Discussion problem assignment:

1. For an LTI system with system function $H(s)=\dfrac{(s+2)}{(s+1)(s+3)}$ **and another system** $g(t)=e^{2t}h(t)$

It is known that the system $g(t)$ **is stable. For the system** $H(s)$**, determine** $h(t)$ **and whether the system is causal or stable.**

解答：

表达式已知，

(1). H(s) has two poles at s = -1 and s = -3, so three possible ROC's.

(2). Find the LT and ROC for $g(t)$ **using LT property as**

$$G(s)=H(s-2)=\frac{s-2+2}{(s-2+1)(s-2+3)}=\frac{s}{(s-1)(s+1)}$$

As the two poles are shifted to s = -1 and +1. The only possible ROC for $g(t)$ **to be stable is** $-1<\mathrm{Re}\{s\}<+1$

(3). $$H(s)=\frac{(s+2)}{(s+1)(s+3)},-3<\mathrm{Re}\{s\}<-1$$

Thus the system is not causal and not stable.

(4). To use inverse LT

$$H(s)=\frac{(s+2)}{(s+1)(s+3)}=\frac{1/2}{(s+1)}+\frac{1/2}{(s+3)},-3<\mathrm{Re}\{s\}<-1$$

$$h(t)=\frac{1}{2}e^{-3t}u(t)-\frac{1}{2}e^{-t}u(-t)$$

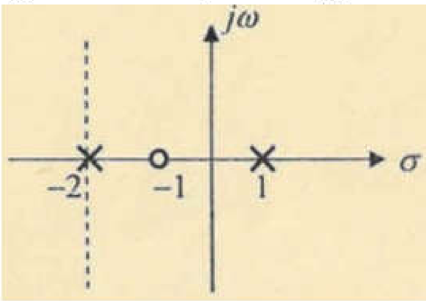
$$\frac{1}{s-\lambda_i}\leftrightarrow\begin{cases}e^{\lambda_i t}u(t), & \text{right ROC}, \quad \text{right-sided signal}\\ -e^{\lambda_i t}u(-t), & \text{left ROC}, \quad \text{left-sided signal}\end{cases}$$

第二题：

2. Consider a causal LTI system with H(s). Its pole-zero plot is given in the figure and $\lim_{t\to 0^+} h(t) = 2$

(a) Determine the system function H(s).

(b) Determine the unit impulse response h(t). Is the system stable?



解答：

Solution:

(1). From the pole-zero plot $H(s) = \dfrac{A(s+1)}{(s+2)(s-1)}$

(2). From the two poles plus causal system, the ROC is thus

$$\mathrm{Re}\{s\} > +1$$

(3). From the initial value theorem,

$$\lim_{t\to 0^+} h(t) = h(0^+) = \lim_{s\to\infty} sH(s) = \lim_{s\to\infty} \frac{As(s+1)}{(s+2)(s-1)} = A = 2$$

$$H(s) = \frac{2(s+1)}{(s+2)(s-1)}, \mathrm{Re}\{s\} > +1$$

(4). The system is not stable. Plus, from inverse LT,

$$H(s) = \frac{2(s+1)}{(s+2)(s-1)} = \frac{2/3}{(s+2)} + \frac{4/3}{(s-1)}, \mathrm{Re}\{s\} > +1$$

$$h(t) = \frac{2}{3}e^{-2t}u(t) + \frac{4}{3}e^{t}u(t)$$