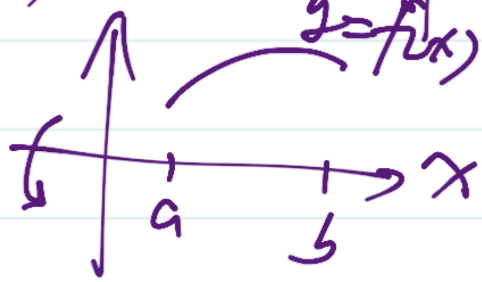
Contur集

$\Downarrow$

Koch-雪花线

关于弧长：

例：求曲线 $y=\left(\frac{x}{2}\right)^{2/3}$ 从 $x=0$ 到 $x=2$ 的弧长.



旋转曲面的表面积：

$$S=\int_a^b 2\pi y \boxed{ds}$$

第五章第七节：一些重要的物理量

Lemma：若 $\vec{r}(t)$ 可微且 $\|r\|$ 为常数，则 $\frac{d\vec{r}}{dt}\cdot\vec{r}=0$

BJS $\longrightarrow$ FRA

若 $\vec{r}$: position

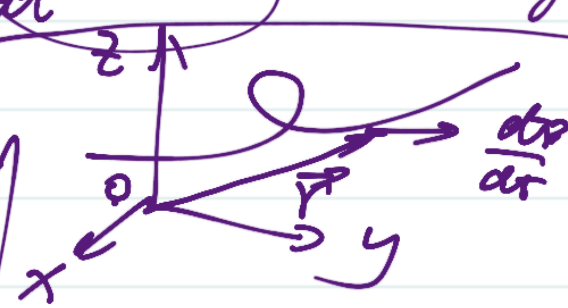
$\frac{d\vec{r}}{dt}=\vec{v}(t)$. velocity

proof. $\|\vec{r}\|^2=K^2$

$\vec{r}\cdot\vec{r}$

$$\Rightarrow \vec{r}\cdot\frac{d\vec{r}}{dt}=\frac{d\vec{r}}{dt}\cdot\vec{r}=0$$

例



1. 切向量 (tangent) $\vec{T}$

$$\boxed{\vec{T} \overset{def}{=} \frac{\vec{v}}{\|\vec{v}\|}} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|}$$

$\boxed{Q: \|\vec{T}\| = 1}$

$$\frac{d\vec{r}}{ds} = \frac{d\vec{r}}{dt}\,\frac{1}{\boxed{\frac{ds}{dt}}} = \frac{\vec{v}}{\|\vec{v}\|} = \vec{T} \qquad \boxed{\vec{T} = \frac{d\vec{r}}{ds}}$$

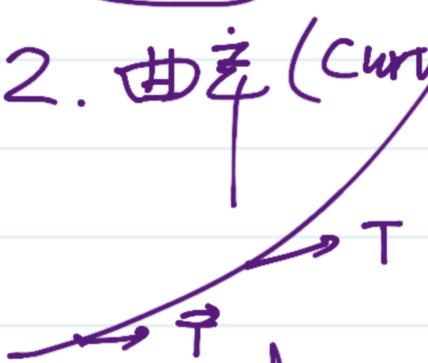
$\|$ 定义

$$\lim \frac{\Delta \vec{r}}{\Delta s}$$

2. 曲率 (curvature)

$$\boxed{\kappa \overset{def}{=} \left\|\frac{d\vec{T}}{ds}\right\|} = \frac{1}{\|\vec{v}\|}\left\|\frac{d\vec{T}}{dt}\right\|$$

(\kappa)



例 ① 直线的曲率

$\vec{r}(t) = \vec{r}_0 + t\vec{v}_0$

$\vec{v}(t) = \vec{v}_0$

② 圆的曲率 $a > 0$

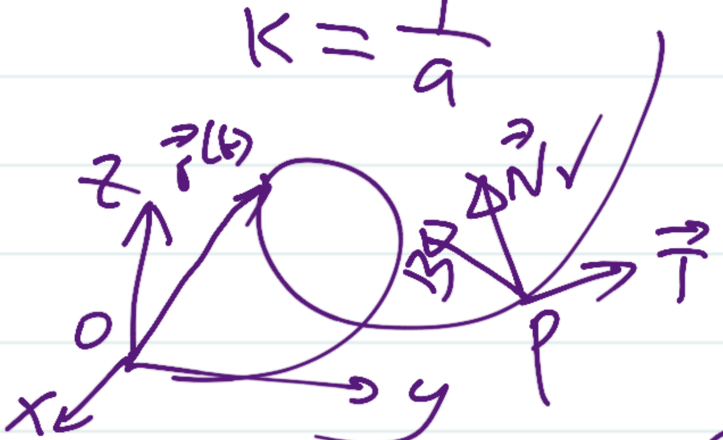
$\vec{r}(t) = (a\cos t, a\sin t), 0 \le t \le 2\pi$

$\kappa = \frac{1}{a}$

$\vec{a} = \frac{d\vec{v}}{dt}$

3. TNB Frame

$\vec{N}$: 主法向量



$\vec{N} \stackrel{def}{=} \frac{1}{k}\frac{d\vec{T}}{ds}$ ($\because \vec{N} \perp \vec{T}$) ($\because \|\vec{T}\| = 1$) $\rightarrow \frac{1}{k}\frac{d\vec{T}/dt}{ds/dt} = \frac{d\vec{T}/dt}{\left|d\vec{T}/dt\right|}$ ✓

(Unit principal normal vector)

$\vec{B}$: 次法向量

$$\vec{B} \stackrel{def}{=} \vec{T} \times \vec{N}$$

(Binomal vector)

例 | 计算Helix的曲率与主法向量:

$\vec{r}(t) = (a\cos t)i + (a\sin t)j + bt\,k$, $a, b \geq 0$, $a^2 + b^2 \neq 0$

例: 证明加速度 $\vec{a} = \frac{dv}{dt}$ 只在 $\vec{T}$, $\vec{N}$ 张成的平面上.