$$① \sum_{n=1}^{\infty} \frac{\sin n}{n^2} \quad \boxed{VS} \quad ①' \sum_{n=1}^{\infty} \frac{\sin n}{n}$$

$$② \sum_{n=2}^{\infty} \frac{\ln n}{n}$$

若 $\lim\limits_{n \to \infty} \frac{a_n}{b_n} = \lambda, \quad 0 < \lambda < \infty$

则 $\exists N$, s.t. $n > N$, 有

$$\left| \frac{a_n}{b_n} - \lambda \right| < \frac{\lambda}{2} \quad (\varepsilon \text{ 取 } \frac{\lambda}{2})$$

$$\Rightarrow \boxed{\frac{\lambda}{2} < \frac{a_n}{b_n} < \frac{3\lambda}{2}} \quad n > N.$$

则 $\begin{cases} 若 \sum a_n 收敛, 则 b_n < \frac{2}{\lambda} a_n, 则 \sum b_n 收敛 \\ \\ 若 \sum a_n 发散, 则 b_n > \frac{2}{3\lambda} a_n, 则 \sum b_n 发散 \end{cases}$

两种中级数:

① $P-$级数 $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ ② $\sum \frac{1}{\alpha^n}, \alpha > 1$ $\left| \sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)} \right.$

$\downarrow$

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当 $p > 1$ $\boxed{VS} \boxed{1 + \int_2^n \frac{1}{x^p} dx}$

例: $\sum\limits_{n=1}^{\infty} a_n$, 其中 $a_n = \begin{cases} n/2^n, & n \text{ 奇} \\ \\ 1/2^n & n \text{ 偶} \end{cases}$

$$\frac{a_{n+1}}{a_n} = \begin{cases} \dfrac{1/2^{n+1}}{n/2^n} & n \text{ 奇} \longrightarrow 0 \\ \dfrac{n+1/2^{n+1}}{1/2^n} & n \text{ 偶} \longrightarrow +\infty \end{cases}$$

$\dfrac{a_{N+n}}{a_n} \to \ell < 1$.

$\sum a_n = a_1 + a_{N+1} + a_{2N+1} + \cdots \checkmark$
$\quad\quad + a_2 + a_{N+2} + a_{2N+2} + \cdots \checkmark$