CHALMERS
EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kursnamn			
DIT 022	Mathematical Foundation for Software Engineering			
Anonymous code Anonym kod		Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
371		07.01.2019	11	~~G~~ VG

* I confirm that I've no mobile or other similar electronic equipment available during the examination.
Jag intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under eximinationen.

Solved task Behandlade uppgifter No/nr		Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
1	✓	8	
2	✓	22	
3	✓	18	
4	✓	10	
5	✓	13	
6	✓	12	
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
Bonus poäng	7		
Total examination points Summa poäng på tentamen		~~84~~ 91	

[redacted]

4

CHALMERS	Anonymous code / Anonym kod: 371	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 1 — Question no. / Uppgift nr: 1

Question 1 - "Languages"

1.1 Answer b) 1

1.2 Answer c) 1

1.3 Answer d) 1

1.4 Answer e) 1

1.5 0

Start → ⟨postalcode⟩ → ⟨forward_or_stationarea⟩ ⟨underscore⟩ ⟨local deliveryunit⟩ → ⟨provarea⟩ ⟨loctype⟩ ⟨letter⟩ _ ⟨digit⟩ ⟨letter⟩ ⟨digit⟩ → A ⟨rural⟩ A _ O ⟨letter⟩ → AOA_O ⟨letter⟩

→ AOA_OA

→ AOA_OB

→ AOA_OC

1.6 ~~Assumming that only positive integers are allowed, and only addition and substraction need to be considered.~~
~~Also assumming that a single number without operand is a allowed expression (e.g. "15")~~

~~⟨term⟩ ::= ⟨positive integer⟩~~
~~⟨positive integer⟩ ::= ⟨non zero digit⟩ 0 | ⟨non zero digit⟩ ⟨number⟩~~
~~⟨non zero digit⟩ ::= 1 | 2 | 3 | ... | 9~~
~~⟨number⟩ ::= ⟨digit⟩ | ⟨digit⟩ ⟨number⟩~~
~~⟨timesbyOperator⟩ ::= · | :~~

See next page!

4

CHALMERS	Anonymous code Anonym kod 371	Points for question (to be filled in by teacher) Poäng på uppgiften (ifylles av lärare)	Consecutive page no. Löpande sid nr 2 Question no. Uppgift nr 1

4

1.6 Assumming that only positive integers are allowed, adding the following production rules need solve the task:

I also assume that a single number with no operand or second number is a valid word.

<expression> ::= <term> <term> <multdiv Operator>

<term> ::= <positive integer>

<multdiv Operator> ::= • | : | • <term> <operator> | : <term> <operator>

<operator> ::= <multdiv Operator> | <addsub Operator>

<addsub Operator> ::= + <term> <operator> | − <term> <operator>

~~<term> ::=~~

<positive integer> ::= 0 | <non zero digit> ~~<number>~~ <digits>

<non zero digit> ::= 1|2|3|...|9

~~<number>~~

<digits> ::= <digit> | <digit> <digits>

These rules added to the already provided ones result in the desired grammar

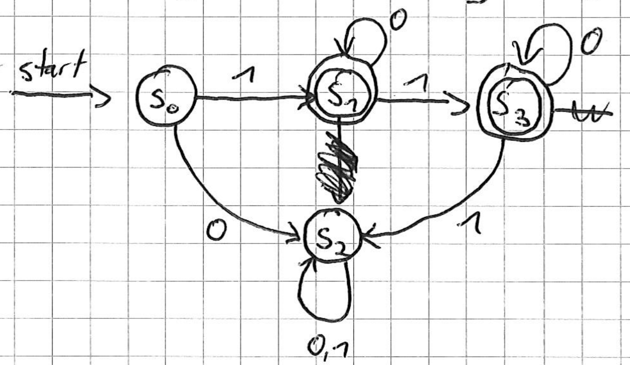
Question 2 - "Automata"

2.1 Answer d) 1

2.2 Answer ~~b)~~ 0

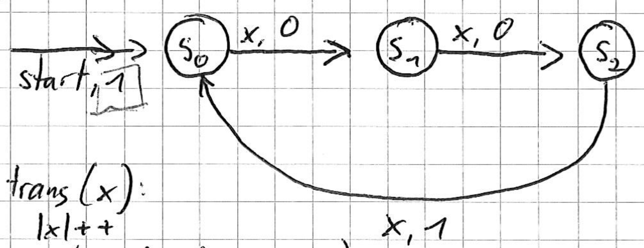
2.3: $S = \{10^n \mid n \geq 0\} \cup \{10^n 10^m \mid n, m \geq 0\}$

7



2.4: Let's define x to be any input symbol.

14



trans(x):
|x|++
return (|x| % 3 == 0)

I am ~~assuming~~ designing the machine to first ~~count~~ count the latest input to the total and then determining the divisibility by 3

The inputs between s_0 and s_1 are ~~every~~ all numbered by the following rule: $i = (3k+1)$, $k \geq 0$

They are therefore not divisible by 3

The inputs between s_1 and s_2 are all numbered by $i = (3k+2)$, $k \geq 0$ => not divisible by 3

And the inputs between s_2 and s_0 are all numbered as $i = 3k$, $k \geq 1$ => divisible by 3

Additionally when starting the ~~program~~ machine, there would be 0 prior inputs and therefore one could argue that the machine should output an initial "1" when starting.

CHALMERS	Anonymous code / Anonym kod: 371	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 4 — Question no. / Uppgift nr: 3

Question 3 – "Logic"

3.1 $\neg[p \wedge (q \vee r) \wedge ((p \wedge q) \to r)] \equiv \neg p \vee (\neg q \wedge \neg r) \vee \neg(\ldots)$

8

p	q	r	Complete expression	$(p \wedge q) \to r$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	1
1	1	0	1	0
1	1	1	0	1

3.2 Answer c 1

3.3 Answer a 1

3.4 i) a compound proposition is a composite of multiple individua logical propositions using ~~conne~~ logical operators (like $\wedge, \vee, \oplus, \to, \leftrightarrow$) to connect them into a single truth-value dependent on the components truth values.
9

ii) A tautology is a compound proposition that is always true (e.g.: $p \vee \neg p$)

iii) A contradiction is a compound proposition which never can be true. (e.g. $p \wedge \neg p$)

4

CHALMERS	Anonymous code / Anonym kod: 371	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 5 — Question no. / Uppgift nr: 3

Question 3 - "Logic"

3.4 Continuation

iv) a contingency is a compound proposition which ~~ca~~ is included in a simpler proposition and can be replaced by it (e.g.: $p \wedge (p \vee q)$ can be replaced with p)
0

v) It is not a contradiction because its ~~truth~~ is true for the following values of p, q, and r:
4

~~p = 0, q = 0~~ $p = 0$, ~~$q \neq 0$~~ $q = 1$, $r = 1$

p	q	r	$p \vee \neg q$	$r \rightarrow (p \vee \neg q)$	$\neg(\ldots)$	$\neg(p \wedge q)$
0	0	0	~~0 0 1~~	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	0	1
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	1	1	0	0
1	1	1	1	1	0	0

vi) $((p \vee \neg q) \vee (r \vee (p \vee \neg q))) \vee p \vee q$
2

This is the same as $\neg q \vee q$ = true.

Question 4 – "Proofs"

6

4.1 $f(x,n) = (1+x)^n$

$g(x,n) = 1+nx$

We want to prove that $f(x,n) \geq g(x,n)$ $\forall n \geq 1$, $\forall x > -1$

Proof by induction:

Base case: Be $n = 1$

$f(x,1) = 1+x$

$g(x,1) = 1+1x = 1+x \Rightarrow f(x,1) = g(x,1)$ ✓

IH: Be $f(x,k) \geq g(x,k)$ already proven.

Induction: $n = k+1$

Then: $f(x,k+1) = (1+x)^{k+1} = (1+x)^k(1+x)$

$= f(x,k)\underbrace{(1+x)}_{>0}$

$\overset{IH}{\geq} g(x,k)(1+x)$

$g(x,k+1) = 1+(k+1)x = 1+kx+x$

$= g(x,k)+x$

$\Rightarrow f(x,k+1) : g(x,k) \geq 1+x$ $\quad | -1$

$g(x,k+1) : g(x,k) = 1+\frac{x}{1+kx}$ $\quad | -1$

$\Rightarrow$ if $1+kx \geq x$ $\forall x>-1, \forall k$

$1+x \overset{!}{\geq} 1+\frac{x}{1+kx}$ $\quad | -1$

$x \geq \frac{x}{1+kx}$ $\quad | \cdot \frac{1}{x}$

$1 \geq \frac{1}{1+kx}$ $\quad | \cdot (1+kx)$

$1+kx$

Continues on page 7

4.1 Continuation

$$f(x, k+1) \geq g(x,k)(1+x)$$
$$g(x, k+1) = g(x,k) + x$$

case $-1 < x < 0$:

$$g(x,k) = 1 + \underbrace{kx}_{<0}$$

$$g(x, k+1) = 1 + \underbrace{(k+1)x}_{<0} < 1$$

$$\cancel{f(x,k) \geq 0}$$
$$f(x, k+1) \geq g(x, k)(1+x)$$

CHALMERS	Anonymous code / Anonym kod: 371	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 8
			Question no. / Uppgift nr: 4

4.2 $f(n) = 2+4+6+\dots+2n$

4 $g(n) = n(n+1) = n^2+n$

~~Proofe~~ Prove that $f(n) = g(n) \quad \forall n > 0$:

Base step: $n = 1$

$f(1) = 2$

$g(1) = 1^2+1 = 2$ ✓

IH: $f(k) = g(k)$

Induction step: $n = k+1$

$f(k+1) = f(k) + 2(k+1) \overset{IH}{=} g(k) + 2k + 2$

$g(k+1) = (k+1)^2 + k + 1 = k^2 + 3k + 2$

$$\begin{aligned} g(k+1) - f(k+1) &= k^2+3k+2 - g(k) - 2k - 2 \\ &= k^2 + k - g(k) \\ &= k(k+1) - g(k) = \underline{\underline{0}} \end{aligned}$$

The difference between two values is zero, if they are the same. Therefore $g(k+1) = f(k+1)$ if $g(k) = f(k)$. It follows from induction that $g(n) = f(n)$ for all $n > 0$ q.e.d.

4

CHALMERS	Anonymous code / Anonym kod: 371	Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)	Consecutive page no. / Löpande sid nr: 9 — Question no. / Uppgift nr: ~~1~~ 5

Question 5

5.1 The complexity is $O(\cancel{n^2})$ 0

5.2 The complexity is $O(n^3)$ 1

5.3 Start: -50 69 5 0 -20 32 1 8 90 103 76

1

i=0 : -50 69 5 0 -20 32 1 8 90 103 76

i=1 : -50 -20 50 69 32 1 8 90 103 76

i=2 : -50 -20 0 5 69 32 1 8 90 103 76

i=3 : -50 -20 0 1 69 32 5 8 90 103 76

i=4 : -50 -20 0 1 5 32 69 8 90 103 76

i=5 : -50 -20 0 1 5 8 69 32 90 103 76

After 6 iterations the array therefore looks as follows:

-50 -20 0 1 5 8 69 32 90 103 76

5.4 The graph in figure 5.4 can not have an Euler circuit as c and f both have an odd number of edges. 1

1

The graph can not have an Euler path for the same reason. An Euler path is possible with a maximum of on vertex having odd degree. 0

4

CHALMERS

Anonymous code / Anonym kod: 371

Points for question (to be filled in by teacher) / Poäng på uppgiften (ifylles av lärare)

Consecutive page no. / Löpande sid nr: 10

Question no. / Uppgift nr: 6

Question 6 – "Statistics"

6.1 a) $\frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$ ✓ 1

b) $8^3 = 512$ (one person can get more than one present)" $\frac{336}{6}$

6.2 I ignore the X-values corresponding to the missing Y-values so they can not bias my results. ✓ 1

$\bar{x} = 651.17$ ✓ 1 $\quad \sum_{i=1}^{6} x_i y_i = 18565.7$

$\bar{y} = 4.75$ ✓ 1 $\quad \sum y_i = 28.5$

$\hat{y} = \alpha + \beta x \quad \sum x_i^2 = 2544143$

$\sum x_i = 3907$

$\cancel{\beta = \frac{\sum x_i y_i - \bar{x} \sum y_i}{\sum x_i^2 - n \bar{x}^2}} = \quad S_i = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$

$S_x = 2.64$ ✓ 1 $\quad S_y = 0.57$ ✓ 1

$\cancel{S_x = 2.41 \quad S_y = 0.522}$

$\alpha = \underline{\underline{-134.52}}$ ✓ 1

$\beta = \cancel{0.2138}$ $\underline{0.214}$ ✓ 1

$\sum (x_i - \bar{x})(y_i - \bar{y}) = 7.45$

$r = \frac{\sum (x_i - x)(y_i - y)}{(n-1) S_x S_y} = \underline{\underline{0.993}}$ ✓ 1

$\hat{y} = \alpha + \beta x$

α = intersection

β = slope ✓

9

Answer only one question on this page. Do not write on the back of this paper
Behandla endast en uppgift på detta blad. Skriv ej på baksidan

CHALMERS	Anonymous code Anonym kod 371	Points for question (to be filled in by teacher) Poäng på uppgiften (ifylles av lärare)	Consecutive page no. Löpande sid nr 11 Question no. Uppgift nr 6

6.3

a) $H_0 :=$ The ~~time~~ true mean of all 5 groups is identical (Null-Hypothesis) ✓

$H_1 :=$ At least one of the 5 groups has a mean different from the others. ✓ 1

b) SST is the total sum of squares.

$$SST = \sum_{i=1}^{k}\sum_{j=i}^{n}(x_{ij} - \bar{x}_t)$$

$$\bar{x}_t = \frac{1}{5}(7.00 + 6.90 + 11.00 + 13.40 + 12.00) = 10.06$$

Counting : 4 5 2×6 2×7 2×8 9 10

Rhyming : 3 4×6 2×7 8 9 11

Adjective : 6 8 10 3×11 3×13 1×14

Imagery : 9 10 3×11 2×12 16 19 23

Intentional : 5 2×10 3×11 2×14 15 19

Total : 3 4(2×5) ~~5×6~~ 7×6 4×7 4×8 2×9 5×10

10×11 2×12 3×13 3×14 15 16 2×19

23

No time to type all of this... ✗ 3

c) $SSB = \Sigma(\bar{x}_i - \bar{x}_t)^2 = 35.152$ ✓ 1

d) $SSW = \Sigma\Sigma(x_{ij} - \bar{x}_i)^2 = ?$ 25.6 made up, no time

e) $f(SST) = 49$, $f(SSB) = 4$, $f(SSW) = 45$ 1

f) $F = \frac{SSB/f(SSB)}{SSW/f(SSW)} = 15.448$ ← used wrong value from before

g) $F^{crit}_{45,4} = 5.71$ 9.08

h) With my made up value $F = 15.448$ the hypothesis holds as it is smaller than F_{crit} 1