CHALMERS
EXAMINATION / TENTAMEN

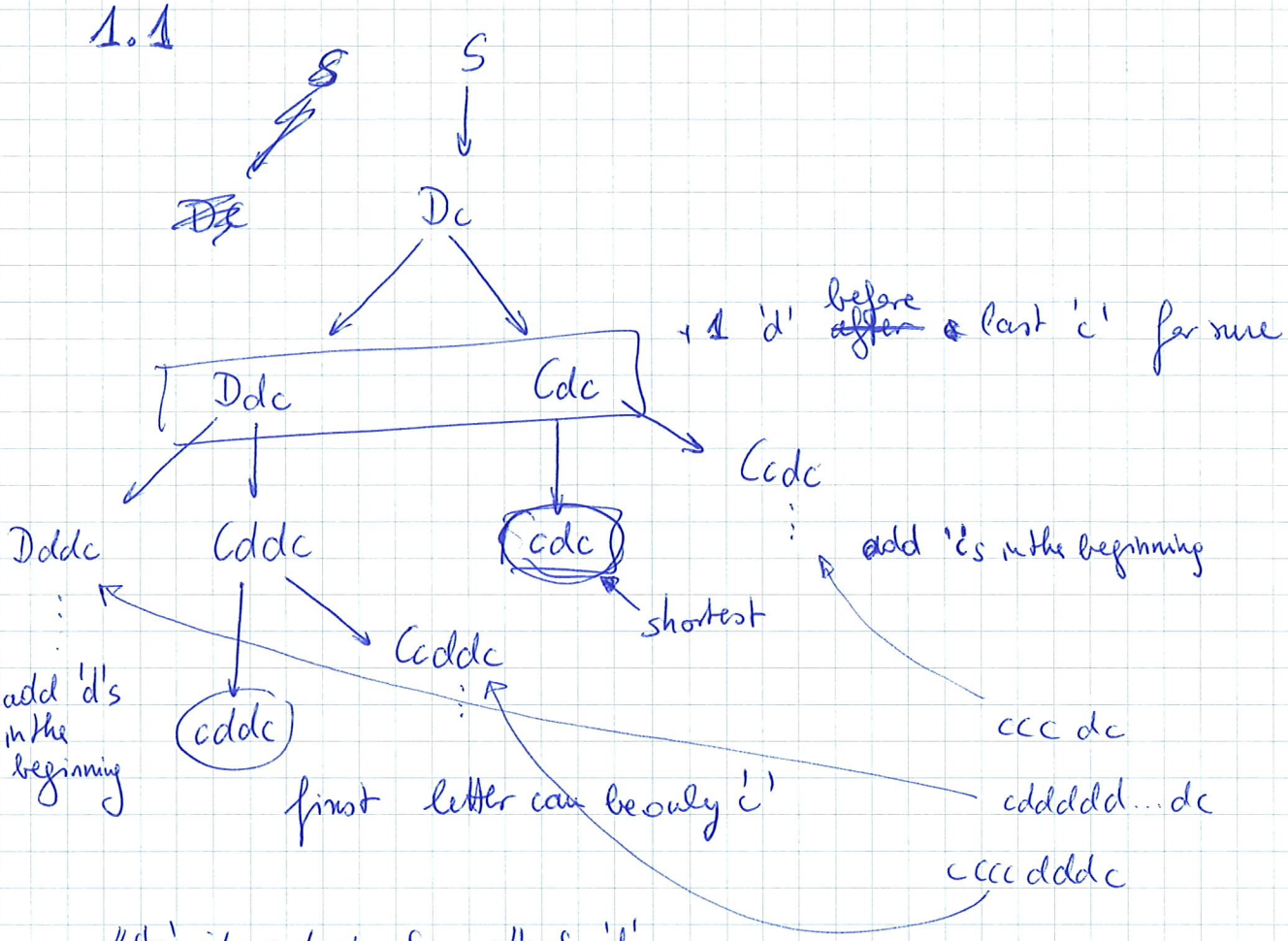
Course code/kurskod	Course name/kursnamn			*
DIT022	Mathematical Foundations			X
Anonymous code Anonym kod		Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
524		30.10.2018	22	VG

* I confirm that I've no mobile or other similar electronic equipment available during the examination.
Jag intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under examinationen.

Solved task Behandlade uppgifter No/nr		Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
1	X	23	
2	X	17	
3	X	10	
4	X	9	
5	X	9	
6	X	23	
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
Bonus credits/ poäng	8	91	
Total examination points Summa poäng på tentamen		99	

3

1.1



#of 'c' independent of # of 'd'

last 'c' is alone

at least ~~one~~ 2 c & one d

a 1

The correct solution is a

b: can be only one 'c' -> ✓

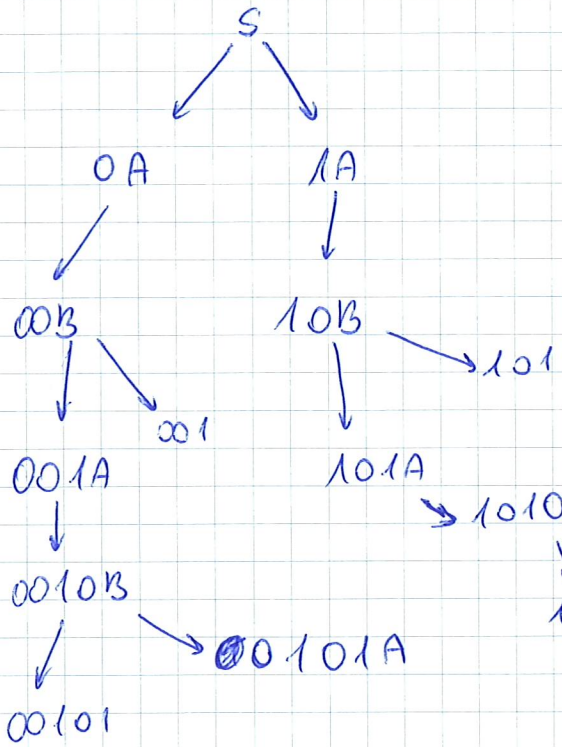
c: # 'c' ≠ # 'd' -> ✓

d: m, n cannot be 0 ✓

e: cccdc acceptable ✓

3

1.2.



a) $11(01)^n \cup 01(01)^m$

↑ cannot happen ↯

b) $1(10)^m \cup 0(01)^n$

↑ cannot happen ↯

c) $00^m 1^n \cup (101)^m$

↑

101101 ↯

cannot happen

d) $0(01)^n \cup 1(01)^m$

sounds good ✓

e) $001(0|1)^n \cup (101(0|1)^m)$

↑

0011 cannot happen ↯

d is the correct answer 1

3

1.3

~~person~~

p ~~person~~ ::= fn + mi + ln

ln ::= ls

mi ::= l

fn ::= ucl | ucl + ls

ls ::= l | ls + l

l ::= lcl | ucl

lcl ::= a | b | ... | z

ucl ::= A | B | ... | Z

First & second words that can be generated are:

1) ~~Abbaucc~~ <Abba><C><zK>

~~spaces between~~ Abba C zK /

2) <Mxm><~~NNm~~> ~~Hm UUm~~ M m NN m /

3

1.4

given

<identifier> ::= <lcletter> <digitorus> <alphanumeric> <alphanumeric> <an> |

<lcletter> <digitorus> <alphn> <alphn> <alphn> <alph.>

<uc letter> ::= A | B | ... | Z

<lcletter> ::= a | b | ... | z

Missing:

<digitorus> ::= <digit> | _ (underscore)

<alphanumeric> ::= <ucletter> | <lcletter> | <digit>

<digit> ::= 0 | 1 | ... | 9 4

~~<alphanumeric> ::= <ucletter> <alphanumeric> |~~

~~<lcletter> <alphanumeric> |~~

~~<digit> <alphanumeric>~~

3

1.5

~~a~~ ~~aab~~

~~{a,b}*~~

a) aa bb ab a

~~b~~) aa bb a aba

~~ab ? ba~~

~~ab~~

d) aba {b,a}* ← one solution not all

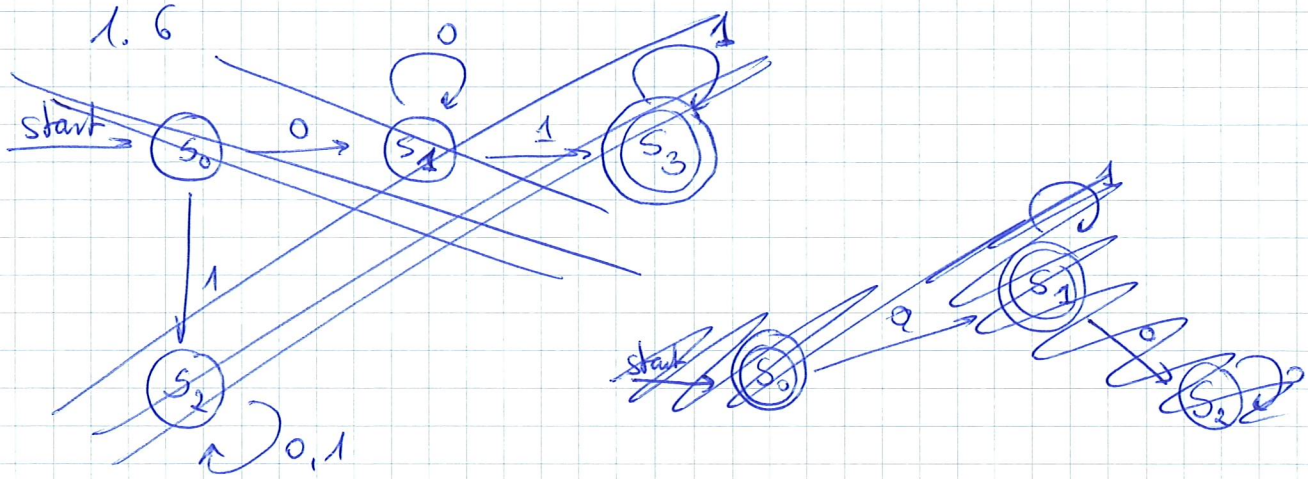
e) {a,b}*

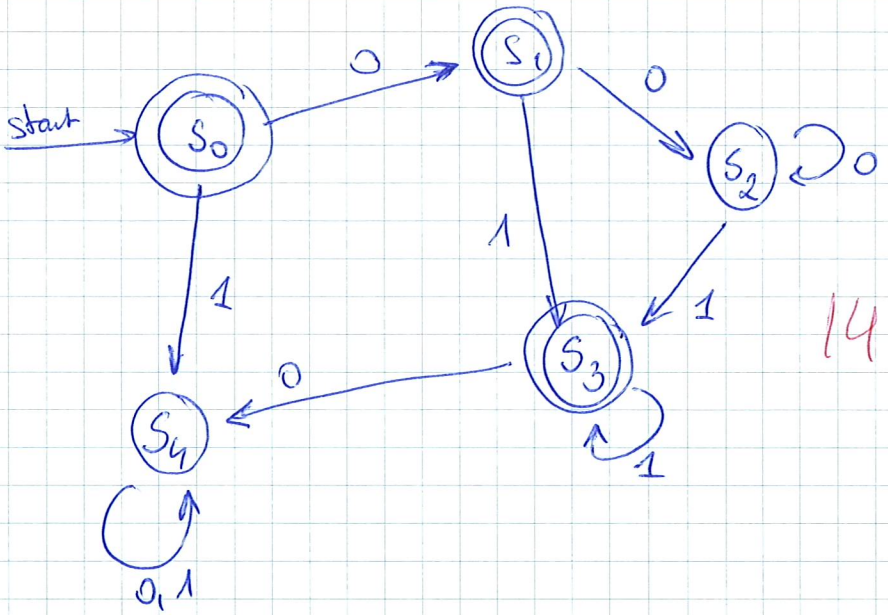
{b* a a* b}* a (a|b)*

a is the correct ~~one~~ solution 1

3

1.6





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	v	p	t	$v \to p$	$(v \to p) \vee t$	$((v \to p) \vee t) \leftrightarrow p$	$t \wedge v$	$v \vee (t \wedge v)$
①	0	0	0	1	1	0	0	0
②	0	0	1	1	1	0	0	0
③	0	1	0	1	1	1	0	0
④	0	1	1	1	1	1	0	0
⑤	1	0	0	0	0	1	0	1
⑥	1	0	1	0	1	0	1	1
⑦	1	1	0	1	1	1	0	1
⑧	1	1	1	1	1	1	1	1

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	$v \to (v \vee (t \wedge v))$	$(((v \to p) \vee t) \leftrightarrow p) \wedge (v \to (v \vee (t \wedge v)))$
①	1	0 ✓
②	1	0 ✓
③	1	1 ✓
④	1	1 ✓
⑤	1	1 ✓
⑥	1	0 ✓
⑦	1	1 ✓
⑧	1	1 ✓

2.2

C is the correct answer 1

2.3

a is the correct answer 1

2.4

i) Tautology is a statement that is always true. 1

ii) Contradiction is a statement that is always false 1

iii)

P	Q	R	(P ∧ Q)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

3

2.4 iii)

P	Q	R	(P ∧ Q)	(P ∧ Q) → ¬R (a)	P ∨ (Q → R) (b)	a ↔ b
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	0	1	0	0
0	1	1	0	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	1	1
1	1	1	1	0	1	0

corresponding column

iii) The table shows that the statement is not always true. 3

iv) The same column shows that the statement is not always false 1

v)

$$((P \wedge Q) \rightarrow \neg R) \vee (P \vee (Q \rightarrow R)) \quad 1$$

3

3.1) 5

$n \in \mathbb{N}$ $n > 1$

n not prime $\Rightarrow$ $n = m \cdot \ell$ $\quad m, \ell \in \mathbb{N}$, $1 < m, \ell < n$

$\Downarrow$?

$2^n - 1$ not prime $\Leftrightarrow$ $(2^n - 1) = k \cdot s$ $\quad k, s \in \mathbb{N}$, $1 < k, s < 2^n - 1$

$$2^{m\ell} - 1 = (2^m - 1)(1 + 2^m + 2^{2m} + \dots + 2^{(\ell-1)m})$$

$1 < (2^m - 1) = k \in \mathbb{N}$ because $m > 1$, $m \in \mathbb{N}$

$(2^m - 1) = k$

$1 < (1 + 2^m + 2^{2m} + \dots + 2^{(\ell-1)m}) \in \mathbb{N}$ because $1 < m, \ell < n$, $m, \ell \in \mathbb{N}$

$2^{m\ell} - 1 > (1 + 2^m + 2^{2m} + \dots + 2^{(\ell-1)m}) = k \cdot s$

$\Rightarrow$ if n not a prime, than $2^n - 1$ not a prime either

if $n = m \cdot \ell$

$\exists\, k, s \in \mathbb{N}$ such that $k \cdot s = 2^n - 1$

where $1 < k, s < 2^n - 1$

3

3.2) 5

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

1) base case $n = 1$.

$$1 = 1^3 \stackrel{?}{=} \frac{1^2(1+1)^2}{4} = \frac{1 \cdot 4}{4} \quad \checkmark \text{ true}$$

2) assume that ~~k = n is~~ $n = k$ is true ~~(k < n)~~

$$1^3 + 2^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$$

3) show for ~~k~~ $n = k+1$

$$\underbrace{1^3 + 2^3 + \dots + k^3}_{\frac{k^2(k+1)^2}{4}} + (k+1)^3 \stackrel{?}{=} \frac{(k+1)^2(k+1+1)^2}{4}$$

↓ inductive ~~hypothesis~~ step

$$\frac{k^2(k+1)^2}{4} + (k+1)^3 \stackrel{?}{=} \frac{(k+1)^2(k+2)^2}{4} \quad /\cdot 4$$

$$k^2(k+1)^2 + 4(k+1)^3 \stackrel{?}{=} (k+1)^2(k+2)^2 \quad /:(k+1)^2$$

$$k^2 + 4(k+1) \stackrel{?}{=} (k+2)^2$$

$$k^2 + 4k + 4 \stackrel{?}{=} k^2 + 4k + 4$$

✓ showed true :)

3

4.1)

int i = 0;

~~while (i < n) {~~

~~}~~

if never found

while (i < n) {

j = 0

n^2

while (j < n) {

if ( ) { break }

j = j + 1

}

if ( ) {

~~break~~ return = break

}

i = i + 1

}

$O(n^2)$

3

4.2)

method 1 :

i = n-1 , i ≥ 0 ; i = i - 3 decreas with constans that is N/3

O(N)

method 2 :

i = 0 ; i < n , i++ ⟵ N

k = 0 , k < n , k++ ⟵ N

Math.log ⟵ 1

O(N²)

method 3 :

i = 0 , i < n , i++ ⟵ n

m1 ⟵ n

m2 ⟵ n²

N x (N + N²) ≈ N³

O(N³)

method 4 : i = 0 , i < n , i++ ⟶ n

k = n-1 , k > 0 , k = k/3 → $\log_3 n$

O(N log N)

~~(Approximately 20 times)~~

Method 3 will print every 3rd element once and the log of each element n times. ←(method 2)

(method 1) N + N/3

3

4.3) 3

i)

$$(n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$$

(removing low terms)

$$n^3 \log n + n^2 \log^2 n + \cancel{17 \log} \; 17 n^3 \log n$$

$$O(n^3 \log n)$$

ii) $(2n + n^2)(n^3 + 3^n)$

$$2n^4 + n^5 + 2n3^n + n^2 3^n$$

$$O(n^2 3^n)$$

iii) $(n^n + n2^n + 5^n)(n! + 5^n)$

$$n^n \cdot n! + n! \cdot n2^n + n! 5^n + (5n)^n + (10)^n \cdot n + 25^n$$

$$O(n^n \cdot n!)$$

3

4.4) 0

~~200 67 4 192 12 94 1 88 9 43~~

before i=0

200 67 4 192 12 94 1 88 9 43

after i=0, before i=1

1 67 200 192 12 94 4 88 9 43

after i=1, before i=2

1 4 200 192 67 94 12 88 9 43

after i=2

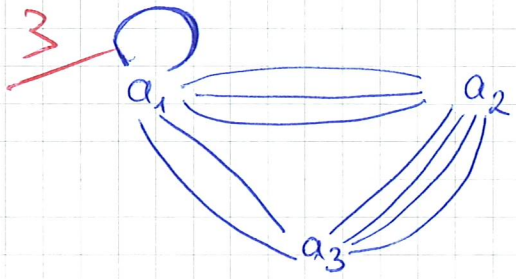
1 4 9 200 192 94 67 88 12 43

Selection sort: swap only once in each iteration.

3

5.1)

3



5.2) 1

No, they are not isomorphic

u_1 u_2 u_3 u_4 u_5 u_6 u_7 u_8 u_9 u_{10}

u_1
u_2
u_3
u_4
u_5
u_6
u_7
u_8
u_9
u_{10}

in G_1 ∃ a vertex with degree of 4 (u_2) that is connected to another vertex with degree of 4 (u_3)

in G_2 no such vertex exist

only v_2 and v_4 have degree of 4, but they are not connected

3

5.3) 3

i) Yes, ~~no vertex has 0~~

there is a ~~path~~ walk between any two vertices

ii)

minimum degree of vertices is 2.

maximum degree of vertices is 4

iii) No reason? (−0.5)

iv) No reason? (−0.5)

5.4) 2

2, 3, 4, 4, 3, 2, 4, 4 ← there are two vertices with odd degrees

Yes it has:

e d c b a h f i j k g l m

3

6.1)

i) ~~4/16~~ $\frac{16}{20} = \frac{4}{5}$ ✓

ii) $\frac{4}{20} = \frac{1}{5}$ ✓

iii) $\frac{1}{20}$ ✓

3

6.2)

~~P(terrorist)~~ $P(\text{terrorist}) = 0.01$ ✓

~~P(T)~~ $P(\text{not terrorist}) = 0.99$

$P(\text{sentence} \mid \text{terrorist}) = 0.95$ ✓

$P(\text{not sentence} \mid \text{not terrorist}) = 0.90 \Rightarrow P(\text{sentence} \mid \text{not terrorist}) = 0.10$

~~P(arrested|terrorist~~

~~P(Terrorist|arrested)~~ =

$$P(T|C) = \frac{P(C|T) \times P(T)}{P(C|T) \times P(T) + P(C|\neg T) \times P(\neg T)} =$$

$$= \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} = \frac{19/2000}{19/2000 + \frac{99}{1000}} \approx 0.088$$ ✓

~~4~~ 3

8.8 %

3

6.3)

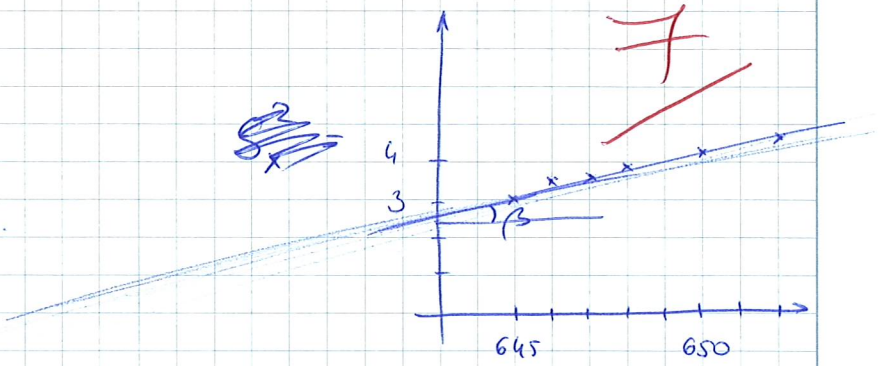
In case of missing y values, the corresponding x also is removed. 1

I do not account for missing values. Forget about them. ✓

X	Y
645	3.0
646	3.4
647	3.5
648	3.9
650	4.1
652	4.6

$\bar{X} = 648$ ✓

$\bar{Y} = 3.75$ ✓ 1



7

$$s_x^2 = \frac{\sum_{i=1}^{6}(x_i - \bar{x})^2}{n-1} = \frac{(-3)^2 + (-2)^2 + (-1)^2 + 0^2 + 2^2 + 4^2}{5} =$$

$$= \frac{9+4+1+4+16}{5} \approx 6.8 \quad ✓ 1$$

$$cov(x,y) = \frac{\sum_{i=1}^{6}(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{(-3)(-0.75) + (-2)(-0.35) + (-1)(-0.25) + 2(0.35) + 4(0.85)}{5} =$$

$$\beta = \frac{cov(x,y)}{s_x^2} = \frac{1.46}{6.8} =$$

$$\approx 0.215 \quad ✓ 1$$

$$= \frac{1.46}{\quad} \quad ✓ 1$$

$Y = -135.38 + 0.215X$ ✓ 1

$$\alpha = \bar{y} - \beta\bar{x} = 3.75 - 0.215 \times 648 = -135.38 \quad ✓ 1$$

β is the slope (degree of the slope), α is where the line cross the y-axis.

If $x = 0$, α is the value of y.

3

6.4)

~~$\bar{x} = 4.78$~~ $\bar{x} = 4.89$

	x_1	x_2	x_3
$\bar{x}_1 = 5.33$	3	3	2
$\bar{x}_2 = 4.667$	5	5	6
$\bar{x}_3 = 4.667$	8	6	6

i) $SST = \sum_{i=1}^{m \times n} (x_i - \bar{x})^2 = (-1.89)^2 \cdot 2 + (-2.89)^2 + (0.11)^2 \cdot 2 +$

$+ (1.11)^2 \cdot 3 + (3.11)^2 =$

$= 7.14 + 8.35 + 0.024 + 3.696 + 9.6721 =$

$= 28.88$ ✓ 1

$SSW = (-2.33)^2 + (0.33)^2 + (2.67)^2 +$

$+ (-1.667)^2 + (0.33)^2 + (1.33)^2 +$

$+ (-2.667)^2 + (1.33)^2 + (1.33)^2 =$

$= 12.667 + 4.667 + 10.667 \approx$

≈ 28 ✓ 1

$SSB = 3(5.33 - 4.89)^2 + 3 \times 2(4.667 - 4.89)^2 \approx$

≈ 0.88 ✓ 1

$SST = SSW + SSB$ ✓ very good

$28.88 = 28 + 0.88$

3

6.4)

(i) SST degree of freedom: $3 \times 3 - 1 = 8$ ✓

SSB degree of freedom: $3 - 1 = 2$ ✓

SSW degree of freedom: $3(3-1) = 6$ ✓

$8 = 2 + 6$ ✓ 3

(iii) H_0 is that $\mu_1 = \mu_2 = \mu_3$ ✓ 1

The null hypothesis is that all means are equal

(iv) $F = \frac{SSB/m-1}{SSW/m(n-1)} = \frac{0.88/2}{28/6} = \frac{0.44}{4.667} \approx 0.094$

✓ 1

$x_{0.1} = 3.46$

$0.094 < 3.46$

$\Rightarrow$ H_0 cannot be rejected ✓

No, the difference is not statistically significant

1

9

6.5)

i) c, b ✓

ii) b ✓

iii) a ✓

1 ✓