

6.2 A

$$\begin{aligned} 3.(6) \iint_{(r)} e^{-y^2} d\sigma &= \int_0^1 dx \int_0^x e^{-y^2} dy = \int_0^1 dy \int_0^y e^{-y^2} dx \\ &= \int_0^1 y e^{-y^2} dy \\ &= \frac{e-1}{2e} \end{aligned}$$

$$\begin{aligned} 5.(4) \text{ 原式} &= \int_0^2 dx \int_{\frac{1}{2}x}^1 f(x,y) dy + \int_1^3 dy \int_0^2 f(x,y) dx \\ &\quad + \int_1^3 dy \int_2^8 dx \int_{\frac{y}{2}}^3 f(x,y) dx \end{aligned}$$

6. (1) 令 $x = \rho \cos \theta$, $y = \rho \sin \theta$, 则 $\Omega: 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \rho \leq 1$

$$\begin{aligned} \therefore \iint_{\Omega} \arctan \frac{y}{x} d\sigma &= \iint_{\Omega} \theta \rho d\rho d\theta \\ &= \int_0^{\frac{\pi}{2}} \theta d\theta \int_0^1 \rho d\rho = \frac{\pi^2}{16} \end{aligned}$$

7.(3) $\sigma: 1 \leq y \leq 2, 0 \leq x \leq y \Rightarrow \Omega: 1 \leq \rho \leq 2, \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$
 $\Rightarrow \Omega: \theta \in [\frac{\pi}{4}, \frac{\pi}{2}], \rho \in [\frac{1}{\sin \theta}, \frac{2}{\sin \theta}]$

$$\begin{aligned} \therefore \int_1^2 dy \int_0^y \frac{x \sqrt{x^2+y^2}}{y} dx &= \iint_{\Omega} \frac{\rho}{\tan \theta} \rho d\rho d\theta \\ &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_{\frac{1}{\sin \theta}}^{\frac{2}{\sin \theta}} \frac{\rho^2}{\tan \theta} d\rho = \frac{7}{9} (2\sqrt{2} - 1) \end{aligned}$$

13.(3) 令 $xy = u, \frac{x}{y} = v \Rightarrow x = \sqrt{uv}, y = \sqrt{\frac{u}{v}}$

$\sigma \Rightarrow \Omega: 1 \leq u \leq 2, \frac{1}{4} \leq v \leq 1$

$$\begin{aligned} \iint_{\sigma} xy d\sigma &\Rightarrow \iint_{\Omega} -\frac{u}{2v} du dv = \int_1^2 u du \cdot \int_{\frac{1}{4}}^1 \left(-\frac{1}{2v}\right) dv \\ &= -\frac{45}{4} \end{aligned}$$