Random Mathematics Homework #3 Fall 2020

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Assigned Date: Sept.30, 2020   Due Date: Oct. 13, 2020

1. Suppose that the probability that a patient recovers from a rare virus is 0.4. If 15 people are known to have contracted this disease, what is the probability that:
   a. At least 10 survive?
   b. From 3 to 8 survive?
   c. Exactly 5 survive?

2. A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the distribution for the number of defectives in terms of :
   a. Probability function (p.f.).
   b. Cumulative distribution function(c.d.f.).
   c. Sketch p.f.
   d. Sketch c.d.f.

3. Suppose that the p.d.f. of a random variable X is as follows:
   $$f(x) = \begin{cases} cx^2 & \text{for } -5 \le x \le 5 \\ 0 & \text{otherwise.} \end{cases}$$
   a. Find the value of the constant c and sketch the p.d.f.
   b. Find the value of $\Pr(X > 4)$.
   c. Find the value of $\Pr(X \geq 5)$.
   d. Find the value of $\Pr(-6 \leq X \leq 2)$.
   e. Find the value of $\Pr(X \leq -5)$.
   f. Find the value of $\Pr(X > -6)$.

4. An ice cream seller takes 20 gallons of ice cream in her truck each day. Let X stand for the number of gallons that she sells. The probability is 0.1 that $X = 20$. If she doesn't sell all 20 gallons, the distribution of X follows a continuous distribution with a p.d.f. of the form
   $$f(x) = \begin{cases} cx & \text{for } 0 < x < 20, \\ 0 & \text{otherwise.} \end{cases}$$
   where c is a constant that makes $\Pr(X < 20) = 0.9$. Find the constant c so that $\Pr(X < 20) = 0.9$ as described above.

5. A civil engineer is studying a left-turn lane that is long enough to hold seven cars. Let X be the number of cars in the lane at the end of a red light that is randomly chosen. The engineer believes that the probability of $X=x$ is proportional to $(x+1)(8-x)$ for $x = 0,1,\ldots,7$ (the possible value of X).
   a. Find the p.f. of X.
   b. Find the probability that X will be at least 5.

6. Suppose that a book with n pages contains on the average x misprints per page. What's the probability that there will be at least m pages that each page contains more than k misprints?

7. Suppose that the p.d.f. of a random variable X is as follows:
$$f(x)=\begin{cases}\frac{1}{8}x & \text{for } 0\le x\le 4,\\ 0 & \text{otherwise.}\end{cases}$$
   a. Find the value of t such that $\Pr(X\le t)=1/4$.
   b. Find the value of t such that $\Pr(X\ge t)=1/2$.

8. Show that there **does not** exit any number c such that the following function $f(x)$ would be a p.d.f.:
$$f(x)=\begin{cases}\frac{c}{1+x} & \text{for } x>0,\\ 0 & \text{otherwise.}\end{cases}$$

9. An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the c.d.f. of T, the number of years to maturity for a randomly selected bond as follows
$$F(t)=\begin{cases}0, t<1\\ \frac{1}{4}, 1\le t<3\\ \frac{1}{2}, 3\le t<5\\ \frac{3}{4}, 5\le t<7\\ 1, t\ge 7\end{cases}$$
   Find:
   a. $\Pr(T=5)$;
   b. $\Pr(T>3)$;
   c. $\Pr(1.4<T<6)$.

10. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous $R.V.$ with c.d.f. as follows;
$$F(x)=\begin{cases}0 & \text{for } x\le 0\\ 1-e^{-8x} & \text{for } x\ge 0.\end{cases}$$
   Find the probability of waiting less than 12 minutes between successive speeders
   a. Using the c.d.f. of X.
   b. Using the p.d.f. of X.