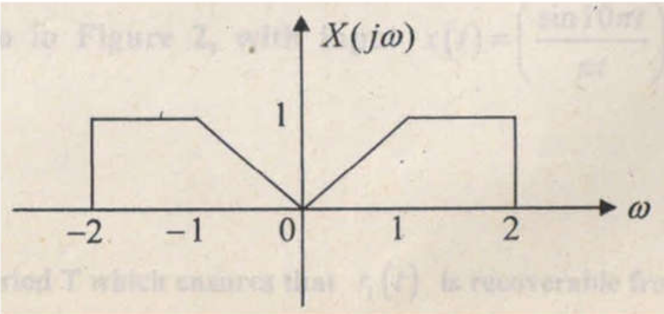
Discussion problem assignment:

第一题：

1. Given the following spectrum, do the following:

(a) Find $\left.\dfrac{dx(t)}{dt}\right|_{t=0}$ **(b) Compute** $\displaystyle\int_{-\infty}^{+\infty} x(t)\frac{\sin t}{\pi t}e^{jt}dt$



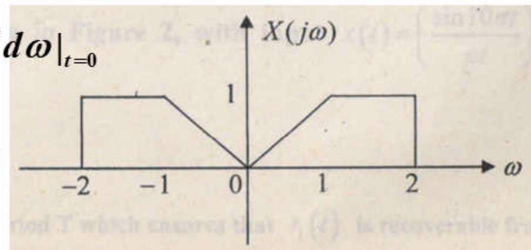
Solution:

1. Given the following spectrum, do the following:

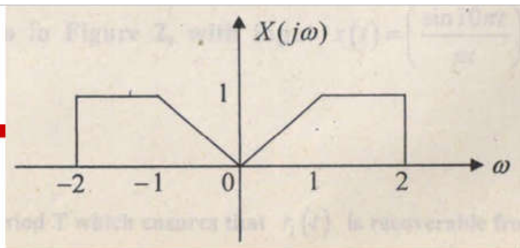
(a) Find $\left.\dfrac{dx(t)}{dt}\right|_{t=0}$

$$y(t)=\frac{dx(t)}{dt},\quad Y(j\omega)=j\omega X(j\omega)$$

$$\begin{aligned}\left.\frac{dx(t)}{dt}\right|_{t=0} &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} j\omega X(j\omega)e^{j\omega t}\,d\omega\Big|_{t=0}\\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} j\omega X(j\omega)d\omega\\ &= 0\end{aligned}$$



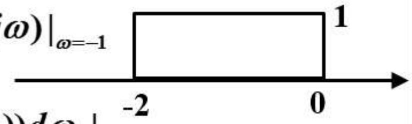
The integral function is an odd function in frequency.



(b) Compute $\int_{-\infty}^{+\infty} x(t)\frac{\sin t}{\pi t}e^{jt}dt$

$$y(t)=x(t)z(t),\quad Y(j\omega)=\frac{1}{2\pi}X(j\omega)*Z(j\omega)$$

$Z(j(-1-\omega_1))$



$$\int_{-\infty}^{+\infty} x(t)\frac{\sin t}{\pi t}e^{jt}dt=\int_{-\infty}^{+\infty} y(t)e^{jt}dt=Y(j\omega)|_{\omega=-1}$$

$$Y(j\omega)|_{\omega=-1}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega_1)Z(j(\omega-\omega_1))d\omega_1|_{\omega=-1}$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega_1)Z(j(-1-\omega_1))d\omega_1 \quad =\frac{1}{2\pi}\frac{3}{2}=\frac{3}{4\pi}$$

Another approach is to take

$$z(t)=\frac{\sin t}{\pi t}e^{jt},\quad \int_{-\infty}^{+\infty}x(t)z(t)dt=\int_{-\infty}^{+\infty}y(t)dt=Y(j\omega)|_{\omega=0}$$

第二题：

For an LTI system, it is known that the input signal

$$x(t)=\delta(t)+e^{-3t}u(t)$$

will generate an output signal $y(t)=e^{-2t}u(t)$. Determine the system's unit impulse response $h(t)$ and write out the linear constant coefficient equation to describe the system.

答案：

$$X(j\omega)=1+\frac{1}{j\omega+3}=\frac{j\omega+4}{j\omega+3}\qquad Y(j\omega)=\frac{1}{j\omega+2}$$

$$H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{1/j\omega+2}{j\omega+4/j\omega+3}=\frac{j\omega+3}{(j\omega+2)(j\omega+4)}=\frac{1/2}{j\omega+2}+\frac{1/2}{j\omega+4}$$

$$h(t)=\frac{1}{2}e^{-2t}u(t)+\frac{1}{2}e^{-4t}u(t)$$

$$((j\omega)^2+6j\omega+8)Y(j\omega)=(j\omega+3)X(j\omega)$$

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=\frac{dx(t)}{dt}+3x(t)$$