习题六

1. C 2. C 3. B

4. -0.5

5. (1) 在A点：$\begin{cases}\cos\omega t_1 = 1 \\ \sin\omega t_1 = 0\end{cases} \Rightarrow \omega t_1 = 0$

$\vec{v}_A = \frac{d\vec{r}}{dt}\Big|_{\omega t_1=0} = -a\omega\sin\omega t\,\vec{i} + b\omega\cos\omega t\,\vec{j}\Big|_{\omega t_1=0} = b\omega\vec{j}$

$\therefore E_{kA} = \frac{1}{2}m\vec{v}_A^2 = \frac{1}{2}mb^2\omega^2$

在B点：$\begin{cases}\cos\omega t_2 = 0 \\ \sin\omega t_2 = 1\end{cases} \Rightarrow \omega t_2 = \frac{\pi}{2}$

$\vec{v}_B = \frac{d\vec{r}}{dt}\Big|_{\omega t_2=\frac{\pi}{2}} = -a\omega\vec{i}$

$E_{kB} = \frac{1}{2}m\vec{v}_B^2 = \frac{1}{2}ma^2\omega^2$

(2) $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = -a\omega^2\cos\omega t\,\vec{i} - b\omega^2\sin\omega t\,\vec{j}$

$\vec{F} = m\vec{a} = -a\omega^2 m\cos\omega t\,\vec{i} + (-b\omega^2 m\sin\omega t\,\vec{j})$

$\vec{\Delta x} = \vec{AB} = (-a, b)$ $\vec{F}_x = (-a\omega^2 m\cos\omega t, 0)$

$\vec{F}_y = (0, -b\omega^2 m\sin\omega t)$

$\cancel{W_x = \vec{F}_x\cdot\vec{\Delta x} = a^2\omega^2\cos\omega t}$

$\cancel{W_y = \vec{F}_y\cdot\vec{\Delta x} = -b^2\omega^2\sin\omega}$

$W_x = \int_a^0 F_x\,dx = \int_0^a \omega^2 m x\,dx = \frac{1}{2}\omega^2 ma^2$

$W_y = \int_0^b F_y\,dy = \int_b^0 \omega^2 m y\,dy = -\frac{1}{2}\omega^2 mb^2$

6. (1) ~~设速度相等时瞬时速度的大小为~~v_1 $m_1v_1 + m_2v_2 = m_2v_0$

$\cancel{W_T = \frac{1}{2}kx_0^2 \quad W_1 = \frac{1}{2}(m_1+m_2)v_1^2 - 0 \Rightarrow v_1 = \frac{1}{2}\sqrt{\frac{kx_0}{m}}}$

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}kx^2 = \frac{1}{2}m_2v_0^2$，$v_1 = v_2$ 解得 $v_1 = v_2 = \frac{3}{4}v_0$

由 $\frac{1}{2}kx_0^2 = \frac{1}{2}\cdot 2mv_0^2$，$v_1 = v_2 = \frac{3}{4}x_0\sqrt{\frac{k}{3m}}$

(2) 由机械能守恒，$x = \frac{1}{2}x_0$

习题七.

1. C　　2. B　　3. 18J ; 6m/s

4. $\sqrt{2gl-\frac{k(l-l_0)^2}{m}}$

5. (1) $\int_0^W dW_f = -\int_0^{l-a} \mu \frac{l-a-x}{l} mg \, dx$

$W_f = -\frac{\mu mg(l-a)^2}{2l}$

(2) $W_G = \frac{a}{l} mg(l-a) + \frac{l-a}{l} mg \cdot \frac{l-a}{2} = \frac{2al+l^2-3a^2}{2l} mg$

$W_G + W_f = \frac{1}{2}mv^2 - 0$

$v = \sqrt{\frac{(2al+l^2-3a^2)g - \mu g(l-a)^2}{l}}$

6. $N = m\frac{v^2}{R}$, $f = -\mu N = ma$. $\therefore -\mu m\frac{v^2}{R} = m\frac{dv}{dt}$

$-\mu v^2 = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} R = \frac{dv}{d\theta} \cdot v$　　$\therefore -\mu v = \frac{dv}{d\theta}$

$\therefore \int_{v_0}^{v} \frac{1}{v} dv = \int_0^{\pi} -\mu d\theta$　解得 $v = v_0 e^{-\mu\pi}$

$\therefore$ 由 $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2(e^{-2\mu\pi} - 1)$

习题八

1. A　　2. A　　3. $\sqrt{2}mv$　西偏南45°

4. $\frac{F\Delta t_1}{m_1+m_2}$　　$\frac{F\Delta t_1}{m_1+m_2}+\frac{F\Delta t_2}{m_2}$

5. $\vec{I}=m(\vec{v}_B-\vec{v}_A)$　$|\vec{I}|=|m(\vec{v}_B-\vec{v}_A)|=20\sqrt{2+\sqrt{2}}\times 0.02\,N\cdot S$

$|\vec{I}|=0.4\sqrt{2+\sqrt{2}}\,N\cdot S$，方向指向左下，与x轴成θ角

其中 $\tan\theta=\frac{\sqrt{2}}{2+\sqrt{2}}$

6. 水平方向动量守恒　$Mv_0=(M+m)v$ ①

由能量守恒　$-\mu mgL_{min}=\frac{1}{2}(M+m)v^2-\frac{1}{2}Mv_0^2$ ②

①②联立解得　$L_{min}=\frac{Mv_0^2}{2(M+m)\mu g}$

习题九

1. C　　2. C

3. (1) $3\times10^{-3}s$　(2) $0.6\,N\cdot S$　(3) $2\times10^{-3}kg$

4. (1) mv_0　(2) 竖直向下

5. 木块下滑时，$Mgl\sin\theta=\frac{1}{2}Mv_0^2$，$v_0=\sqrt{2gl\sin\theta}$，沿斜面向下

子弹射中木块过程中，在沿斜面方向动量守恒

设沿斜面向下方向为正

$$Mv_0-mv\cos\theta=(M+m)v_1$$

$$v_1=\frac{M\sqrt{2gl\sin\theta}-mv\cos\theta}{M+m}$$

6. (1) M与m组成的系统在水平方向动量守恒

设水平向右方向为正

$mv_1-MV=0$ ①　$mgR=\frac{1}{2}mv_1^2+\frac{1}{2}MV^2$ ②

解得 $\begin{cases}V=\sqrt{\frac{2m^2gR}{M^2+Mm}}\\ v_1=\sqrt{\frac{2M^2gR}{M^2+Mm}}\end{cases}$

$\cancel{v_0}$ $v = v_1 + V = (M+m)\sqrt{\frac{2gR}{M^2+Mm}}$

(2) $N - mg = m\frac{v^2}{R}$

$N = mg + \frac{m(M+m)^2 2g}{M^2+Mm}$