

6.7 A

$$2.(4) \text{ 直线段: } \begin{cases} x=1 \\ y=1+2t \\ z=1+3t \end{cases}, t: 0 \rightarrow 1$$

$$\begin{aligned} & \therefore \int_{(C)} x dx + y dy + (x+y-1) dz \\ &= \int_0^1 1 \cdot 0 dt + \int_0^1 (1+2t) \cdot 2 dt + \int_0^1 (1+2t) \cdot 3 dt \\ &= 10 \end{aligned}$$

$$(6) (C): \begin{cases} x=2\cos\theta \\ y=2\sin\theta \\ z=2-\cos\theta+\sin\theta \end{cases}, \theta \text{ 从 } 0 \rightarrow -2\pi$$

$$\begin{aligned} & \therefore \oint_{(C)} (z-y) dx + (x-z) dy + (x-y) dz \\ &= \int_0^{-2\pi} (2-\cos\theta) \sin\theta d\theta + \int_0^{-2\pi} (2\cos\theta-2-\sin\theta) \cos\theta d\theta \\ & \quad + \int_0^{-2\pi} (\cos\theta-\sin\theta) \cdot (\cos\theta+\sin\theta) d\theta \\ &= -2\pi \end{aligned}$$

$$\begin{aligned} 11. \iint_{(S)} \vec{F} d\vec{s} &= \iint_{(S)} y dy \wedge dz - x dz \wedge dx + z^2 dx \wedge dy \\ &= - \iint_{D_{yz}} y dy dz + \iint_{D_{xz}} x dx dz - \iint_{D_{xy}} (x^2+y^2) dx dy \\ &= - \int_0^2 y dy \cdot \int_0^1 dz + \int_0^2 x dx \int_0^1 dz - \int_0^{\frac{\pi}{4}} d\theta \int_0^1 \rho^2 \cdot \rho d\rho \\ &= \frac{\pi}{16} \end{aligned}$$

$$\Sigma_1: x=0, \Sigma_2: y=0, \Sigma_3: z=0, \Sigma_4: x+y+z=1$$

$$\begin{aligned} 12.(2) \iint_{(S)} xy \, dy \wedge dz &= \iint_{\Sigma_1} xy \, dy \wedge dz + \iint_{\Sigma_4} xy \, dy \wedge dz \\ &= 0 + \int_0^1 dy \int_0^1 dz \int_0^{1-z} (y - y^2 - zy) \, dy \\ &= \frac{1}{24} \end{aligned}$$

$$\text{同理, } \iint_{(S)} yz \, dz \wedge dx = \frac{1}{24}, \quad \iint_{(S)} zx \, dx \wedge dy = \frac{1}{24}$$

$$\therefore \oint_{(S)} xy \, dy \wedge dz + yz \, dz \wedge dx + zx \, dx \wedge dy = \frac{1}{8}$$

$$(6) \iint_{(S)} z^2 \, dx \wedge dy = \oint x^2 + y^2 + z^2 = 2z \quad z = \pm \sqrt{1-x^2-y^2}$$

$$\therefore \iint_{(S)} z^2 \, dx \wedge dy = \iint_{D_{xy}} (1 \pm \sqrt{1-x^2-y^2})^2 \, dx \, dy$$

$$= \iint_{D_{xy}} (2 - x^2 - y^2 \pm 2\sqrt{1-x^2-y^2}) \, dx \, dy$$

$$D_{xy}: x^2 + y^2 \leq 1 \Rightarrow 0 \leq \rho \leq 1, \quad 0 \leq \theta \leq 2\pi$$

$$\begin{aligned} \therefore \text{原积分} &= \iint_{D_{xy}} (2 - x^2 - y^2 + 2\sqrt{1-x^2-y^2}) \, dx \, dy \\ &\quad - \iint_{D_{xy}} (2 - x^2 - y^2 - 2\sqrt{1-x^2-y^2}) \, dx \, dy \end{aligned}$$

$$= 4 \iint_{D_{xy}} \sqrt{1-x^2-y^2} \, dx \, dy$$

$$= 4 \iint_{D_{xy}} \sqrt{1-\rho^2} \, \rho \, d\rho \, d\theta$$

$$= 4 \int_0^{2\pi} d\theta \int_0^1 \sqrt{1-\rho^2} \, \rho \, d\rho = \frac{8}{3} \pi$$