7.3 A

4.(6) 中心点：$x_0=-\frac{1}{2}$

$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\frac{1}{3}$ ∴收敛区间为 $(-\frac{5}{6},-\frac{1}{6})$

当 $x=-\frac{1}{6}$ 时，$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=2$，$\lim_{n\to\infty}=+\infty$ ∴发散.

$x=-\frac{5}{6}$ 时，发散. ∴收敛域为 $(-\frac{5}{6},-\frac{1}{6})$

B. 3. ∵ $\sum_{n=1}^{\infty}a_n$ 收敛 ∴ $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}<1$ ∴ $R=\lim_{n\to\infty}\frac{a_n}{a_{n+1}}>1$

在 $[-1,1]$ 上，$f(x)=\sum_{n=0}^{\infty}a_nx^n$ 一致收敛.

在 $(-1,1)$ $f(x)$ 收敛.

7.4. A

5.(4) $f(x)$ 满足 Dirichlet 条件.

$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=1$

$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx=\frac{1}{\pi}(0+\int_0^{\pi}\cos nx\,dx)=0$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx=\frac{1}{\pi}(0+\int_0^{\pi}\sin nx\,dx)=\frac{-(-1)^n+1}{n\pi}$

$\therefore f(x)=\frac{1}{2}+\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}$

6.(3) $f(x)=\begin{cases}x+2, & x\in(-4,0)\\ 2-x, & x\in[0,4]\end{cases}$ 满足 Dirichlet 条件.

$\therefore a_0=\frac{1}{4}\int_{-4}^{4}f(x)dx=0$

$a_n=\frac{1}{4}\int_{-4}^{4}f(x)\cos nx\,dx$

$=\frac{1}{4}\int_{-4}^{0}(x+2)\cos nx\,dx+\frac{1}{4}\int_0^4\cos nx\,(2-x)dx$

$=\frac{1}{2}\int_0^4(2-x)\cos nx\,dx$

$=\frac{1}{2}\left[\frac{1}{n}\sin nx(2-x)\Big|_0^4-\int_0^4(-\frac{1}{n}\sin nx)dx\right]$

$=\frac{1}{2}\left[\frac{-2\sin 4n}{n}-\frac{\cos 4n-1}{n^2}\right]$

$=\frac{1-2n\sin 4n-\cos 4n}{2n^2}$

$b_n=\frac{1}{4}\int_{-4}^{4}f(x)\sin nx\,dx=0$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{1-2n\sin 4n-\cos 4n}{2n^2} \cos nx$$

7.(3) $f(x)$ 奇延拓后，F 为 $[-\pi, \pi]$ 上的奇函数. $T=2\pi$.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} F(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\frac{\pi}{3}} \frac{3}{2}x\sin nx\,dx + \frac{1}{\pi}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{\pi}{2}\cdot\sin nx\,dx + \frac{1}{\pi}\int_{\frac{2}{3}\pi}^{\pi} \frac{3}{2}(\pi-x)\sin nx\,dx$$

$$= \frac{3}{2\pi}\left[\frac{-1}{n}x\cos nx\Big|_{0}^{\frac{\pi}{3}} + \int_{0}^{\frac{\pi}{3}}\frac{1}{n}\cos nx\right] + \frac{1}{2}\left(-\frac{1}{n}\cos nx\right)\Big|_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$$

$$+ \frac{3}{2\pi}\left[\frac{1}{n}(x-\pi)\cos nx\Big|_{\frac{2}{3}\pi}^{\pi} - \int_{\frac{2}{3}\pi}^{\pi}\frac{1}{n}\cos nx\right]$$

$$= \frac{3}{2\pi n^2}\sin\frac{\pi}{3}n\left(1+2\cos\frac{\pi}{3}n\right)$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{3}{2\pi n^2}\sin\frac{\pi}{3}n\left(1+2\cos\frac{\pi}{3}n\right)\sin nx, \quad x\in[0,\pi]$$