Interpretation of confidence Interval:

the calculated interval is: [2926.32, 2991.68]

Example 3, page 38/54 of my notes (slides)

this particular interval comes from a random interval that will on repeated use capture "μ" 95% of the time. (Note: with each use we will get different data and hence different calculated confidence intervals. However, we would expect 95% of them will contain "μ".

unfortunately, after any experiment we do not know if we have captured "μ" or not. the probability is now "0" or "1" that "μ" lies in the calculated interval [2926.32, 2991.68].

Examples: (Page 40/54 of slides)

$$\begin{cases} \bar{x}_n = 18.97 \text{ weeks} \\ s_n = \sqrt{10.7} \text{ weeks} \end{cases}, \quad \chi^2_{13-1;0.05} = 5.226$$

$$1-\alpha = 0.9 \Rightarrow \alpha/2 = 0.05 \qquad \chi^2_{13-1;1-0.05} = 21.026$$

A 90% C.I. for σ^2:

$$\left[\frac{(n-1)s_n^2}{\chi^2_{13-1;1-0.05}}, \frac{(n-1)s_n^2}{\chi^2_{13-1;1-0.05}} \right] = \left[\frac{(13-1)\times 10.7}{21.026}, \frac{(13-1)\times 10.7}{5.226} \right]$$

$$= [6.107, 24.569]$$

Interpretation is similar to what I have on Page 1.

Example 6: (page 41/54 of slides)

$$\begin{cases} \bar{x}_n = 33, \quad n = 64 \\ s_n = 16 \end{cases}$$

$1-\alpha = 0.9 \Rightarrow \alpha/2 = 0.05 \Rightarrow z_{0.05} = 1.645$

An approximate 90% confidence interval for μ:

$$\left[\bar{x}_n - z_{\frac{\alpha}{2}} \cdot \frac{s_n}{\sqrt{n}}, \ \bar{x}_n + z_{\frac{\alpha}{2}} \cdot \frac{s_n}{\sqrt{n}}\right]$$

$$= \left[33 - 1.645\sqrt{\frac{256}{64}}, \ 33 + 1.645 \times \sqrt{\frac{256}{64}}\right]$$

$$= [29.71, 36.29]$$

Interpretation is similar to the previous examples.

Example 7 : (Page 42/54 of slides)

$$\begin{cases} \bar{x}_n = 167.1 \\ s_n = 24.3 \\ \bar{y}_m = 14.9 \\ s_m = 17.9 \end{cases} \qquad n = m = 30$$

$$1-\alpha = 0.95 \Rightarrow \alpha/2 = 0.025 \Rightarrow z_{0.025} = 1.96$$

An approximate 95% C.I. for $\mu_1 - \mu_2$:

$$\left[(\bar{x}_n - \bar{y}_n) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{s_n^2}{n} + \frac{s_m^2}{m}},\ (\bar{x}_n - \bar{y}_n) + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{s_n^2}{n} + \frac{s_m^2}{m}} \right]$$

$$= \left[(167.1 - 14.9) \mp 1.96 \sqrt{\frac{(24.3)^2}{30} + \frac{(17.9)^2}{30}} \right]$$

$$= [15.4632, 36.9369]$$

Interpretation is similar to the previous examples.

Example 8: (Page 43/54 of slides)

$$\hat{p}_n = \bar{x}_n = \frac{560}{1000} = 0.560 \quad \text{[sample proportion]}$$

$$1-\alpha = 0.99 \Rightarrow \alpha/2 = 0.005 \Rightarrow z_{0.005} = 2.58$$

An approximate 99% C.I. for p:

$$\left[0.56 - 2.58 \times \sqrt{\frac{0.56 \times 0.44}{1000}}\ ,\ 0.56 + 2.58 \times \sqrt{\frac{0.56 \times 0.44}{1000}}\right]$$

$$= [0.5195, 0.6005]$$

Interpretation is similar to the previous examples.

Example 9: [Page 44/54 of slides]

$$\begin{cases} n = 150 \\ m = 250 \\ \hat{p}_1 = \dfrac{95}{150} \\ \hat{p}_2 = \dfrac{105}{250} \end{cases}, \quad 1-\alpha = .95 \Rightarrow \alpha = 0.025$$

$$z_{0.025} = 1.96$$

An approximate 95% C.I. for $p_1 - p_2$:

$$\left[(\hat{p}_1 - \hat{p}_2) \mp 1.96 \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{m}} \right]$$

$$= \left[\left(\frac{95}{150} - \frac{105}{250} \right) \mp 1.96 \sqrt{\frac{\frac{95}{150}\left(1-\frac{95}{150}\right)}{150} + \frac{\frac{105}{250}\left(1-\frac{105}{250}\right)}{250}} \right]$$

$$= [0.1149, 0.3118]$$