

6.8 A

1. (1) ~~正确~~ 错误, 积分方向取反了, 正确答案应为 $\frac{\pi}{4}$

(2) 解法一 错误, $I + \int_{\text{DBA}} \frac{-ydx + xdy}{x^2 + y^2} = 0$, 故 ~~解法~~ $I = -\pi$

解法二 正确

2. (5) 设 \vec{OA} , 则 $\oint_{\vec{AO} + \vec{OA}} (e^x \sin y - my) dx + (e^x \cos y - m) dy$
 $= \iint_{\Omega} m ds$, 其中 Ω 为 $x^2 + y^2 = ax$ 上半圆围与 x 轴围成的面.

$$\therefore \text{原式} = m \iint_{\Omega} ds = m \cdot \frac{1}{2} \pi \left(\frac{1}{2}a\right)^2 = \frac{1}{8} m \pi a^2$$

$$\therefore \int_{\vec{OA}} (e^x \sin y - my) dx + (e^x \cos y - m) dy = 0$$

$$\therefore \int_{(C)} (e^x \sin y - my) dx + (e^x \cos y - m) dy = \frac{1}{8} m \pi a^2$$

9. (1) $\nabla \times \vec{A} = (0, 0, 0) \therefore A$ 为有势场.

$$\therefore \frac{\partial u}{\partial x} = 2x \cos y - y^2 \sin x, \quad \frac{\partial u}{\partial y} = 2y \cos x - x^2 \sin y$$

$$\therefore u = x^2 \cos y + y^2 \cos x + \varphi(x, y)$$

$$\therefore \frac{\partial \varphi}{\partial y} = 0 \quad \therefore \varphi(x, y) = \varphi(x) = C$$

$$\therefore u = x^2 \cos y + y^2 \cos x + C$$

$$\begin{aligned}
 16(5) \quad \text{原式} &= \int_0^c 2\pi ab \left(1 - \frac{z^2}{c^2}\right) dz - \left[- \iint_{(D)} (x+y+1) dx dy \right] \\
 &= 2\pi abc + ab \int_0^{2\pi} d\theta \int_0^1 \rho (1 + a\rho \cos\theta + b\rho \sin\theta) d\rho \\
 &= ab\pi(2c+1)
 \end{aligned}$$