

6.3 A

$$4.(3) \text{ 令 } x = \rho \cos \theta, y = \rho \sin \theta, \iiint_V \frac{e^z}{\sqrt{x^2+y^2}} dV = \iiint_V \frac{1}{\rho} e^z \rho d\rho d\theta dz$$

$$\therefore \text{原积分} = \iiint_V e^z d\rho d\theta dz = \int_0^{2\pi} d\theta \int_0^2 d\rho \int_1^2 e^z dz$$

$$= \int_0^{2\pi} d\theta \cdot \int_1^2 z e^z dz = 2\pi e^2$$

(10) (V): $x^2 + y^2 + z^2 \leq R^2$ 与 $x^2 + y^2 + (z-R)^2 \leq R^2$ 的公共区域.

$$\text{令 } x = \rho \cos \theta, y = \rho \sin \theta, \text{ 则区域为 } \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq \rho \leq \sqrt{R^2 - z^2} \text{ 和 } 0 \leq \rho \leq \sqrt{2Rz - z^2} \\ \frac{1}{2}R \leq z \leq R \end{cases}$$

$$\therefore \iiint_V z^2 dV = \int_0^{2\pi} d\theta \int_0^{\sqrt{R^2 - z^2}} \rho d\rho \int_{\frac{1}{2}R}^R z^2 dz + \int_0^{2\pi} d\theta \int_0^{\sqrt{2Rz - z^2}} \rho d\rho \int_{\frac{1}{2}R}^R z^2 dz$$

$$= (\frac{1}{15} - \frac{1}{192}) R^5$$

(11) 设 $x = r \sin \varphi \cos \theta, y = r \sin \varphi \sin \theta, z = r \cos \varphi$

$$\therefore \iiint r^5 \sin^3 \varphi \cos \varphi \sin \theta \cos \theta d\theta d\varphi dr.$$

$$= \int_0^1 r^5 dr \cdot \int_0^{\frac{\pi}{2}} \sin^3 \varphi \cos \varphi d\varphi \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta$$

$$= \frac{1}{6} \cdot \int_0^1 \sin^3 \varphi d(\sin \varphi) \cdot \int_0^1 \sin \theta d(\sin \theta)$$

$$= \frac{1}{6} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{48}$$

5.(2) 积分域: $-3 \leq x \leq 3$

$$\Omega: \begin{cases} -\sqrt{9-x^2} \leq y \leq \sqrt{9-x^2} \\ 0 \leq z \leq \sqrt{9-x^2-y^2} \end{cases} \Rightarrow z = \sqrt{9-x^2-y^2} \text{ 与 } xOy \text{ 平面所围}$$

设 ~~$x = \rho \cos \theta, y = \rho \sin \theta$~~ $x = \rho \sin \varphi \cos \theta, y = \rho \sin \varphi \sin \theta, z = \rho \cos \varphi$

$$\iiint_{\Omega} z \sqrt{x^2 + y^2 + z^2} d\sigma = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{2}} \sin \varphi \cos \varphi d\varphi \int_0^3 \rho^4 d\rho$$

$$= \frac{243}{5} \pi.$$