

5.6 A

$$4.(4) S = \int_0^{\frac{\pi}{2}} \sqrt{e^{2t}(\sin t + \cos t)^2 + e^{2t}(\cos t - \sin t)^2} dt = \sqrt{2} \int_0^{\frac{\pi}{2}} e^t dt = \sqrt{2}(e^{\frac{\pi}{2}} - 1)$$

$$5.(2) S = \int_0^2 \sqrt{(2t)^2 + (-2t)^2} dt$$

$$S = \int_0^2 \sqrt{2^2 + (2t)^2 + (-2t)^2} dt = 2 \int_0^2 \sqrt{1 + 2t^2} dt$$

$$= 2 \left[\frac{1}{2} + \sqrt{1 + 2t^2} + \frac{\sqrt{2}}{2} \ln |4t + 2\sqrt{2(1 + 2t^2)}| \right] \Big|_0^2$$

$$= 6 + \sqrt{2} \ln(\sqrt{2} + 3)$$

$$10.(3) \vec{n} = \nabla F \quad \text{令 } F(x, y, z) = x^3 + y^3 + z^3 + xyz - 6 = 0$$

$$\vec{n} = (F_x, F_y, F_z) = (3x^2 + yz, 3y^2 + xz, 3z^2 + xy)$$

$$\therefore \text{在点 } P_0(1, 2, -1) \text{ 处, } \vec{n}_0 = (F_x(P_0), F_y(P_0), F_z(P_0)) = (1, 11, 5)$$

$$\therefore \text{切平面: } (x-1) + 11(y-2) + 5(z+1) = 0$$

$$\text{法线: } \frac{x-1}{1} = \frac{y-2}{11} = \frac{z+1}{5}$$

$$11. \text{曲线中, 令 } t=y, \text{ 则 } \begin{cases} y^2=x \\ z=3(y-1) \end{cases} \text{ 可化为 } \vec{r}(t) = (t^2, t, 3t-3)$$

$$\text{在 } y=1 \text{ (即点 } (1, 1, 0) \text{)} \text{ 的切线为 } \frac{x-1}{2} = y-1 = \frac{z}{3}, \text{ 法向量为 } \vec{n}$$

$$\text{设平面为 } a(x-1) + b(y-1) + cz = 0, \text{ 法向量 } \vec{n}_1 = (a, b, c)$$

$$\therefore \text{平面过曲线的切线 } \therefore \vec{n} \cdot \vec{n}_1 = 2a + b + 3c = 0$$

$$\text{曲面: } \vec{r} = (x, y, z) \quad F(x, y, z) = x^2 + y^2 - 4z = 0$$

$$\therefore \text{曲面的法向量 } \vec{n}_2 = (F_x, F_y, F_z) = (2x, 2y, -4)$$

$$\therefore \text{曲面与平面相切 } \therefore \vec{n}_1 = k\vec{n}_2 \quad \text{即 } (a, b, c) = (2kx, 2ky, -4k)$$

$$\therefore 2a + b + 3c = 4kx + 2ky - 12k = 0 \quad \therefore 4x + 2y - 12 = 0 \quad (1)$$

$$\therefore a(x-1) + b(y-1) + cz = 2kx(x-1) + 2ky(y-1) - 4kz = 0$$

$$\text{即 } x(x-1) + y(y-1) - 2z = 0 \quad (2)$$

$$\text{又 } x^2 + y^2 = 4z \quad (3)$$

$$(1)(2)(3) \text{ 联立解得 } \begin{cases} x=2 \\ y=2 \\ z=2 \end{cases} \text{ 或 } \begin{cases} x=\frac{12}{5} \\ y=\frac{6}{5} \\ z=\frac{9}{5} \end{cases}$$

$$\therefore \text{平面为 } x + y - z = 2 \text{ 或 } 6x + 3y - 5z = 9$$