

## Random Mathematics Homework #1 Fall 2020

Instructor: Dr. Jing Liang

Assigned Date: Sept.10, 2020 Due Date: Sept.17, 2020

1. Consider two events  $A$  and  $B$  such that  $\Pr(A)=1/2$  and  $\Pr(B)=1/4$ . Determine the value of  $A^c \cap B$  for each of the following conditions:
  - (a)  $A$  and  $B$  are disjoint;
  - (b)  $A \supset B$ ;
  - (c)  $\Pr(A \cap B) = 1/5$ .
2. If three dice are rolled, what is the probability that each of the three numbers that appear will be different?
3. Prove:
  - (a) De Morgan's laws (Theorem 1.4.9);
  - (b) The distributive laws (Theorem 1.4.10).
4. If 6 different balls are thrown at random into 10 different boxes, what is the probability that no box will receive more than one ball?
5. A restaurant has  $n$  items on its menu. During a particular day,  $k$  customers will arrive and each one will choose one item. The manager wants to count how many different collections of customer choices are possible without regard to the order in which the choices are made. (For example, if  $k = 3$  and  $a_1, \dots, a_n$  are the menu items, then  $a_1 a_3 a_1$  is not distinguished from  $a_1 a_1 a_3$ .) Prove that the number of different collections of customer choices is  $C_{n+k-1, k}$ . *Hint:* Assume that the menu items are  $a_1, \dots, a_n$ . Show that each collection of customer choices, arranged with the  $a_1$ 's first, the  $a_2$ 's second, etc., can be identified with a sequence of  $k$  zeros and  $n - 1$  ones, where each 0 stands for a customer choice and each 1 indicates a point in the sequence where the menu item number increases by 1. For example, if  $k = 3$  and  $n = 5$ , then  $a_1 a_1 a_3$  becomes 0011011.
6. If  $n$  people are seated in a random manner in a row containing  $2n$  seats, what is the probability that no two people will occupy adjacent seats?
7. Suppose that 20 books will be randomly packed into three boxes. Suppose the first box will contain 10 books, the second box will contain 6 books, the remaining box will contain 4 books. What's the probability that two particular books  $A$  and  $B$  will be in the same box?
8. A deck of 52 cards contains four aces. If the cards are shuffled and distributed in a random manner to four players so that each player receives 13 cards, what is the probability that all four aces will be received by the same player?

9. Suppose that two boys named Davis, three boys named Jones, and four boys named Smith are seated at random in a row containing nine seats. What's the probability that the Davis boys will occupy the first two seats in a row, the Jones boys will occupy the next three seats, and the Smith boys will occupy the last four seats?
10. Let  $A_1$ ,  $A_2$  and  $A_3$  be three arbitrary events. Show that the probability that exactly one of these three events will occur is
- $$\Pr(A_1) + \Pr(A_2) + \Pr(A_3) - 2\Pr(A_1 \cap A_2) - 2\Pr(A_1 \cap A_3) - 2\Pr(A_2 \cap A_3) + 3\Pr(A_1 \cap A_2 \cap A_3).$$