98	1.10	51.67	55.12	46.96	37.00	32.76
Big	1.03	0.99	0.89	0.84	0.89	51.92	61.51	43.03	35.96	27.75
	R^2					$s(e)$				
Small	0.67	0.70	0.68	0.65	0.61	4.46	3.76	3.55	3.56	3.92
2	0.79	0.79	0.76	0.76	0.71	3.34	2.96	2.85	2.59	3.25
3	0.84	0.84	0.80	0.79	0.74	2.65	2.28	2.33	2.26	2.90
4	0.89	0.90	0.87	0.80	0.76	2.01	1.73	1.84	2.21	2.83
Big	0.89	0.92	0.84	0.79	0.69	1.66	1.35	1.73	1.95	2.69

- The market factor leaves much of the variation in stock returns unexplained (particularly for small-stock and high-BE/ME portfolios).

Regression on SMB and HML

Table 5

Regressions of excess stock and bond returns (in percent) on the mimicking returns for the size (SMB) and book-to-market equity (HML) factors: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + sSMB(t) + hHML(t) + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity

Size quintile	Book-to-market equity (BE/ME) quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	s					$t(s)$				
Small	1.93	1.73	1.63	1.59	1.67	22.52	21.38	21.88	22.30	22.16
2	1.52	1.46	1.35	1.18	1.40	17.23	17.68	17.08	15.47	16.42
3	1.28	1.12	1.05	0.93	1.16	14.43	13.89	13.42	12.13	13.45
4	0.86	0.82	0.77	0.72	0.95	10.16	9.64	9.29	8.57	10.02
Big	0.28	0.35	0.22	0.29	0.44	3.70	4.39	2.79	3.69	5.02
	h					$t(h)$				
Small	0.95	0.57	− 0.35	− 0.18	0.01	− 9.72	− 6.19	− 4.10	− 2.20	0.16
2	− 1.23	− 0.66	− 0.38	− 0.16	0.00	− 12.25	− 7.02	− 4.20	− 1.82	0.05
3	− 1.09	− 0.65	− 0.31	− 0.11	− 0.01	− 10.84	− 7.07	− 3.43	− 1.23	− 0.12
4	− 1.11	− 0.65	− 0.36	− 0.11	− 0.01	− 11.43	− 6.69	− 3.80	− 1.12	− 0.09
Big	− 1.07	− 0.65	− 0.42	− 0.06	0.08	− 12.46	− 7.07	− 4.64	− 0.66	0.81
	R^2					$s(e)$				
Small	0.65	0.60	0.60	0.60	0.59	4.57	4.31	3.98	3.79	4.01
2	0.59	0.53	0.49	0.42	0.44	4.68	4.41	4.20	4.06	4.53
3	0.51	0.43	0.37	0.31	0.35	4.71	4.31	4.19	4.10	4.60
4	0.43	0.30	0.24	0.18	0.23	4.53	4.55	4.40	4.48	5.06
Big	0.34	0.18	0.08	0.04	0.06	4.02	4.27	4.20	4.19	4.69

- In the absence of competition from the market factor, SMB and HML typically capture substantial time variation in stock returns.
- However, they leave common variation in stock returns that is picked up by the market (especially for large-size portfolios).

Regression on 3 FF Factors

Table 6

Regressions of excess stock and bond returns (in percent) on the excess market return ($RM-RF$) and the mimicking returns for the size (SMB) and book-to-market equity (HML) factors: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity

Size quintile	Book-to-market equity (BE/ME) quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	b					$t(b)$				
Small	1.04	1.02	0.95	0.91	0.96	39.37	51.80	60.44	59.73	57.89
2	1.11	1.06	1.00	0.97	1.09	52.49	61.18	55.88	61.54	65.52
3	1.12	1.02	0.98	0.97	1.09	56.88	53.17	50.78	54.38	52.52
4	1.07	1.08	1.04	1.05	1.18	53.94	53.51	51.21	47.09	46.10
Big	0.96	1.02	0.98	0.99	1.06	60.93	56.76	46.57	53.87	38.61
	s					$t(s)$				
Small	1.46	1.26	1.19	1.17	1.23	37.92	44.11	52.03	52.85	50.97
2	1.00	0.98	0.88	0.73	0.89	32.73	38.79	34.03	31.66	36.78
3	0.76	0.65	0.60	0.48	0.66	26.40	23.39	21.23	18.62	21.91
4	0.37	0.33	0.29	0.24	0.41	12.73	11.11	9.81	7.38	11.01
Big	− 0.17	− 0.12	− 0.23	− 0.17	− 0.05	− 7.18	− 4.51	− 7.58	− 6.27	− 1.18
	h					$t(h)$				
Small	− 0.29	0.08	0.26	0.40	0.62	− 6.47	2.35	9.66	15.53	22.24
2	− 0.52	0.01	0.26	0.46	0.70	− 14.57	0.41	8.56	17.24	24.80
3	− 0.38	0.00	0.32	0.51	0.68	− 11.26	0.05	9.75	16.88	19.39
4	− 0.42	0.04	0.30	0.56	0.74	− 12.51	1.04	8.83	14.84	17.09
Big	− 0.46	0.00	0.21	0.57	0.76	− 17.03	0.09	5.80	18.34	16.24

- SMB captures shared variation in returns missed by the market and HML.
- HML captures shared variation in returns missed by the market and SMB.
- Addition of SMB and HML causes the market βs to collapse toward 1.

Table 6 cont'd

	R^2					$s(e)$			
Small	0.94	0.96	0.97	0.97	0.96	1.94	1.44	1.16	1.12
2	0.95	0.96	0.95	0.95	0.96	1.55	1.27	1.31	1.16
3	0.95	0.94	0.93	0.93	0.93	1.45	1.41	1.43	1.32
4	0.94	0.93	0.91	0.89	0.89	1.46	1.48	1.49	1.63
Big	0.94	0.92	0.88	0.90	0.83	1.16	1.32	1.55	1.36

- Addition of SMB and HML causes large increases in R^2.

Cross-Section of Average Returns

- Average-return tests centre on the intercepts in the time-series regressions.

If the premiums associated with any set of factors suffice to describe the cross-section of average returns, the intercepts in the time-series regressions of excess returns on the factor-mimicking portfolio returns should be indistinguishable from zero.

Intercepts from Time-Series Regressions

(ii) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + e(t)$

Small	− 0.22	0.15	0.30	0.42	0.54	− 0.90	0.73	1.54	2.19	2.53
2	− 0.18	0.17	0.36	0.39	0.53	− 1.00	1.05	2.35	2.79	3.01
3	− 0.16	0.15	0.23	0.39	0.50	− 1.12	1.25	1.82	3.20	3.19
4	− 0.05	− 0.14	0.12	0.35	0.57	− 0.50	− 1.50	1.20	2.91	3.71
Big	− 0.04	− 0.07	− 0.07	0.20	0.21	− 0.49	− 0.95	− 0.70	1.89	1.41

(iii) $R(t) - RF(t) = a + sSMB(t) + hHML(t) + e(t)$

Small	0.24	0.46	0.49	0.53	0.55	0.97	1.92	2.24	2.52	2.49
2	0.52	0.58	0.64	0.58	0.64	2.00	2.40	2.76	2.61	2.56
3	0.52	0.61	0.52	0.60	0.66	2.00	2.58	2.25	2.66	2.61
4	0.69	0.39	0.50	0.62	0.79	2.78	1.55	2.07	2.51	2.85
Big	0.76	0.52	0.43	0.51	0.44	3.41	2.23	1.84	2.20	1.70

(iv) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$

Small	− 0.34	− 0.12	− 0.05	0.01	0.00	− 3.16	− 1.47	− 0.73	0.22	0.14
2	− 0.11	− 0.01	0.08	0.03	0.02	− 1.24	− 0.20	1.04	0.51	0.34
3	− 0.11	0.04	− 0.04	0.05	0.05	− 1.42	0.47	− 0.47	0.71	0.56
4	0.09	− 0.22	− 0.08	0.03	0.13	1.07	− 2.65	− 0.99	0.33	1.24
Big	0.21	− 0.05	− 0.13	− 0.05	− 0.16	3.27	− 0.67	− 1.46	− 0.69	− 1.41

- The 3 factors do a good job explaining the cross-section of stock returns.
- The size and book-to-market factors can explain the differences in average returns across stocks, but the market factor is needed to explain why stock returns are on average above the one-month bill rate.