Discussion problem assignment:

第一题:

1. For the signal $x(t) = \delta(t) + \delta(t-1)$, find its Laplace transform, determine the poles and zeros plus sketching the pole-zero plot.

解答:

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt = \int_{-\infty}^{+\infty} (\delta(t) + \delta(t-1))e^{-st}dt = 1 + e^{-s}$$
ROC: entire s-plane
No poles on the finite s-plane.

Zeros: $1 + e^{-s} = 0$

$$e^{s} = (-1)$$
For complex s, $e^{s} = (-1) = e^{j(2n+1)\pi}$
Many zeros with $s = j(2n+1)\pi$

第二题:

2. For the following Laplace transform *X*(*s*) with signal *x*(*t*) to be absolutely <u>integrable</u>, determine the signal *x*(*t*) from inverse Laplace transform. Find the Fourier transform of the signal and confirm the inverse Fourier transform is the same as from LT.

$$X(s) = \frac{-4}{s^2 - 4}$$

解答:

$$X(s) = \frac{-4}{s^2 - 4} = \frac{-4}{(s+2)(s-2)} = \frac{1}{s+2} - \frac{1}{s-2} \qquad -2 < \text{Re}\{s\} < +2$$

The ROC is determined by the two poles plus the fact that the signal is absolutely integrable.

$$x(t) = e^{-2t}u(t) + e^{t}u(-t) = e^{-2|t|}$$

While,
$$X(j\omega) = \frac{-4}{(j\omega)^2 - 4} = \frac{-4}{-\omega^2 - 4} = \frac{4}{\omega^2 + 4}$$

The inverse FT can be obtained as from Example 4.2 with the same result as above.