Chapter 2
Conditional Probability
Exercises

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单选题 20分

设置

EX1-a. A box contains 5 coins. $\Pr(H|C_i)$: the probability of a head when the ith coin is tossed.

$\Pr(H|C_1) = 0,\ \Pr(H|C_2) = 1/4,\ \Pr(H|C_3) = 1/2,$
$\Pr(H|C_4) = 3/4,\ \ \Pr(H|C_5) = 1.$

Suppose that one coin is selected at random from the box and it is tossed once, a head is obtained. What is the posterior probability that the 3rd coin was selected.

A 0.1

C 0.2

B 0.3

D 0.4

提交

Ex1 (Book Section 2.3 Exercise 7)

A box contains 5 coins.

$P(H|C_i)$: the probability of a head when the ith coin is tossed

$P(H|C_1) = 0$,$P(H|C_2) = 1/4$,$P(H|C_3) = 1/2$,

$P(H|C_4) = 3/4$,$P(H|C_5) = 1$.

a. Suppose that one coin is selected at random from the box and when it is tossed once, a head is obtained. What is the posterior probability that the i th coin was selected ($i = 1, \ldots, 5$).

Sol:

a: Using Bayes' Theorem. Since coin is selected at random, the prior probability of each coin $\Pr(C_i) = 1/5$.

$$\Pr(C_i|H) = \frac{\Pr(C_i)\Pr(H|C_i)}{\sum_{j=1}^{5} \Pr(C_i)\Pr(H|C_i)}$$

$\Pr(C_1|H) = 0$, $\Pr(C_2|H) = 0.1$, $\Pr(C_3|H) = 0.2$,

$\Pr(C_4|H) = 0.3$,$\Pr(C_4|H) = 0.4$.

单选题 20分

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EX1-b. A box contains 5 coins. $\Pr(H|C_i)$: the probability of a head when the ith coin is tossed.

$\Pr(H|C_1) = 0$, $\Pr(H|C_2) = 1/4$, $\Pr(H|C_3) = 1/2$, $\Pr(H|C_4) = 3/4$, $\Pr(H|C_5) = 1$.

Suppose that one coin is selected at random from the box and it is tossed once, a head is obtained. If the same coin were tossed again, what would be the probability of obtaining another head?

A 1/4

C 3/4

B 1/2

D 1

提交

Ex1 (Book Section 2.3 Exercise 7)

A box contains 5 coins.

$P(H|C_i)$: the probability of a head when the ith coin is tossed

$P(H|C_1) = 0$,$P(H|C_2) = 1/4$,$P(H|C_3) = 1/2$,

$P(H|C_4) = 3/4$,$P(H|C_5) = 1$.

b. If the same coin were tossed again, what would be the probability of obtaining another head?

Sol: B_i: the first time a head was obtained, i th coin was selected.

$P(B_i) = P(C_i|H)$.

H_2: the second time a head was obtained

Since tossing a coin is an independent event, $P(H_2|B_i) = P(H|C_i)$

b: $P(H_2) = \sum_{i=1}^{5} P(B_i)P(H_2|B_i) = \sum_{i=1}^{5} P(C_i|H)P(H|C_i) = 3/4$

Computation of Probabilities in More Than One Stage

单选题 20分

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EX2. A box contains 2 coins. One is fair, the other with a head on each side. One coin is selected randomly and is tossed, a head is obtained. Then, the same coin is tossed and a head is obtained again. What's the prob. that this coin is fair?

- A 1/2
- B 1/3
- C 1/4
- D 1/5

提交

Ex2 (Book Section 2.3 Exercise 7)

A box contains 2 coins. One is fair, the other with a head on each side. One coin is selected randomly and is tossed, a head is obtained. Then, the same coin is tossed and a head is obtained again. What's the prob. that this coin is fair?

Sol: Let B_1 be the event that the coin is fair, and B_2 be the event that the coin has two heads. Let H_i be the event that a head is obtained after tossing the coin ith time, i=1,2. We want

$$\Pr(B_1|H_1 \cap H_2) = \frac{\Pr(B_1)\Pr(H_1 \cap H_2|B_1)}{\Pr(B_1)\Pr(H_1 \cap H_2|B_1) + \Pr(B_2)\Pr(H_1 \cap H_2|B_2)}$$

$$= \frac{\Pr(B_1)\Pr(H_1|B_1)\Pr(H_2|B_1)}{\Pr(B_1)\Pr(H_1|B_1)\Pr(H_2|B_1) + \Pr(B_2)\Pr(H_1|B_2)\Pr(H_2|B_2)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times 1\times 1} = \frac{1}{5}$$

Computation of Posterior Probabilities in More Than One Stage

Ex2 (Book Section 2.3 Exercise 7)

Conditional version of Bayes' Theorem:

$$\Pr(B_i \mid A \cap C) = \frac{\Pr(B_i \mid C)\Pr(A \mid B_i \cap C)}{\sum_{j=1}^{k} \Pr(B_j \mid C)\Pr(A \mid B_j \cap C)}.$$

Sol2: Let B_1 be the event that the coin is fair, and B_2 be the event that the coin has two heads. Let H_i be the event that a head is obtained after tossing the coin ith time, i=1,2.

$$\Pr(B_1|H_1 \cap H_2) = \frac{\Pr(B_1|H_1)\Pr(H_2|B_1 \cap H_1)}{\Pr(B_1|H_1)\Pr(H_2|B_1 \cap H_1) + \Pr(B_2|H_1)\Pr(H_2|B_2 \cap H_1)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times 1}$$

$$= \frac{1}{5}$$

主观题 10分

设置

EX3. Three cards are drawn in succession without replacement from a thoroughly shuffled the deck of 52 cards. Find the prob. that the first card is a red ace, the second card is a 10 or a jack, and the third card is greater than 3 but less than 7.

作答

Ex3

Three cards are drawn in succession without replacement from a thoroughly shuffled the deck of 52 cards. Find the prob. that the first card is a red ace, the second card is a 10 or a jack, and the third card is greater than 3 but less than 7.

Sol:

Let A_1 be the event that the first card is a red ace.

Let A_2 be the event that second card is a 10 or a jack.

Let A_3 be the event the third card is greater than 3 but less than 7.

We will determine

$$\Pr(A_1 \cap A_2 \cap A_3) = \Pr(A_1)\Pr(A_2|A_1)\Pr(A_3|A_1 \cap A_2)$$

$$= \left(\frac{2}{52}\right)\left(\frac{8}{51}\right)\left(\frac{12}{50}\right)$$

$$= \frac{8}{5525}$$

单选题 20分

设置

You know that a certain letter is equally like to be in any one of three different folders. Let α_i be the prob. that you will find your letter upon making a quick examination of folder i of the letters, i=1,2,3. Suppose you look in folder 1 and do not find the letter. 1)What's the prob. that the letter is in folder 1? 2)What's the prob. that the letter is in folder 3?

A $\frac{1-\alpha_1}{3-\alpha_1},\frac{1}{3-\alpha_1}$

C $\frac{\alpha_1}{3-\alpha_1},\frac{1-\alpha_1}{3-\alpha_1}$

B $\frac{1}{3-\alpha_1},\frac{1-\alpha_1}{3-\alpha_1}$

D $\frac{1-\alpha_1}{3-\alpha_1},\frac{\alpha_1}{3-\alpha_1}$

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Ex4

You know that a certain letter is equally like to be in any one of three different folders. Let α_i be the prob. that you will find your letter upon making a quick examination of folder i of the letters, i=1,2,3. Suppose you look in folder 1 and do not find the letter.

1)What's the prob. that the letter is in folder 1?

Sol: Let F_i, i=1,2,3 be the event that the letter is in folder i. Let E be the event that a search of folder 1 does not come up with the letter. We desire $\Pr(F_1|E)$.

$$\Pr(F_1 \mid E) = \frac{\Pr(F_1)\Pr(E \mid F_1)}{\sum_{j=1}^{3}\Pr(F_j)\Pr(E \mid F_j)}$$

$$= \frac{(1-\alpha_1)\frac{1}{3}}{(1-\alpha_1)\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \frac{1-\alpha_1}{3-\alpha_1}$$

2)What's the prob. that the letter is in folder 3?

$$\Pr(F_3 \mid E) = \frac{\Pr(F_3)\Pr(E \mid F_3)}{\sum_{j=1}^{3}\Pr(F_j)\Pr(E \mid F_j)} = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times(1-\alpha_1)+\frac{1}{3}\times 1+\frac{1}{3}\times 1} = \frac{1}{3-\alpha_1}$$

投票 最多可选1项

设置

How about the complexity of our homework?

A Very difficult

B Some questions are easy, but most of them are difficult

C Some questions are difficult, but most of them are easy

D Very easy

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设置

How about the amount of our homework?

A Too much

B A little bit more than my expectation

C Moderate

D Not enough

提交