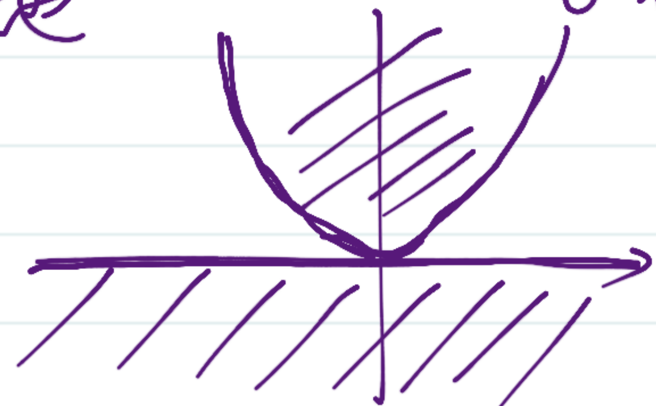
关于方向导数:

可微 $\Longrightarrow$ $\forall$方向导数都存在 $\left(\frac{\partial f}{\partial l} = \nabla f \cdot l\right)$

$\Downarrow$?

连续

例: $f(x,y) = \begin{cases} 0, & 0<y<x^2, \quad -\infty<x<\infty \\ 1, & \text{其他} \end{cases}$



例: $z=f(u,x,y)$, $u=xe^y$, 求 z_{yx}.

$z_y = f_1\frac{\partial u}{\partial y} + f_3 = xe^y f_1 + f_3$ $\quad f_1(u,x,y)$, $f_3(u,x,y)$

$z_{yx} = e^y f_1 + e^y x\frac{\partial f_1}{\partial x} + \frac{\partial f_3}{\partial x}$

$= e^y f_1 + xe^y\left(f_{11}\underbrace{\frac{\partial u}{\partial x}}_{e^y} + f_{12}\cdot 1\right) + \left(f_{31}\frac{\partial u}{\partial x} + f_{32}\cdot 1\right)$

$$= e^{y}\underline{f_1} + xe^{2y}\underline{f_{11}} + xe^{y}\underline{f_{12}} + e^{y}\underline{\underline{f_{31}}}$$
$$+ \underline{\underline{f_{32}}}$$

$d^2y \neq (dy)^2 \qquad dx^2 = (dx)^2$