Chapter 3
Homework-3 Solutions

Instructor : Dr. Jing Liang
Department of Electronic Engineering
liangjing@uestc.edu.cn

Q1

n	k	$p=0.4$
15	0	.0005
	1	.0047
	2	.0219
	3	.0634
	4	.1268
	5	.1859
	6	.2066
	7	.1771
	8	.1181
	9	.0612
	10	.0245
	11	.0074
	12	.0016
	13	.0003
	14	.0000
	15	.0000

Suppose that the probability that a patient recovers from a rare virus is 0.4. If 15 people are known to have contracted this disease. What's the prob. that:

a. At least 10 survive?

b. From 3 to 8 survive?

c. Exactly 5 survive?

Sol: Let X be the number of people who survive

a. $\Pr(X \geq 10) = 1 - \Pr(X < 10) = 1 - \sum_{x=0}^{9} b(x; 15, 0.4) = 1 - 0.9662 = 0.0338$

b. $\Pr(3 \leq X \leq 8) = \sum_{x=3}^{8} b(x; 15, 0.4) = \sum_{x=0}^{8} b(x; 15, 0.4) - \sum_{x=0}^{2} b(x; 15, 0.4) = 0.8779$

c. $\Pr(X = 5) = b(5; 15, 0.4) = 0.1859$

Q2-1

A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the distribution for the number of defectives in terms of :

a. p.f.; b. c.d.f; c. sketch p.f.; d.sketch c.d.f.

Sol: Let *X* be the possible numbers of defective computers purchased by the school.

$$f(0)=\Pr(X=0)=\frac{\binom{3}{0}\binom{17}{2}}{\binom{20}{2}}=\frac{68}{95}\qquad f(1)=\Pr(X=1)=\frac{\binom{3}{1}\binom{17}{1}}{\binom{20}{2}}=\frac{51}{190}$$

$$f(2)=\Pr(X=2)=\frac{\binom{3}{2}\binom{17}{0}}{\binom{20}{2}}=\frac{3}{190}$$

x	0	1	2
$f(x)$	$68/95$	$51/190$	$3/190$

Q2-2

The p.f. of X is

$$f(x)=\begin{cases}\dfrac{\binom{3}{x}\binom{17}{2-x}}{\binom{20}{2}} & x=0,1,2\\ 0 & \text{otherwise}\end{cases}$$

The c.d.f. of X is

$$F(x)=\begin{cases}0 & x<0\\ 68/95 & 0\le x<1\\ 187/190 & 1\le x<2\\ 1 & x\ge 2\end{cases}$$

Theorem 5.3.1 The Hypergeometric Distribution has the p.f.

$$f(x|A,B,n)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$$

for $\max\{0,n-B\}\le x\le\min\{n,A\}$, otherwise $f(x|A,B,n)=0$.

Q2-3

Definition 5.5.2 Geometric Distribution A *R.V.* has the geometric distribution with parameter p ($0<p<1$) if X has a discrete distribution for which the p.f. is

$$f(x) = \begin{cases} p(1-p)^x & \text{for } x = 0,1,2,\cdots \\ 0 & \text{otherwise.} \end{cases}$$

Q3-1

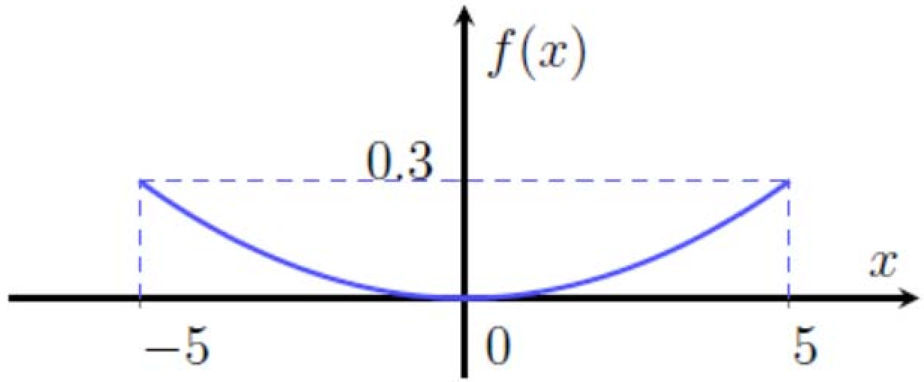
- Suppose that the p.d.f. of a *R.V. X* is as follows:

$$f(x)=\begin{cases} cx^2 & \text{for } -5\le x\le 5 \\ 0 & \text{otherwise.}\end{cases}$$

a. Find the value of the constant c and sketch the p.d.f.

Sol: a. we must have

$$\int_{-\infty}^{\infty} f(x)dx=\int_{-5}^{5} cx^2 dx = 250c/3 = 1.\ \ \text{Therefore, } c=3/250.$$



Q3-2

b. Find the value of $\Pr(X > 4)$.
c. Find the value of $\Pr(X \geq 5)$.
d. Find the value of $\Pr(-6 \leq X \leq 2)$.
e. Find the value of $\Pr(X \leq -5)$.
f. Find the value of $\Pr(X > -6)$.

Sol:

$$\Pr(X > 4) = \int_4^5 f(x)dx = 61/250$$

$$\Pr(X \geq 5) = \int_5^\infty f(x)dx = 0$$

$$\Pr(-6 \leq X \leq 2) = \int_{-6}^2 f(x)dx = \int_{-5}^2 f(x)dx = 1 - \int_2^5 f(x)dx = 133/250$$

$$\Pr(X \leq -5) = \int_{-\infty}^{-5} f(x)dx = 0 \qquad \Pr(X > -6) = \int_{-6}^{\infty} f(x)dx = 1$$

Q4

- An ice cream seller takes 20 gallons of ice cream in her truck each day. Let X stand for the number of gallons that she sells. The probability is 0.1 that $X = 20$. If she doesn't sell all 20 gallons, the distribution of X follows a continuous distribution with a p.d.f. of the form

$$f(x) = \begin{cases} cx, & \text{for } 0 < x < 20 \\ 0, & \text{otherwise.} \end{cases}$$

where c is a constant that makes $\Pr(X < 20) = 0.9$. Find the constant c so that $\Pr(X < 20) = 0.9$ as described above.

Sol: $\Pr(X<20)=\int_0^{20} cx\,dx = \frac{1}{2}cx^2\Big|_0^{20} = 200c = 0.9.$

$\therefore c = 0.0045.$

Q5

- A civil engineer is studying a left-turn lane that is long enough to hold seven cars. Let X be the number of cars in the lane at the end of a red light that is randomly chosen. The engineer believes that the probability of $X=x$ is proportional to $(x+1)(8-x)$ for $x=0,1,\ldots,7$ (the possible value of X).

  a. Find the p.f. of X.

  b. Find the probability that X will be at least 5.

Sol: a. If $x=0,1,\ldots,7$, make $f(x)=c(x+1)(8-x)$. We know that

$\sum_{x=0}^{7} f(x)=1$. So $c=1/[\sum_{x=0}^{7}(x+1)(8-x)]=1/120$.

b. $\Pr(X\geq 5)=\sum_{x=5}^{7}\frac{(x+1)(8-x)}{120}=\frac{1}{3}$.

Q6

- Suppose that a book with *n* pages contains on the average *x* misprints per page. What's the probability that there will be at least *m* pages that each page contains more than *k* misprints?

Sol: Let *Y* denote the number of misprints on a given page. Then *Y* follows Poisson distribution with the parameter *x*.

$$p = \Pr(Y > k) = \sum_{i=k+1}^{\infty} \frac{\exp(-x)x^i}{i!}. \quad 1 - p = \sum_{i=0}^{k} \frac{\exp(-x)x^i}{i!}.$$

Let *Z* denote the number of pages that contain more than *k* misprints among the *n* pages of the book.

$$\Pr(Z \geq m) = \sum_{z=m}^{n} \binom{n}{z} p^z (1-p)^{n-z}$$

Q7

- Suppose that the p.d.f. of a *R.V.* X is as follows:

$$f(x)=\begin{cases}\frac{1}{8}x & \text{for } 0\le x\le 4,\\ 0 & \text{otherwise.}\end{cases}$$

a. Find the value of t such that $\Pr(X\le t)=1/4$.

b. Find the value of t such that $\Pr(X\ge t)=1/2$.

Sol:

a. $$\Pr(X\le t)=\int_0^t f(x)dx=t^2/16=1/4$$

Thus, $t=2$.

b. $$\Pr(X\ge t)=\int_t^4 f(x)dx=1-t^2/16=1/2$$

Thus, $t=\sqrt{8}$.

Q8

- Show that there does not exit any number c such that the following function f(x) would be a p.d.f.:

$$f(x) = \begin{cases} \dfrac{c}{1+x} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Sol: If f(x) is the p.d.f, it must satisfy $\int_0^\infty f(x)dx = 1.$

However,

$$\int_0^\infty f(x)dx = c\int_0^\infty 1/(1+x)dx = c[\ln(1+x)]|_{x=0}^\infty = \infty$$

Thus, there does not exit any number c that satisfies the condition.

Q9

An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the c.d.f. of T, the number of years to maturity for a randomly selected bond as follows

$$F(t)=\begin{cases} 0, t<1 \\ \frac{1}{4}, 1\le t<3 \\ \frac{1}{2}, 3\le t<5 \\ \frac{3}{4}, 5\le t<7 \\ 1, t\ge 7 \end{cases}$$

Sol: X is a discrete $R.V.$

a. $\Pr(T=5)= F(5)-F(5^-)=\frac{3}{4}-\frac{1}{2}=\frac{1}{4}$

b. $\Pr(T>3)=1-F(3)=1-1/2=1/2$

c. $\Pr(1.4<T<6)= F(6^-)-F(1.4)$

$$=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$$

Find: a. $\Pr(T=5)$; b. $\Pr(T>3)$; c. $\Pr(1.4<T<6)$.

Q10

The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous *R.V.* with c.d.f. as follows

$$F(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 1 - e^{-8x} & \text{for } x \ge 0. \end{cases}$$

Find the probability of waiting less than 12 minutes between successive speeders:

a. Using the c.d.f. of *X*; b. Using the p.d.f. of *X*.

Sol: 12 minutes = 0.2 hour, as *X* is continuous, we have

$$\Pr(X < 0.2) = F(0.2) = 1 - e^{-1.6} = 0.7981$$

$$f(x) = F'(x) = \begin{cases} 8e^{-8x} & \text{for } x > 0, \\ 0 & \text{for } x \le 0. \end{cases}$$

$$\Pr(X < 0.2) = 8\int_0^{0.2} e^{-8x}\,dx = -\,e^{-8x}\Big|_0^{0.2} = 0.7981$$