

Random Mathematics Homework #2 Fall 2020

Instructor: Dr. Jing Liang

Assigned Date: Sept.17, 2020 Due Date: Sept.22, 2020

1. Three students A, B, C are enrolled in the same class. Suppose A attends class 40% of time, B attends 80% of time, and C attends 90% of time. If these students attend class independently of each other, what is (a) the probability that at least one of them will be in class on a particular day and (b) the probability that exactly one of them will be in class on a particular day?
2. A and B are events such that $\Pr(A) = 1/4$, $\Pr(B) = 1/2$.
 - (1) If $\Pr(A|B) + \Pr(B|A) = 3/5$. What is the value of $\Pr(A^c \cup B^c)$?
 - (2) A and B are independent. What is the value of $\Pr(A \cup B)$?
 - (3) A and B are disjoint. What is the value of $\Pr(A|B)$?
3. Suppose A, B, C are events such that A and B are independent. $\Pr(A \cap B \cap C) = 0.02$, $\Pr(C | A \cap B) = 0.4$, and $\Pr(A) = 5 \Pr(B)$. What is the value of $\Pr(A \cup B)$?
4. Two boys A and B throw a ball at a target. Suppose that the probability that boy A will hit the target on any throw is $1/3$ and the probability that boy B will hit the target on any throw is $1/4$. Suppose also that boy A throws first and the two boys take turns throwing. Determine the probability that the target will be hit for the first time on the fourth throw of boy B.
5. A box contains 20 white balls, 30 blue balls, and 50 red balls. Suppose that 10 balls are selected at random one at a time, with replacement; that is, each selected ball is replaced in the box before the next selection is made. Determine the probability that at least one color will be missing from the 10 selected balls.
6. Suppose that 30 percent of the bottles produced in a certain plant are defective. If a bottle is defective, the probability is 0.9 that an inspector will notice it and remove it from the filling line. If a bottle is not defective the probability is 0.2 that the inspector will think that it is defective and remove it from the filling line. (a) If a bottle is removed from the filling line, what is the probability that it is defective? (b) If a customer buys a bottle that has not been removed from the filling line, what is the probability that it is defective?
7. Suppose that a family has exactly n children ($n \geq 2$). Assume that the probability that any child will be a girl is $1/2$ and that all births are independent. Given that the family has at least one girl, determine the probability that the family has at least one boy.
8. Suppose that we have two baskets labeled A and B. Basket A contains 5 apples and 5 pears; B contains 7 apples and 3 pears. A biased coin is flipped, for which $\Pr(\text{Head}) = 1/4$. If the coin comes up a head, we take one fruit from box A, and if it comes up a tail, we take a fruit from box B.
 - (1) What is the probability of obtaining an apple?
 - (2) If you are told that the fruit obtained is a pear, what is the probability that the coin is head up?

9. Input one of the letters from A, B, C into a system. The probability that the output letter will be the same as the input is α , and the probability that the output will be a different letter is $(1-\alpha)/2$. Suppose that the input of the system will be one of the following strings: AAAA, BBBB, CCCC with the probability of p_1 , p_2 , and p_3 , respectively. $p_1 + p_2 + p_3 = 1$. Assume that the system transmit each letter independently, and if the output of the system is ABCA, what is the probability that the input string is AAAA?
10. There are 8 blue balls and 2 red balls in a box.
- (1) Suppose we randomly choose 2 balls from the box without replacement. What is the probability of the event that both of the selected balls are blue.
 - (2) We put the selected 2 balls in (1) back to the box and randomly choose 2 balls again. What is the probability of the event that one selected ball is red and the other is blue.
 - (3) We put the selected 2 balls in (2) back to the box and select the first ball randomly **without replacement**, and then select the second ball **without replacement**. What is the probability of the event that the first ball is red and the second ball is blue?
 - (4) Now we put the selected 2 balls in (3) back to the box and select the first ball randomly **with replacement**, and then select the second ball randomly. What is the probability of the event that the second ball is red?