

CHALMERS

EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kursnamn		
DIT 022	Mathematical Foundation for Software Engineering		
Anonymous code Anonym kod		Examination date Tentamensdatum	Number of pages Antal blad
371		07.01.2019	11 VG

* I confirm that I've no mobile or other similar electronic equipment available during the examination.
 Jag intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under
 eximinationen.

Solved task Behandlade uppgifter No/nr	Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylltes av lärare.
1 ✓	8	
2 ✓	22	
3 ✓	18	
4 ✓	10	
5 ✓	13	
6 ✓	12	
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
Bonus poäng	7	
Total examination points Summa poäng på tentamen	84	91

Question 1 - "Languages"

1.1 Answer b) 11.2 Answer c) 11.3 Answer d) 11.4 Answer e) 11.5 0

Start $\rightarrow \langle \text{postalcode} \rangle \rightarrow \langle \text{forward_or_stationarea} \rangle \langle \text{underscore} \rangle \langle \text{local_deliverunit} \rangle \rightarrow \langle \text{provarea} \rangle \langle \text{locotype} \rangle \langle \text{letter} \rangle - \langle \text{digit} \rangle \langle \text{letter} \rangle$
 $\langle \text{digit} \rangle \rightarrow A \langle \text{rural} \rangle A - O \langle \text{letter} \rangle \rightarrow AOA - OA \langle \text{letter} \rangle$

$\rightarrow \underline{\underline{AOA - OA}}$

$\rightarrow \underline{\underline{AOA - OB}}$

$\rightarrow \underline{\underline{AOA - OC}}$

1.6 Assuming that only positive integers are allowed. (and only addition and subtraction need to be considered).
Also assuming that a single number without operand is a allowed expression (e.g. "15")

$\langle \text{term} \rangle ::= \langle \text{number} \times \text{positive integer} \rangle$

$\langle \text{positive integer} \rangle ::= \langle \text{non zero digit} \rangle 0 | \langle \text{non zero digit} \rangle \langle \text{number} \rangle$

$\langle \text{non zero digit} \rangle ::= 1 | 2 | 3 | \dots | 9$

$\langle \text{number} \rangle ::= \langle \text{digit} \rangle | \langle \text{digit} \rangle \langle \text{number} \rangle$

$\langle \text{times by operator} \rangle ::= \cdot | : | /$

See next page

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	Anonym kod 371		Löpande sid nr 2

1.6 Assuming that only positive integers are allowed,
adding the following production rules need solve the task:

I also assume that a single number with no
 operand or second number is a valid word.

$\langle \text{expression} \rangle ::= \langle \text{term} \rangle \langle \text{term} \rangle \langle \text{multdiv Operator} \rangle$

$\langle \text{term} \rangle ::= \langle \text{positive integer} \rangle$

$\langle \text{multdiv Operator} \rangle ::= \cdot | : | \cdot \langle \text{term} \rangle \langle \text{operator} \rangle | : \langle \text{term} \rangle \langle \text{operator} \rangle$

$\langle \text{operator} \rangle ::= \langle \text{multdiv Operator} \rangle | \langle \text{addsub Operator} \rangle$

$\langle \text{addsub Operator} \rangle ::= + \langle \text{term} \rangle \langle \text{operator} \rangle | - \langle \text{term} \rangle \langle \text{operator} \rangle$

$\langle \text{Term} \rangle ::=$

$\langle \text{positive integer} \rangle ::= 0 | \langle \text{non zero digit} \rangle \langle \text{number} \rangle \langle \text{digits} \rangle$

$\langle \text{non zero digit} \rangle ::= 1 | 2 | 3 | \dots | 9$

$\langle \text{number} \rangle ::=$

$\langle \text{digits} \rangle ::= \langle \text{digit} \rangle | \langle \text{digit} \rangle \langle \text{digit} \rangle \dots$

These rules added to the already provided ones
 result in the desired grammar

bottom up against setting sun half permanent
 almost like beach color without any problem
 on this stone stone is half covered with
 brownish red reddish brownish red

$$\langle \text{bottom yellow} \rangle \langle \text{red} \rangle \langle \text{red} \rangle + \langle \text{brown} \rangle$$

$$\langle \text{yellow} \rangle \langle \text{red} \rangle = \langle \text{red} \rangle$$

$$\langle \text{brown} \rangle \langle \text{red} \rangle + \langle \text{brown} \rangle \langle \text{red} \rangle + \langle \text{brown} \rangle \langle \text{brown} \rangle$$

$$\langle \text{brown} \rangle \langle \text{brown} \rangle + \langle \text{brown} \rangle \langle \text{brown} \rangle = \langle \text{brown} \rangle$$

$$\langle \text{brown} \rangle \langle \text{red} \rangle - \langle \text{brown} \rangle \langle \text{red} \rangle + \langle \text{brown} \rangle \langle \text{brown} \rangle$$

$$\langle \text{brown} \rangle \langle \text{brown} \rangle = \langle \text{brown} \rangle$$

$$\langle \text{left} \rangle \langle \text{right} \rangle \langle \text{left} \rangle \langle \text{right} \rangle + \langle \text{right} \rangle \langle \text{left} \rangle \langle \text{right} \rangle$$

$$P_1 + P_2 = \langle \text{right} \rangle \langle \text{left} \rangle$$

$$\langle \text{right} \rangle \langle \text{right} \rangle + \langle \text{left} \rangle \langle \text{left} \rangle = \langle \text{right} \rangle$$

zero delivery which will not be color test

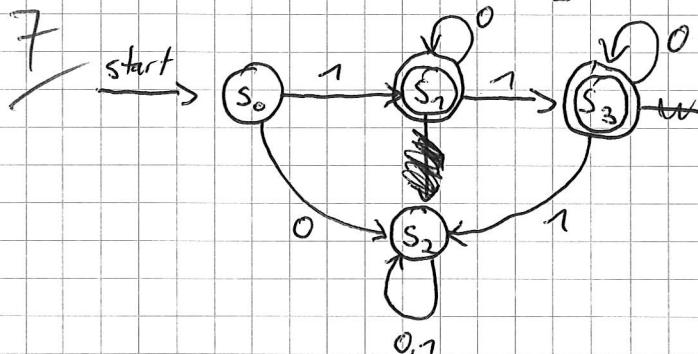
color test which will be the same

Question 2 - "Automata"

2.1 Answer d) 1

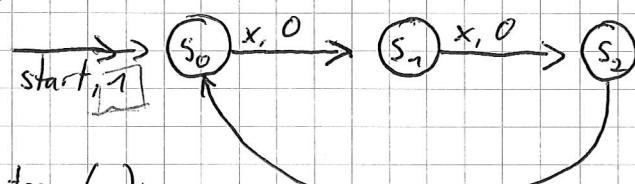
2.2 Answer b) 0

$$2.3: S = \{10^n \mid n \geq 0\} \cup \{10^n 10^m \mid n, m \geq 0\}$$



2.4: Let's define x to be any input symbol.

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trans(x):

$lx++$
return ($lx \% 3 == 0$)

I am assuming the machine to first count the latest input to the total and then determining the divisibility by 3

The inputs between s_0 and s_1 are every all numbered by the following rule: $i = (3k+1)$, $k \geq 0$

They are therefore not divisible by 3

The inputs between s_1 and s_2 are all numbered by $i = (3k+2)$, $k \geq 0 \Rightarrow$ not divisible by 3

And the inputs between s_2 and s_0 are all numbered as $i = 3k$, $k \geq 1 \Rightarrow$ divisible by 3

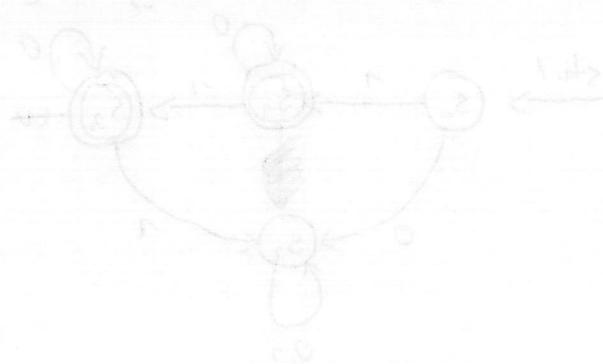
Additionally when starting the program machine, there would be 0 prior inputs and therefore one could argue that the machine should output an initial "1" when starting.

"monotonic" - I understand

that $\{x_n\}$ is increasing (b) "weak" if $x_n \rightarrow x$

(d) "converges" if $x_n \rightarrow x$

$$\{\phi_n\} \text{ and } \{\psi_n\} \subset \{\phi_n \in \mathcal{H}^*\} = \mathbb{R} \quad \text{and}$$

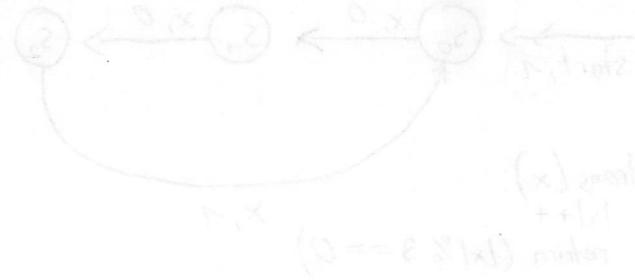


Inductive step in ϕ_n and ψ_n with respect to the

number of points

and ϕ_{n+1} and ψ_{n+1} with respect to the number of points

with respect to the number of points



(x) and
+ or -

(0 = -8.8)x)

because the sum of two wanted steps will

$\phi_n + \psi_n$, $(\phi + \psi)_n = 1$ when parallel with ϕ_n

$\phi_n + \psi_n$ and when the two steps are not

parallel then the sum of two wanted steps will

$\phi_n + \psi_n$ $\leq \phi_n + \psi_n$, $(\phi + \psi)_n = 0$

because the sum of two wanted steps will be

$\phi_n + \psi_n \leq \phi_n + \psi_n$, $\phi_n + \psi_n = 0$

and parallel mapping with parallel paths the probability

is zero and we expect the steps to be zero

and when "x" steps are higher than $\phi_n + \psi_n$ with

Question 3 - "Logic"

$$3.1 \neg [p \wedge (q \vee r) \wedge ((p \wedge q) \rightarrow r)] = \neg p \vee (\neg q \wedge \neg r) \vee \neg (...)$$

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<u>p</u>	<u>q</u>	<u>r</u>	<u>Complete expression</u>	$(p \wedge q) \rightarrow r$
0	0	0	1.	1
0	0	1	1.	1
0	1	0	1.	1
0	1	1	1.	1
1	0	0	1.	1
1	0	1	0.	1
1	1	0	1	0
1	1	1	0.	1

3.2 Answer c 13.3 Answer a 1

3.4 i) a compound proposition is a composite of multiple individual logical propositions using connective operators (like \wedge , \vee , \oplus , \rightarrow , \leftrightarrow) to connect them into a single truth-value dependent on the components truth values.

ii) A tautology is a compound proposition that is always true (e.g.: $p \vee \neg p$)

iii) A contradiction is a compound proposition which never can be true. (e.g. $p \wedge \neg p$)

"aged" → 2 notes

(i) $\neg \rightarrow (\neg p \wedge q) \vee q$, $\neg (\neg p \wedge q) \vee (\neg p \wedge q)$

$\neg (\neg p \wedge q)$	maximally satisifed	$\neg p \wedge q$
0	0	0
1	1	0
2	1	0
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	0	0
15	0	0

→ 2 notes 5.8

→ 2 notes 8.8

which shows the sequence is not logically knowable (i) 3.8

(ii) 8.8 (iii) 13.8 (iv) 14.8 (v) 15.8

however consider that sequence is not knowable (i) 3.8

and sequence is not logically knowable is not knowable (ii) 8.8

($q \wedge q \rightarrow p \wedge q$)

which shows the sequence is not logically knowable (iii) 13.8

($q \wedge q \rightarrow p \wedge q$) (iv) 14.8 (v) 15.8

Question 3 - "Logic"

3.4 Continuation

- iv) a contingency is a compound proposition which is included in a simpler proposition and can be replaced by it (e.g.: $p \wedge (p \vee q)$ can be replaced with p)
- v) It is not a contradiction because its truth is true for the following values of p , q , and r :
- $$\begin{array}{l} p=0 \quad p=0, \quad q=0 \quad q=1, \quad r=1 \\ q=1 \end{array}$$

p	q	r	$p \vee \neg q$	$r \rightarrow (p \vee \neg q)$	$\neg(\dots)$	$\neg(p \wedge q)$
0	0	0	0	1	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	1
0	1	1	0	1	1	1
<hr/>						
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	1	1	0	0
1	1	1	1	1	0	0

vi) $((p \vee \neg q) \vee (r \vee (p \vee \neg q))) \vee p \vee q$

2 This is the same as $\neg q \vee q = \text{true}$.

Question 4 - "Proofs"

$$4.1 \quad f(x, n) = (1+x)^n \quad \checkmark$$

$$g(x, n) = 1+nx$$

We want to prove that $f(x, n) \geq g(x, n) \quad \forall n \geq 1 \quad \forall x > -1$

Proof by induction:

Base case: Be $n=1$

$$f(x, 1) = 1+x$$

$$g(x, 1) = 1+1x = 1+x \Rightarrow f(x, 1) = g(x, 1) \quad \checkmark$$

IH: Be $f(x, k) \geq g(x, k)$ already proven.

Induction: $n=k+1$

$$\begin{aligned} \text{Then: } f(x, k+1) &= (1+x)^{k+1} = (1+x)^k (1+x) \\ &\stackrel{\text{IH}}{=} f(x, k) \underbrace{(1+x)}_{\geq 0} \\ &\stackrel{\text{IH}}{=} g(x, k) \underbrace{(1+x)}_{\geq 0} \end{aligned}$$

$$\begin{aligned} g(x, k+1) &= 1+(k+1)x = 1+kx+x \\ &= g(x, k)+x \end{aligned}$$

$$\Rightarrow f(x, k+1) \geq g(x, k) \geq 1+x$$

~~$$g(x, k+1) : g(x, k) = 1 + \frac{x}{1+kx}$$~~

~~if $1+kx \geq x \quad \forall x > -1, \forall k$~~

$$\begin{aligned} 1+x &\geq 1+\frac{x}{1+kx} \\ x &\geq \frac{x}{1+kx} \\ 1 &\geq \frac{1}{1+kx} \\ 1+kx &\geq 1 \end{aligned}$$

$$\begin{aligned} 1 &\geq \frac{1}{1+kx} \\ 1 \cdot (1+kx) &\geq 1 \\ 1+kx &\geq 1 \end{aligned}$$

Continues on
Page

"foot" = 4 notes and

$$(x+t) \in (a, b)$$

$$x+t \in (a, b)$$

$(a, b) \subseteq (a, t)$ tells us that all

notes in (a, t) are "foot"

$$x \in (a, t)$$

$$x+t \in (a, b)$$

$$(a, b) \subseteq (a, t) \cap (t, b) = (a, b)$$

missing words: $(a, b) \subseteq (a, t) \cap (t, b)$

$$(x+t) \cap (x+t) = \emptyset \quad (x+t) \cap (t, b) = \text{not}$$

$$(x+t) \cap (a, t) =$$

$$(x+t) \cap (a, t) = \emptyset$$

$$x + x + t = x + (t + t) = (x + t) + t = (x + t, t)$$

$$x + (t, t) =$$

$$x + (t, t) = x + t + t = (x + t) + t = (x + t, t)$$

$$x + (t, t) = x + t + t = (x + t) + t = (x + t, t)$$

no control
no error

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	Anonym kod		
		Poäng på uppgiften (ifylls av lärare)	Question no. Uppgift nr

4.7 Continuation

$$f(x, h+1) \geq g(x, h)(1+x)$$

$$g(x, h+1) = g(x, h) + x$$

case $-1 < x < 0$:

$$g(x, h) = 1 + \underbrace{hx}_{< 0}$$

$$g(x, h+1) = 1 + \underbrace{(h+1)x}_{< 0} < 1$$
 ~~$f(x, h) \neq 2022$~~

$$f(x, h) \geq g(x, h) (1+x)$$

$$(x+\alpha)(\beta, \gamma)_P \leq (\alpha, \beta, \gamma)_P$$

$$\alpha + (\beta, \gamma)_P = (\alpha, \beta, \gamma)_P$$

$$x > \underbrace{\gamma(\beta, \gamma)}_{\geq 0} + \alpha = (\alpha, \beta, \gamma)_P$$

$$(x+\alpha)(\beta, \gamma)_P \leq (\alpha, \beta, \gamma)_P$$

$$4.2 \quad f(n) = 2+4+6+\dots+2n$$

$$\underline{g(n)} = n(n+1) = n^2+n$$

Prove that $f(n) = g(n) \quad \forall n > 0$:

Base step: $n=1$

$$f(1) = 2$$

$$g(1) = 1^2+1 = 2 \quad \checkmark$$

$$IH: f(k) = g(k)$$

Induction step: $n=k+1$

$$f(k+1) = f(k) + 2(k+1) \stackrel{IH}{=} g(k) + 2k+2$$

$$g(k+1) = (k+1)^2 + k+1 = k^2 + 3k + 2$$

$$\begin{aligned} g(k+1) - f(k+1) &= k^2 + 3k + 2 - g(k) - 2k - 2 \\ &= k^2 + k - g(k) \\ &= k(k+1) - g(k) = \underline{\underline{0}} \end{aligned}$$

The difference between two values is zero, if they are the same. Therefore $g(k+1) = f(k+1)$ if $g(k) = f(k)$. It follows from induction that $g(n) = f(n)$ for all $n > 0$ q.e.d.

$$as + \dots + ds + s = (a+d)s$$

$$as^2 + a = (a+n)a = (a)b$$

$$as^2 + a = (a)b = (a) + \text{left side}$$

$$as^2 + a = n + \text{left side}$$

$$as^2 + a = (a) + \text{left side}$$

$$as^2 + a = (a)b = (a)$$

$$(a)b = (a)c : \text{H1}$$

$$n + a = n : \text{left side}$$

$$s + as + (a)b = (a + a)s + (a)b = (a + a)b$$

$$s + as + a = s + a + a(a + a) = (a + a)b$$

$$s + as - a(a)b - s + as + a^2a = (a + a)b - (a + a)b$$

$$(a)b - a + a^2a =$$

$$\underline{\underline{0}} = (a)b - (a)a =$$

This was an example that we wanted to show that
 $(a+b)c = (a+c)b$ is not true since left side will
 fail induction and condition $(a)b = (b)a$
 also fails since $0 < a$. Hence $(a)b = (b)a$

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			Question no. Uppgift nr

Question 5

5.1 The complexity is $O(n^2)$ 2

5.2 The complexity is $O(n^3)$ 1

5.3 Start: -50 69 5 0 -20 32 1 8 90 103 76

i=0: -50 69 5 0 -20 32 1 8 90 103 76

i=1: ~~-50~~ -50 -20 50 69 32 1 8 90 103 76

i=2: -50 -20 0 5 69 32 1 8 90 103 76

i=3: -50 -20 0 1 69 32 5 8 90 103 76

i=4: -50 -20 0 1 5 32 69 8 90 103 76

i=5: -50 -20 0 1 5 8 69 32 90 103 76

After 6 iterations the array therefore looks as follows:
-50 -20 0 1 5 8 69 32 90 103 76

5.4 The graph in figure 5.4 can not have an Euler circuit as c and f both have an odd number of edges. 1

The graph can not have an Euler path for the same reason. An Euler path is possible with a maximum of one vertex having odd degree. 0

3 postcard

2 (290) 21 May 1905 att 22

2 (291) 21 May 1905 att 22

2 (292) 21 May 1905 att 22

2 (293) 21 May 1905 att 22

2 (294) 21 May 1905 att 22

2 (295) 21 May 1905 att 22

2 (296) 21 May 1905 att 22

2 (297) 21 May 1905 att 22

2 (298) 21 May 1905 att 22

2 (299) 21 May 1905 att 22

2 (300) 21 May 1905 att 22

2 (301) 21 May 1905 att 22

2 (302) 21 May 1905 att 22

2 (303) 21 May 1905 att 22

2 (304) 21 May 1905 att 22

2 (305) 21 May 1905 att 22

2 (306) 21 May 1905 att 22

2 (307) 21 May 1905 att 22

2 (308) 21 May 1905 att 22

2 (309) 21 May 1905 att 22

2 (310) 21 May 1905 att 22

2 (311) 21 May 1905 att 22

2 (312) 21 May 1905 att 22

2 (313) 21 May 1905 att 22

2 (314) 21 May 1905 att 22

2 (315) 21 May 1905 att 22

Question 6 - "Statistics"

$$6.1 \text{ a) } \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336 \checkmark 1$$

$$\text{b) } 8^3 = 512 \text{ (one person can get more than one present)} \quad \frac{336}{6}$$

6.2 I ignore the X -values corresponding to the missing Y -values so they can not bias my results. $\checkmark 1$

$$\bar{x} = 651.17 \checkmark 1$$

$$\sum_{i=1}^6 x_i y_i = 18565.7$$

$$\bar{y} = 4.75 \checkmark 1$$

$$\sum y_i = 28.5$$

$$\hat{y} = \alpha + \beta x$$

$$\begin{aligned} \sum x_i^2 &= 2544143 \\ \sum x_i &= 3907 \end{aligned}$$

$$\hat{\alpha} = \frac{\sum x_i y_i - \bar{x} \sum y_i}{\sum x_i^2 - n \bar{x}} =$$

$$S_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$S_x = 2.64 \checkmark 1 \quad S_y = 0.57 \checkmark 1$$

$$\hat{\beta} = 2.42 \quad S_y = 0.522$$

$$\alpha = -134.52 \checkmark 1$$

$$\beta = 0.2788 \quad 0.214 \checkmark 1$$

~~at~~

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 7.45$$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1) S_x S_y} = 0.993 \checkmark 1$$

$$\hat{y} = \alpha + \beta x$$

α = intersection

β = slope

g

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"estimated" - 3' instead

$$\Delta V_{\text{eff}} = 24.8 - \frac{1}{10} (6 + 3)$$

so with min lag we need $\Delta t = 5.2 \times 10^{-3}$ s

or 3×10^{-3} s (assuming

minimum of 3 min between each sample)

✓ Vietnam went to me with similar

$$+ 2.33285 = \sqrt{x^2 - 3} \quad 1.444,283 = x$$

$$2.33 = \sqrt{3}$$

$$+ 2.33285 = \sqrt{3} \quad 1.444,283 = x$$

$$\boxed{1.444}$$

$$\sqrt{3} - 1.444,283$$

$$1.444 = 2 - \sqrt{3} = 2$$

$$1.444 = \sqrt{3} = 2$$

$$1.444 = \sqrt{3} = 2$$

$$\Delta V_{\text{eff}} = 24.8 - \sqrt{3} = 2$$

$$24.8 = (x - 2)(x - 2)3$$

$$\Delta V_{\text{eff}} = \frac{(x - 2)(x - 2)3}{(x - 2)(x - 2)} = 3$$

$$x - 2 = 3$$

so $x = 5$

sampled

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6.3

a) H_0 := The true mean of all 5 groups is identical (Null-Hypothesis) ✓

H_1 := At least one of the 5 groups has a mean different from the others. ✓

b) SST is the total sum of squares.

$$SST = \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{x}_i)^2$$

$$\bar{x}_i = \frac{1}{5} (7.00 + 6.90 + 11.00 + 13.40 + 12.00) = 10.06$$

Counting: 45 ✓ 2x6 ✓ 2x7 ✓ 2x8 ✓ 9 ✓ 10 ✓

Rhyming: 3 4x6 ✓ 2x7 ✓ 8 ✓ 9 ✓ 11 ✓

Adjective: 6 ✓ 8 ✓ 10 ✓ 3x11 ✓ 3x13 ✓ 1x14 ✓

Imagery: 9 ✓ 10 ✓ 3x11 ✓ 2x12 ✓ 16 ✓ 19 ✓ 23 ✓

Intentional: 5 ✓ 2x10 ✓ 3x11 ✓ 2x14 ✓ 15 ✓ 19 ✓

Total: 3 4(2x5) 5x8 7x6 4x7 4x8 2x9 5x10
10x11 2x12 3x13 3x14 15 16 2x19

23

3 ✓

No time to type all of this. ✗

c) $SSB = \sum (\bar{x}_i - \bar{x})^2 = 35.152$ ✓ 1

d) $SSW = \sum \sum (x_{ij} - \bar{x}_i)^2 = ?$ 25.6, ↙ made up, no time

e) $f(SSB) = 4$ ✓ , $f(SSW) = 4$ ✓ , $f(SSW) = 45$ ✓ 1

f) $F = \frac{SSB/f(SSB)}{SSW/f(SSW)} = 75.4648$ ↙ used wrong value from bda

g) $F_{crit} = 5.71$ ✓ 9.08

h) With my made up value $F = \frac{1}{75.4648}$ the hypothesis holds as it is smaller than F_{crit}

