第二章扩展作业（第二部分）

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目录

1 第一题

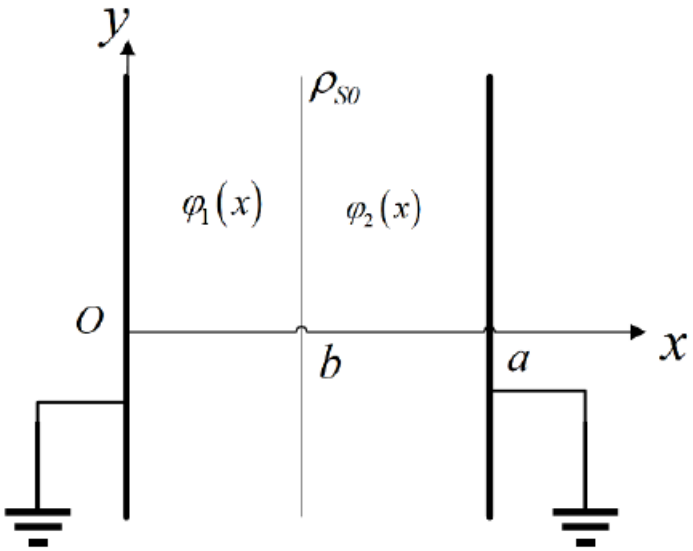

图 1: 平板与坐标系

在 $x = b$ 两侧有电位函数：

$$\begin{aligned} \varphi_1 &= \varphi_1(x) \\ \varphi_2 &= \varphi_2(x) \end{aligned} \tag{1}$$

由于在分界面上没有自由电荷，所以泊松方程退化为拉普拉斯方程。

$$\begin{aligned} \nabla^2\varphi_1(x) &= 0 \\ \nabla^2\varphi_2(x) &= 0 \end{aligned} \tag{2}$$

方程的解为：

$$\begin{aligned}\varphi_1(x) &= C_1 x + D_1 \\ \varphi_2(x) &= C_2 x + D_2\end{aligned} \tag{3}$$

利用边界条件，可知

$$\begin{aligned}x = 0 \;\; & \varphi_1(0) = 0 \\ x = a \;\; & \varphi_2(a) = 0 \\ x = b \;\; & \varphi_1(b) = \varphi_2(b) \\ & \left[\frac{\partial \varphi_2(x)}{\partial x} - \frac{\partial \varphi_1(x)}{\partial x}\right]_{x=b} = -\frac{\rho_{S0}}{\epsilon_0}\end{aligned} \tag{4}$$

由上面四个式子，将 $\varphi_1(x)$ 与 $\varphi_2(x)$ 带入解得

$$\begin{aligned}C_1 &= -\frac{\rho_{S0}(b-a)}{\epsilon_0 a} x \\ C_2 &= -\frac{\rho_{S0} b}{\epsilon_0 a}, \\ D_1 &= 0 \\ D_2 &= \frac{\rho_{S0} b}{\epsilon_0}\end{aligned} \tag{5}$$

最后得到

$$\begin{aligned}\varphi_1(x) &= \frac{\rho_{S0}(a-b)}{\epsilon_0 a} x \\ \varphi_2(x) &= \frac{\rho_{S0} b}{\epsilon_0 a}(a-x) \\ \vec{E}_1(x) &= -\nabla \varphi_1(x) = -\frac{\rho_{S0}(a-b)}{\epsilon_0 a} \cdot \vec{e}_x \\ \vec{E}_2(x) &= -\nabla \varphi_2(x) = \frac{\rho_{S0} b}{\epsilon_0 a} \cdot \vec{e}_x\end{aligned} \tag{6}$$