

11. (5) 不正确. $a_n = \frac{1}{n}$, $\sum_{n=1}^{\infty} a_n$ 发散; $\sum_{n=1}^{\infty} a_n^2 = \sum_{n=1}^{\infty} \frac{1}{n^2}$ 收敛

12. (6) $\frac{n^3 [2 + (-1)^n]^n}{3^n} < \frac{n^3 \cdot (\frac{5}{2})^n}{3^n} = n^3 \cdot (\frac{5}{6})^n$

$\lim_{n \rightarrow \infty} \frac{(\frac{5}{6})^{n+1} (n+1)^3}{(\frac{5}{6})^n (n)^3} = \frac{5}{6} < 1 \therefore \sum_{n=1}^{\infty} n^3 (\frac{5}{6})^n$ 收敛 \therefore 原级数收敛

(10) $n \rightarrow \infty$, $n \sin \frac{\pi}{3^n} < n \cdot \frac{\pi}{3^n}$

$\therefore \lim_{n \rightarrow \infty} \frac{(n+1) \frac{\pi}{3^{n+1}}}{n \cdot \frac{\pi}{3^n}} = \frac{1}{3} < 1 \therefore \sum_{n=1}^{\infty} n \frac{\pi}{3^n}$ 收敛 $\therefore \sum_{n=1}^{\infty} n \sin \frac{\pi}{3^n}$ 收敛.

14. (6) $n \rightarrow \infty$, $\tan \frac{1}{\sqrt{n}} \sim \frac{1}{\sqrt{n}}$

① 若 $x = -1$, $\therefore \tan \frac{1}{\sqrt{n+1}} < \tan \frac{1}{\sqrt{n}}$ 且 $\lim_{n \rightarrow \infty} \tan \frac{1}{\sqrt{n}} = 0$ 此时收敛.
 $\sum_{n=1}^{\infty} |u_n| = \sum_{n=1}^{\infty} \tan \frac{1}{\sqrt{n}} \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ 且 $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ 发散 $\therefore \sum_{n=1}^{\infty} |u_n|$ 发散
 \therefore 条件收敛.

② 若 $-1 < x < 1$, 原级数 $= \sum_{n=1}^{\infty} (-1)^n |x|^n \tan \frac{1}{\sqrt{n}} \therefore |x| \tan \frac{1}{\sqrt{n+1}} < |x| \tan \frac{1}{\sqrt{n}}$
 $\sum_{n=1}^{\infty} |u_n| = \sum_{n=1}^{\infty} |x|^n \tan \frac{1}{\sqrt{n}}$ $\lim_{n \rightarrow \infty} \frac{|x|^{n+1} \tan \frac{1}{\sqrt{n+1}}}{|x|^n \tan \frac{1}{\sqrt{n}}} = |x| < 1 \therefore \sum_{n=1}^{\infty} |u_n|$ 收敛

\therefore 绝对收敛.

③ 若 $|x| \geq 1$ 发散; $x < -1$ 发散

18. (1) $\left| \frac{\cos(n!)}{n\sqrt{n}} \right| \leq \frac{1}{n\sqrt{n}}$ 而 $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ 收敛 $\therefore \sum_{n=1}^{\infty} \left| \frac{\cos(n!)}{n\sqrt{n}} \right|$ 收敛 \therefore 原级数绝对收敛.

(4) $n \rightarrow \infty$ 时 $\left| (-1)^{n+1} \frac{\ln(2+\frac{1}{n})}{\sqrt{9n^2-4}} \right| \sim \frac{1}{n} \therefore \sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{\ln(2+\frac{1}{n})}{\sqrt{9n^2-4}} \right|$ 发散. \therefore 原级数条件收敛