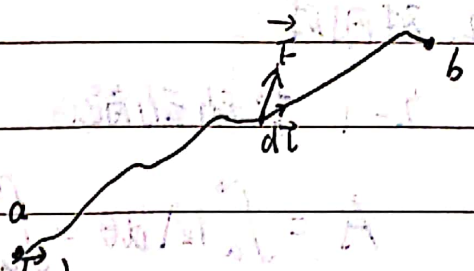
3.2 动能定理与机械能守恒定律

一、1. $dA=\vec{F}\cdot d\vec{l}$



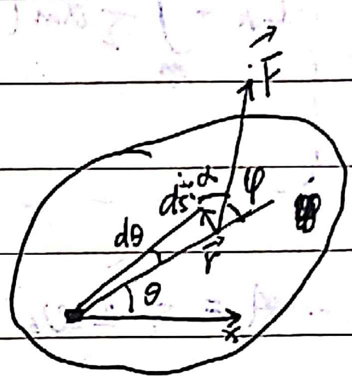
$$A=\int_a^b \vec{F}\cdot d\vec{l}$$

$$=\int_a^b (F_x\vec{i}+F_y\vec{j}+F_z\vec{k})\cdot(dx\vec{i}+dy\vec{j}+dz\vec{k})$$

$$=\int_a^b (F_xdx+F_ydy+F_zdz)$$

$$=A_x+A_y+A_z$$

2. $d\vec{s}=rd\theta\,\vec{\tau}$



$$dA=\vec{F}\cdot d\vec{s}=Fr\,d\theta\cdot\cos\alpha$$

$$=Fr\,d\theta\cdot\sin\varphi$$

$$=M\cdot d\theta$$

$$=\vec{M}\cdot d\vec{\theta}$$ （方向都与旋转轴相同）

$$A=\int_i^f M\cdot d\theta$$

二、功率

$$P=\begin{cases}\dfrac{dA}{dt}=\vec{F}\cdot\dfrac{d\vec{l}}{dt}=\vec{F}\cdot\vec{v}\\ \dfrac{dA}{dt}=\vec{M}\cdot\dfrac{d\vec{\theta}}{dt}=\vec{M}\,\vec{\omega}\end{cases}$$

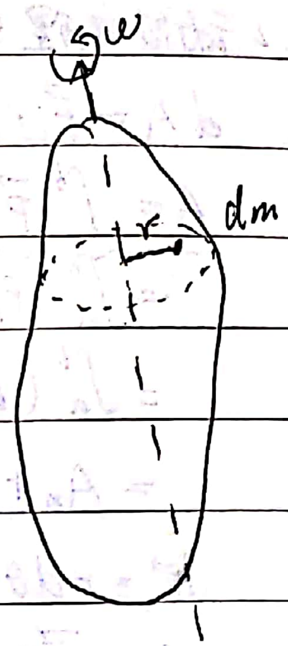
三、动能定理

1. 对质点： $A=\int_a^b\vec{F}\cdot d\vec{l}=\int_a^b m\frac{d\vec{v}}{dt}\cdot d\vec{l}=\int_a^b m\boxed{\vec{v}\,d\vec{v}}=\int_a^b m\boxed{v\,dv}$

$$A=\frac{1}{2}mv_b^2-\frac{1}{2}mv_a^2=\Delta E_k$$

2. 对刚体

2-1 转动动能



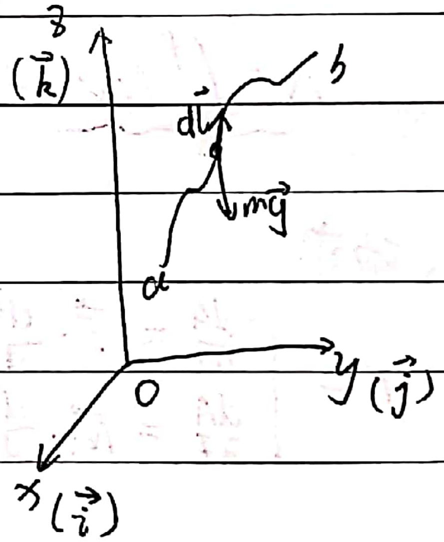
$$A=\int_a^b M\,d\theta=\int_a^b I\frac{d\omega}{dt}d\theta=\int_a^b I\omega\,d\omega$$

$$\boxed{E_k=\int_m \frac{1}{2}dm(r\omega)^2=\frac{1}{2}\left(\int_m r^2 dm\right)\omega^2=\frac{1}{2}I\omega^2}$$

$$\therefore A=\frac{1}{2}I\omega_b^2-\frac{1}{2}I\omega_a^2=\Delta E_k$$

四、势能与保守力

1. 重力势能



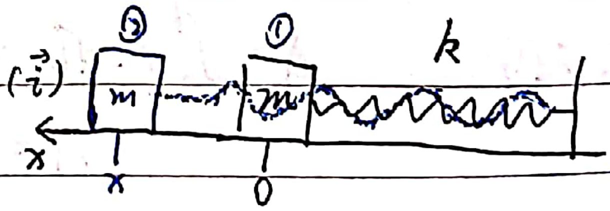
$$A=\int_a^b m\vec{g}\cdot d\vec{l}=\int_a^b(-mg\vec{k})\cdot d\vec{l}$$

$$=\int_a^b -mg\,d\vec{z}=-\Delta E_p$$

$$E_p=mgz$$

2. 弹性势能

$$\vec{f}=-kx\vec{i}$$



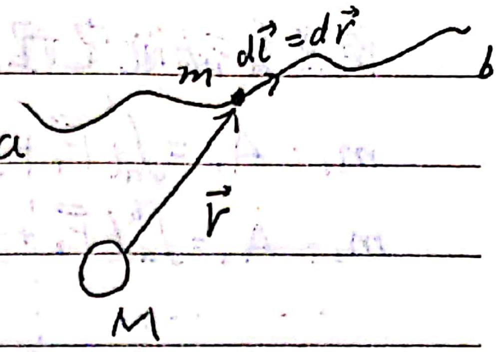
$$A=\int_①^② \vec{f}\,d\vec{x}=\int_0^② -kx\,dx=-\frac{1}{2}kx^2$$

$$=-\Delta E_p$$

$$E_p=\frac{1}{2}kx^2$$

3. 引力势能

$$\vec{F} = -G\frac{Mm}{r^3}\vec{r}$$

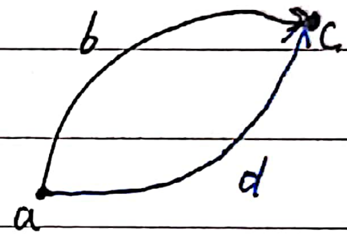


$$A=\int_a^b -G\frac{Mm}{r^3}\boxed{\vec{r}\cdot \underbrace{d\vec{r}}_{d\vec{l}}} = \int_a^b -G\frac{Mm}{r^3}\boxed{r\,dr}$$

$$= -G\frac{Mm}{r_a} - \left(-G\frac{Mm}{r_b}\right) = -\Delta E_p$$

$$E_p = -G\frac{Mm}{r}$$

▲ 对保守力 F：做功与路径无关.



$$\int_{abc}\vec{F}\cdot d\vec{l} = \int_{adc}\vec{F}\cdot d\vec{l}$$

$$\int_{abc}\vec{F}\cdot d\vec{l} - \int_{adc}\vec{F}d\vec{l} = \int_{abc}\vec{F}d\vec{l} + \int_{cda}\vec{F}d\vec{l} = \oint_{abcda}\vec{F}\cdot d\vec{l} = 0 \quad (\text{回路积分})$$

$$\Updownarrow$$

$$\oint_L \vec{F}\cdot d\vec{l} = \iint_{S(L)}(\nabla\times\vec{F})d\vec{s} = 0 \Rightarrow \nabla\times\vec{F} = 0$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \quad (\text{旋度})$$

$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$dA = \vec{F}\cdot d\vec{l} = -dE_p$$

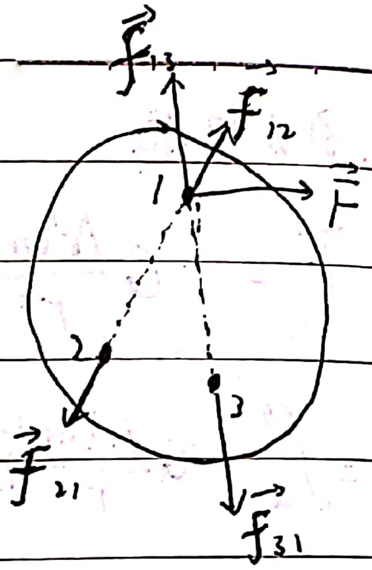
$$= F_l\,dl = -dE_p \qquad F_l = -\frac{dE_p}{dl} = -\frac{\partial E_p}{\partial l}$$

故在直角坐标中，$F_x = -\frac{\partial E_p}{\partial x}$，$F_y = -\frac{\partial E_p}{\partial y}$，$F_z = -\frac{\partial E_p}{\partial z}$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = -\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)E_p = -\nabla E_p$$

(梯度)

质点系动能定理.



$m_1: A_1=\int_a^b(\vec{F}_1+\vec{f}_{12}+\vec{f}_{13}+\cdots)d\vec{l}_1=\Delta E_{k1}$

$m_2: A_2=\int_a^b(\vec{F}_2+\vec{f}_{21}+\vec{f}_{23}+\cdots)dl_2=\Delta E_{k2}$

$\cdots\cdots \quad \cdots\cdots \qquad =0,$

但由于 $d\vec{l}_n$ 不同，不能抵消

$A_{外}+A_{内力}=\sum_i \Delta E_{ki}$