Math 324

"single sample Tests"

1) $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

σ^2 unknown

Test statistic: $T = \dfrac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}}$

under H_0: $T \sim t(n-1)$

(small sample)

[~~If σ is known, replace S_n by σ~~]

Rejection Region:

(i) $\begin{cases} H_0: \mu \leq \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$ $\quad T \geq t_{(n-1)}^{(\alpha)}$

(ii) $\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$ $\quad T \leq -t_{(n-1)}^{(\alpha)}$

(iii) $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$ $\quad |T| \geq t_{n-1}\left(\frac{\alpha}{2}\right)$

2) $X_1, X_2, \ldots, X_n$ are iid

σ^2 unknown ; $n \geq 30$

Test statistic $Z = \dfrac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}}$

under H_0: $Z \overset{approx}{\sim} N(0,1)$

(Large Sample)

Rejection Region

(i) $\begin{cases} H_0: \mu \leq \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$ $\quad Z \geq z_\alpha$

(ii) $\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$ $\quad Z \leq -z_\alpha$

(iii) $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$ $\quad |Z| \geq z_{\alpha/2}$

3) $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

μ is unknown

Test statistic: $Q = \dfrac{(n-1)S_n^2}{\sigma_0^2}$

under H_0: $Q \sim \chi^2_{n-1}$

Rejection Region

(i) $\begin{cases} H_0: \sigma^2 \leq \sigma_0^2 \\ H_1: \sigma^2 > \sigma_0^2 \end{cases}$ $\quad Q \geq \chi^2_{(n-1;\alpha)}$

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3) Continued ...

(ii) $\begin{cases} H_0: \sigma^2 \geq \sigma_0^2 \\ H_1: \sigma^2 < \sigma_0^2 \end{cases}$ Rejection Region: $Q \leq \chi^2_{(n-1; 1-\alpha)}$

(iii) $\begin{cases} H_0: \sigma^2 = \sigma_0^2 \\ H_1: \sigma^2 \neq \sigma_0^2 \end{cases}$ $Q \geq \chi^2_{(n-1; \frac{\alpha}{2})}$

or $Q \leq \chi^2_{(n-1; 1-\frac{\alpha}{2})}$

4) $X_1, X_2, \ldots, X_n \overset{iid}{\sim} Ber(p)$

such that $\sum_{i=1}^{n} X_i = X \sim Bin(n, p)$, $\hat{p}_n = \frac{X}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$

$(n \geq 30)$

Test statistic: $Z = \frac{\hat{p}_n - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

Rejection Region

(i) $\begin{cases} H_0: p \leq p_0 \\ H_1: p > p_0 \end{cases}$ $Z \geq z_\alpha$

under H_0: $Z \overset{approx.}{\approx} N(0, 1)$

(ii) $\begin{cases} H_0: p \geq p_0 \\ H_1: p < p_0 \end{cases}$ $Z \leq -z_\alpha$

(iii) $\begin{cases} H_0: p = p_0 \\ H_1: p \neq p_0 \end{cases}$ $|Z| \geq z_{\frac{\alpha}{2}}$.

"Two-Sample Tests"

5) $X_1, X_2, \ldots, X_m \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$Y_1, Y_2, \ldots, Y_n \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$

the two samples are indep.

σ_1^2 & σ_2^2 are unknown

& $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (i.e. they are equal)

- Test statistic: $T = \dfrac{\bar{X}_m - \bar{Y}_n}{S_{Pooled}\sqrt{\frac{1}{m} + \frac{1}{n}}}$

$$S^2_{Pooled} = \frac{(m-1)S_m^2 + (n-1)S_n^2}{m+n-2}.$$

- under H_0: $T \sim t_{(m+n-2)}$

		Rejection Region
(i)	$H_0: \mu_1 \leq \mu_2$ $H_1: \mu_1 > \mu_2$	$T \geq t_{m+n-2}^{(\alpha)}$
(ii)	$H_0: \mu_1 \geq \mu_2$ $H_1: \mu_1 < \mu_2$	$T \leq -t_{m+n-2}^{(\alpha)}$
(iii)	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	$\lvert T \rvert \geq t_{m+n-2}^{(\alpha/2)}$

6) $X_1, X_2, \ldots, X_m$ are iid with μ_1, σ_1^2

$Y_1, Y_2, \ldots, Y_n$ are iid with μ_2, σ_2^2

the samples are indep. , $m \geq 30$

σ_1^2 & σ_2^2 are unknown $n \geq 30$

Test statistic: $Z = \dfrac{\bar{X}_m - \bar{Y}_n}{\sqrt{\frac{S_m^2}{m} + \frac{S_n^2}{n}}}$

under H_0: $Z \underset{\text{approx.}}{\approx} N(0,1)$

		Rejection Region
(i)	$H_0: \mu_1 \leq \mu_2$ $H_1: \mu_1 > \mu_2$	$Z \geq z_\alpha$
(ii)	$H_0: \mu_1 \geq \mu_2$ $H_1: \mu_1 < \mu_2$	$Z \leq -z_\alpha$
(iii)	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	$\lvert Z \rvert \geq z_{\frac{\alpha}{2}}$

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7) $X_1, X_2, \ldots, X_m \overset{iid}{\sim} Ber(p_1)$ where $X = \sum_{i=1}^{n} X_i \sim Bin(m, p_1)$

$Y_1, Y_2, \ldots, Y_n \overset{iid}{\sim} Ber(p_2)$ where $Y = \sum_{i=1}^{n} Y_i \sim Bin(n, p_2)$

The two samples are indep. Denote $\hat{p}_1 = \frac{X}{m}$, $\hat{p}_2 = \frac{Y}{n}$

$m \geq 30$, $n \geq 30$, $\hat{p} = \frac{X+Y}{n+m}$

Test statistic: $Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{m}+\frac{1}{n}\right)}}$

under H_0: $Z \overset{appx.}{\approx} N(0,1)$

Rejection Region

(i) $\begin{cases} H_0: p_1 \leq p_2 \\ H_1: p_1 > p_2 \end{cases}$ $Z \geq z_\alpha$

(ii) $\begin{cases} H_0: p_1 \geq p_2 \\ H_1: p_1 < p_2 \end{cases}$ $Z \leq -z_\alpha$

(iii) $\begin{cases} H_0: p_1 = p_2 \\ H_1: p_1 \neq p_2 \end{cases}$ $|Z| \geq z_{\frac{\alpha}{2}}$.