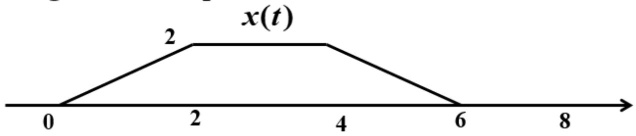
In-class quiz:

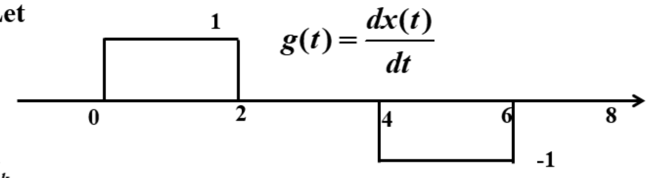
The continuous-time periodic signal $x(t)$ with $T = 8$ is defined as the following within one period as



Find the FS a_k of $x(t)$.

Solution one: Let

$$\omega_0 = \frac{\pi}{4}$$



$$g(t) = \frac{dx(t)}{dt}$$

$$x(t) \xleftrightarrow{FS} a_k$$

$$g(t) \xleftrightarrow{FS} b_k$$

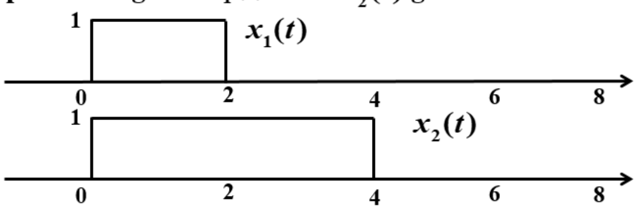
利用例题3.5周期方波信号的FS结果：$\dfrac{\sin(k\omega_0)}{k\pi}$

$$b_k = \frac{\sin(k\omega_0)}{k\pi}(e^{-jk\omega_0} - e^{-j5k\omega_0}) \qquad b_k = jk\omega_0 a_k$$

$$a_k = \frac{\sin(k\omega_0)}{(k\pi)(jk\omega_0)}(e^{-jk\omega_0} - e^{-j5k\omega_0}) = \frac{4\sin(k\omega_0)}{j(k\pi)^2}(e^{-jk\omega_0} - e^{-j5k\omega_0})$$

Solution two: Take the signal $x(t)$ as the periodic convolution between two periodic signals $x_1(t)$ and $x_2(t)$ given as

$$\omega_0 = \frac{\pi}{4}$$



$$a_k = Ta_{1,k}a_{2,k} = T\frac{\sin(k\omega_0)}{(k\pi)}e^{-jk\omega_0}\frac{\sin(2k\omega_0)}{(k\pi)}e^{-j2k\omega_0}$$

$$= \frac{8\sin(k\omega_0)\sin(2k\omega_0)}{(k\pi)^2}e^{-j3k\omega_0}$$

可以确认两种方法结果一致

结果不好。大部分同学都是用分析公式求积分。唉。

Discussion problem assignment:

Prove the following result:

$$\sum_{n=-\infty}^{+\infty}\delta(t-nT) = \frac{1}{T} + \sum_{k=1}^{+\infty}\frac{2}{T}\cos\left(\frac{2k\pi t}{T}\right)$$

第一题：

答案：

Prove: periodic, and fundamental frequency

$$\omega_0 = \frac{2\pi}{T}$$

$$\sum_{n=-\infty}^{+\infty} \delta(t-nT) = \sum_{k=-\infty}^{+\infty} \frac{1}{T}\exp\left(jk\omega_0 t\right) = \sum_{k=-\infty}^{+\infty} \frac{1}{T}\exp\left(j\frac{2k\pi t}{T}\right)$$

$$= \frac{1}{T} + \sum_{k=1}^{+\infty} \frac{1}{T}\exp\left(j\frac{2k\pi t}{T}\right) + \frac{1}{T}\exp\left(-j\frac{2k\pi t}{T}\right)$$

第二题：

Suppose that the unit impulse response of an LTI system is

$$h(t) = 1,\ \text{for}\ -2 < t < +2;\quad 0,\ \text{otherwise}$$

1. Determine the system's frequency response.

2. Find system's output for a periodic input signal $x(t) = 1 + \cos(\pi t)$

答案：

Solution:

1. Determine the system's frequency response.

$$H(j\omega) = \int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt = \int_{-2}^{+2} e^{-j\omega t}dt = \frac{1}{-j\omega}e^{-j\omega t}\Big|_{-2}^{+2}$$

$$= \frac{e^{-j2\omega} - e^{+j2\omega}}{-j\omega} = \frac{-2j\sin(2\omega)}{-j\omega} = \frac{2\sin(2\omega)}{\omega}$$

2. Find system's output for a periodic input signal .

$$x(t) = 1 + \cos(\pi t) = 1 + \frac{1}{2}\{e^{j\pi t} + e^{-j\pi t}\}$$

The output is the sum from each term. Using frequency response,

$$1 = e^{j0\times t} \to H(j0)e^{j0\times t} = H(j0) = 4,\quad e^{\pm j\pi t} \to H(\pm j\pi)e^{\pm j\pi t} = 0$$

So, the output is $y(t) = 4$ $\quad H(j0) = \int_{-2}^{+2} e^{-j0t}dt = \int_{-2}^{+2} dt = 4$