Discussion problem assignment:

第一题：

对以下两个系统，确定其六个性质，并给出相应的证明。

(a) $y(t) = x(t-2) + x(2-t)$

(b) $y(t) = \cos[3t]x(t)$

结果：

(a) $y(t) = x(t-2) + x(2-t)$

(1) Not memoryless, since y(t) depends on the input at times other than t.

(2) Not invertible, since the following two input signals have the same output, $x_1(t) = 1$ and $x_2(t) = 2u(t)$.

(3) Not causal, since y(0) = x(-2) + x(2).

(4) Stable. Assume that |x(t)| ≤ M, then |y(t)| ≤ 2M.

(5) Not time invariant

$$x(t) \to y(t) = x(t-2) + x(2-t)$$

Solution:
$$x_1(t) \to y_1(t) = x_1(t-2) + x_1(2-t)$$
$$x_2(t) \to y_2(t) = x_2(t-2) + x_2(2-t)$$

Let $x_2(t) = x_1(t - t_0)$ **be the new input, what is the output?**

$$x_2(t) \to y_2(t) = x_2(t-2) + x_2(2-t) = x_1(t-2-t_0) + x_1(2-t-t_0)$$
$$y_1(t-t_0) = x_1(t-t_0-2) + x_1(2-(t-t_0)) = x_1(t-t_0-2) + x_1(2-t+t_0)$$

In general, for any input signal $x_1(t) \to y_1(t)$ **, we have**

$$x_1(t-t_0) \to x_1(t-2-t_0) + x_1(2-t-t_0) \neq y_1(t-t_0)$$

That is, the system is NOT time invariant.

(6) Linear

$$x(t) \to y(t) = x(t-2) + x(2-t)$$

Solution:
$$x_1(t) \to y_1(t) = x_1(t-2) + x_1(2-t)$$
$$x_2(t) \to y_2(t) = x_2(t-2) + x_2(2-t)$$
$$x_3(t) \to y_3(t) = x_3(t-2) + x_3(2-t)$$

Let $x_3(t) = ax_1(t) + bx_2(t)$ **be the new input, what is the output?**

$$\begin{aligned} x_3(t) \to y_3(t) &= x_3(t-2) + x_3(2-t) \\ &= (ax_1(t-2) + bx_2(t-2)) + (ax_1(2-t) + bx_2(2-t)) \\ &= a(x_1(t-2) + x_1(2-t)) + b(x_2(t-2) + x_2(2-t)) \end{aligned}$$
$$ay_1(t) + by_2(t) = a(x_1(t-2) + x_1(2-t)) + b(x_2(t-2) + x_2(2-t))$$

In general, for any input signal $x_1(t) \to y_1(t)$ $x_2(t) \to y_2(t)$ **,**

$$ax_1(t) + bx_2(t) \to ay_1(t) + by_2(t)$$

That is, the system is linear.

(b) $y(t) = \cos[3t]x(t)$

(1) Memoryless

(2) Not invertible, since the following two input signals have the same output, $x_1(t)=\delta(t-\frac{\pi}{6})$ and $x_2(t)=\delta(t+\frac{\pi}{6})$.

(3) Causal since it is memoryless.

(4) Stable. Assume that |x(t)| ≤ M, then |y(t)| ≤ |cos[3t]| |x(t)| ≤ M.

(5) Not time invariant.

$$x(t)\to y(t)=[\cos 3t]x(t)$$

Solution:
$$x_1(t)\to y_1(t)=\cos[3t]x_1(t)$$
$$x_2(t)\to y_2(t)=\cos[3t]x_2(t)$$

Let $x_2(t)=x_1(t-t_0)$ be the new input, what is the output?

$$x_2(t)\to y_2(t)=\cos[3t]x_2(t)=\cos[3t]x_1(t-t_0)$$
$$y_1(t-t_0)=\cos[3(t-t_0)]x_1(t-t_0)$$

In general, for any input signal $x_1(t)\to y_1(t)$, we have

$$x_1(t-t_0)\to\cos[3t]x_1(t-t_0)\neq y_1(t-t_0)$$

That is, the system is NOT time invariant.

(6) Linear since

$$x(t)\to y(t)=[\cos 3t]x(t)$$

Solution:
$$x_1(t)\to y_1(t)=\cos[3t]x_1(t)$$
$$x_2(t)\to y_2(t)=\cos[3t]x_2(t)$$
$$x_3(t)\to y_3(t)=\cos[3t]x_3(t)$$

Let $x_3(t)=ax_1(t)+bx_2(t)$ be the new input, what is the output?

$$x_3(t)\to y_3(t)=\cos[3t]x_3(t)=\cos[3t](ax_1(t)+bx_2(t))$$
$$=\cos[3t]ax_1(t)+\cos[3t]bx_2(t)=a\cos[3t]x_1(t)+b\cos[3t]x_2(t)$$
$$ay_1(t)+by_2(t)=a\cos[3t]x_1(t)+b\cos[3t]x_2(t)$$

In general, for any input signal $x_1(t)\to y_1(t)$ $x_2(t)\to y_2(t)$,

$$ax_1(t)+bx_2(t)\to ay_1(t)+by_2(t)$$

That is, the system is linear.