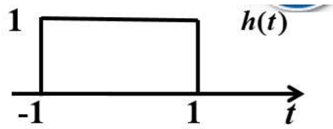
Discussion problem assignment:

问题一：

已知 LTI 系统的单位冲激响应为$h(t)=u(t+1)-u(t-1)$，

对以下两个输入信号，分别计算对应的输出。$x_1(t)=e^{j\pi t}, x_2(t)=e^{j\frac{\pi}{2}t}$

A：

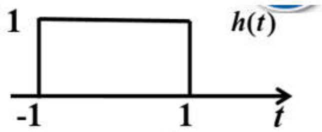


$$e^{j\pi t}\rightarrow e^{j\pi t}*h(t)=\int_{-\infty}^{+\infty}h(\tau)e^{j\pi(t-\tau)}d\tau=e^{j\pi t}\int_{-1}^{+1}e^{-j\pi\tau}d\tau$$

$$=e^{j\pi t}\frac{e^{-j\pi\tau}}{-j\pi}\Big|_{\tau=-1}^{+1}=e^{j\pi t}\frac{e^{-j\pi}-e^{+j\pi}}{-j\pi}=e^{j\pi t}\frac{-1-(-1)}{-j\pi}=0$$

$$e^{j\frac{\pi}{2}t}\rightarrow e^{j\frac{\pi}{2}t}*h(t)=\int_{-\infty}^{+\infty}h(\tau)e^{j\frac{\pi}{2}(t-\tau)}d\tau=e^{j\frac{\pi}{2}t}\int_{-1}^{+1}e^{-j\frac{\pi}{2}\tau}d\tau$$

$$=e^{j\frac{\pi}{2}t}\frac{e^{-j\frac{\pi}{2}\tau}}{-j\frac{\pi}{2}}\Big|_{\tau=-1}^{+1}=e^{j\frac{\pi}{2}t}\frac{e^{-j\frac{\pi}{2}}-e^{+j\frac{\pi}{2}}}{-j\frac{\pi}{2}}=e^{j\frac{\pi}{2}t}\frac{-j-(-j)}{-j\frac{\pi}{2}}=\frac{4}{\pi}e^{j\frac{\pi}{2}t}$$



$$e^{j\pi t}\rightarrow e^{j\pi t}*h(t)=\int_{-\infty}^{+\infty}h(\tau)e^{j\pi(t-\tau)}d\tau=e^{j\pi t}\int_{-1}^{+1}e^{-j\pi\tau}d\tau$$

$$=e^{j\pi t}\frac{e^{-j\pi\tau}}{-j\pi}\Big|_{\tau=-1}^{+1}=e^{j\pi t}\frac{e^{-j\pi}-e^{+j\pi}}{-j\pi}=e^{j\pi t}\frac{-1-(-1)}{-j\pi}=0$$

$$e^{j\frac{\pi}{2}t}\rightarrow e^{j\frac{\pi}{2}t}*h(t)=\int_{-\infty}^{+\infty}h(\tau)e^{j\frac{\pi}{2}(t-\tau)}d\tau=e^{j\frac{\pi}{2}t}\int_{-1}^{+1}e^{-j\frac{\pi}{2}\tau}d\tau$$

$$=e^{j\frac{\pi}{2}t}\frac{e^{-j\frac{\pi}{2}\tau}}{-j\frac{\pi}{2}}\Big|_{\tau=-1}^{+1}=e^{j\frac{\pi}{2}t}\frac{e^{-j\frac{\pi}{2}}-e^{+j\frac{\pi}{2}}}{-j\frac{\pi}{2}}=e^{j\frac{\pi}{2}t}\frac{-j-(-j)}{-j\frac{\pi}{2}}=\frac{4}{\pi}e^{j\frac{\pi}{2}t}$$

学生容易做到这两步后，就不再继续计算了。

第二题：

Given an LTI system with $y(t)=\int_{-\infty}^{t}e^{-(t-\tau)}x(\tau-2)d\tau$, try to determine the unit impulse response.

答案一：

Solution one:

$$h(t)=\int_{-\infty}^{t}e^{-(t-\tau)}\delta(\tau-2)d\tau=\int_{-\infty}^{t-2}e^{-(t-2-\tau')}\delta(\tau')d\tau'$$

$$=e^{-(t-2)}\int_{-\infty}^{t-2}\delta(\tau')d\tau'=\begin{cases}e^{-(t-2)}, & t>2\\ 0, & t<2\end{cases}=e^{-(t-2)}u(t-2)$$

$$x(t)\delta(t-t_0)=x(t_0)\delta(t-t_0)$$

$$e^{-(t-2-\tau')}\delta(\tau'-0)=e^{-(t-2-0)}\delta(\tau')$$

答案二：

Solution two:

$$y(t)=\int_{-\infty}^{t}e^{-(t-\tau)}x(\tau-2)d\tau=\int_{+\infty}^{0}e^{-\tau'}x(t-\tau'-2)(-d\tau')(\tau'=t-\tau)$$

$$=\int_{0}^{+\infty}e^{-\tau'}x(t-\tau'-2)d\tau'=\int_{2}^{+\infty}e^{-(\tau-2)}x(t-\tau)d\tau(\tau=\tau'+2)$$

$$=\int_{-\infty}^{+\infty}e^{-(\tau-2)}u(\tau-2)x(t-\tau)d\tau=\int_{-\infty}^{+\infty}h(\tau)x(t-\tau)d\tau$$

$$h(\tau)=e^{-(\tau-2)}u(\tau-2)$$

$$h(t)=e^{-(t-2)}u(t-2)$$