1. $N(33,30) \approx 42$

2. $N_x > 0$, $N_y < 0$.

① The gradient vector $\nabla N(45,30)$ is always perpendicular to its contour at $N(45,30)$

② The gradient vector $\nabla N(45,30) = (N_x, N_y)$ always points from a lower function value to a higher function value, which is from $N(x,y) = 40$ to $N(x,y) = 45$ in this case.

Based on reason ① and reason ②, it can be concluded that the gradient vector $\nabla N(45,30) = (N_x, N_y)$ points into the Fourth Quadrant of the x-y coordinate system. Therefore, the sign of N_x is positive and the sign of N_y is negative.

3. $n = 70$.

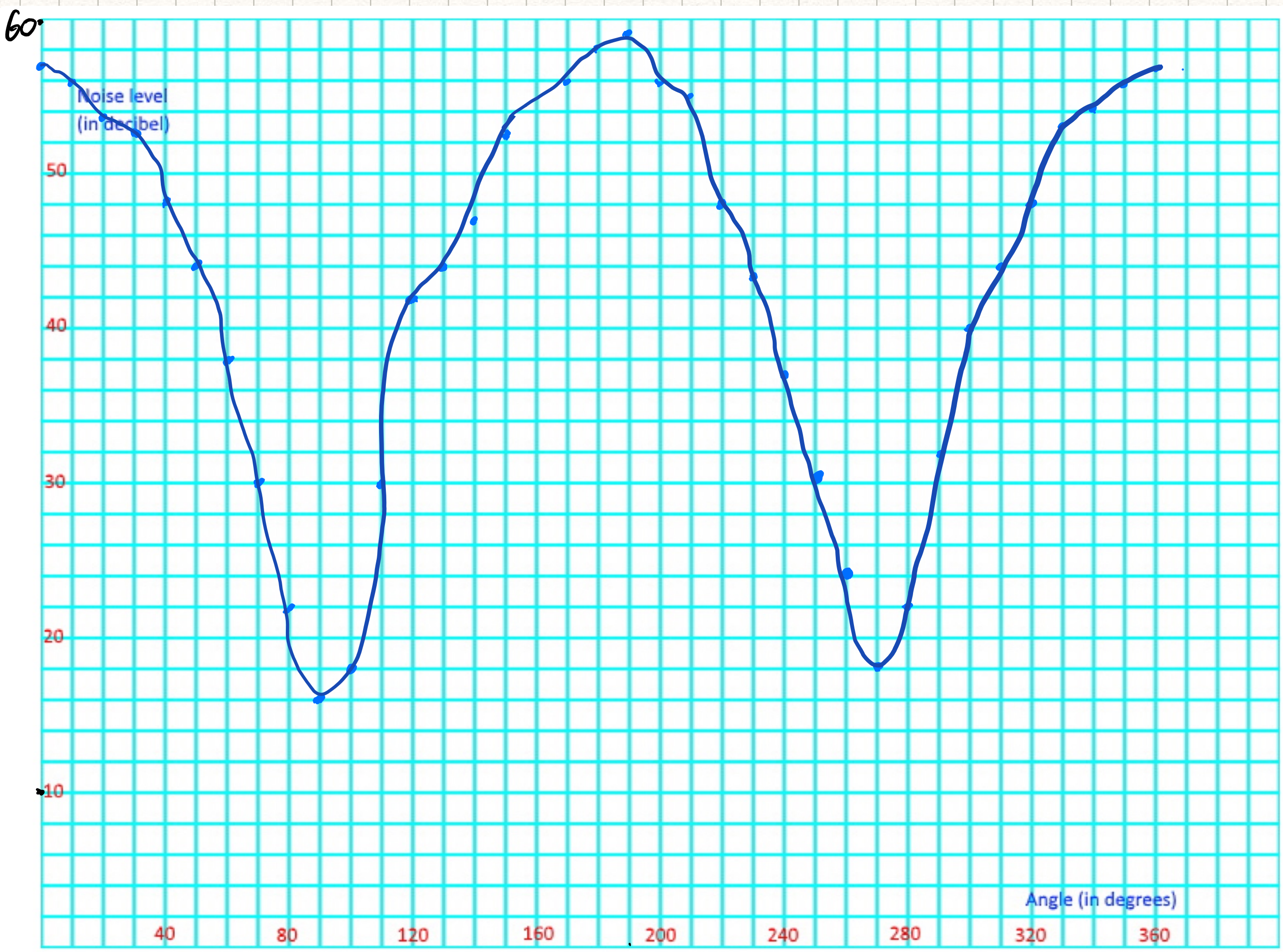
Each level curve differs by 5 dB.

The level curve with a noise value of n is located next to the level curve with value of 65 and the level curve with value 70.

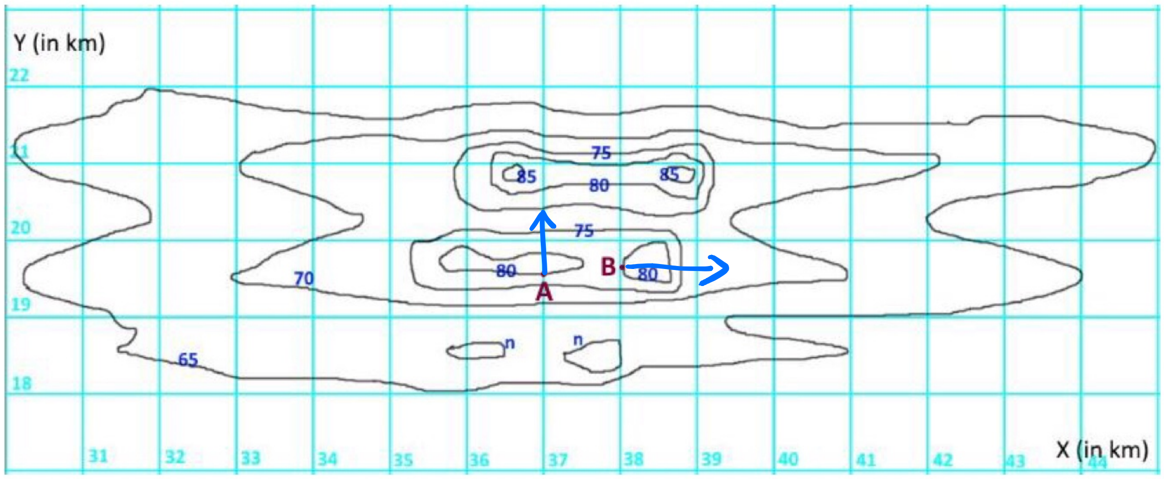
According to the level curve with value of 65, n should be 65 ± 5. While according to the level curve with value of 70 ± 5, n shoud be 70 ± 5. Therefore, $n = 70$.

4. Sketch the graph of the noise levels versus the x-axis for $y = 18$ (use both Fig.1 and Fig.2). Use the plot provided below. [4 points]

5.



6. In the figure below, sketch ∇N at the point A of coordinate $(37, 19.5)$ and the point B of coordinate $(38, 19.6)$. [2 points]



7. The ∇N at point A has a larger magnitude.

The contours at point A are closer than those at point B.

8.

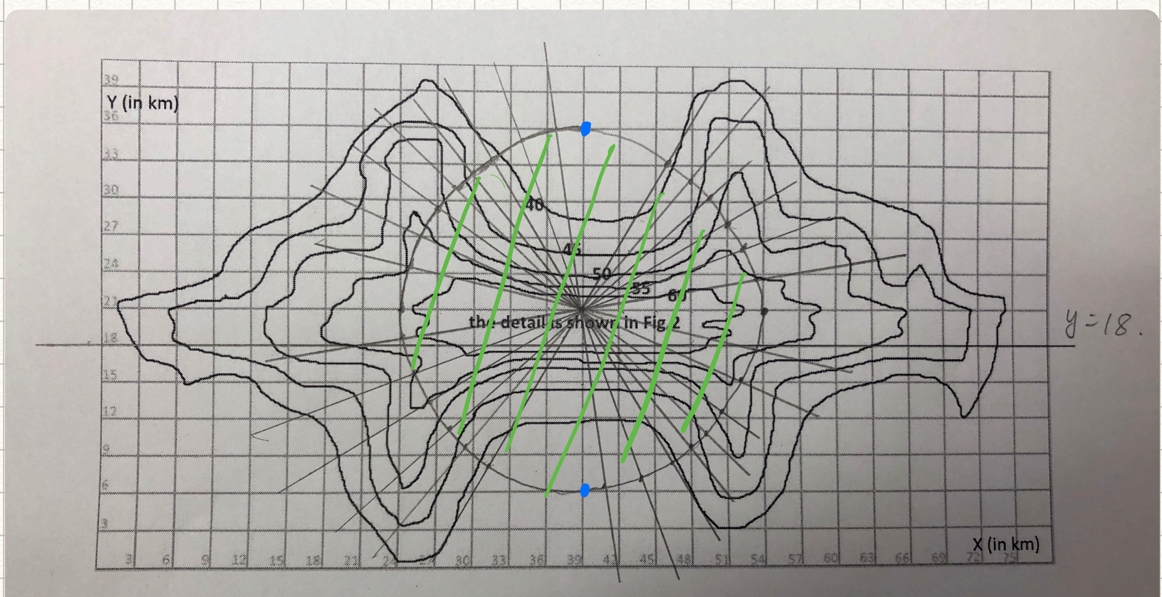


Figure 1: Level curves of $N(x, y)$ for the airport. See text for details.

The red point has the highest noise values.

The blue point has the lowest noise values.

Based on Fig. 1 and the graph in Question 5, the red point marked in Fig. 1 has a noise value higher than 55 dB and it is the closest point to the contour of 60 dB.

The blue point marked in Fig. 1 is the furtherest from the contour line where $N = 40$ (further by more than 6 km). The second furtherest point is at (39, 6) and is further from the contour line where $N = 40$ by slightly less than 6 km.

9.

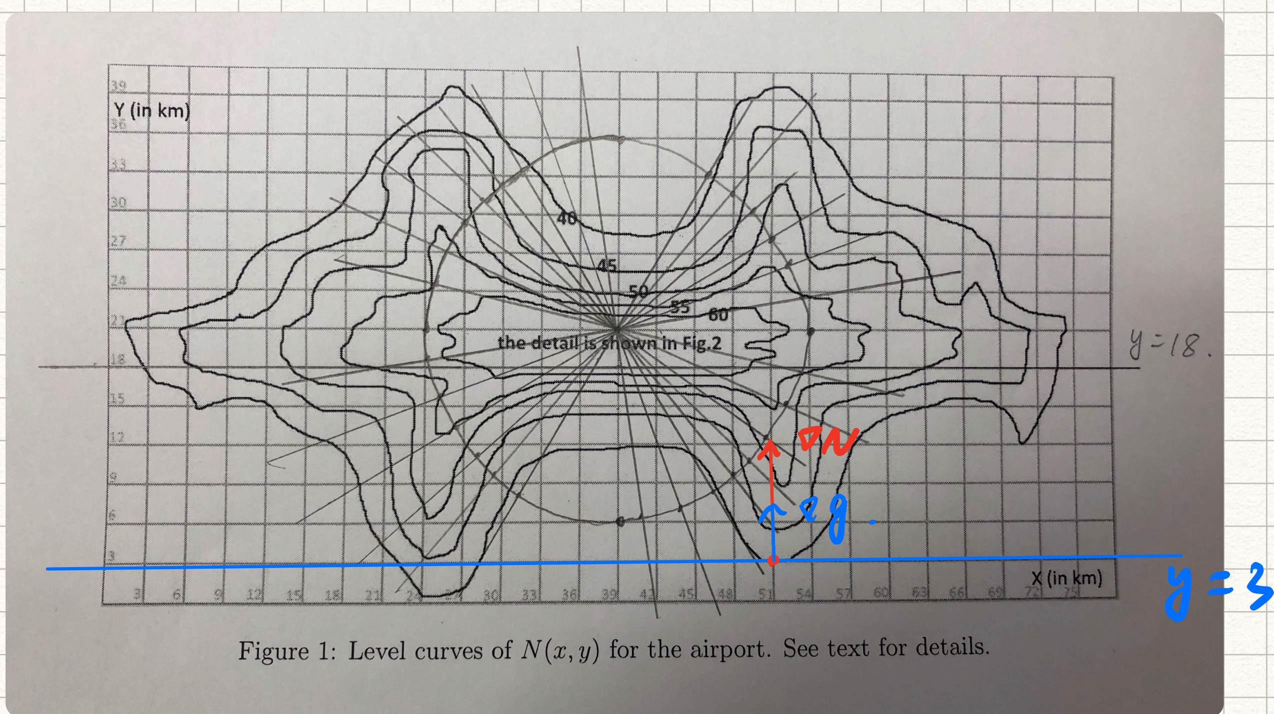


Figure 1: Level curves of $N(x, y)$ for the airport. See text for details.

The red point marked in the Figure 1 above satisfies the Lagrange gradient condition.

$g(x,y) = y - 3$

∇N is sketched in red and ∇g is sketched in blue

10. a local maximum.

By Lagrange Multipliers, ∇N and ∇g are parallel at the marked red point, thus the red point is a local optimum.

The gradient $\nabla N(x_0, y)$ has a positive component in the direction where $\nabla N(x_0, y)$ moves from left to right. Thus, as $\nabla N(x_0, y)$ keeps moving from left to right until it reaches the marked red point, where $N(x_0, y)$ has a local maximum along the constraint of $y=3$.

11.

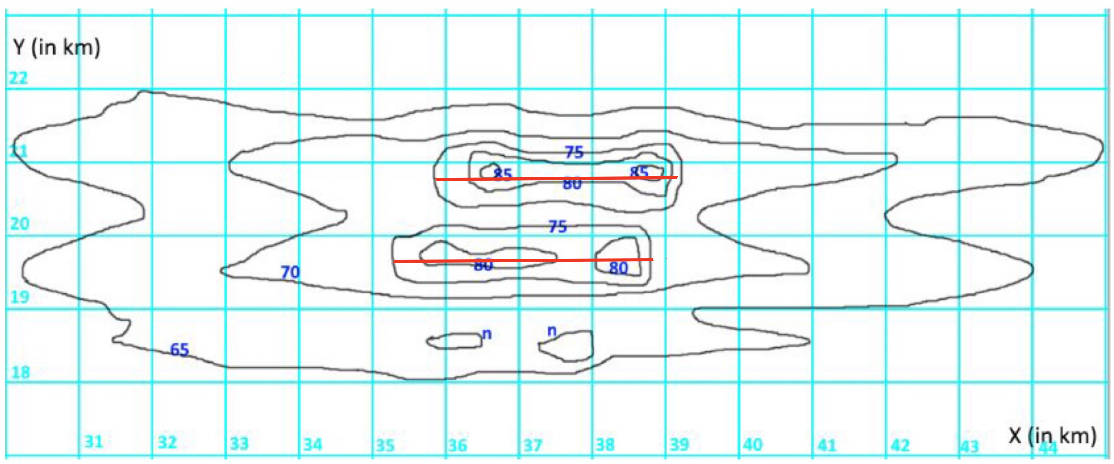


Figure 2: Level curves of $N(x, y)$ for the airport (zoomed in). See text for details.

The 2 red lines marked above are 2 main runways of the airport because the landing and taking off make the loudest noise in the airport while the 2 red lines go through the regions with the highest noise value given in the figure.

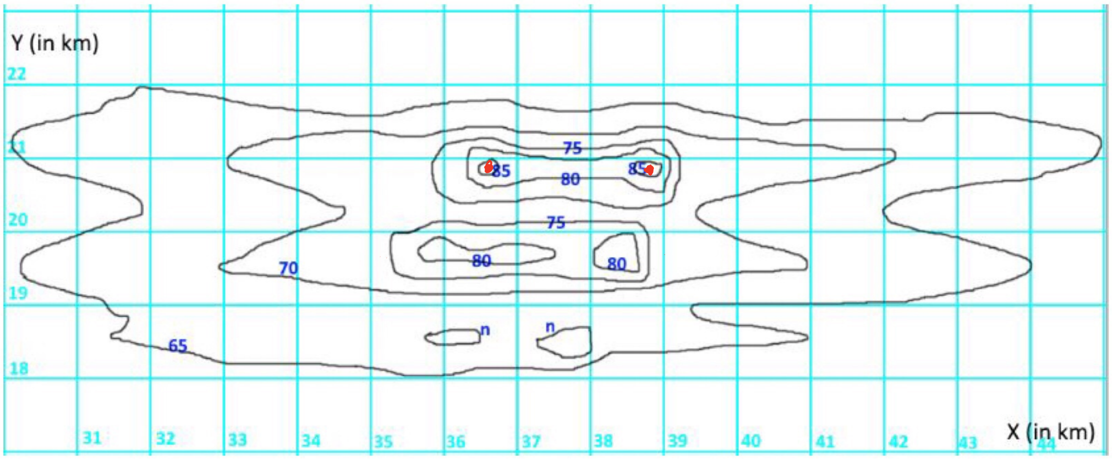


Figure 2: Level curves of $N(x, y)$ for the airport (zoomed in). See text for details.