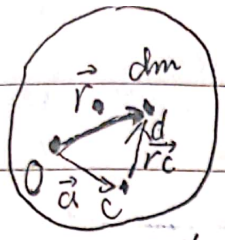


C为质心

$I_c = \int_m r_0^2 \, dm$

$\vec{r} = \vec{a} + \vec{r}_c$

$\int \vec{r}_c \, dm$

$= \int x\,dm \cdot \vec{i} + \int y\,dm \cdot \vec{j} + \int z\,dm \cdot \vec{k}$

$= x_c m \vec{i} + y_c m \cdot \vec{j} + z_c m \cdot \vec{k}$

质心坐标

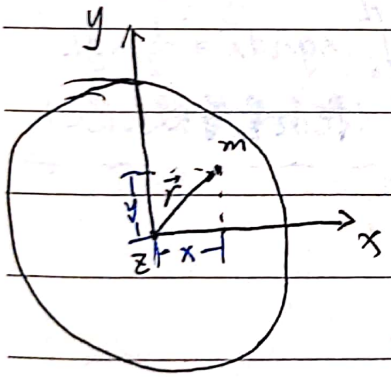
当以质心为原点时 $\int \vec{r}_c \, dm = 0$

$I_o = \int r^2 dm = \int \vec{r} \cdot \vec{r} \, dm$

$= \int (|\vec{a}|^2 + 2\vec{a} \cdot \vec{r}_c + |\vec{r}_c|^2) dm$

$= a^2 \int dm + 0 + \int r_c^2 \, dm$

$= ma^2 + I_c$



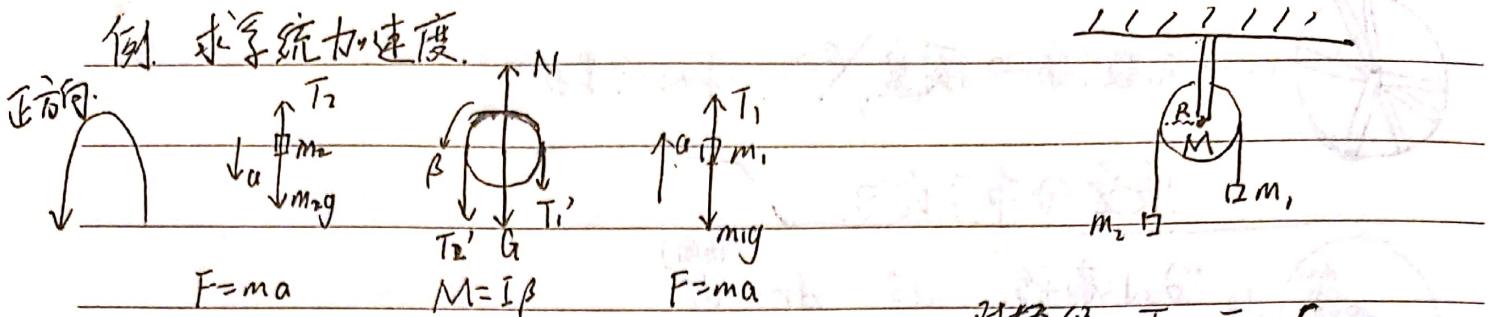
$I_z = \int r^2 dm = \int (x^2 + y^2) dm$

$= \int x^2 dm + \int y^2 dm$

$= I_y + I_x$

垂直轴定理

例. 求系统加速度.



$F=ma$ $\quad M=I\beta$ $\quad F=ma$

对轴线：$T_2-T_1-f=0a$

$m_2g-T_2=m_2a$ ①

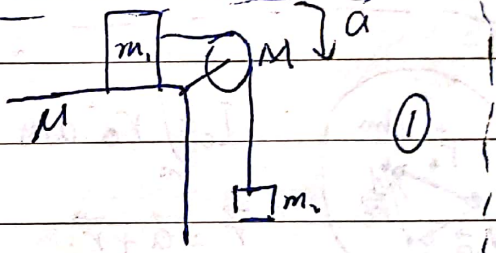
$m_1g-T_1=-m_1a$ ③

$(T_2-T_1)R=(\frac{1}{2}MR^2)\cdot\frac{a}{R}$ ②

①+②−③ $\Rightarrow$ $a=\dfrac{m_2-m_1}{m_1+m_2+\frac{1}{2}M}g$

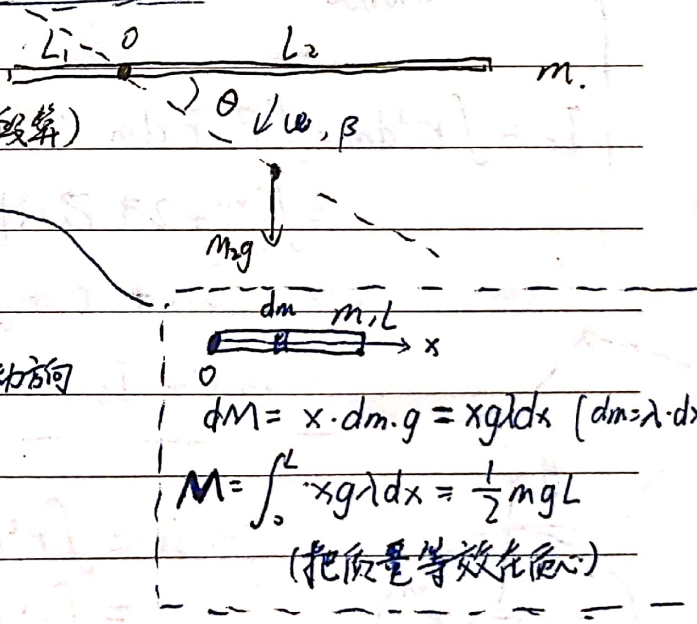
$$a=\frac{\text{动力}}{\text{惯性体现}}$$

类似情况



① $a=\dfrac{m_2g-\mu m_1g}{m_1+m_2+\frac{1}{2}M}$

例1 如图，求系统在竖直位置的角速度



$I=\frac{1}{3}m_2L_2^2+\frac{1}{3}m_1L_1^2$ $\quad (m=m_1+m_2)$(分段算)

$M=\frac{1}{2}m_2gL_2\cos\theta-\frac{1}{2}m_1gL_1\cos\theta$

$M=I\beta=I\dfrac{d\omega}{d\theta}\dfrac{d\theta}{dt}=I\omega\dfrac{d\omega}{d\theta}$ → 保证转动方向

$M\,d\theta=I\omega\,d\omega$

$\int_0^\theta M\,d\theta=\int_0^\omega I\omega\,d\omega$

$\frac{1}{2}\omega^2=\dfrac{(m_2L_2-m_1L_1)g}{2I}\sin\theta$

$\frac{1}{2}I\omega^2=\frac{1}{2}(m_2L_2-m_1L_1)g\sin\theta$

势能变化

$dM=x\cdot dm\cdot g=xg\lambda dx$ $\quad (dm=\lambda\cdot dx)$

$M=\int_0^L xg\lambda\,dx=\frac{1}{2}mgL$

(把质量等效在质心)

例4. 电风扇开启时经 t_1 时间达到转速 ω_0，关闭时经 t_2 停止. 设风扇转动惯量为 I，且电磁力矩与摩擦力矩都为恒量.

求：电磁力矩

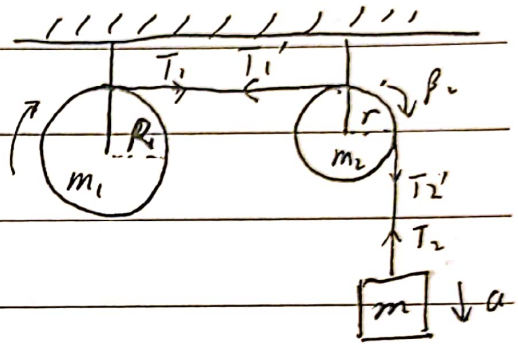
加速：$M - M_f = I\beta_1$ ① $\quad 0 \xrightarrow{t_1} \omega_0 \quad \beta_1 = \frac{\omega_0}{t_1}$

减速：$-M_f = I\beta_2$ ② $\quad \omega_0 \xrightarrow{t_2} 0 \quad \beta_2 = -\frac{\omega_0}{t_2}$

① - ② 得 $M = I\omega_0\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$

例6. $m_1 = 24kg$，$m_2 = 5kg$，$m = 10kg$

求当 m 下落 $h = 0.5m$ 时，物体的速度及张力：



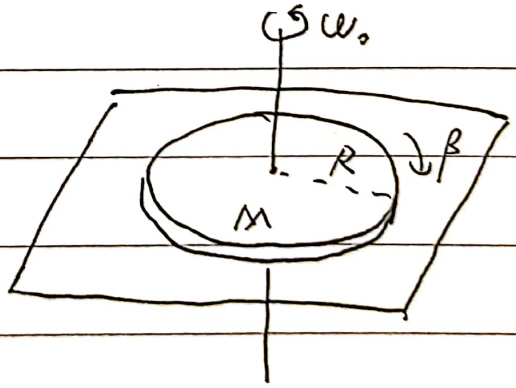
$$a = \frac{mg}{\frac{1}{2}m_1 + \frac{1}{2}m_2 + m}$$

由 $v^2 - v_0^2 = 2as$ $\quad v = \sqrt{\frac{2mgh}{\frac{1}{2}m_1 + \frac{1}{2}m_2 + m}}$

例7. 圆盘 m，R，初速度 ω_0，摩擦（盘与桌面）系数 μ.

求：圆盘经多少时间，转几圈停下来？

分成圆环



$dG = g\sigma\, ds = g\sigma\, 2\pi r\, dr$

$dM_f = -\mu\, dG\, r = -\mu g\sigma\, 2\pi r\, dr \cdot r$

$M_f = -\int_0^R \mu g\sigma\, 2\pi r^2\, dr$

$= -\frac{2}{3}\mu mg R$ → 小扇形质心在 $\frac{2}{3}R$ 处

$M_f = I\beta \Rightarrow \beta = -\frac{4\mu g}{3R}$

由 $\omega_0 + \beta t = \omega = 0$

$$t = \frac{3\omega_0 R}{4\mu g}$$