Discussion problem assignment:

第一题：

2. Find the signal for the following z-transform $X(z)$ and given the fact that the signal has DTFT.

$$X(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-2z^{-1})}$$

解答：

Solutions:

1) Find the poles and the results are z = - ½ and z = 2.

2) For the three possible ROC, the one with unit circle for the signal to have DTFT is

$$\frac{1}{2} < |z| < 2$$

3) Find the partial fraction expansion as

$$X(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-2z^{-1})} = \frac{1/5}{(1+\frac{1}{2}z^{-1})} + \frac{4/5}{(1-2z^{-1})}$$

4) For inverse z-transform

$$X(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-2z^{-1})} = \frac{1/5}{(1+\frac{1}{2}z^{-1})} + \frac{4/5}{(1-2z^{-1})} \qquad \frac{1}{2} < |z| < 2$$

$$\frac{1}{1-\alpha_i z^{-1}} \leftrightarrow \begin{cases} \alpha_i^n u[n], & \text{ROC outside the pole} \\ -\alpha_i^n u[-n-1], & \text{ROC inside the pole} \end{cases}$$

$$x[n] = \frac{1}{5}\left(-\frac{1}{2}\right)^n u[n] - \frac{4}{5} 2^n u[-n-1]$$

第二题：

1. Find the poles for the following $X(z)$, then compute the signal $x[n]$.

$$X(z)=\frac{1}{1-z^{-4}},|z|>1$$

解答：

1. Find the poles for the following $X(z)$, then compute the signal $x[n]$.

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Solution 1:

首先，求Z变换表达式的极点，有 $X(z)=\dfrac{z^4}{z^4-1}$

因为 z 是复数，所以结果是四个成共轭对的极点，具体为

$$z=\pm 1,\pm j \qquad 或者 \qquad z=e^{j\frac{\pi}{2}k},k=0,1,2,3$$

其次，反变换由 $X_1(z)=\dfrac{1}{1-z^{-1}},|z|>1 \xleftrightarrow{Z} x_1[n]=u[n]$

由离散时间信号时域补零拓展性质，有

$$\cancel{X(z)=X_1(z^4) \qquad x[n]=x_{1(4)}[n]}$$

Solution 2:

如果未发现可以直接使用性质，也可以按照常规的部分分式展开

$$X(z)=\frac{1}{1-z^{-4}}=\frac{1}{(1-z^{-1})(1+z^{-1})(1-jz^{-1})(1+jz^{-1})}$$

$$=\frac{1/4}{1-z^{-1}}+\frac{1/4}{1+z^{-1}}+\frac{1/4}{1-jz^{-1}}+\frac{1/4}{1+jz^{-1}}$$

再结合ROC完成反变换，有

$$x[n]=\frac{1}{4}u[n]+\frac{1}{4}(-1)^n u[n]+\frac{1}{4}(j)^n u[n]+\frac{1}{4}(-j)^n u[n]$$

虽然形式不同，但是对信号仔细分析可以验证两种方法的结果一致。

$$x[n]=x_{1(4)}[n]$$