

EX1 Set Operations1. A student selected from a class will be either a boy or a girl. If the probability that a boy will be selected is 0.3, what is the probability that a girl will be selected.



Ex1- solution

◆ **Set Operations1.** A student selected from a class will be either a boy or a girl. If the probability that a boy will be selected is 0.3, what is the probability that a girl will be selected.

Sol: Let B be the event that a boy is selected, and let G be the event that a girl is selected.

We are told that $B \cup G = S$, so $G = B^c$.

Since $\Pr(B) = 0.3$, it follows that $\Pr(G) = 0.7$.



Set Operations2. If A, B, C are three events such that

$$\Pr(A \cup B \cup C) = 0.7$$

What's $\Pr(A^c \cap B^c \cap C^c)$?

☐ A 0.1

☒ B 0.3

☐ C 0.5

☐ D 0.7

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Ex2 - Solution

◆ **Set Operations2.** If A, B, C are three events such that

$$\Pr(A \cup B \cup C) = 0.7$$

What's $\Pr(A^c \cap B^c \cap C^c)$?

Sol: Based on De Morgan's Laws,

$$\Pr(A^c \cap B^c \cap C^c) = \Pr[A \cup B \cup C]^c = 1 - \Pr[A \cup B \cup C] = 0.3.$$



Set Operations3. If A , B , C are three events. Please use set operations to describe following cases:

- (1) A occurs but B and C don't occur.
- (2) None of the events occurs.
- (3) At least one of the events occurs.
- (4) At most one of the events occur.

Ex3-Solution

◆ **Set Operations3.** If A, B, C are three events. Please Using set operations to describe following cases:

- (1) A occurs but B and C don't occur.
- (2) None of the events occurs.
- (3) At least one of the events occurs.
- (4) At most one of the events occur.

Sol: (1) $A \cap B^c \cap C^c$

(2) $A^c \cap B^c \cap C^c$

(3) $A \cup B \cup C$

(4) $(A^c \cap B^c \cap C^c) \cup (A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$



EX4-Set Operations4. If A_1, A_2, A_3 are events such that $\Pr(A_1) = \Pr(A_2) = \Pr(A_3) = 0.3$, $\Pr(A_1 \cap A_3) = \Pr(A_2 \cap A_3) = 0.1$, A_1 and A_2 are mutually exclusive (disjoint). What is the $\Pr(A_1 \cup A_2 \cup A_3)$?

- ☐ A 0.1
- ☐ B 0.3
- ☐ C 0.5
- ☒ D 0.7

Ex4 - Solution

◆ **Set Operations4.** If A_1, A_2, A_3 are three events such that $\Pr(A_1) = \Pr(A_2) = \Pr(A_3) = 0.3$, $\Pr(A_1 \cap A_3) = \Pr(A_2 \cap A_3) = 0.2$, A_1 and A_2 are mutually exclusive (disjoint). What is the $\Pr(A_1 \cup A_2 \cup A_3)$?

Sol: Based on theorem 1.10.1, we have

$$\begin{aligned}\Pr(A_1 \cup A_2 \cup A_3) &= \Pr(A_1) + \Pr(A_2) + \Pr(A_3) \\ &\quad - [\Pr(A_1 \cap A_2) + \Pr(A_2 \cap A_3) + \Pr(A_1 \cap A_3)] \\ &\quad + \Pr(A_1 \cap A_2 \cap A_3).\end{aligned}$$

Because A_1 and A_2 are disjoint, we have $\Pr(A_1 \cap A_2) = 0$, and $(A_1 \cap A_2 \cap A_3) \subset (A_1 \cap A_2)$, thus $\Pr(A_1 \cap A_2 \cap A_3) \leq \Pr(A_1 \cap A_2)$, $\Pr(A_1 \cap A_2 \cap A_3) = 0$.

$$\Pr(A_1 \cup A_2 \cup A_3) = 0.3 + 0.3 + 0.3 - (0.1 + 0.1) = 0.7$$



EX5 A box contains 24 light bulbs of which four are defective. If one person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs, what is the probability that all four defective bulbs will be obtained by the same person?

☒ A $(C_{20,6} + C_{20,10}) / C_{24,10}$

☐ B $(P_{20,6} + P_{20,10}) / P_{24,10}$

☐ C $(C_{20,6} C_{20,10}) / C_{24,10}$

☐ D $(P_{20,6} P_{20,10}) / P_{24,10}$

提交



Ex5-Solution

◆ A box contains 24 light bulbs of which four are defective. If one person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs, what is the probability that all four defective bulbs will be obtained by the same person?

Sol:

$$p = \frac{\binom{20}{6} + \binom{20}{10}}{\binom{24}{10}} = 0.1140$$



EX6- Suppose 100 students are divided into 5 groups, each containing 20 students. Awards are to be given to 10 of these students. If each student is equally likely to receive an award, what is the prob. that exactly two students in each group will receive awards.



Ex6-Solution

◆ Suppose 100 students are divided into 5 groups, each containing 20 students. Awards are to be given to 10 of these students. If each student is equally likely to receive an award, what is the prob. that exactly two students in each group will receive awards.

Sol:

$$p = \frac{\binom{20}{2}^5}{\binom{100}{10}} = 0.0143$$



EX7- Playing Cards. A deck of 52 cards containing 4 aces is shuffled thoroughly and the cards are then distributed among 4 players so that each player receives 13 cards.

Determine the prob. that each player will receive 1 ace.

Ex7(Book Ex 1.8.9&1.8.10)

◆ **Playing Cards.** A deck of 52 cards containing 4 aces is shuffled thoroughly and the cards are then distributed among 4 players so that each player receives 13 cards. Determine the prob. that each player will receive 1 ace.

Sol 1:

$$p = \frac{13^4 \times 4!}{P_{52,4}} = 0.1055$$

Sol 2:

$$p = \frac{13^4}{C_{52,4}} = 0.1055$$

If we count the same elements in different orders as different outcomes when counting the outcomes in S , we must do the same when counting the elements of E .

