

1. 证明: 在  $\mathbb{R}^2$  中 <sup>无数个</sup> 边长为 1 的正方形紧密排列, ~~成~~ 不是闭集

5.2. A

$$3.(3) \quad |3x - 4y - 1| = |3x - 9 - (4y - 8)| \leq 3|x - 3| + 4|y - 2| < 4(|x - 3| + |y - 2|)$$

令  $\delta = \frac{\varepsilon}{8}$ , 当  $|x - 3| < \delta, |y - 2| < \delta$  时, 原式  $< \varepsilon$ .

$$\therefore \lim_{(x,y) \rightarrow (3,2)} (3x - 4y) = 1$$

$$4.(2) \quad \text{令 } y = -x + kx^2 \quad \therefore \frac{xy}{x+y} = -\frac{1}{k} + x$$

$$\therefore \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x+y} = \lim_{x \rightarrow 0} \left(-\frac{1}{k} + x\right) = -\frac{1}{k}, \text{ 极限值随 } k \text{ 变化而变化.}$$

$\therefore$  极限不存在.

$$5.(3) \quad \text{设 } y=2. \text{ 则 } \lim_{(x,y) \rightarrow (0,2)} \frac{\sin 4x}{x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} \frac{2\cos 2x}{1} = 2$$

$$5.(4) \quad \text{设 } y=x>0$$

$$\begin{aligned} \text{则 } \lim_{(x,y) \rightarrow (0,0)} x^2 y^2 \ln(x^2 + y^2) &= \lim_{x \rightarrow 0} x^4 \ln(2x^2) = \lim_{x \rightarrow 0} x^4 \ln 2 + \lim_{x \rightarrow 0} x^4 \ln 2x \\ &= \lim_{x \rightarrow 0} \frac{2 \ln x}{\frac{1}{x^4}} = \lim_{x \rightarrow 0} \left(-\frac{1}{2}\right) x^2 = 0 \end{aligned}$$