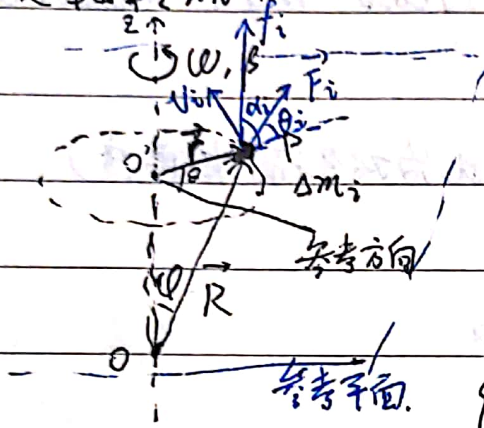
# 刚体转动定律

刚体：特殊的质点系，具有相对性

平动：刚体上所有质点运动都相同

转动：刚体上所有质点都绕某一轴作半径不同作半径不同的圆周运动

定轴转动：




$v = r\omega = R\sin\varphi\,\omega$

$$\begin{cases}\vec{v} = \vec{\omega} \times \vec{r} \\ \vec{v} = \vec{\omega} \times \vec{R}\end{cases}$$

$$\begin{cases}a_\tau = r\beta \\ a_n = \frac{v^2}{R} = \omega^2 R\end{cases}$$

匀加速：$\beta = \frac{d\omega}{dt} = const$ $d\omega = \beta dt$ $\int d\omega = \int \beta dt$

$\Downarrow$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\beta t^2 \Leftarrow \omega = \omega_0 + \beta t$

$\beta = \frac{d\omega}{dt} = \frac{d\theta}{dt}\cdot\frac{d\omega}{d\theta} = \omega\frac{d\omega}{d\theta} \Rightarrow \beta d\theta = \omega d\omega \therefore \int_{\omega_0}^{\omega}\omega d\omega = \int_{\theta_0}^{\theta}\beta d\theta$

得 $\omega^2 - \omega_0^2 = 2\beta(\theta - \theta_0)$

力矩 $M = F_i \cdot \sin\theta \cdot r_i$ $\vec{M_i} = \vec{r_i} \times \vec{F_i}$

▲ $F_i$ 是在参考平面里的分量，$r_i$ 也是.

# 刚体的定轴转动定律

设外力是F，内力是f.(任一质点)

$$\Delta m_i \cdot a_\tau = F_i \sin\theta_i + f_i \sin\alpha_i$$

上式同乘 $r_i$ 并对 $i$ 求和

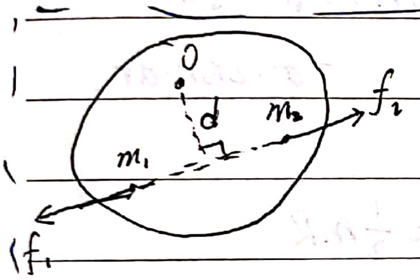
I(转动惯量)

$$左 = \sum_i \Delta m_i r_i a_\tau = \sum_i \Delta m_i r_i^2 \beta = \beta \boxed{\sum_i \Delta m_i r_i^2}$$

切向加速度 $a_\tau = \beta r_i$

合外力矩 $M_合$

$$右 = \boxed{\sum_i F_i r_i \sin\theta_i} + \sum_i f_i \sin\alpha_i r_i$$




$f_1$, $f_2$ 相互作用力(等大反向)

内力矩 $f_1 d$, $f_2 d$ 不为0.

但 $\sum_i M_{内i} = 0$

$\therefore \sum_i f_i r_i \sin\alpha_i = 0$ (合内力矩恒为0)

$\therefore \boxed{\vec{M}_合 = I\vec{\beta}}$ 即 $\sum_i F_i r_i \sin\theta_i = \sum_i (\Delta m_i r_i^2)\beta$

## 求子转动惯量

$$I = \sum \Delta m_i r_i^2 \text{ (离散)}$$

$$= \boxed{\int_m r_i^2 \, dm} \text{ (连续)}$$

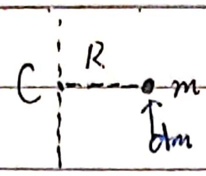
$$= \begin{cases} \iiint r^2 \rho \, dV \\ \iint r^2 \sigma \, ds \\ \int r^2 \lambda \, dl \end{cases}$$

$$dm = \begin{cases} \underset{体密度}{\rho} \cdot dV \text{ (体)} \\ \underset{面密度}{\sigma} \cdot ds \text{ (面)} \\ \underset{线密度}{\lambda} \cdot dl \text{ (线)} \end{cases}$$

I与 ~~质量分布~~ $m$, $\begin{pmatrix} \rho \\ \sigma \\ \lambda \end{pmatrix}$, $r$ 有关

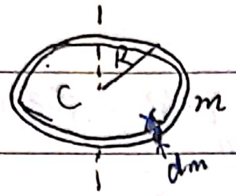
(三要素)

质点:



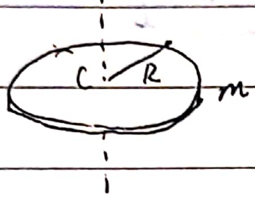

$$I=\int r^2 dm=\int R^2 dm=R^2\int dm=mR^2$$

均匀圆环:

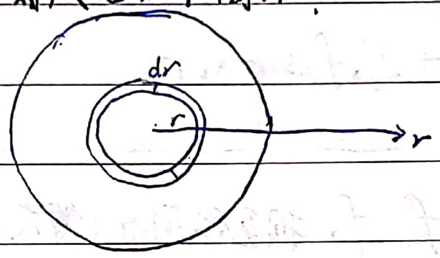



$$I=\int R^2 dm=R^2\int dm=mR^2$$

均匀圆盘




看成无数个圆环




$$dS=2\pi r\cdot dr$$

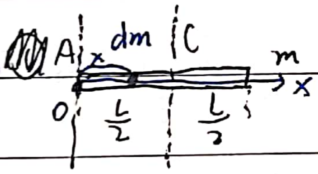
$$dm=\sigma\cdot dS$$

$$=\sigma\cdot 2\pi r\cdot dr$$

$$dI=dm\,r^2=\sigma\cdot 2\pi r^3 dr$$

$$I=\int_0^R \sigma 2\pi r^3 dr=\frac{1}{2}\sigma\pi R^4=\frac{1}{2}\underbrace{\sigma\pi R^2}_{m}\cdot R^2=\frac{1}{2}mR^2$$

均匀细棒:



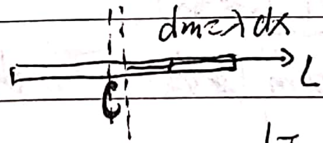

① 以A为轴:

$$dm=\lambda\cdot dx$$

$$dI_A=x^2\lambda\cdot dx$$

$$I_A=\int_0^L \lambda x^2 dx=\frac{1}{3}\lambda L^3=\frac{1}{3}\underbrace{\lambda L}_{m}\cdot L^2=\frac{1}{3}mL^2$$

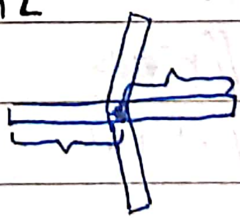
② 以质心(C)为轴:

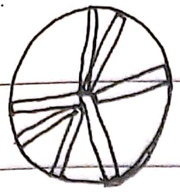



$$dI=\lambda x^2 dx$$

$$I_C=\int_{-\frac{L}{2}}^{\frac{L}{2}}\lambda x^2 dx=\frac{1}{12}mL^2$$

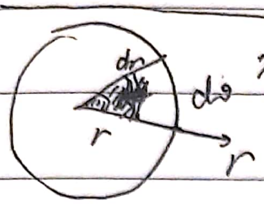
$$\int_{-\frac{L}{2}}^{\frac{L}{2}}=\int_0^{\frac{L}{2}}+\int_0^{\frac{L}{2}}$$





……无数个条 ⇒ 圆盘 ✗  中心会累积

质量分布不均匀.




分小扇形. $dS = dr \cdot r d\theta$ (小扇面)

$$dm = \sigma \cdot r dr \cdot d\theta$$

$$dI_\theta = \int \sigma r^3 dr \cdot d\theta$$

(小扇形) $I_\theta = \int_0^R \sigma r^3 dr \cdot d\theta = \frac{1}{4}(\sigma R^2 d\theta) R^2$

$d\theta \to 2\pi$. $I = \frac{1}{2} m R^2$ (圆盘)