6.6 A

1.(6) 设 $x=\cos\theta$, $y=\cos\theta$, $z=\sqrt{2}\sin\theta$

$$\oint_C |y|\,ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\cdot\sqrt{2}\,d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos\theta)\sqrt{2}\,d\theta$$

$$= 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$$

3.(2) 设 $x=\frac{a}{2}\cos\theta+\frac{a}{2}$, $y=\frac{a}{2}\sin\theta$

$$\oint_C \sqrt{x^2+y^2}\,ds = \int_0^{2\pi} \sqrt{\cos\theta+1}\cdot\frac{\sqrt{2}a^2}{4}\,d\theta$$

$$= \frac{\sqrt{2}}{2}a^2\int_0^{\pi}\sqrt{2\cos^2\frac{\theta}{2}}\,d\left(\frac{\theta}{2}\right)$$

$$= 2a^2$$

10.(1) (S): $z=4-2x-\frac{4}{3}y$, 其中 (x,y) 在区域 $\sigma: x\geq 0, y\geq 0, 3x+2y\leq 6$ 中.

$$\therefore \iint_{(S)}\left(2x+\frac{4}{3}y+z\right)ds = \iint_{(\sigma)} 4\sqrt{1+z_x^2+z_y^2}\,d\sigma$$

$$= \frac{4\sqrt{61}}{3}\iint_{\sigma} d\sigma = 4\sqrt{61}\cdot\frac{1}{3}\int_0^3 dy\int_0^{2-\frac{2}{3}y} dx$$

$$= 4\sqrt{61}$$

(8) 令 $x=\rho\cos\theta$, $y=\rho\sin\theta$, $z=\rho$. (σ): $-\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$, $0\leq\rho\leq 2a\cos\theta$

$$\therefore \iint_{(S)}(xy+yz+zx)\,ds = \iint_{(\sigma)}\sqrt{2}\rho\left(\rho^2\sin\theta\cos\theta+\rho^2\sin\theta+\rho^2\cos\theta\right)d\rho\,d\theta$$

$$= \int_0^{2a\cos\theta}\sqrt{2}\rho^3\,d\rho\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin\theta\cos\theta+\sin\theta+\cos\theta)\,d\theta$$

$$= \frac{14\sqrt{2}}{15}a^4$$