CHALMERS
EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kurskod		*
DIT022	Mathematical Foundations for Software Eng.		

Anonymous code Anonym kod	Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
586	29/10/19	17	VG

* I confirm that I've no mobile or other similar electronic equipment available during the examination.
Jag intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under examinationen.

Solved task Behandlade uppgifter No/nr		Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
1	x	9	
2	x	13	
3	x	14	
4	x	9	
5	x	11	
6	x	27	
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
Bonus credits/ poäng	8		
Total examination points Summa poäng på tentamen		~~83~~ 91	

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~~1.1~~ $S \to X \to aX \to aAB$
$\to bX$

6

1.1 b. 1

1.2 a. 1

1.3 d. 1

1.4 e. 1

$S \to aaSb \mid \lambda$
$\to aaaaSbb.$
$\to aaaaaaSbbb$

$S \to 1S \mid 1A$ ~~0000A~~ .
$A \to 00A$
$A \to 0.$

$11A$
$11A.$
$1100A.$
$11000.$

1.5

exp → term
term → factor
factor → constant
constant → integer
integer → 0.

~~exp →~~

exp → term
term → ~~factor~~ term / factor
~~f~~ → factor / facior
→ constant / constant
→ integer / integer
→ 1/9.

∴ 0 1, 1/9 1.

Answer only one question on this page. Do not write on the back of this paper

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Anonymous code Anonym kod	Points for question (to be filled in by teacher) Poäng på uppgiften (ifylles av lärare)	Consecutive page no. Löpande sid nr	Question no. Uppgift nr
DIT022 586		2	1

3

1.6

~~<term> ::= x | x^2 | x^3 | x^4~~

~~<sign><term> ::=~~

~~<sign> :: + | -~~

~~<term> :: = x | x^<digit> |<digit>~~

~~<sign> :: + | -~~

This should've been another terminal

<term> :: = x | x^<power> |<digit>

<power> :: = <digit>

<sign> :: = + | - .

3

CHALMERS	Anonymous code / Anonym kod: DIT022 586	Points for question (to be filled in by teacher) / Poäng pa uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 3 — Question no. / Uppgift nr: 2

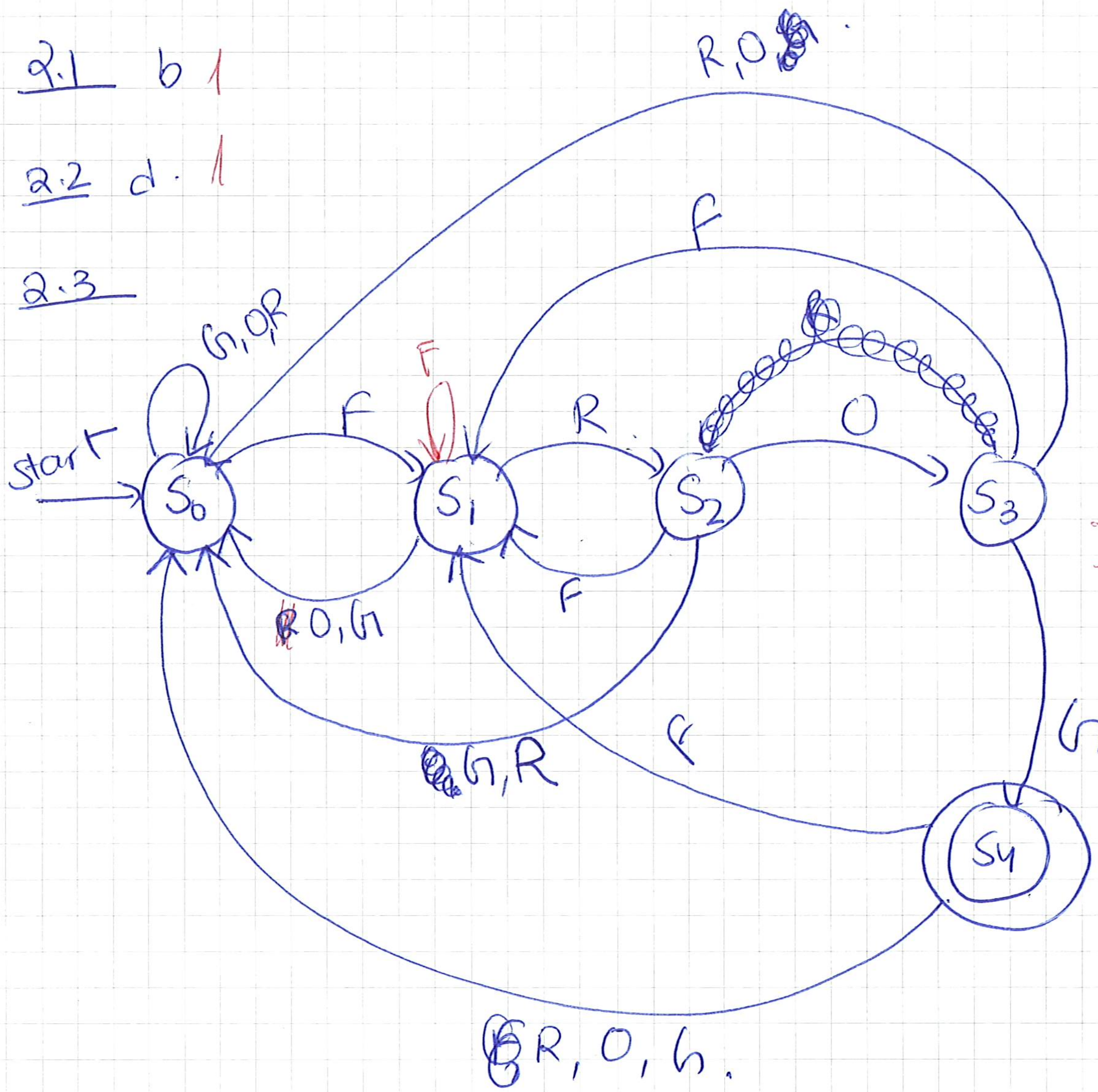
3

5

2.1 b 1

2.2 d. 1

2.3



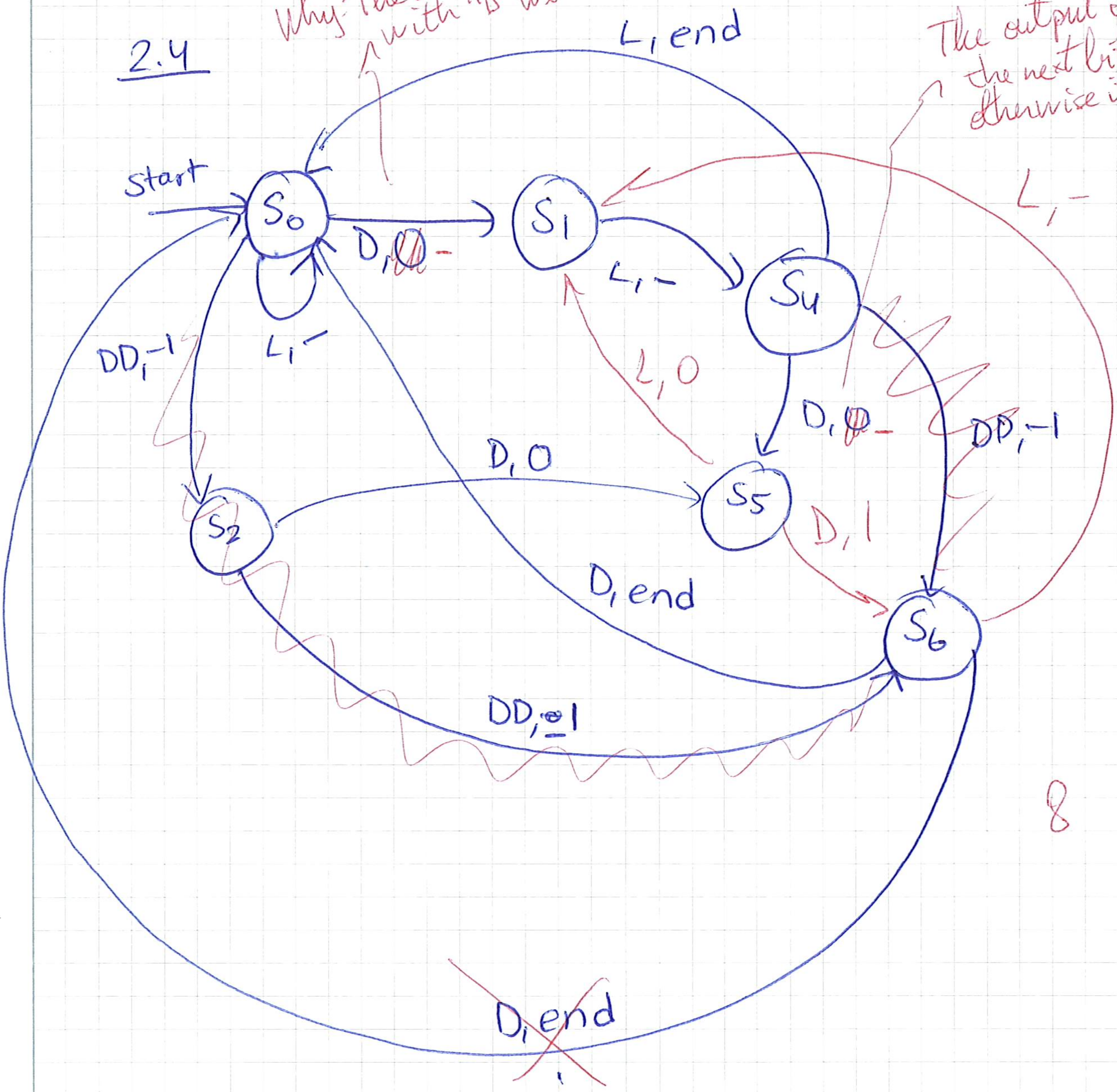
CHALMERS	Anonymous code / Anonym kod	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr
3	DIT022 586		4
			Question no. / Uppgift nr 2

8/

Why? The word could start with "D" without any "L"

The output is only "0" if the next bit is "L", otherwise it is "1"

2.4



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Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare): 4

Consecutive page no. / Löpande sid nr: 5

Question no. / Uppgift nr: 3

3.1

p	q	r	$p \wedge \neg p$	$(p \wedge \neg p) \vee \neg r$	$((p \wedge \neg p) \vee \neg r) \to \neg q \wedge \neg r$	$(\neg q \wedge \neg r)$	$((p \wedge \neg p) \vee \neg r) \to (\neg q \wedge \neg r) \leftrightarrow p$
0	0 1	0 1	0	1	1	1	0
0	0 1	1 0	0	0	1	0	0
0	1 0	0 1	0	1	0	0	1
0	1 0	1 0	0	0	1	0	0
1	0 1	0 1	0	1	1	1	1
1	0 1	1 0	0	0	1	0	1
1	1 0	0 1	0	1	0	0	0
1	1 0	1 0	0	0	1	0	1

8

3.2 c 1

$x+1 > 2x$

$1+1 > 2(1)$ X $x=1$

$2+1 > 2(2)$

$3 > 4$ X $x=2$

$x+1 > 2x$

$-x > -1$

$x < 1$

3.3 b 0

~~3.4~~

CHALMERS | Anonymous code / Anonym kod: DIT022 586 | Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare) | Consecutive page no. / Löpande sid nr: 6 | Question no. / Uppgift nr: 3

3

5

3.4

p	q	r	$p \to r$	$q \vee r$	$p \wedge q$	r
0	0	0	1	0	0	1
0	0	1	1	0 1	0	1
0	1	0	1	0 1	0 1	
0	1	1	1	1	0 1	
1	0	0	0	0	0 1	
1	0	1	1	1	0 1	
1	1	0	0	0 1	1	0
1	1	1	1	1	1	1

~~$(p \to r) \vee$~~

$$\underbrace{((p \to r) \to (q \vee r))}_{\text{always true}} \vee ((p \wedge q) \to r)$$ 5

true $\vee$ whatever $\equiv$ true.

Contradiction? :(

Answer only one question on this page. Do not write on the back of this paper

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Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)

Consecutive page no. / Löpande sid nr: 7

Question no. / Uppgift nr: 4

4.1 ii) ~~Th~~. The theorem is only tested. It is not a direct proof. ✓ 1

i) The counter examples given are irrelevant to the ~~question~~ given proof. 0

$2^k \bmod 3 \neq 0$ never equals 0, if $k > 1$. ~~and then~~

At first, they did not prove the first ~~of~~ statement.

which means? F→F≡T so the claim is true

4.2 $p \rightarrow$ no largest even number

$\neg p \rightarrow$ finite even numbers.

Assume $\neg p$ is true,

If m is ~~a~~ the largest even number, then there exist a k, ~~w~~ such that ✓ 1

$$m = 2k.$$

If we multiply m by 2, then an even number p is generated which is

$p = 2m$. ✓ 1 $p > m$.

~~s~~ which means that m is not the largest even number. ✓ 1

$\neg p$ does not hold

Therefore, p must hold.

4

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3

CHALMERS	Anonymous code / Anonym kod: DIT022 586	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 8 — Question no. / Uppgift nr: 4

4.3

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\cdots\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1} \qquad n>0$$

Proof by induction,

Basic step : $n=1$

$$\frac{1}{1\times3}=\frac{1}{2(1)+1}$$

$$\frac{1}{3}=\frac{1}{2+1}=\frac{1}{3}$$ ✓1

~~Proof~~

Inductive step, $\boxed{n=k}$

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\cdots\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$$ ✓1

$$n=k+1 \longrightarrow \frac{k+1}{2(k+1)+1}=\frac{k+1}{2k+2+1}=\frac{k+1}{2k+3}$$

$$\underbrace{\frac{1}{1\times3}+\frac{1}{3\times5}+\cdots+\frac{1}{(2k-1)(2k+1)}}+\frac{1}{(2(k+1)-1)(2(k+1)+1)}$$ ✓1

$$\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$$ ✓1 / addition

/ ~~cutti~~

$$\frac{k(2k+3)}{(2k+1)(2k+3)}$$

$$=\frac{k}{(2k+1)}$$

4

CHALMERS	Anonymous code / Anonym kod: DIT022 586	Points for question (to be filled in by teacher) / Poäng pa uppgiften (ifylles av larare)	Consecutive page no. / Löpande sid nr: 9 — Question no. / Uppgift nr: 4

4.3 continued

$$\frac{k}{2k+1} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$\frac{k}{2k+1} + \frac{1}{(2k+2-1)(2k+2+1)}$$

$$\cancel{\frac{k}{(2k+1)}}$$

$$\frac{k}{(2k+1)} + \frac{1}{(2k+1)(2k+3)}$$

$$\frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$\frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$\frac{2k^2+2k+k+1}{(2k+1)(2k+3)}$$

$$\frac{2k(k+1)+1(k+1)}{(2k+1)(2k+3)}$$

$$\frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$$

$$\frac{k+1}{2k+3} \quad //$$ proved.

1

1

Answer only one question on this page. Do not write on the back of this paper

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3

~~5.~~

5.1 $O(n^2)$ ✓1 $\qquad n \times n + n \times n$

5.2 ~~O(n lo~~ $O(\cancel{n}\log n)$. // $\qquad F(n) = F(n-1) + F(n-1)$

5.3 $O(n\log n)$ ✓1

5.4

i=0

6	-4	9	13	31	7	-1	27	42
-4	6	9	13	31	7	-1	27	42
-4	6	9	13	7	31	-1	27	42
-4	6	9	13	7	-1	31	27	42
-4	6	9	13	7	-1	27	31	42

i=1

-4	6	9	7	13	-1	27	31	42
-4	6	9	7	-1	13	27	31	42

i=2

-4	6	9	7	-1	13	27	31	42
-4	6	7	9	-1	13	27	31	42
-4	6	7	-1	9	13	27	31	42

i=3.

-4	6	7	-1	9	13	27	31	42
-4	6	-1	7	9	13	27	31	42
-4	-1	6	7	9	13	27	31	42

✓ 5

7

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			Question no. / Uppgift nr: 5

3

5.5

i) $\deg^+(c) = 1$ ✓ 0,5

$\deg^-(c) = 2$ ✓ 0,5

ii)

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 1 & 0 & 0 \\ b & 1 & 0 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 1 & 0 \\ d & 0 & 1 & 0 & 0 & 1 \\ e & 0 & 0 & 1 & 1 & 0 \end{array}$$

✓ ✓ ✓ ✓ ✓ 2

$\deg^+(a) = 2$, 1

(b) = 2, 2

(c) = 1, 2

(d) = 2, = 2

(e) = 2, 2

iii) ~~d b a c d e~~

0

e d b a b e c d e ∤ a->c

IV) No Euler circuit. Because all the vertices do not have even degree. ✓ 1

4

3

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6.1

Mean, $\bar{x} = 19$ V

mean, $\bar{y} = 10.275$ V

$\sigma_x^2 = 72$

$s_x^2 = 82.2$

$\sigma_y^2 = 50.5$

$s_y^2 = 57.7$

$r = 0.97839$ V

$b = 0.81979$

$$r = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\sum(y_i-\bar{y})^2}}$$ V

$x_i y_i$	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$\hat{y}$
0.5	-14	-10.175	142.45	196	103.5	-1.2
3	-9	-9.975	89.775	81	99.5	2.8969
'	-4	-2.475	9.9	16	6.125	6.99585
'	-2	-0.375	0.75	4	'	8.63543
'	2	0.125	0.25	4	'	11.914
'	5	3.925	19.625	25	'	14.374
	9	9.225	83.025	81	'	17.653
640	13	9.725	126.425	169	'	20.932
			472.2	576	404	17.27

$$r = \frac{472.2}{\sqrt{576 \times 404}}$$

$= 0.978$ V

$$b = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n(\bar{x})^2} = \frac{2034 - 8(19 \times 10.275)}{3464 - 8(19)^2}$$

$= 0.81979.$ V

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3

6.1 continued

$y = a + bx$

$\bar{y} - b\bar{x} = a$ . .

$a = 10.275 - (0.81979 \times 19)$

$= -5.301$

$\therefore\ y = -5.301 + 0.81979x$.

$$\varepsilon = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}$$

$$= \sqrt{\frac{17.27}{6}}$$

$$= 1.69 .$$

Critical value : 0.582 ?

~~0.582~~

~~T~~ Since the value is greater than critical, the correlation is statistically significant

6

good!

6.2

P(beneficial) = 0.85   P(B)
P(not beneficial) = 0.15   P(B').

$P(0 \text{ vampires} \mid \text{beneficial}) = \frac{e^{-2}2^0}{0!}$ V
$P(V|B)$
$= 0.13534$

$P(0 \text{ vampires} \mid \text{not beneficial}) = \frac{e^{-4}4^0}{0!}$ V
$P(V|B')$.
$= 0.0183$.

~~P(beneficial | 0 vampires) =~~
Writing it shortly,
B = beneficial   B' = not beneficial
V = 0 vampires.   ~~V' = no zero~~

$$\therefore P(B|V) = \frac{P(V|B)P(B)}{P(V|B)P(B) + P(V|B')P(B')}$$ V

$$= \frac{0.13534 \times 0.85}{(0.13534 \times 0.85) + (0.0183 \times 0.15)}$$

$$= 0.9767$$

6
great!

6.3

2 $H_0 : \mu_1 = \mu_2 = \mu_3$ 1

$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 .$ 1

3 mean $= 22.8.$

$$SST = (10-22.8)^2 + (11-22.8)^2 + (13-22.8)^2 + (25-22.8)^2 + (33-22.8)^2 + (20-22.8)^2 + (17-22.8)^2 + (11-22.8)^2 + (26-22.8)^2 + (8-22.8)^2 + (34-22.8)^2 + (45-22.8)^2 + (35-22.8)^2 + (21-22.8)^2 + (34-22.8)^2$$

$mn-1$

$= 1813.8$

$$SSW = (10-18.4)^2 + (11-18.4)^2 + (13-18.4)^2 + (25-18.4)^2 + (33-18.4)^2 + (20-16.4)^2 + \ldots + (34-33.8)^2$$

$= 411.2 + 205.2 + 290.8$

$= 907.2 .$

$m(n-1)$

6.3 continued

$$SSB = 5(18.4 - 22.8)^2 + 5(16.4 - 22.8)^2 + 5(33.8 - 22.8)^2$$

$$= 906.6 .$$ ✓

m-1

$$\cancel{F = \frac{SSB}{mn-1}}$$

$$F = \frac{\frac{SSB}{m-1}}{\frac{SSW}{m(m-1)}} = \frac{\frac{906.6}{2}}{\frac{907.2}{3 \times 4}} = 5.996 .$$ ✓

2 12

5/ Reject null hypothesis if ~~critical v~~
$F >$ ~~Fco~~ critical value.

Do not reject otherwise.

6/ $F_{2,12,0.05} = 3.89 .$ ✓

~~Since~~

~~The results are significantly d~~

The null hypothesis is rejected. Their
~~There is differe sig~~ means are not
~~There~~ equal.

great! 10

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3

6.4

$$S = \sqrt{\frac{(11-1)\,6.1^2 + (19-1)\,7.5^2}{(11-1)+(19-1)}}$$

$$= 7.032.$$ ✓

$$t = \frac{22.8 - 19.2}{7.032\sqrt{1/11 + 1/19}}$$

$$= 1.351$$ ✓

$$df = 11+19-2 = 28.$$ ✓

For $\alpha = 0.1$,

~~critical value = 1.313~~ ~~= 1.701~~. ~~= 1.313~~.

– correct! $\alpha = 20\%$

critical value $= 2.048$

For $\alpha = 0.2$

~~critical value = 2.467~~. ~~1.313~~ 0.855

critical value $= 1.701$. $\alpha = 10\%$ correct

Since, the t-statistic is less than the critical values for both $\alpha = 0.1$ & $\alpha = 0.2$, the null hypothesis ~~is~~ ~~not~~ cannot be rejected.

∴ The means are equal.

5

some minor errors.