CHALMERS
EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kursnamn			
DAT060	Logic in Computer Science.			
Anonymous code Anonym kod		Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
DAT060-0006-RZK		2023/08/14	14	4

confirm that I've no mobile or other similar electronic equipment available during the examination.
ıg intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under
ximinationen.

ılved task ehandlade uppgifter No/nr		Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
1	√	6	
2	√	3.25.	
3	√	6.5.	
4	√	9	
5	√	1	
6	√	0	
7	√	6	
8	√	4	
9	√	2	
10	√	5	
11	BP	4.1	
12		~~[illegible]~~	46,85 → 47.
13			
14			
15			
16			
17			
Bonus poäng			
Total examination points Summa poäng på tentamen			

Lemma List

$\neg p \vee q \vdash p \to q$.

1.	$\neg p \vee q$	~~assume~~ premise
2.	p	assume.
3.	$\neg p$	assume
4	$\bot$	$\neg e$ 2 3
5	q	q
6	q	assume
7	q	copy 6
8	q	$\vee e$ 1, 3-5, 6-7
9	$p \to q$	$\to i$ 2-8.

$p \to q \vdash \neg p \vee q$.

1.	$p \to q$	Premise
2.	$p \vee \neg p$	LEM
3.	p	assume
4	q	$\to e$ 1,3
5	$\neg p \vee q$	$\vee i_2$ 4
6	$\neg p$	assume
7	$\neg p \vee q$	$\vee i_1$ 6
8	$\neg p \vee q$	$\vee e$ 2 3-5, 6-7.

$\neg(p \to q) \vdash \neg(\neg p \vee q)$

1.	$\neg(p \to q)$	premise
2.	$\neg p \vee q$	assume.
3	$p \to q$	$\neg p \vee q \vdash p \to q$
4	$\bot$	$\neg e$ 1,3
5	$\neg(\neg p \vee q)$	$\neg i$ 2-4.

$\neg(p \wedge q) \vdash \neg p \vee \neg q$.

1.	$\neg(p \wedge q)$	premise.
2.	p	assume.
3.	q	assume.
4	$p \wedge q$	$\wedge i$ 2 3.
5	$\bot$	$\neg e$ 1,4
6	$\neg q$	$\neg i$ 3-5
7	$p \to \neg q$	$\to i$ 2-6.
8	$\neg p \vee \neg q$	$\phi \to \psi \vdash \neg \phi \vee \psi$.

$\neg \exists x. P(x) \vdash \forall x. \neg P(x)$

1.	$\neg \exists x. P(x)$	premise
2.	x_0	fresh.
3	$P(x_0)$	assume
4	$\exists x. P(x)$	$\exists i$ 3
5	$\bot$	$\neg e$ 1,4
6	$\neg P(x_0)$	$\neg i$ 3-5
7	$\forall x. \neg P(x)$	$\forall i$ 2-6.

$\neg \forall x. P(x) \vdash \exists x. \neg P(x)$

1.	$\neg \forall x. P(x)$	premise
2.	$\neg \exists x. \neg P(x)$	assume
3.	$\forall x. \neg\neg P(x)$	$\neg \exists x. \phi \vdash \forall x. \neg \phi$.
4	x_0	fresh
5	$\neg\neg P(x_0)$	$\forall e$ 3 with 4
6	$P(x_0)$	$\neg\neg e$ 5
7	$\forall x. P(x)$	$\forall i$ 4-6
8	$\bot$	$\neg e$ 1,7
9	$\exists x. \neg P(x)$	PBC. 2-8.

$\neg(p \vee q) \vdash \neg p \wedge \neg q$

1.	$\neg(p \vee q)$	premise
2.	$\neg(\neg p \wedge \neg q)$	assume
3.	$\neg\neg p \vee \neg\neg q$	$\neg(\phi \wedge \psi) \vdash \neg \phi \vee \neg \psi$).
4	$\neg\neg p$	assume
5	p	$\neg\neg e$ 4
6	$p \vee q$	$\vee i_1$ 5
7	$\bot$	$\neg e$ 1,6
8	$\neg\neg q$	assume.
9	q	$\neg\neg e$
10	$p \vee q$	$\vee i_2$ 9
11	$\bot$	$\neg e$ 1 10
12	$\bot$	$\vee e$ 3, 4-7, 8-11
13	$\neg p \wedge \neg q$	PBC.

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Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 6

Consecutive page no. / Löpande sid nr: 2

Question no. / Uppgift nr: 1

1.

(a) $(p \to q) \to p \vdash p$

1.	$(p \to q) \to p$	premise
2.	$\neg p$	assume
3.	$\neg(p \to q)$	MT 1,2.
4	$\neg p \vee q$	$\vee_i$ 2.
5	$p \to q$	$\neg\phi \vee \psi \vdash \phi \to \psi$ check Lemma list page.
6	$\bot$	$\neg e$ 3 5
7	p	PBC 2-6.

(b) $\vdash ((p \to q) \to q) \to (q \to p) \to p$

$\neg(p \to q)$
$\Leftrightarrow \neg(\neg p \vee q)$.
$\Leftrightarrow \neg\neg p \wedge \neg q$.

1	$((p \to q) \to q$	assume.
2.	$(q \to p)$	assume
3	$\neg p$	assume.
4	$\neg q$	MT 2,3.
5	$\neg(p \to q)$	MT 1,4
6	$\neg p \vee q$	$\vee_i$, 3
7	$p \to q$	$\neg\phi \vee \psi \vdash \phi \to \psi$ check lemma list page.
8	$\bot$	$\neg e$ 5,7
9	p	PBC 3-8
10	$(q \to p) \to p$	$\to_i$ 2-9
11	$((p \to q) \to q) \to (q \to p) \to p$	$\to_i$ 1-10.

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3

2.

① when ~~a~~ v(p), v(s), v(r), v(p) are all equal to true, the valuation of the formula is also true

②. Since the formula is in conjunctive form, all three sub-formulas have to ~~the~~ have the valuation of true, to make the whole formula true.

Starting by tackling the second formula, $((q \vee p) \to (s \wedge r))$, to make this formula true, there are several choices at our disposal. Let's starting by assuming they all have the valuation of true. $v(q) \vee v(p) = true$, $v(s) \wedge v(r) = true$. Hence $v((q \vee p) \to (s \wedge r)) = true$.

wrong notation

Let's propogate these valuations to other subformula.

* ① $p \vee (s \to r)$

$v(p) = true$ $v(s \to r) = true$. Hence $v(p \vee (s \to r)) = true$.

② $s \vee (r \to p) \vee q$.

$v(s) = true$

$v(r \to p) = true$

$v(q) = true$

Hence. $(s \vee (r \to p) \vee q) = true$.

Hence, all three subformula have the valuation of true. The whole formula has the valuation of true.

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3.

(a) $\vdash \forall x \forall y \forall z . \forall w . x = z \to y = w \to R(x,y) \to R(z,w)$.

1.	$\neg \forall x \forall y \forall z \forall w . x=z \to y=w \to R(x,y) \to R(z,w)$	assume
2.	$\exists x . \neg \forall y \forall z \forall w . x=z \to y=w \to R(x,y) \to R(z,w)$	$\neg \forall x.\phi \vdash \exists x. \neg\phi$. ~~the~~ check lemma page.
3	x_0	fresh
4	$\neg \forall y \forall z \forall w . x_0=z \to y=w \to R(x_0,y) \to R(z,w)$	assume.
5	$\exists y . \neg \forall z \forall w . x_0=z \to y=w \to R(x_0,y) \to R(z,w)$	$\neg \forall x.\phi \vdash \exists x.\neg\phi$.
6	y_0	fresh
7	$\neg \forall z \forall w . x_0=z \to y_0=w \to R(x_0,y_0) \to R(z,w)$	assume.
8	$\exists z . \neg \forall w . x_0=z \to y_0=w \to R(x_0,y_0) \to R(z,w)$	$\neg \forall x.\phi \vdash \exists x.\neg\phi$.
9	z_0	fresh
10	$\neg \forall w . x_0=z_0 \to y_0=w \to R(x_0,y_0) \to R(z_0,w)$	assume.
11	$\exists w . \neg (x_0=z_0 \to y_0=w \to R(x_0,y_0) \to R(z_0,w))$	$\neg \forall x.\phi \vdash \exists x.\neg\phi$.
12	w_0	fresh
13	$\neg (x_0=z_0 \to y_0=w_0 \to R(x_0,y_0) \to R(z_0,w_0))$	assume.
14	$\neg (\neg(x_0=z_0) \vee (y_0=w_0 \to R(x_0,y_0) \to R(z_0,w_0)))$	$\neg(\phi\to\psi) \vdash \neg(\neg\phi \vee \psi)$
15	$\neg\neg x_0=z_0 \wedge \neg (y_0=w_0 \to R(x_0,y_0) \to R(z_0,w_0))$	$\neg(\phi \vee \psi) \vdash \neg\phi \wedge \neg\psi$.
16	$\neg\neg x_0=z_0$	$\wedge e_1$ 15
17	$\neg (y_0=w_0 \to R(x_0,y_0) \to R(z_0,w_0))$	$\wedge e_2$ 15
18	$x_0=z_0$	$\neg\neg e$ 16.
19	$\neg(\neg(y_0=w_0) \vee (R(x_0,y_0) \to R(z_0,w_0)))$	$\neg(\phi\to\psi) \vdash \neg(\neg\phi \vee \psi)$
20	$\neg\neg y_0=w_0 \wedge \neg(R(x_0,y_0) \to R(z_0,w_0))$	$\neg(\phi \vee \psi) \vdash \neg\phi \wedge \neg\psi$.
21	$\neg\neg y_0=w_0$	$\wedge e_1$ 20
22	$\neg(R(x_0,y_0) \to R(z_0,w_0))$	$\wedge e_2$ 20
23	$y_0=w_0$	$\neg\neg e$.
24	$\neg(\neg R(x_0,y_0) \vee R(z_0,w_0))$	$\neg(\phi\to\psi) \vdash \neg(\neg\phi \vee \psi)$
25	$\neg\neg R(x_0,y_0) \wedge \neg R(z_0,w_0)$	$\neg(\phi \vee \psi) \vdash \neg\phi \wedge \neg\psi$.
26	$\neg\neg R(x_0,y_0)$	$\wedge e_1$ 25
27	$\neg R(z_0,w_0)$	$\wedge e_2$ 25
28	$R(x_0,y_0)$	$\neg\neg e$ 26.
29	$R(z_0,y_0)$	$=e$ with $\phi \equiv$ ~~with~~ $x_0=z_0$.
30	$R(z_0,w_0)$	$=e$ with $\phi = y_0=w_0$.
31	$\perp$	$\neg e$ 27, 30
32	$\perp$	$\exists e$ 11, 12–~~13~~31
33	$\perp$	$\exists e$ 8, 9–32
34	$\perp$	$\exists e$ 5, 6–33.
35	$\perp$	$\exists e$ 2, 3–34.
36	$\forall x \forall y \forall z \forall w . x=z \to y=w \to R(x,y) \to R(z,w)$	PBC.

1.5

this is an unnecessarily complicated solution.

?

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~~Give~~ (b) $\forall x.(P(x) \to Q(x)) \vdash \exists x. P(x) \to \forall x. Q(x)$

LHS RHS

Q^M?

Given a model.

$A = \{1,2\}$.

$P^M = \{1\}$

$Q^M = \{1\}$.

In this model, LHS is satisfied as $1 \in P^M$ and $1 \in P^M$. ?

Since $2 \notin P^M$. ~~it~~ ~~no need to~~ whether or not $2 \in Q^M$ is irrelevant.

not clear

2

However, for the RHS, since $1 \in P^M$, it's necessary for both $1, 2 \in Q^M$. In this case $2 \notin Q^M$. Hence, RHS is not satisfied

should be better explained.

(c) $\vdash (\neg\neg\forall x. P(x)) \to \forall x. \neg\neg P(x)$.

1.	$\neg\neg\forall x. P(x)$	assume
2.	$\forall x. P(x)$	$\neg\neg e$
3	x_0	fresh
4	$P(x_0)$	$\forall e$ 2 with 3
5	$\neg\neg P(x_0)$	$\neg\neg i$ 4
6	$\forall x. \neg\neg P(x)$	$\forall i$ 3-5
7	$\neg\neg\forall x. P(x) \to \forall x. \neg\neg P(x)$	$\to i$ 1-6.

3

(d) $\vdash \exists x. (P(x) \to \forall y. P(y))$.

Given a model

$A = \{1,2\}$

$P^M = \{1\}$.

In this model, the formula is not satisfied.

As, exist an element 1, $1 \in P^M$, but not every element in A, such as 2, $a \in P^M$.

Hence, this formula is not satisfied.

3

CHALMERS

Anonymous code / Anonym kod: DAT060-0006-RZK

Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 9

Consecutive page no. / Löpande sid nr: 6

Question no. / Uppgift nr: 4

4(a).

The model for this language consists of an non-empty domain A, and a binary predicate function R^M: ~~$A \times A \to Bool$~~. $R^M \subseteq A \times A$

1

Answer only one question on this page. Do not write on the back of this paper
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3

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4(b) $\exists x. \forall y. R(x,y) \models \forall y. \exists x. R(x,y)$.

LHS translates

~~for every element a~~

exists at least an element a in A, such that for every element b in A, such that

$$M \models_{[x \mapsto a,\ y \mapsto b]} R(x,y) \text{ holds.}$$

RHS translates

for every element b in A, such that exists at least an element a in A, such that

$$M \models_{[x \mapsto a,\ y \mapsto b]} R(x,y).$$

2

~~Given a model~~ ~~In this model, the LHS is satisfied, as exists an element 1, $(1,x) \in R^M$, for every x in A.~~

~~$A = \{1,2,3\}$.~~

~~$R^M = \{(1,2),(1,3),(1,1)\}$.~~ ~~However,~~

Since the LHS guarantees that exists an element a in A, $(a,x) \in R^M$ for every x in A, for every element x in A, $(a,x) \in A$. ?

Hence, the ~~formula~~ entailment holds.

could be explained better.

3

CHALMERS

Anonymous code / Anonym kod: DAT060-0006-RZLC

Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): —

Consecutive page no. / Löpande sid nr: 8

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4 (ii) $\forall y. \exists x. R(x,y) \models \exists x. \forall y. R(x,y)$

LHS translates

for every element a in A, exists an element b in A. $M \models_{[x \mapsto b][y \mapsto a]} R(x,y)$ holds.

RHS translates

exists at least one element b in A, for every element a in A. $M \models_{[x \mapsto b][y \mapsto a]} R(x,y)$ holds.

3.

Given a model

$A = \{1,2,3\}$

$R^M = \{(1,3), (2,1), (3,2)\}$.

Hence, this entailment doesn't hold.

In this model, the LHS is satisfied, as for every element $\tilde{a}$ in A, they all have ~~an~~ corresponding element b, such that $(b,a) \in R^M$. However, ~~there is no~~ ~~an~~ it doesn't exist an element b which $(b,c) \in R^M$ for every ~~b~~ c in A.

(iii) $\forall x. \forall y. R(x,y) \models \neg \forall x. \neg R(x,x)$

LHS translates

for every element a in A, for every element b in A. $M \models_{[x \mapsto a][y \mapsto b]} R(x,y)$ holds.

RHS translates to.

it's not the case that, for element a in A, it is not the case that $M \models_{[x \mapsto a]} R(x,x)$ holds.

$\Updownarrow$ semantically equal to.

3.

exists an element a in A. it's not the case that it's not the case that $M \models_{[x \mapsto a]} R(x,x)$ holds

$\Updownarrow$ semantically equal to

exists an element a in A. $M \models_{[x \mapsto a]} R(x,x)$ holds.

LHS ensures that for every element a in A. $(a,b) \in R^M$ for every b in A. including a itself. Hence, it's definitely the case that exists an element a, ~~R(a,a)~~ $(a,a) \in R^M$

Hence this entailment holds.

CHALMERS

Anonymous code / Anonym kod: DAT060-0006-RZLC

Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 1

Consecutive page no. / Löpande sid nr: 9

Question no. / Uppgift nr: 5

5 ~~$A = \{1, 2, 3\}$.~~

~~$R^M = \{(1,2), (2,3), ($~~ the semantic of R is less than

$A =$ ~~A~~. ~~Real time~~ Both negative and positive integers

$\forall x. \neg R(x,x)$. In this model, every ~~natural number~~ integers is not less than itself.

$\forall x \exists y\ R(x,y)$ for every ~~number natural number,~~ integer it always exists another ~~natural~~ integer ~~number~~ greater than itself.

$\forall x \forall y \forall z. (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$ less than in ~~natural number~~ integer is transmitive. $a<b$ and $b<c$, $a<c$ for every $a, b, c \in$ ~~Ns~~. integers

~~Q~~. $\neg(\forall x \exists y. R(y,x))$ ~~in~~ for every integer i, there is always another integer (i-1) which is less than i

$R^M = \{(a,b) \mid a<b,\ a,b \in \text{Integer}\}$. ?

CHALMERS	Anonymous code / Anonym kod: DAT060-0006-RZK	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 0	Consecutive page no. / Löpande sid nr: 10
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6. Herbrand theorem states that it ~~a~~ every subsets of an infinite theory has a model, the infinite theory also has a model.

finite

Since it's not infinite ~~is~~ in every subset, it's still finite. every subset has a model.

Hence, the infinite theory has a model.

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3

7. $F(X) = X \cup A - B$

$F(X \cup A - B) = X \cup A - B.$

2

Since union and difference function on set are monotone function.

function F is also monotone.

(b) $F(\phi) = \phi \cup A - B$

$= A - B.$

2 The least fix point of this function is $A - B$.

$F(A-B) = (A-B) \cup A - B$

$= A - B.$

$F(S) = S \cup A - B$

$= S - B.$

2 The greastest fix point of this function is $S - B$.

$F(S-B) = (S-B) \cup A - B.$

$= S - B$

3

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8. $G(Fp \wedge Fq) \to GF(p \wedge q)$.

2

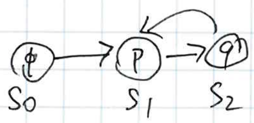
LHS

For an arbitrary path π, it's always the case that exists a future p holds and exists a future q holds

RHS

for an arbitrary path π, it's always the case that exists a future p and q both hold.

Given the model



In this model, since p and q alternatively becomes ~~tot~~ valid, it satisfies the LHS. However, it doesn't exist a state n. $\pi, n \models p \wedge q$.

Hence, it doesn't satisfy the RHS.

Hence, the formula is not valid.

$GFp \wedge FGq \to GF(p \wedge q)$.

LHS

For an arbitrary path π, it's always the case that exists a future p holds, and it's has a future that q holds from now on.

RHS

for an arbitrary path π, it's always the case that exists a future p and q both hold

LHS states that.

2

for an arbitrary path π, exists a state n, $\pi, n \models p$, and exists a state m, $\pi, i \models \cancel{p}\ q$ for every $i \geq m$.

Then, exists a state j, $j = \max(n, m)$, $\pi, j \models p \wedge q$.

This meets the definition of RHS

Hence this formula is valid.

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9 (a) a CTL model is given by a transition system, $S, \rightarrow$, in which for every state s, there exists ~~at least~~ s', $s \rightarrow s'$. and a labeling function $L(s)$ is the set of all atomic formula valid in that state.

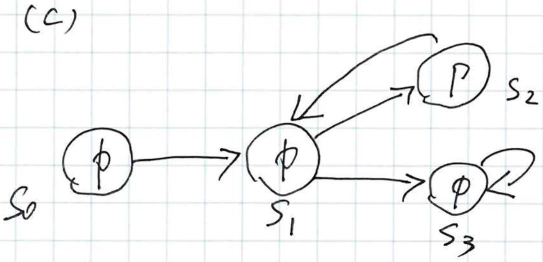
1

(b) we can do it by checking whether or not exists ~~an~~ at least an path in which p holds ~~at~~ all the time

?

0

(c)



~~AG(EFp) tr~~

~~$S_0 \models AG(EFp)$ translates to for every path~~

In this model, it exists a path $\pi = S_0 \rightarrow S_1 \rightarrow S_3 \cdots$, in which p never hold.

1

However, for pathes start at S_0, exists at least a path has a future where p holds.

AG(EF p) not valid at

EF p not valid at S_3

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Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 5

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6.

(a) $M \models AG(q \rightarrow AF(AG\,p))$

for every pathe π, it's always the case that if ~~$\pi \models q$~~ $\pi^u \models q$ then for every pathes starts at u, it has a future that for every pathe starts at m, p always holds.

2.5

~~(a)~~ Yes, for pathes start at S_2, they ~~all~~ always end in ~~S_4~~ S_4 and S_0, in which p holds constantly.

(b) in the given model, all states in S satisfy the LTL formula, as all pathes either end up in S_0, or S_4. In both states, p holds constantly.

2.5