Financial Derivatives FINE 448

2. Commodity Futures: Basis Risk & Cross Hedging

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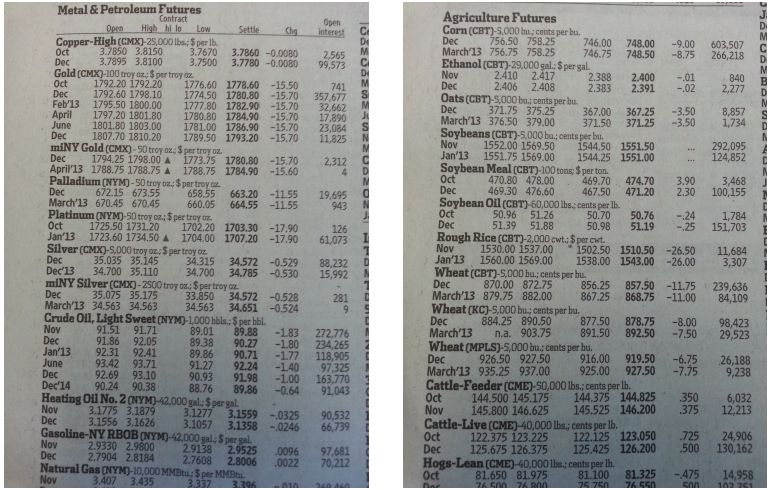
Outline

Commodity Forwards and Futures



Metal & Petroleum Futures

	Open	High	Low	Settle	Chg	Open Interest
Copper-High (CMX)-25,000 lbs.; $ per lb.						
Oct	3.7850	3.8150	3.7670	**3.7860**	-0.0080	2,565
Dec	3.7895	3.8100	3.7500	**3.7780**	-0.0080	99,573
Gold (CMX)-100 troy oz.; $ per troy oz.						
Oct	1792.20	1792.20	1776.60	**1778.60**	-15.50	741
Dec	1792.60	1798.10	1774.50	**1780.80**	-15.70	357,677
Feb'13	1795.50	1800.00	1777.80	**1782.90**	-15.70	32,662
April	1797.20	1801.80	1780.80	**1784.90**	-15.70	17,890
June	1801.80	1803.00	1781.00	**1786.90**	-15.70	23,084
Dec	1807.70	1810.20	1789.50	**1793.20**	-15.70	11,825
miNY Gold (CMX) - 50 troy oz.; $ per troy oz.						
Dec	1794.25	1798.00 ▲	1773.75	**1780.80**	-15.70	2,312
April'13	1788.75	1788.75 ▲	1788.75	**1784.90**	-15.60	4
Palladium (NYM) - 50 troy oz.; $ per troy oz.						
Dec	672.15	673.55	658.55	**663.20**	-11.55	19,695
March'13	670.45	670.45	660.05	**664.55**	-11.55	943
Platinum (NYM)-50 troy oz.; $ per troy oz.						
Oct	1725.50	1731.20	1702.20	**1703.30**	-17.90	126
Jan'13	1723.60	1734.50 ▲	1704.00	**1707.20**	-17.90	61,073
Silver (CMX)-5,000 troy oz.; $ per troy oz.						
Dec	35.035	35.145	34.315	**34.572**	-0.529	88,232
Dec'13	34.700	35.110	34.700	**34.785**	-0.530	15,992
miNY Silver (CMX) - 2500 troy oz.; $ per troy oz.						
Dec	35.075	35.175	33.850	**34.572**	-0.528	281
March'13	34.563	34.563	34.563	**34.651**	-0.524	9
Crude Oil, Light Sweet (NYM)-1,000 bbls.; $ per bbl.						
Nov	91.51	91.71	89.01	**89.88**	-1.83	272,776
Dec	91.86	92.05	89.38	**90.27**	-1.80	234,265
Jan'13	92.31	92.41	89.86	**90.71**	-1.77	118,905
June	93.42	93.71	91.27	**92.24**	-1.40	97,325
Dec	92.69	93.10	90.93	**91.98**	-1.00	163,770
Dec'14	90.24	90.38	88.76	**89.86**	-0.64	91,043
Heating Oil No. 2 (NYM)-42,000 gal.; $ per gal.						
Nov	3.1775	3.1879	3.1277	**3.1559**	-.0325	90,532
Dec	3.1556	3.1626	3.1057	**3.1358**	-.0246	66,739
Gasoline-NY RBOB (NYM)-42,000 gal.; $ per gal.						
Nov	2.9330	2.9800	2.9138	**2.9525**	.0096	97,681
Dec	2.7904	2.8184	2.7608	**2.8006**	.0022	70,212
Natural Gas (NYM)-10,000 MMBtu.; $ per MMBtu.						
Nov	3.407	3.435	3.337	**3.396**	...	240,460

Agriculture Futures

Corn (CBT)-5,000 bu.; cents per bu.						
Dec	756.50	758.25	746.00	**748.00**	-9.00	603,507
March'13	756.75	758.25	746.75	**748.50**	-8.75	266,218
Ethanol (CBT)-29,000 gal.; $ per gal.						
Nov	2.410	2.417	2.388	**2.400**	-.01	840
Dec	2.406	2.408	2.383	**2.391**	-.02	2,277
Oats (CBT)-5,000 bu.; cents per bu.						
Dec	371.75	375.25	367.00	**367.25**	-3.50	8,857
March'13	376.50	379.00	371.50	**371.25**	-3.50	1,734
Soybeans (CBT)-5,000 bu.; cents per bu.						
Nov	1552.00	1569.50	1544.50	**1551.50**	...	292,095
Jan'13	1551.75	1569.00	1544.25	**1551.00**	...	124,852
Soybean Meal (CBT)-100 tons; $ per ton.						
Oct	470.80	478.00	469.70	**474.70**	3.90	3,468
Dec	469.30	476.60	467.50	**471.20**	2.30	100,155
Soybean Oil (CBT)-60,000 lbs.; cents per lb.						
Oct	50.96	51.26	50.70	**50.76**	-.24	1,784
Dec	51.39	51.88	50.98	**51.19**	-.25	151,703
Rough Rice (CBT)-2,000 cwt.; $ per cwt.						
Nov	1530.00	1537.00	▼ 1502.50	**1510.50**	-26.50	11,684
Jan'13	1560.00	1569.00	1538.00	**1543.00**	-26.00	3,307
Wheat (CBT)-5,000 bu.; cents per bu.						
Dec	870.00	872.75	856.25	**857.50**	-11.75	239,636
March'13	879.75	882.00	867.25	**868.75**	-11.00	84,109
Wheat (KC)-5,000 bu.; cents per bu.						
Dec	884.25	890.50	877.50	**878.75**	-8.00	98,423
March'13	n.a.	903.75	891.50	**892.50**	-7.50	29,523
Wheat (MPLS)-5,000 bu.; cents per bu.						
Dec	926.50	927.50	916.00	**919.50**	-6.75	26,188
March'13	935.25	937.00	925.00	**927.50**	-7.75	9,238
Cattle-Feeder (CME)-50,000 lbs.; cents per lb.						
Oct	144.500	145.175	144.375	**144.825**	.350	6,032
Nov	145.800	146.625	145.525	**146.200**	.375	12,213
Cattle-Live (CME)-40,000 lbs.; cents per lb.						
Oct	122.375	123.225	122.125	**123.050**	.725	24,906
Dec	125.675	126.375	125.425	**126.200**	.500	130,162
Hogs-Lean (CME)-40,000 lbs.; cents per lb.						
Oct	81.650	81.975	81.100	**81.325**	-.475	14,958
Dec	76.500	76.800	75.750	**76.550**	.500	102,351

Figure 1: Listing of various currency futures contracts from the *Wall Street Journal*, October 6-7, 2012.

Introduction to Commodity Forwards and Futures

- Financial forward prices are described by the general formula

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

- At a general level, commodity forward prices can be described by the same formula. There are, however, important differences:
  - For financial assets δ is the dividend yield, whereas for commodities δ is the **commodity lease rate**
  - While the dividend yield for a financial asset can typically be observed directly, the lease rate for a commodity can be estimated **only by observing the forward price**
- The formula for a commodity forward price is

$$F_{0,T} = S_0 e^{(r-\delta_l)T} \tag{1}$$

Forward Prices and The Lease Rate

- When we observe the forward price, we can infer the lease rate. Specifically, if the forward price is $F_{0,T}$, the annualized lease rate is

continuous

$$\delta_l = r - \frac{1}{T} \ln\left(\frac{F_{0,T}}{S_0}\right) \tag{2}$$

- If instead we use an effective annual interest rate, the effective annual lease rate is

concrete

$$\delta_l = \frac{1+r}{(F_{0,T}/S_0)^{1/T}} - 1 \tag{3}$$

Suppose that on June 6, 2001, the gold spot price is \$265.7 and the gold future price with maturity in December is \$269. The June to December interest rate (annualized, effective) is 3.9917%. What is the annualized 6-month gold lease rate?

- Using equation (3), the annualized 6-month lease rate is

$$\text{6-month lease rate } = \frac{1+0.039917}{(269/265.7)^{1/0.5}} - 1 = 1.456\%$$

The Cost of Carry

$$F_{0,T} = S_0 e^{(r-\delta_l)T} \tag{4}$$

- Commodities can be subject to significant storage costs
- Ownership of the physical commodity provides benefits that are not obtained by holders of futures contracts
  - For an oil refiner, the crude oil in inventory is an essential input
  - For a food producer, the corn in inventory is an essential input
- One can thus write

$$\delta_l = \text{storage cost} + \text{convenience yield} = -u + y \tag{5}$$

- The futures price is then

$$F_{0,t} = S_0 e^{(r+u-y)} = S_0 e^{(c-y)T} \tag{6}$$

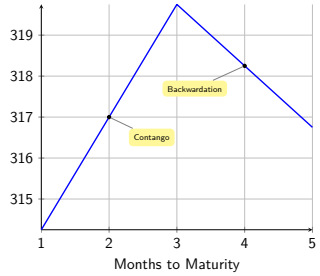
where $c = r + u$ is the **cost of carry**.

The Forward Curve

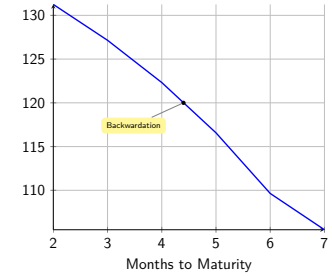
- Commodities are complex because every commodity market differs in the details. For example:
  - Storage is not possible for electricity
  - Gold is durable and relatively inexpensive to store (compared to its value)
  - Some commodities feature seasonality in production (for example, corn in the United States is harvested primarily in the fall)
- One way to observe and understand this heterogeneity is to build the **forward curve** (or the **forward strip**), i.e., the set of prices for different expiration dates for a given commodity
  - If on a given date the forward curve is upward-sloping, then the market is in **contango**. storage high
  - If the forward curve is downward sloping, the market is in **backwardation** convenience high
  - Forward curves can have portions in backwardation and portions in contango

Forward Curves for Various Commodities, May 5, 2004

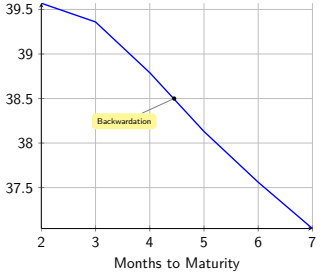
Corn Futures Price (cents per bushel)



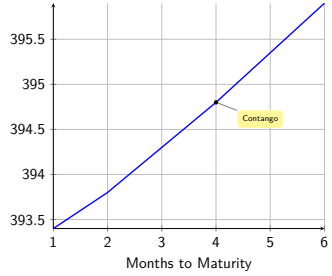
Gasoline Futures Price (cents per gallon)



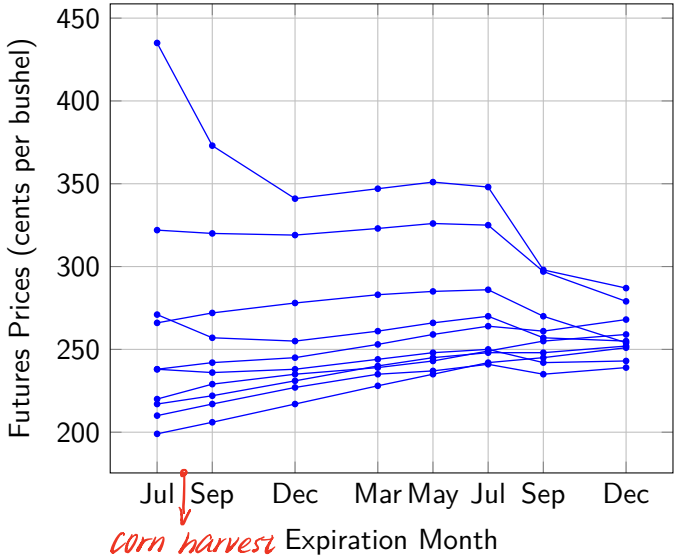
Crude Oil Futures Price ($ per barrel)



Gold Futures Price ($ per ounce)



Futures Prices for Corn for the first Wednesday in June, 1995-2004



market expect the corn prices to converge

Basis Risk

- Exchange-traded commodity futures contracts call for delivery of the underlying commodity at specific locations and specific dates.
- The actual commodity to be bought or sold may reside at a different location and the desired delivery date may not match that of the futures contract.
- Additionally, the **grade** of the deliverable under the futures contract may not match the grade that is being delivered.
- This general problem of the futures or forward contract not representing exactly what is being hedged is called **basis risk**

Cross Hedging

- **Cross hedging** = the use of a derivative on one asset to hedge another asset.
- **Example:** Jet fuels do not exist (or they are very illiquid) in United States, but firms sometimes hedge jet fuel with crude oil futures, or heating oil futures.
- If we own a quantity of jet fuel and hedge by holding N crude oil futures contracts, our mark-to-market profit depends on the change in the jet fuel price and the change in the futures price:

$$(S_t - S_{t-1}) + N(F_t - F_{t-1}) \tag{7}$$

  where S_t is the price of jet fuel and F_t the heating oil futures price.
- What is the **optimal number** of futures contracts for hedging, N^*?

$$\min_N \ \Delta S + N \Delta F$$

Cross Hedging Example

- An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.
- Define:
  - ΔS = change in the jet fuel price **per gallon**
  - ΔF = change in the heating oil futures price price **per gallon**
- The usual formulas for calculating standard deviations and correlations give (see Excel file *"CorssHedging.xlsx"*)

$$\sigma_F = 0.0313, \quad \sigma_S = 0.0263, \quad \text{and } \rho = 0.9284 \tag{8}$$

- Consider the linear relationship:

$$\Delta S = a + h^* \times \Delta F + \epsilon \tag{9}$$

- $h^* = \rho \frac{\sigma_S}{\sigma_F}$ is the **minimum variance hedge ratio**
- $R^2 = \rho^2$ is the **hedge effectiveness**
- In this example, $h^* = 0.7777$ and $R^2 = 0.8619$

(Without basis risk, these numbers would be $h^* = 1$ and $R^2 = 1$.)

Cross Hedging Example

- To calculate the number of contracts that should be used in hedging, define:
  - Q_A: Size of position being hedged (units)
  - Q_F: Size of one futures contract (units)
  - N^*: Optimal number of futures contracts for hedging
- The futures contracts should be $h^* Q_A$ units of the asset. The number of futures contracts required is therefore given by

$$N^* = \frac{h^* Q_A}{Q_F} \tag{10}$$

- Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil. Thus, the optimal number of contracts is

$$N^* = \frac{0.7777 \times 2,000,000}{42,000} = 37.03 \tag{11}$$

or, rounding to the nearest whole number, 37.

Cross Hedging Example: Tailing the Hedge

- When **futures** are used for hedging, a small adjustment, known as **tailing the hedge**, can be made to allow for the impact of daily settlement. ③ margin

  (Go back to the S&P 500 futures contract to understand why.)

- In practice, this means that the optimal number of contracts becomes:

$$N^* = \frac{h^* Q_A S_0}{Q_F F_{0,T}} \tag{12}$$

  where S_0 is the spot price and $F_{0,T}$ is the futures price.

- Suppose that in our example the spot price and the futures price are 1.94 and 1.99 dollars per gallon, respectively. Then

$$N^* = \frac{0.7777 \times 2{,}000{,}000 \times 1.94}{42{,}000 \times 1.99} = 36.10 \tag{13}$$

  or, rounding the the nearest whole number, 36.

More on Forwards and Futures

- If you want to learn more about forwards and futures, read chapter 5, *"Determination of forward and futures prices"* in **Hull** (eight edition) and chapters 5, *"Financial forwards and futures"* and 6, *"Commodity Forwards and Futures"* in **McDonald** (third edition)
- In particular, you will also discover forward and futures contracts on currencies:

$$F_{0,T} = x_0 e^{(r-r^*)T} \tag{14}$$

where x_0 is the exchange rate today and r^* is the foreign denominated interest rate.