Active Management

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Alpha

Definition

- Alpha is the average return in excess of a benchmark:

$$\begin{aligned} \alpha &\equiv \frac{1}{T}\sum_{t=1}^{T} r_t^{ex} \\ &= \frac{1}{T}\sum_{t=1}^{T} \left(r_t - r_t^{bmk}\right) \end{aligned}$$

- If the benchmark is passive (i.e., can be produced without any particular investment knowledge), r_t^{ex} is referred to as active returns:

$$r_t = \underbrace{\left(r_t - r_t^{bmk}\right)}_{\text{Active Return}} + \underbrace{r_t^{bmk}}_{\text{Benchmark Return}}$$

- The benchmark returns, r_t^{bmk} are typically the result of a strategic asset allocation decision.
- The active returns arise when asset classes in the benchmark are timed (called tactical asset allocation) or when securities are picked in each asset class (called security selection).

Tracking Error

Definition

- Tracking error is the standard deviation of the excess return r_t^{ex}; it measures how disperse the manager's returns are relative to the benchmark.

$$\text{Tracking Error} = \sigma\left(r_t^{ex}\right)$$

- Tracking error constraints are often imposed to ensure a manager does not stray too far from the benchmark.
- If the benchmark is risk-adjusted (e.g., using an asset pricing model like the CAPM), the tracking error is referred to as idiosyncratic volatility.

Information Ratio

Definition

- The information ratio is the ratio of alpha to tracking error:

$$\text{Information Ratio} = IR = \frac{\alpha}{\sigma\left(r_t^{ex}\right)}$$

- Alpha can, for example, be produced by a manager by taking large amounts of risk.
- The information ratio divides the alpha by the risk taken so it is the average excess return per unit of risk.
- When the benchmark is the risk free rate r_t^f, the alpha is the average return in excess of the risk free rate and the information ratio coincides with the Sharpe ratio:

$$\begin{aligned} \alpha &\equiv \overline{r_t - r_t^f} \\ \text{Sharpe Ratio} &= \text{IR} = \frac{\overline{r_t - r_t^f}}{\sigma\left(r_t\right)} \end{aligned}$$

Benchmarks

The concept of alpha requires first defining a benchmark against which alpha can be measured.

- Example: Martingale Asset Management's Low Volatility Strategy based on Russell 1000 universe of large stocks.
  - CAPM regression implies

$$r_t - r_t^f = 0.0344 + 0.7272 \left[r_t^{R1000} - r_t^f \right] + \varepsilon_t$$

  - $\Rightarrow \alpha = 3.44\%$ per year is the average excess return of the strategy relative to a market-adjusted portfolio.
  - If we assume a naive benchmark of just the Russell 1000, the alpha is only 1.50% per year:

$$r_t = 0.0150 + r_t^{R1000} + \varepsilon_t$$

  - The naive benchmark falsely assumes that the beta of the strategy is 1 when, in fact, it is 0.73.

Ideal Benchmarks

1. Well defined: verifiable and free of ambiguity about its contents.
2. Tradeable: Otherwise the computed alphas do not represent implementable returns on investment strategies.
3. Replicable: by both the asset owner and the funds manager.
4. Adjusted for risk: The particular risk adjustment used can make a big difference in the alpha and information ratio of a strategy.