正项级数：$\sum a_n,\ a_n \geq 0$

$$S_n \overset{\text{def}}{=} a_1 + a_2 + \cdots + a_n \nearrow$$

EX: $\displaystyle\sum_{n=2}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$ VS $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^{3/2}}$

收敛

$\displaystyle\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$

EX: ① $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

② $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n}$ or $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n}$ $\nearrow$ $\displaystyle\int_1^{\infty} \frac{\sin x}{x}\,dx$

Remark: 关于 Laibniz 定理：

① $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots$

② $-\frac{1}{3} - \frac{1}{3^2} - \frac{1}{3^3} - \cdots -$

$\Rightarrow \frac{1}{2} - \frac{1}{3} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{2^3} - \frac{1}{3^3} + \cdots\cdots$

$\sum (-1)^{n+1} u_n,$

$u_n = \begin{cases} \frac{1}{2^n}, \\ \frac{1}{3^n}, \end{cases}$

Thm. 若 $\sum u_n$ ~~绝对~~收敛，则 $\sum u_n^+$ 与 $\sum u_n^-$ 都收敛；若 $\sum u_n$ 条件收敛，则 $\sum u_n^+$ 与 $\sum u_n^-$ 都发散到 $+\infty$.

级数的重排.

Riemann定理：若 $\sum u_n$ 条件收敛，则对 $\forall a \in \mathbb{R}$，$\exists \sum u_n$ 的重排级数 $\sum \tilde{u}_n$，s.t.

$$\sum \tilde{u}_n = a.$$

如 $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots$

证明思路：

$$\sum u_n^+ = +\infty$$

$$\sum u_n^- = +\infty$$