Solutions to Random Mathematics Homework #3 Fall 2020

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Assigned Date: Sept.30, 2020   Due Date: Oct.13, 2020

H3.1

Let X be the number of people who survive.

$$\Pr(X = x) = \begin{pmatrix} 15 \\ x \end{pmatrix} \times 0.4^x \times 0.6^{15-x}$$

The p.f. of X will be

$$f(x) = \begin{cases} \begin{pmatrix} 15 \\ x \end{pmatrix} \times 0.4^x \times 0.6^{15-x} & x = 0, 1, \dots, 15 \\ 0 & \text{otherwise} \end{cases}$$

a.

$$\begin{aligned} \Pr(X \geq 10) &= 1 - \Pr(X < 10) = 1 - \sum_{x=0}^{9} b(x; 15, 0.4) \\ &= 1 - 0.9662 = 0.0338 \end{aligned}$$

b.

$$\begin{aligned} \Pr(3 \leq X \leq 8) &= \sum_{x=3}^{8} b(x; 15, 0.4) = \sum_{x=0}^{8} b(x; 15, 0.4) - \sum_{x=0}^{2} b(x; 15, 0.4) \\ &= 0.9050 - 0.0271 = 0.8779 \end{aligned}$$

c.

$$\Pr\,(X = 5) = b(5; 15, 0.4) = 0.1859$$

H3.2

Let X be a random variable whose values x are the possible numbers of defective computers purchased by the school. Then x can only take the numbers 0, 1, and 2.

a.

$$f(0) = \Pr(X = 0) = \frac{\begin{pmatrix} 3 \\ 0 \end{pmatrix}\begin{pmatrix} 17 \\ 2 \end{pmatrix}}{\begin{pmatrix} 20 \\ 2 \end{pmatrix}} = \frac{68}{95}$$

$$f(1) = \Pr(X = 1) = \frac{\begin{pmatrix} 3 \\ 1 \end{pmatrix}\begin{pmatrix} 17 \\ 1 \end{pmatrix}}{\begin{pmatrix} 20 \\ 2 \end{pmatrix}} = \frac{51}{190}$$

$$f(2) = \Pr(X = 2) = \frac{\binom{3}{2}\binom{17}{0}}{\binom{20}{2}} = \frac{3}{190}$$

Thus, the probability distribution of X is

x	0	1	2
$f(x)$	$68/95$	$51/190$	$3/190$

The p.f. of X is as follows:

$$f(x) = \begin{cases} \dfrac{\binom{3}{x}\binom{17}{2-x}}{\binom{20}{2}} & x = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

b.

$$F(x) = \begin{cases} 0 & x < 0 \\ 68/95 & 0 \le x < 1 \\ 187/190 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

c. The p.f. has the appearance of Fig.1.

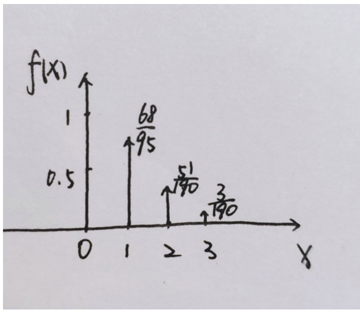

Figure 1: p.f. for H3.2.

d. The c.d.f. has the appearance of Fig.2.

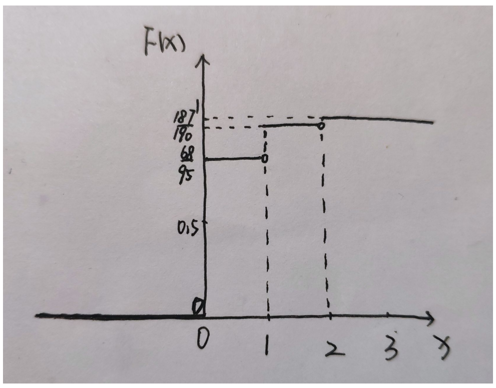

Figure 2: c.d.f. for H3.2.

H3.3

a. We must have

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-5}^{5} cx^2 dx = 250c/3 = 1.$$

Therefore, $c = 3/250$. The p.d.f has the appearance of Fig.3.

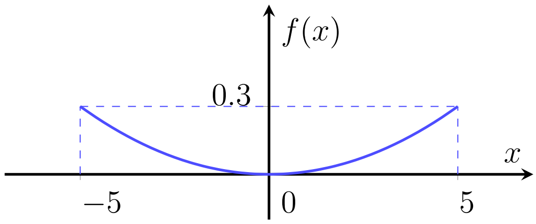

Figure 3: p.d.f. for H3.3.a.

b.

$$\Pr(X > 4) = \int_{4}^{5} f(x)dx = 61/250$$

c.

$$\Pr(X \geq 5) = \int_{5}^{\infty} f(x)dx = 0$$

d.

$$\Pr(-6 \leq X \leq 2) = \int_{-6}^{2} f(x)dx = \int_{-5}^{2} f(x)dx = 1 - \int_{2}^{5} f(x)dx = 133/250$$

e.

$$\Pr(X \leq -5) = \int_{-\infty}^{-5} f(x)dx = 0$$

f.

$$\Pr(X > -6) = \int_{-6}^{\infty} f(x)dx = 1$$

H3.4

We find $\Pr(X < 20) = \int_0^{20} cxdx = 200c$. Setting this equal to 0.9 yields $c = 0.0045$.

H3.5

a. The p.f of X is $f(x) = c(x+1)(8-x)$ for $x = 0, ..., 7$ where c is chosen so that $\sum_{x=0}^{7} f(x) = 1$. So, c is $1/\sum_{x=0}^{7}(x+1)(8-x)$, which sum equals 120. Thus, $c = 1/120$, then the p.f of X is

$$f(x) = \begin{cases} (x+1)(8-x)/120 & x = 0, 1, \ldots, 7 \\ 0 & \text{otherwise} \end{cases}$$

b. $\Pr(X \geq 5) = [(5+1)(8-5) + (6+1)(8-6) + (7+1)(8-7)]/120 = 1/3$.

H3.6

Let Y denote the number of misprints on a given page. Then the probability p that a given page will contain more than k misprints is

$$p = \text{Pr}(Y > k) = \sum_{i=k+1}^{\infty} f(i|x) = \sum_{i=k+1}^{\infty} \frac{\exp(-x)x^i}{i!}.$$

Therefore,

$$1 - p = \sum_{i=0}^{k} f(i|x) = \sum_{i=0}^{k} \frac{\exp(-x)x^i}{i!}.$$

Now let Z denote the number of pages, among the n pages in the book, on which there are more than k misprints. Then for $z = 0, 1, ..., n$,

$$\text{Pr}(Z = z) = \binom{n}{z} p^z (1-p)^{n-z}$$

and

$$\text{Pr}(Z \geq m) = \sum_{z=m}^{n} \binom{n}{z} p^z (1-p)^{n-z}$$

H3.7

a.

$$\text{Pr}(X \leq t) = \int_0^t f(x)dx = t^2/16 = 1/4$$

Thus, $t = 2$.

b.

$$\text{Pr}(X \geq t) = \int_t^4 f(x)dx = 1 - t^2/16 = 1/2$$

Thus, $t = \sqrt{8}$.

H3.8

If $f(x)$ is a p.d.f, the function must satisfy that $\int_0^\infty f(x)dx = 1$. However,

$$\int_0^\infty f(x)dx = c\int_0^\infty 1/(1+x)dx = c[\ln(1+x)]|_{x=0}^{\infty} = \infty$$

Thus, there does not exist any number c that satisfies the condition.

H3.9

a.

$$\text{Pr}(T = 5) = F(5) - F(5^-) = 3/4 - 1/2 = 1/4$$

b.

$$\text{Pr}(T > 3) = 1 - F(3) = 1 - 1/2 = 1/2$$

c.

$$\text{Pr}(1.4 < T < 6) = F(6^-) - F(1.4) = 3/4 - 1/4 = 1/2$$

H3.10

a. 12 minutes=0.2 hour, as X is continuous, we have

$$\text{Pr}(X < 0.2) = F(0.2) = 1 - e^{-1.6} = 0.7981$$

b.

$$f(x) = F'(x) = \begin{cases} 8e^{-8x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Therefore,

$$\text{Pr}(X < 0.2) = 8\int_0^{0.2} e^{-8x} dx = -\left. e^{-8x} \right|_0^{0.2} = 0.7981$$