Active Management: Stock Market Prediction

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Outline

Appendix

Stock Market Prediction

- Academic research has provided compelling theoretical and empirical evidence that the return on the aggregate stock market is predictable (e.g. Fama and French (1988, 1989), Campbell and Shiller (1988), Cochrane (2008), ...)
- Numerous predictor variables have been proposed:
  - Valuation ratios (e.g., dividend-price ratio, earnings-price ratio, book-to-market ratio)
  - Interest rates (e.g., short-term T-bill rate, yield spread between long and short term Treasury bonds, default spread between corporate and Treasury bonds)
  - Corporate issuing activity (e.g., net equity expansion, percent equity issuing)
  - Inflation
  - Investment to capital ratio

Is the Market Return Predictable?

- Comprehensively re-examine the performance of variables that have been suggested to be good predictors of the equity premium, via a regression framework

$$r_{t+1} = \delta_0 + \delta_1 x_t + \varepsilon_{t+1}$$

- Emphasize Out-Of-Sample (OOS) performance as indicative of whether an investor can profitably time the market (OOS forecast uses only data available up to the time at which the forecast is made)
- Commonly used OOS statistic:

$$R^2 = 1 - \frac{MSE_A}{MSE_N} = 1 - \frac{\sum e_A^2}{\sum e_N^2}$$

where e_N and e_A denote the vector of rolling OOS forecast errors from the historical mean model and the OLS model, respectively.

Predictive Performance

			Full Sample									1927–2005
				Forecasts begin 20 years after sample				Forecasts begin 1965				Sample
			IS	IS for	OOS			IS for	OOS			IS
	Variable	Data	$\bar{R}^2$	OOS $\bar{R}^2$	$\bar{R}^2$	ΔRMSE	Power	OOS $\bar{R}^2$	$\bar{R}^2$	ΔRMSE	Power	$\bar{R}^2$
Full Sample, Not Significant IS												
dfy	Default yield spread	1919–2005	−1.18		−3.29	−0.14			−4.15	−0.12		−1.31
infl	Inflation	1919–2005	−1.00		−4.07	−0.20			−3.56	−0.08		−0.99
svar	Stock variance	1885–2005	−0.76		−27.14	−2.33			−2.44	+0.01		−1.32
d/e	Dividend payout ratio	1872–2005	−0.75		−4.33	−0.31			−4.99	−0.18		−1.24
lty	Long term yield	1919–2005	−0.63		−7.72	−0.47			−12.57	−0.76		−0.94
tms	Term spread	1920–2005	0.16		−2.42	−0.07			−2.96	−0.03		0.89
tbl	Treasury-bill rate	1920–2005	0.34		−3.37	−0.14			−4.90	−0.18		0.15
dfr	Default return spread	1926–2005	0.40		−2.16	−0.03			−2.82	−0.02		0.32
d/p	Dividend price ratio	1872–2005	0.49		−2.06	−0.11			−3.69	−0.09		1.67
d/y	Dividend yield	1872–2005	0.91		−1.93	−0.10			−6.68	−0.31		**2.71***
ltr	Long term return	1926–2005	0.99		−11.79	−0.76			−18.38	−1.18		0.92
e/p	Earning price ratio	1872–2005	1.08		−1.78	−0.08			−1.10	0.11		**3.20***

Predictive Performance Cont'd

			Full Sample									1927–2005
			Forecasts begin 20 years after sample					Forecasts begin 1965				Sample
			IS	IS for	OOS			IS for	OOS			IS
Variable		Data	$\bar{R}^2$	OOS $\bar{R}^2$	$\bar{R}^2$	ΔRMSE	Power	OOS $\bar{R}^2$	$\bar{R}^2$	ΔRMSE	Power	$\bar{R}^2$
Full Sample, Significant IS												
b/m	Book to market	1921–2005	**3.20***	1.13	−1.72	−0.01	42 (67)	−7.29	−12.71	−0.77	40 (61)	**4.14***
i/k	Invstmnt capital ratio	1947–2005	**6.63****	−0.25	−1.77	0.07	47 (77)		Same			Same
ntis	Net equity expansion	1927–2005	**8.15*****	−4.21	−5.07	−0.26	57 (78)	0.96	−6.79	−0.32	53 (72)	Same
eqis	Pct equity issuing	1927–2005	**9.15*****	2.81	**2.04****	0.30	72 (85)	3.64	−1.00	0.12	66 (77)	Same
all	Kitchen sink	1927–2005	**13.81****	2.62	−139.03	−5.97	- (-)	−20.91	−176.18	−6.19	- (-)	Same
Full sample, no IS equivalent (caya, ms) or Ex-Post Information (cayp)												
cayp	Cnsmptn, wlth, incme	1945–2005	**15.72*****	20.70	**16.78*****	1.61	- (-)		Same			Same
caya	Cnsmptn, wlth, incme	1945–2005	-	-	−4.33	−0.14	- (-)		Same			Same
ms	Model selection	1927–2005	-	-	−22.50	−1.69	- (-)	-	−23.71	−1.79	- (-)	Same
1927-2005 Sample, Significant IS												Full Sample
d/y	Dividend yield	1927–2005	**2.71***					−0.35	−6.44	−0.30	30 (71)	0.91
e/p	Earning price ratio	1927–2005	**3.20***					−0.94	−3.15	−0.05	39 (64)	1.08
b/m	Book to market	1927–2005	**4.14***					−8.65	−19.46	−1.26	45 (64)	3.20*

Representing OOS Performance Grapically

- Plot the cumulative squared prediction errors of the prevailing mean minus the cumulative squared prediction errors of the predictive variable from the linear historical regression.
- Whenever the line increases, the ALTERNATIVE (predictor variable) predicted better.
- Whenever the line decreases, the NULL predicted better.

Note: The units of the graph are not intuitive, but the time series pattern allows diagnosis of years with good or bad performance.

Predictive Performance: Graphical Illustration

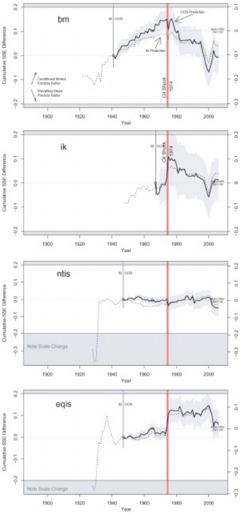

Figure 2
Annual performance of predictors that are not in-sample significant
Explanation: See Figure 1.

Predictive Performance: Graphical Illustration Cont'd

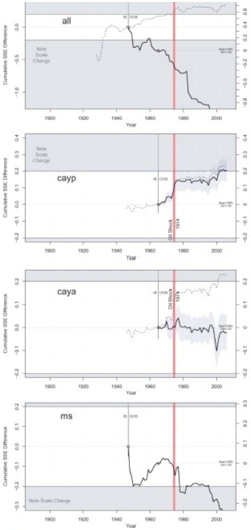

Figure 2 Continued

Sign Restrictions on Slope Coefficients and Forecasts

- Rolling OOS regressions are typically estimated over short sample periods, and can easily generate perverse results (e.g., a negative coefficient when theory suggests it should be positive).
- Explore the impact of imposing sensible restrictions on the OOS forecasting exercise:
  1. Set the regression coefficient to zero whenever it has the 'wrong'sign (different from the theoretically expected sign estimated over the full sample).
  2. Assume investors rule out a negative equity premium, and set forecast to zero whenever it is negative.
  3. Impose first the sign restriction on the coefficient, and then the sign restriction on the forecast.

Predictive Performance

Table 1
Excess return prediction with regression constraints

	Sample Begin	Forecast Begin	In-Sample t-statistic	In-Sample R-squared	Out-of-Sample R-squared with Different Constraints: Unconstrained	Positive Slope	Pos. Forecast
A: Monthly Returns							
Dividend-price ratio	1872m2	1927m1	1.25	1.13%	−0.65%	0.05%	0.07%
Earnings-price ratio	1872m2	1927m1	2.29	0.71	0.12	0.18	0.14
Smooth earnings-price ratio	1881m2	1927m1	1.85	1.36	0.33	0.42	0.38
Book-to-market	1926m6	1946m6	1.96	0.61	−0.43	−0.43	0.00
ROE	1936m6	1956m6	0.36	0.02	−0.93	−0.06	−0.93
T-Bill rate	1920m1	1940m1	2.44	0.86	0.52	0.51	0.57
Long-term yield	1870m1	1927m1	1.46	0.19	−0.19	−0.19	0.20
Term spread	1920m1	1940m1	2.16	0.65	0.46	0.47	0.45
Default spread	1919m1	1939m1	0.74	0.10	−0.19	−0.19	−0.19
Inflation	1871m5	1927m1	0.39	0.06	−0.22	−0.21	−0.18
Net equity issuance	1927m12	1947m12	1.74	0.48	0.34	0.34	0.50
Consumption-wealth ratio	1951m12	1971m12	4.57	2.60	−1.36	−1.36	0.27
B: Annual Returns							
Dividend-price ratio	1872m2	1927m1	2.69	10.8	5.53	5.53	5.63
Earnings-price ratio	1872m2	1927m1	2.84	6.78	4.93	4.93	4.94
Smooth earnings-price ratio	1881m2	1927m1	3.01	13.57	7.89	7.89	7.85
Book-to-market	1926m6	1946m6	1.98	8.26	−3.38	−3.38	1.39
ROE	1936m6	1956m6	0.35	0.32	−8.60	−0.03	−8.35
T-Bill rate	1920m1	1940m1	1.77	4.26	5.54	5.54	7.47
Long-term yield	1870m1	1927m1	0.91	0.77	−0.15	−0.15	2.26
Term spread	1920m1	1940m1	1.72	3.10	4.79	4.79	4.74
Default spread	1919m1	1939m1	0.07	0.01	−3.81	−3.81	−3.81
Inflation	1871m5	1927m1	0.17	0.07	−0.71	−0.71	−0.71
Net equity issuance	1927m12	1947m12	0.54	0.35	−4.27	−4.27	−2.38
Consumption-wealth ratio	1951m12	1971m12	3.76	19.87	−7.75	−7.75	−1.48

- The restrictions never worsen and almost always improve the performance of the OOS regressions.
- However, the R^2 statistics are small in magnitude raising the question of whether they are economically meaningful.

How Large an R^2 Should We Expect?

- Consider the following predictive regression:

$$r_{t+1} = \mu + x_t + \varepsilon_{t+1}$$

  where r_{t+1} is the excess return on the market over the risk free interest rate, μ is the unconditional average excess return, x_t is a predictor variable with mean zero and variance σ_x^2, and ε_{t+1} is a random shock with mean zero and variance σ_ε^2.
- Suppose an investor has a single-period horizon and mean-variance preferences:

$$U = E\left(r_p\right) - \frac{\gamma}{2}\sigma_p^2$$

  where γ is the coefficient of relative risk aversion.

Example Cont'd: Investor does not observe x

- In this case, the investor chooses a portfolio weight in the risky asset

$$\omega_t = \omega = \left(\frac{1}{\gamma}\right)\left(\frac{\mu}{\sigma_x^2 + \sigma_\varepsilon^2}\right)$$

- She earns an average excess return of

$$\left(\frac{1}{\gamma}\right)\left(\frac{\mu^2}{\sigma_x^2 + \sigma_\varepsilon^2}\right) = \frac{S^2}{\gamma}$$

Example Cont'd: Investor observes x

- In this case, the investor chooses a portfolio weight in the risky asset

$$\omega_t = \left(\frac{1}{\gamma}\right)\left(\frac{\mu + x_t}{\sigma_\varepsilon^2}\right)$$

- She earns an average excess return of

$$\left(\frac{1}{\gamma}\right)\left(\frac{\mu^2 + \sigma_x^2}{\sigma_\varepsilon^2}\right) = \left(\frac{1}{\gamma}\right)\left(\frac{S^2 + R^2}{1 - R^2}\right)$$

where

$$R^2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2}$$

Example Cont'd

- The difference between the two expected returns is

$$\left(\frac{1}{\gamma}\right)\left(\frac{R^2}{1-R^2}\right)\left(1+S^2\right) > \frac{R^2}{\gamma}$$

  [Note:] the difference is close to $\frac{R^2}{\gamma}$ when the time interval is short and R^2 and S^2 are both small.

- The proportional increase in the expected return from observing x_t is

$$\left(\frac{R^2}{1-R^2}\right)\left(\frac{1+S^2}{S^2}\right) > \frac{R^2}{S^2}$$

  [Note:] the difference is close to $\frac{R^2}{S^2}$ when the time interval is short and R^2 and S^2 are both small.

- $\Rightarrow$ the correct way to judge the magnitude of R^2 is to compare it with the squared Sharpe ratio S^2.
- If R^2 is large relative to S^2, the investor can use the information in the predictive regression to obtain a large proportional increase in expected return.
- The absolute increase in portfolio return depends on risk aversion.

Example Cont'd

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Exploiting Predictability: Increase in Average Returns

- The monthly Sharpe ratio for stocks is .108 $\Rightarrow S^2 = .012 = 1.2\%$
- The monthly OOS R^2 for, say, smoothed earnings-price ratio is .43%.
- $\Rightarrow$ a mean-variance investor can use this variable to increase average monthly portfolio return by proportional factor of $.43/1.2 = 36\%$.
- The absolute increase in portfolio return depends on the investor's risk aversion:
  - If $\gamma = 1$, the absolute increase in portfolio return is .43% or 5.2% per year.
  - If $\gamma = 3$, the absolute increase in portfolio return is 1.7% per year.

Exploiting Predictability: Welfare Gains

- The investor who observes x_t earns a higher portfolio return in part by taking on greater risk.
- $\Rightarrow$ the increase in average return is not a pure welfare gain for a risk-averse investor.
- We calculate the change in utility, relative to investing with the historical mean forecast of the equity premium, for an investor with $\gamma = 3$ (the welfare benefits generated by optimally trading on each predictor variable).
  - Constrain α_t to lie between 0 and 1.5 (realistic portfolio constraints for average investors).
  - Assume investor estimates variance using a rolling five-year window of monthly data.

Note: Utility differences have units of expected annualized returns $\Rightarrow$ they can be interpreted as transactions costs or management fees that investors would be willing to pay each year to exploit the information in the predictor variable.

Welfare Gains: Example

Table 4
Portfolio choice

	Sample: 1927–1956			Sample: 1956–1980			Sample: 1980–2005		
	Unconstrained	Pos. Intercept, Bounded Slope	Fixed Coefs	Unconstrained	Pos. Intercept, Bounded Slope	Fixed Coefs	Unconstrained	Pos. Intercept, Bounded Slope	Fixed Coefs
A: Monthly Returns									
Dividend/price	−0.03%	0.43%	0.11%	1.93%	0.92%	1.46%	−3.69%	−0.22%	−1.32%
Earnings/price	0.86	1.42	1.80	0.28	0.12	−0.51	−0.80	0.07	0.74
Smooth earnings/price	0.53	1.14	1.89	0.75	0.05	−0.04	−2.73	0.13	0.46
Dividend/price + growth	−0.28	0.64	0.43	0.46	0.46	0.64	0.22	0.22	0.34
Earnings/price + growth	−1.05	0.36	1.60	0.08	0.08	−0.21	0.00	0.00	0.19
Smooth earnings/price + growth	−0.47	0.77	0.89	0.04	0.04	0.13	0.04	0.06	0.24
Book-to-market + growth				0.43	0.15	1.38	0.08	−0.02	0.23
Dividend/price + growth − real rate	0.42	0.96	−0.74	0.01	0.01	1.49	0.04	0.04	0.18
Earnings/price + growth − real rate	−0.31	0.61	0.79	−0.17	−0.17	0.73	−0.09	−0.09	0.17
Smooth earnings/price + growth − real rate	0.36	1.14	−0.20	−0.38	−0.38	1.07	0.04	0.04	0.21
Book-to-market + growth − real rate				0.38	0.05	1.94	−0.04	−0.10	−0.11
B: Annual Returns									
Dividend/price	0.55	0.76	0.34	1.56	0.59	0.95	−4.74	0.34	−0.95
Earnings/price	1.29	1.30	0.78	1.41	0.41	0.40	−1.44	0.53	0.68
Smooth earnings/price	1.81	1.91	1.42	1.53	0.44	0.44	−3.06	0.81	0.81
Dividend/price + growth	−0.18	0.34	1.04	0.35	0.35	0.70	0.19	0.17	0.34
Earnings/price + growth	−0.14	0.28	1.52	0.15	0.15	0.49	0.04	0.04	0.24
Smooth earnings/price + growth	0.05	0.53	1.34	0.28	0.25	0.51	0.41	0.36	0.50
Book-to-market + growth				−0.01	0.14	1.27	−0.10	−0.13	0.26
Dividend/price + growth − real rate	0.44	0.66	0.02	0.01	0.01	1.06	0.06	0.06	0.23
Earnings/price + growth − real rate	0.34	0.49	0.60	−0.09	−0.09	0.85	−0.02	−0.02	0.15
Smooth earnings/price + growth − real rate	0.68	0.87	0.39	−0.20	−0.18	0.94	0.24	0.24	0.36
Book-to-market + growth − real rate				0.25	0.13	1.93	−0.17	−0.14	0.26

This table presents out-of-sample portfolio choice results. The numbers are the change in average utility from forecasting the market with the predictor instead of the historical mean. All numbers are annualized, so we multiply the monthly numbers by 12. "Unconstrained" indicates that we use the unconstrained OLS predictor of the equity premium. "Pos. Intercept, Bounded Slope" indicates that we use the forecast with the intercept bounded above zero and the slope bounded between zero and one. "Fixed Coefs" indicates that we use the forecast that sets the intercept to zero and the slope to one. The utility function is $E(Rp) - (\gamma/2)\mathrm{Var}(Rp)$, where Rp is the portfolio return and $\gamma = 3$. All utility changes are annualized, so we multiply monthly utility changes by 12.

Insight

- Small R^2 statistics can generate large benefits for investors
- $\Rightarrow$ we should expect predictive regressions to have only modest explanatory power.
- Monthly regressions with large R^2 statistics would be too profitable to believe.
- S^2 increases in proportion to the investment horizon $\Rightarrow$ larger R^2 statistics are believable at longer horizons.
- Academic research has shown that R^2 statistics increase strongly with the horizon when the predictor variable is present.