5.3 A.

31.(1) $F(x,y)=\ln\sqrt{x^2+y^2}-\arctan\frac{y}{x}=0$

$F_x=\frac{x+y}{x^2+y^2}$ $F_y=\frac{y-x}{x^2+y^2}$

$\therefore \frac{dy}{dx}=-\frac{F_x}{F_y}=\frac{x+y}{x-y}$

$\frac{d^2y}{dx^2}=\frac{2x^2-4xy-2y^2}{(x-y)^3}$

32.(2) $F(x,y,z)=x^2-2y^2+z^2-4x+2z-5$

$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=\frac{4-2x}{2z+2}=\frac{2-x}{z+1}$, $\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=\frac{4y}{2z+2}=\frac{2y}{z+1}$

$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{-(z+1)^2-(2-x)^2}{(z+1)^3}$ $\frac{\partial^2 z}{\partial^2 y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=\frac{2(z+1)^2-4y^2}{(z+1)^3}$

$\frac{\partial^2 z}{\partial y\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{2(x-2)y}{(z+1)^3}$ $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{2(x-2)y}{(z+1)^3}$

36.(2) $F(x,y,z)=x^2+y^2+z^2-yf\left(\frac{z}{y}\right)$

$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=\frac{2x}{f'\left(\frac{z}{y}\right)-2z}$ $\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=\frac{2y-f'\left(\frac{z}{y}\right)+\frac{z}{y}f'\left(\frac{z}{y}\right)}{f'\left(\frac{z}{y}\right)-2z}$

$\therefore dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$

$=\frac{2x}{f'\left(\frac{z}{y}\right)-2z}dx+\frac{2y-f'\left(\frac{z}{y}\right)+\frac{z}{y}f'\left(\frac{z}{y}\right)}{f'\left(\frac{z}{y}\right)-2z}dy$