5.3 A

1.(9) $\frac{\partial u}{\partial x} = z e^{\sin(yz)}$ $\frac{\partial u}{\partial y} = xz^2 e^{\sin(yz)} \cdot \cos yz$

$\frac{\partial u}{\partial z} = x(e^{\sin(yz)} + yz e^{\sin(yz)} \cos yz)$

6(1) 设方向向量 $\vec{e_l} = (\cos\theta_1, \cos\theta_2)$

则方向导数 $\frac{\partial f}{\partial l}\Big|_{(0,0)} = \lim_{t\to 0} \frac{\sqrt[3]{t^2\cos\theta_1\cos\theta_2}}{t} = \lim_{t\to 0} \frac{1}{\sqrt[3]{t}} \sqrt[3]{\cos\theta_1\cos\theta_2} = \infty$

∴只在沿坐标轴正负方向存在方向导数.

(2) $\lim_{(x,y)\to(0,0)} \sqrt[3]{xy} = f(0,0) = 0$ ∴$f(x,y)$在$(0,0)$连续

24.(4) $\frac{\partial T}{\partial t} = -\frac{1}{4a\sqrt{\pi t^3}} e^{-\frac{(x-a)^2}{4a^2t}} + \frac{1}{2a\sqrt{\pi t}} e^{-\frac{(x-a)^2}{4a^2t}} \cdot \frac{(x-a)^2}{4a^2} \cdot \frac{1}{t^2}$

$\frac{\partial T}{\partial x} = \frac{1}{2a\sqrt{\pi t}} e^{-\frac{(x-a)^2}{4a^2t}} \cdot \frac{1}{4a^2 t} \cdot (-2x+2a) = -\frac{e^{-\frac{(x-a)^2}{4a^2t}}}{4a^3\sqrt{\pi t^3}}(x-a)$

$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial^2 T}{\partial x^2} = -\frac{1}{4a^3\sqrt{\pi t^3}} e^{-\frac{(x-a)^2}{4a^2t}} + \frac{(x-a)^2}{8a^5\sqrt{\pi t}\cdot t^2} \cdot e^{-\frac{(x-a)^2}{4a^2t}}$

∴ $\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}$

26.(6) $\frac{\partial z}{\partial x} = f'_x\left(\frac{x}{y}\right) + g\left(\frac{y}{x}\right) - \frac{y}{x} g'_x\left(\frac{y}{x}\right)$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{1}{y} f''_x\left(\frac{x}{y}\right) + \left(-\frac{y}{x^2}\right) \cdot g'_x\left(\frac{y}{x}\right) + \frac{y}{x^2}\left[g'_x\left(\frac{y}{x}\right) + \frac{y}{x} g''_x\left(\frac{y}{x}\right)\right]$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = -\frac{x}{y^2} \frac{\partial f_x\left(\frac{x}{y}\right)}{\partial y} + \frac{1}{x} g_y\left(\frac{y}{x}\right) - \frac{1}{x} g_x\left(\frac{y}{x}\right) - \frac{y}{x^2} \frac{\partial g_x\left(\frac{x}{y}\right)}{\partial y}$

B. 2. $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$, $\vec{e_1} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$, $\vec{e_2} = (-1, 0)$, $\vec{e_3} = \left(\frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13}\right)$

$$\begin{cases} \frac{\partial f(\vec{x})}{\partial l_1}\Big|_{P_0} = \nabla f(x,y) \cdot \vec{e_1} = 1 & ① \\ \frac{\partial f(\vec{x})}{\partial l_2}\Big|_{P_0} = \nabla f(x,y) \cdot \vec{e_2} = -3 & ② \end{cases}$$

解①②得 $\nabla f(x,y) = (3, 3-\sqrt{2})$

∴ $\frac{\partial f(\vec{x})}{\partial l} = \nabla f(x,y) \cdot \vec{e_3} = \frac{9\sqrt{13}}{13} + \frac{6\sqrt{13}}{13} - \frac{2\sqrt{26}}{13} = \frac{15\sqrt{13} - 2\sqrt{26}}{13}$