6.2 A

3.(6) $\iint\limits_{(\sigma)} e^{-y^2} d\sigma = \cancel{\int_0^1 dx \int_0^x e^{-y^2} dy} = \int_0^1 dy \int_0^y e^{-y^2} dx$

$= \int_0^1 y e^{-y^2} dy$

$= \frac{e-1}{2e}$

5.(4) 原式 $= \int_0^2 dx \int_{\frac{1}{2}x}^1 f(x,y) dy + \int_1^3 dy \int_0^2 f(x,y) dx$

$+ \cancel{\int_1^3 dy} \int_2^{\sqrt{8}} dx \int_{\frac{x^2}{2}}^3 f(x,y) dx$

6.(4) 令 $x = \rho\cos\theta$, $y = \rho\sin\theta$, 则 $\Omega: 0 \le \theta \le \frac{\pi}{2}$, $0 \le \rho \le 1$

$\therefore \iint\limits_{\sigma} \arctan\frac{y}{x} d\sigma = \iint\limits_{\Omega} \theta\rho \, d\rho \, d\theta$

$= \int_0^{\frac{\pi}{2}} \theta d\theta \int_0^1 \rho \, d\rho = \frac{\pi^2}{16}$

7.(3) $\sigma: 1 \le y \le 2$, $0 \le x \le y \Rightarrow \cancel{\Omega: 1 \le \rho \le \sqrt{2}, \frac{\pi}{4} \le \theta \le \frac{\pi}{2}}$

$\Rightarrow \Omega: \theta \in [\frac{\pi}{4}, \frac{\pi}{2}]$, $\rho \in [\frac{1}{\sin\theta}, \frac{2}{\sin\theta}]$

$\therefore \int_1^2 dy \int_0^y \frac{x\sqrt{x^2+y^2}}{y} dx = \iint\limits_{\Omega} \frac{\rho}{\tan\theta} \rho \, d\rho \, d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_{\frac{1}{\sin\theta}}^{\frac{2}{\sin\theta}} \frac{\rho^2}{\tan\theta} d\rho = \frac{7}{9}(2\sqrt{2}-1)$

13.(3) 令 $xy = u$, $\frac{x}{y} = v \Rightarrow x = \sqrt{uv}$, $y = \sqrt{\frac{u}{v}}$

$\sigma \Rightarrow \Omega: 1 \le u \le 2$, $\frac{1}{4} \le v \le 1$

$\iint\limits_{(\sigma)} xy \, d\sigma \Rightarrow \iint\limits_{\Omega} -\frac{u}{2v} du dv = \int_1^2 u du \cdot \int_{\frac{1}{4}}^1 (-\frac{1}{2v}) dv$

$= \frac{45}{4}$