Discussion problem assignment:

第一题：

1. From Example 4.2, we know that $x(t)=e^{-a|t|}, a>0 \xleftrightarrow{\text{FT}} X(j\omega)=\dfrac{2a}{a^2+\omega^2}$. Write the signal $x(t)$ as the sum of one right-sided signal and one left-sided signal. Find the Fourier transform of the two signals and confirm the FT pair of Example 4.2.

2. Find the Fourier transform of $e^{-a|t|}\operatorname{sgn}(t), a>0$ with $\operatorname{sgn}(t)=\begin{cases}+1, & \text{if } t>0\\ -1, & \text{if } t<0\end{cases}$

Solution:

1. $x(t)=e^{-a|t|}=e^{-at}u(t)+e^{at}u(-t)$

Use Example 4.1, $e^{-at}u(t), a>0 \xleftrightarrow{\text{FT}} \dfrac{1}{a+j\omega}$

Then use time reversal property, $e^{at}u(-t), a>0 \xleftrightarrow{\text{FT}} \dfrac{1}{a-j\omega}$

So, $x(t)=e^{-a|t|}, a>0 \xleftrightarrow{\text{FT}} X(j\omega)=\dfrac{1}{a+j\omega}+\dfrac{1}{a-j\omega}=\dfrac{2a}{a^2+\omega^2}$

2. $e^{-a|t|}\operatorname{sgn}(t), a>0 = e^{-at}u(t)-e^{at}u(-t) \xleftrightarrow{\text{FT}} \dfrac{1}{a+j\omega}-\dfrac{1}{a-j\omega}=\dfrac{-2j\omega}{a^2+\omega^2}$

第二题：

Question: assume that $f(t) \xleftrightarrow{\text{FT}} F(j\omega)$

Define the n-th order moment $m_n=\int_{-\infty}^{+\infty} t^n f(t)dt$

Prove that $(-j)^n m_n = \left.\dfrac{d^n F(j\omega)}{d\omega^n}\right|_{\omega=0}$

答案：

$$F(j\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

$$\frac{dF(j\omega)}{d\omega}=\frac{d}{d\omega}\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt=\int_{-\infty}^{+\infty}\frac{d}{d\omega}(f(t)e^{-j\omega t})dt$$

$$=\int_{-\infty}^{+\infty} f(t)\frac{d}{d\omega}(e^{-j\omega t})dt=\int_{-\infty}^{+\infty}(-jt)f(t)e^{-j\omega t}dt$$

$$\frac{d^n F(j\omega)}{d\omega^n}=\int_{-\infty}^{+\infty}(-jt)^n f(t)e^{-j\omega t}dt=(-j)^n\int_{-\infty}^{+\infty} t^n f(t)e^{-j\omega t}dt$$

然后，代入 w=0 可得。