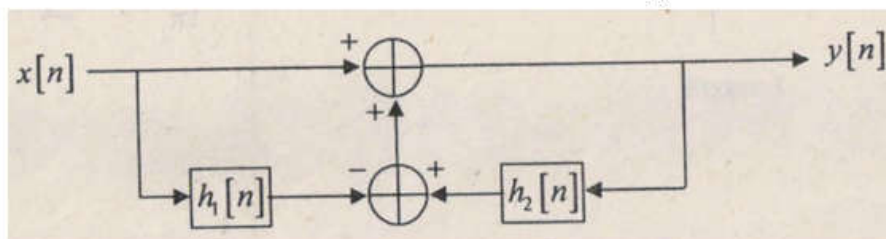


Discussion problem assignment:

第一题:

1. Consider a causal discrete-time system shown in the figure with



$$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$h_2[n] = \frac{1}{2} \delta[n-1]$$

(a) Find the system function  $H(z)$  and its ROC

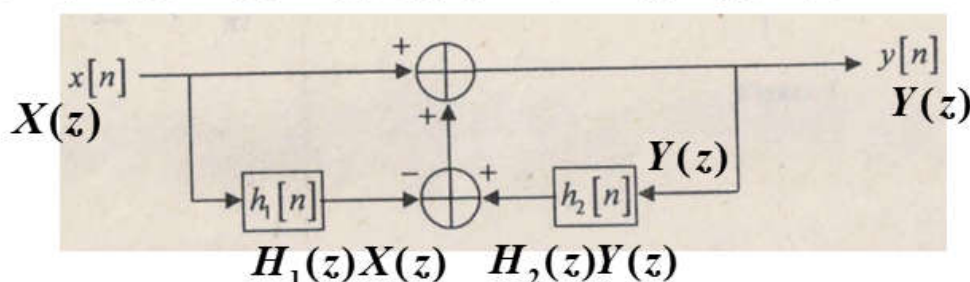
(b) Determine the unit impulse response  $h[n]$ .

(c) Compute the output of this system if the input signal is

$$x[n] = \cos(\pi n)$$

解答:

(a) 确定系统函数，需要确定各子系统的输入输出关系



$$Y(z) = X(z) - H_1(z)X(z) + H_2(z)Y(z)$$

$$H(z) = \frac{1 - H_1(z)}{1 - H_2(z)} \quad H_1(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, |z| > \frac{1}{2} \quad H_2(z) = \frac{1}{2}z^{-1}, |z| > 0$$

代入有

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})^2} = \frac{z^2}{(z - \frac{1}{2})^2}, |z| > \frac{1}{2}$$

(b) 系统单位冲激响应可以由系统函数的反变换得到。由二阶极点的常用反变换结果有

$$g[n] = n\left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{Z} G(z) = \frac{\frac{1}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)^2}$$

再由时移性质，有

$$H(z) = 2zG(z), h[n] = 2g[n+1] = 2(n+1)\left(\frac{1}{2}\right)^{n+1} u[n+1]$$

注意，系统是因果的，可由ROC判断，同时，虽然 $h[n]$ 看似从 $n=-1$ 时刻开始，但其实 $h[-1]=0$

(c) 求系统输出，关键在于  $x[n] = \cos(\pi n) = (-1)^n$

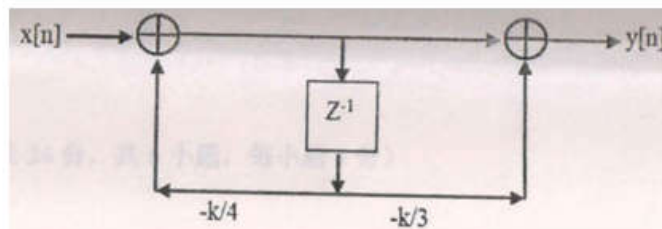
同时， $z = -1$ 在系统函数的ROC内，有

$$y[n] = H(z = -1)(-1)^n = \frac{4}{9}(-1)^n$$

注意，这个输入信号没有Z变换

第二题：

**2. Given a causal digital filter structure shown in the figure where  $k$  is a real constant**



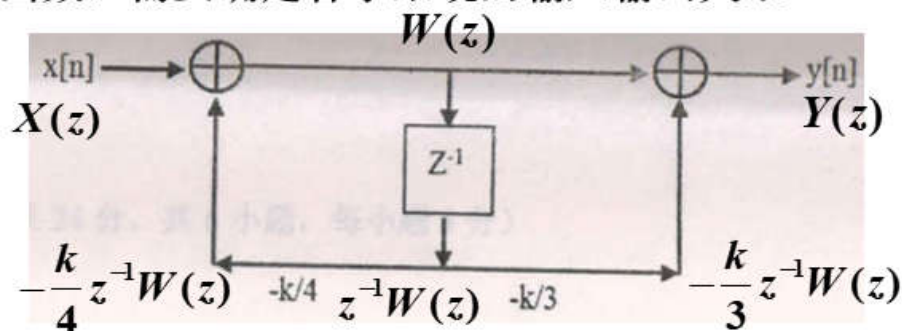
**(a) Find the system function  $H(z)$  and its ROC**

**(b) Determine the unit impulse response  $h[n]$  if  $k = 1$ .**

**(c) For what value of  $k$ , is the system stable?**

解答：

(a) 确定系统函数，需要确定各子系统的输入输出关系



$$W(z) = X(z) - \frac{k}{4} z^{-1} W(z)$$

$$Y(z) = (1 - \frac{k}{3} z^{-1}) W(z)$$

$$X(z) = (1 + \frac{k}{4} z^{-1}) W(z)$$

代入有系统函数，ROC可由因果性可知

$$H(z) = \frac{1 - \frac{k}{3} z^{-1}}{1 + \frac{k}{4} z^{-1}} = \frac{z - \frac{k}{3}}{z + \frac{k}{4}}, |z| > \frac{|k|}{4}$$

(b) 系统单位冲激响应可以由系统函数的反变换得到，当  $k=1$  时

$$H(z) = \frac{1 - \frac{1}{3} z^{-1}}{1 + \frac{1}{4} z^{-1}} = \frac{1}{1 + \frac{1}{4} z^{-1}} - \frac{\frac{1}{3} z^{-1}}{1 + \frac{1}{4} z^{-1}}, |z| > \frac{1}{4}$$

$$(-\frac{1}{4})^n u[n] \xleftrightarrow{z} \frac{1}{1 + \frac{1}{4} z^{-1}}, |z| > \frac{1}{4}$$

其中第二项由时移性质，有  $h[n] = (-\frac{1}{4})^n u[n] - \frac{1}{3} (-\frac{1}{4})^{n-1} u[n-1]$

(c) 如果系统稳定，需要ROC包含单位圆，为此所有极点需要在单位圆内，因此有  $\frac{|k|}{4} < 1, \therefore |k| < 4$