10.004: ADVANCED MATHEMATICS II

1D PROJECT
STUDY AND RESEARCH ON AIRPLANE NOISE

Cohort Class 03
Group 3

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Individual Contributions

As a group, Part II on Math Modelling was done together where each member contributed their part to complete the Math Modelling. The work was equally distributed and each member took turns completing the various sections in the Executive Report and Analysis Report of Math Modelling.

Qiao Yingjie:

Completed the first 11 questions in Part I, proposed solution method for the math modelling, collected online datasets, wrote the python code to retrieve the data and plotted the regression graphs. Formatted the final report.

Han Xing Yi:

Double-checked all the answers in Part I, came up with the solution for math modelling, wrote and calculated for the report and summary, produced the preference indicator.

Hoo Yong Wei, Gerald:

Assist in completion of question 12 in Part I, contributed to creating preference indicator and writing the executive summary and math modelling report.

Ling Chun Yi Andrea:

Completed question 12 in Part I, proposed the basis for the analysis, strengths, and weaknesses of math modelling. Assisted in the completion of the Executive Summary for math modelling.

Chung Wah Kit:

Assisted in collecting online datasets, as well as writing the python code to retrieve the data to plot the regression line graphs.

Table of Content

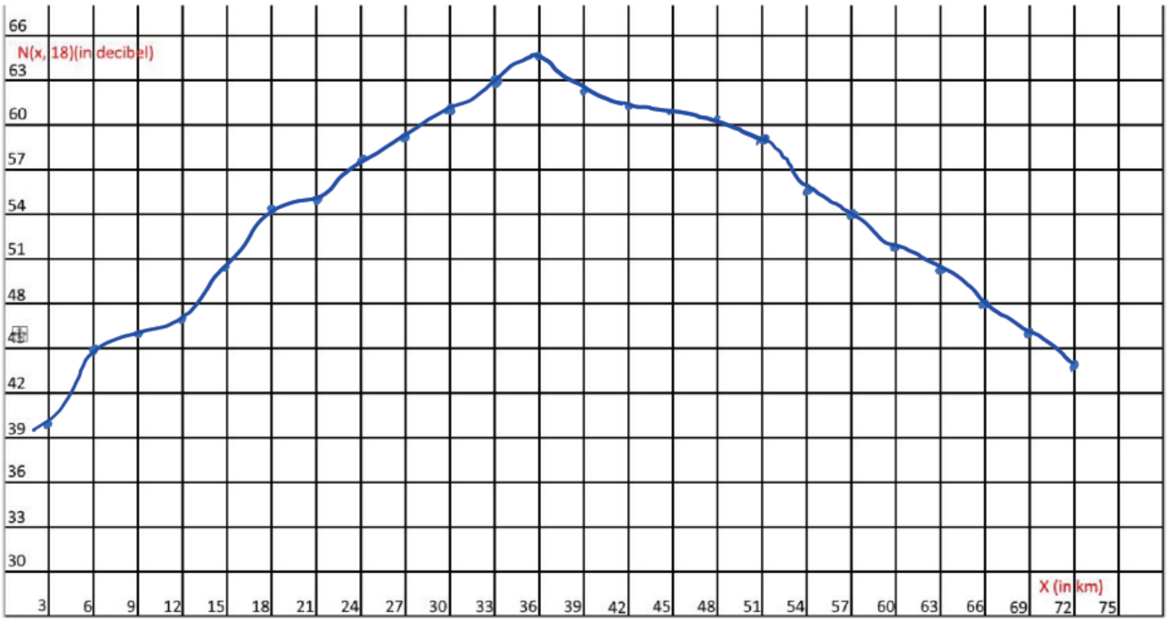
PART I -- Case Study: Noise Pollution at Airports

Question 1. N(33,30) ≈ 42

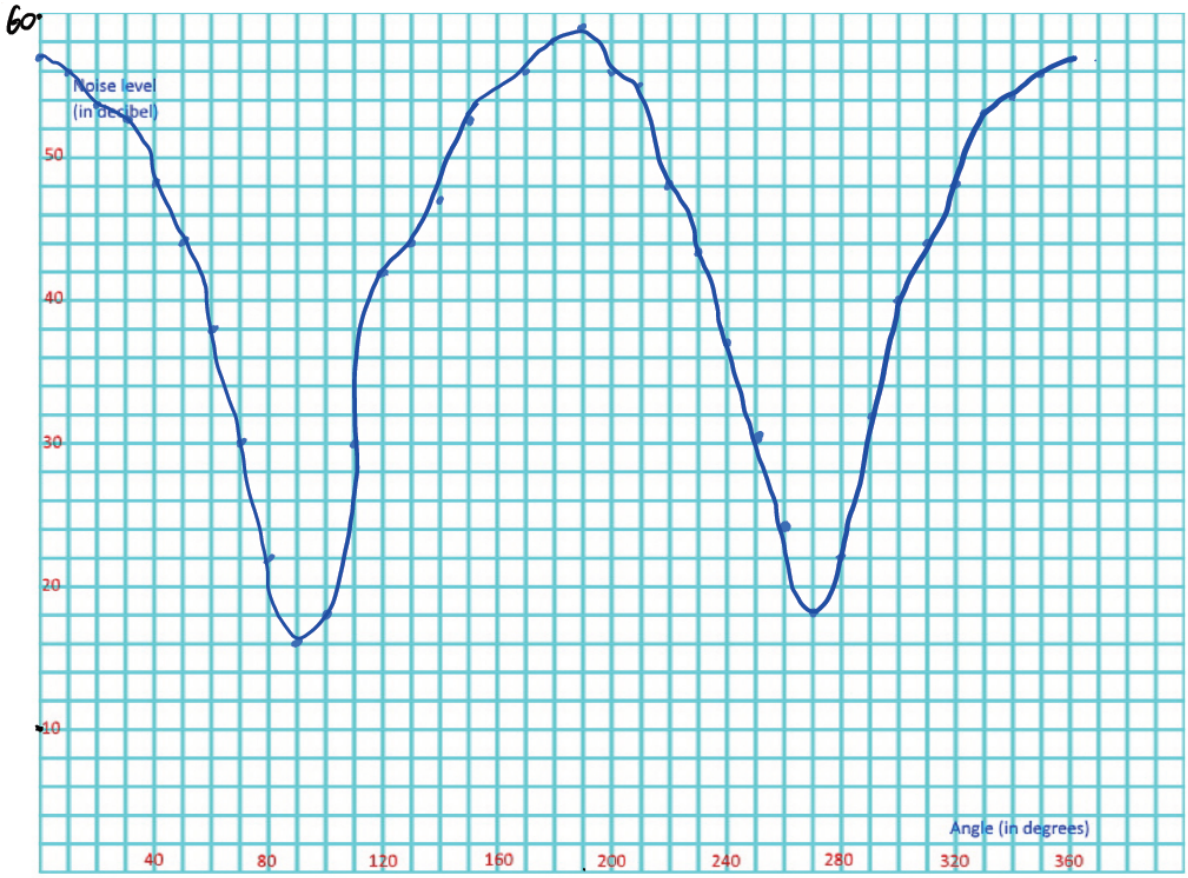
Question 2. Nx>0 , Ny<0. The gradient vector ∇N(45,30) is always perpendicular to its contour at N(45,30). The gradient vector ∇N(45,30)=(Nx,Ny) always points from a lower function value to a higher function value, which is from N(x,y)=40 to N(x,y)=45 in this case. Based on these two reasons, it can be concluded that the gradient vector ∇N(45,30)=(Nx,Ny) points into the fourth quadrant of the x-y coordinate system. Therefore, the sign of Nx is positive and the sign of Ny is negative.

Question 3. n=65 or 70. Each level curve differs by 5dB. The level curve with a noise value of n is located next to the level curve with a value of 65 and the level curve with value 70. According to the level curve with value 65, n should be 65 ± 5. While according to the level curve with value 70, n should be 70 ± 5. Therefore, n=65 or 70.

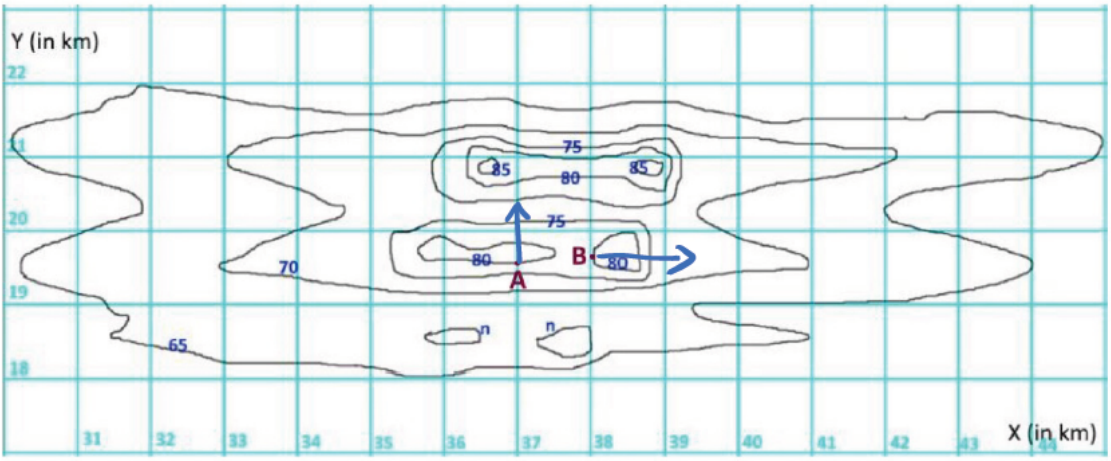
Question 4.



Question 5.



Question 6.



Question 7.

The ∇N at point A has a larger magnitude. The contours at point A are denser than those at point B.

Question 8.

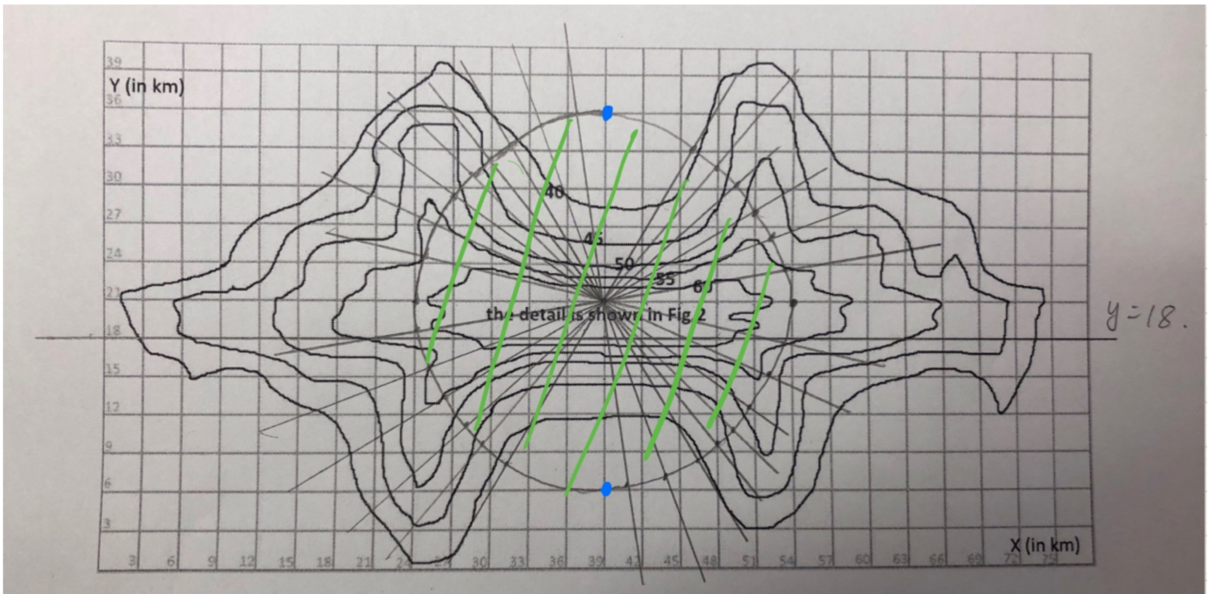


Figure 1: Level curves of $N(x, y)$ for the airport. See text for details.

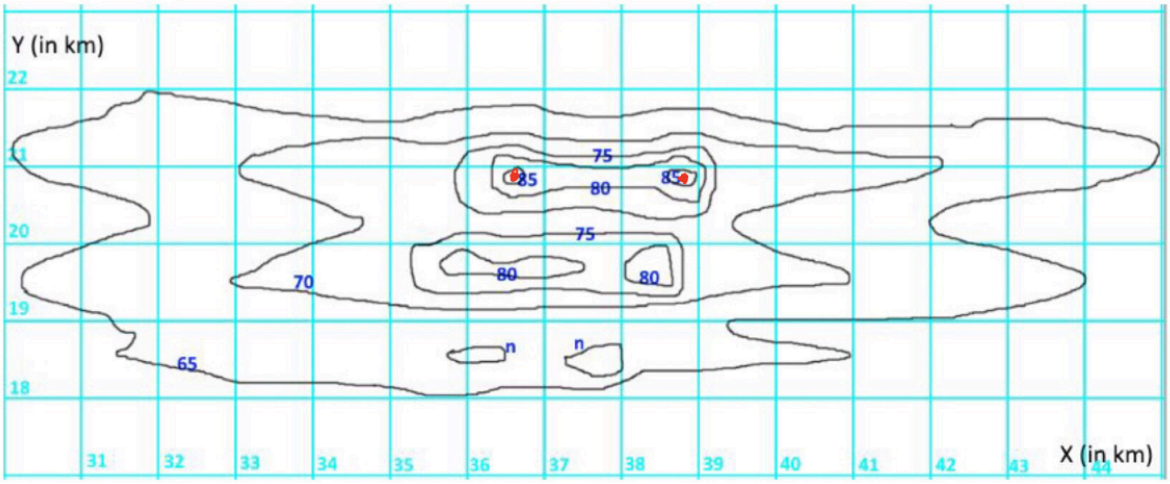


Figure 2: Level curves of $N(x, y)$ for the airport (zoomed in). See text for details.

As described in question 8 "**within** 15km radius", thus, we are analysing the area shaded in green in the Fig. 1 above.

The blue points, (39,6) and (39,36) have the lowest noise values. The blue points marked in Fig.1 are the farthest from the contour line where N=40 compared to other points. Since the contour lines are not given at the two points, the exact value of the 2 points cannot be confirmed and thus both blue points can have the lowest noise value.

The 2 red points marked in the Fig. 2 above have the highest noise values. Since we are working on all the points within the shaded region, the 2 red points are within the contour with the highest noise value, N = 85. Therefore, the 2 red points have the highest noise values.

Question 9.

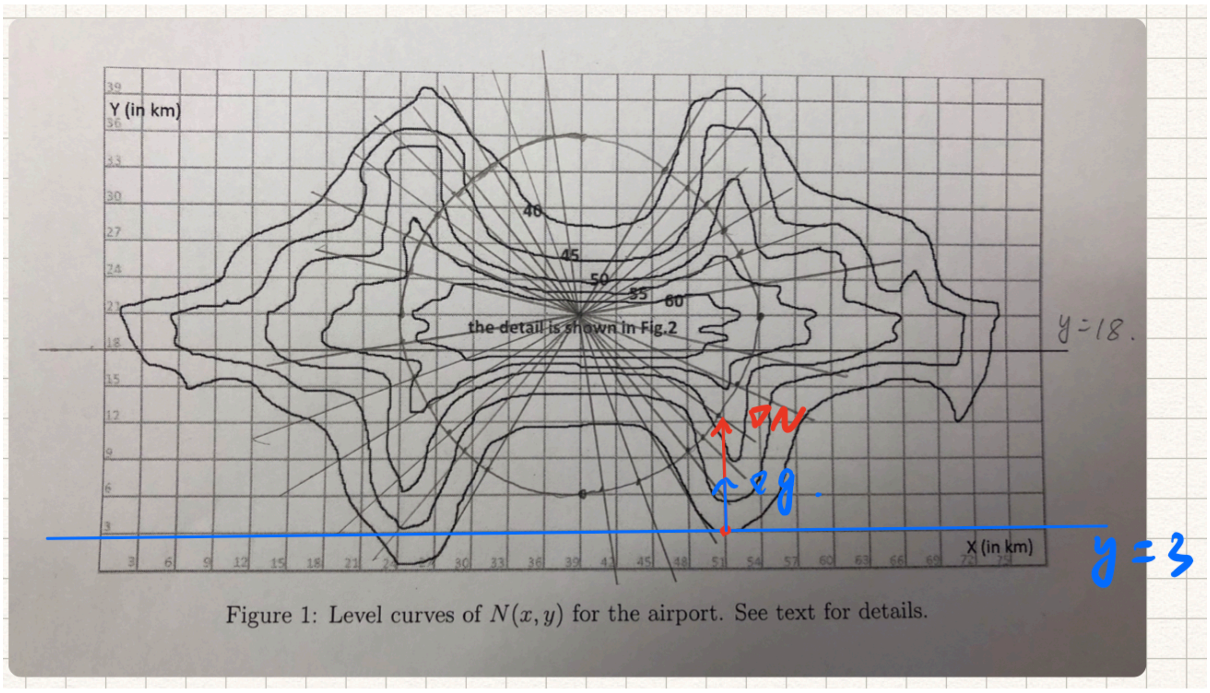


Figure 1: Level curves of $N(x, y)$ for the airport. See text for details.

The red point, (51,3), marked in Fig.1 above satisfies the Lagrange gradient condition.

g(x,y)=y-3

∇N is sketched in red and ∇g is sketched in blue.

Question 10.

It is going to be a local maximum.

By Lagrange Multipliers, ∇N and ∇g are parallel at the marked red point, thus the red point is a local optimum. The gradient ∇N(x,y) has a positive component in the direction where ∇N(x,y) moves from left to right or from right to left along the constraint line y = 3. Thus, as ∇N(x,y) keeps moving until it reached the marked red point, where ∇N(x,y) has a local maximum along the constraint of y=3.

Question 11.

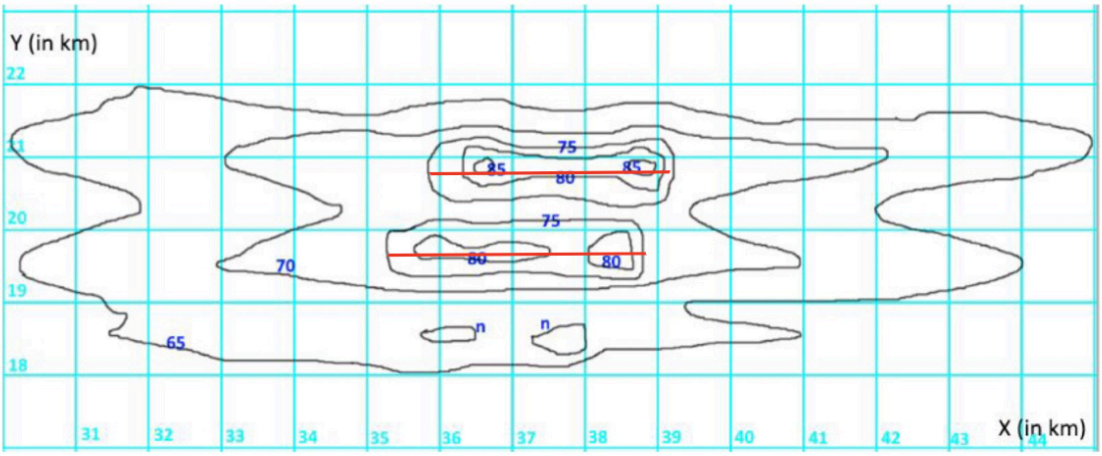


Figure 2: Level curves of $N(x, y)$ for the airport (zoomed in). See text for details.

The 2 red lines marked above are the 2 main runways of the airport as the landing and taking off make the loudest noise in the airport as the engines are running at maximum power, and the 2 red lines go through the regions with the highest noise values given in the figure.

Question 12(a). We can see that from the centre of the runway, when the airplane moves about 5km (in the x-axis) from the centre, the noise contour increases suddenly resulting in the U-shape and inverted U-shape in the middle of Fig 1. This is very likely to be due to obstruction such as buildings and trees which are able to block out/ absorb noises from the airplane, hence noise does not spread out in that region, resulting in the noise contour graphs as seen in Fig 1. We can also see that noise is almost constant within about 5km (in the x-axis) of the centre of the runway, which suggest that the obstruction is about 10km long and absorbs noise almost consistently throughout this distance. This supports our hypothesis that the obstruction is a building because a typical building is cuboid shaped and because of its cuboid shape, it absorbs noise consistently as seen in the graph in Fig 1.

Question 12(b). The shape of an airplane is largely symmetrical having almost symmetrical engine parts on both sides of the plane. This would mean that both sides of the airplane would produce noise that is almost symmetrical towards both its sides (vertical axis), hence the symmetry in the vertical axis. As we can see from the graph, the runway is parallel to the x-axis which means that airplanes fly parallel to the x-axis. Since airplanes in the graph largely fly parallel to the x-axis, it would also result in the symmetry in the horizontal axis for the noise contour.

PART II -- MATH MODELLING

IMPACT OF AIRPLANE NOISE ON SINGAPOREANS

Executive Summary

Known as the "Little Red Dot", Singapore's land scarcity means that airports and military air bases are always situated near residential areas. A recent study found that Singapore's average outdoor sound level throughout the day is 69.4 decibels, which exceeds the National Environmental Agency's recommendation of no more than 67 decibels averaged over an hour [1]. As can be seen, noise pollution due to airplanes is inevitable for Singaporeans. This is further exacerbated if residents live in close proximity to these facilities. As such, we were tasked by the Aviation Studies Institute (ASI) at SUTD to model the impact of airplane noise on Singaporeans. Hence, we will investigate the extent to which airplane noise affects Singaporeans' choice of housing location in the regions of Tengah and Paya Lebar Air Base, which would provide a good representation of how airplane noise affects housing prices across Singapore (from West to East).

Our model runs on the main assumptions that airplane noise is the main factor that influences housing prices of the houses near air bases in Singapore and that housing prices are only affected within the radius of 10km from the specified air base. We also assumed that the noise created by airplanes is inversely proportional to the distance away from the air base and that the choice of housing is directly proportional to the purchasing price of houses in that location.

For our model, we devised equations to show how the independent variable, airplane noise level (N) affects Singaporeans' choice of housing area (C). Since we have previously assumed that N decreases as the distance from an air base (d) increases and that Singaporeans' increased preference for a housing location (C) is showcased by the increase in purchase price of housing (P), we are able to plot a graph of P against d from gathering online data. The graphs plotted then show the correlation between airplane noise level (N) and choice of housing area (C), where a decrease in N would result in an increase in C. To quantify this, we calculate C by multiplying P and d, and we are able to obtain a preference indicator to show how likely Singaporeans would choose to buy a house in relation to its distance from an air base. This allows us to effectively measure the extent to which Singaporeans' choice of housing area is being affected by airplane noise, and we have concluded the impact of airplane noise on Singaporeans' housing location preference to be of a large extent.

However, there are limitations to our model as well. The accuracy of our model is mildly compromised as we have averaged out the prices in that district and the distances from the air base are taken from the general centre of that district. Meanwhile, there is also the presence of other factors that influence housing prices which we did not take into consideration. Furthermore, the flight paths of military aircrafts may result in erratic noise levels within our limit radius.

Despite these weaknesses, our equation-based model depicts the relationship between airplane noise levels and choice of housing area as accurately as possible, which is justified through the quantifiable relationship between the purchase price of housing and its distance away from an air base. This makes our model scalable and easily implemented across other air bases for future analysis. By doing a comparison of the data across multiple air bases, it also allows us to validate our results, making it more reliable.

[1] News on noise in Singapore: https://www.straitstimes.com/singapore/housing/sounds-awful-cant-sleep-cant-talk-because-of-noise

[2] Government data on the resale prices: https://data.gov.sg/dataset/resale-flat-prices.

Problem definition:

<u>To what extent does airplane noise affect Singaporeans' choice of housing area near the vicinities of Tengah and Paya Lebar Air Base?</u>

We chose both Tengah and Paya Lebar Air Base because we took into account that military aircraft that take off and land from these air bases would create more noise disturbance as compared to commercial planes. Also, Tengah is located in the West and Paya Lebar is located in the East, giving a good representation on how airplane noise affects housing prices across Singapore.

Assumptions:

1. Airplane noise is the main factor that influences housing prices in Singapore.
2. As distance from the air base increases, the noise level due to said air base decreases inversely proportional up to a certain radius, where airplane plane noise can be said to be negligible.
3. Singaporean's choice of housing area is linearly proportional to the purchasing price of a house in that area.
4. Housing prices are only affected by airplane noise within the limit radius (L) of 10km. After this limit radius, other factors become more significant in influencing housing prices than airplane noise, hence not considered.

Model Variables:

	VARIABLES	MATH NOTATION	UNITS
INDEPENDENT	Airplane Noise Level	N	dB
DEPENDENT	Choice of Housing Area	C	N.A.
MODEL PARAMETERS	Limit Radius (10km)	L	km

C denotes how likely is a Singaporean to choose that housing location.

Solution:

To model our problem statement, we hypothesised that as airplane noise level (N) increases, the choice of housing area (C) decreases, which makes intuitive sense since Singaporeans would generally prefer a quieter place for rest. Hence, we are able to derive the following equation where C is inversely proportional to the N.

$$C \propto {}^{1}/_{N} \rightarrow C = {}^{\alpha}/_{N} \ \textit{where } \alpha \textit{ is a constant} - (1)$$

Since we have previously assumed that N is inversely proportional to the distance (d) and C is directly proportional to Purchase Price (P),

$$N \propto {}^{1}/_{d} \rightarrow N = {}^{k_1}/_{d} \ \textit{where } k_1 \textit{ is a constant} - (2)$$

$$C \propto P \rightarrow C = k_2 P \ \textit{where } k_2 \textit{ is a constant} - (3)$$

Substituting (2) and (3) into (1), we get:

$$P = \frac{\alpha}{k_1 k_2} d - (4)$$

Equation (4) indicates that P is directly proportional to d, of which both variables are quantifiable and hence to validate our model, we gathered data online (from https://data.gov.sg/dataset/resale-flat-prices) to plot the graph of P against d. To improve accuracy, we have removed locations that are near to another airport/air base other than our stipulated air base as such locations may experience more fluctuations of housing prices.

After filtering and analysing the data, we plotted the graphs shown below:

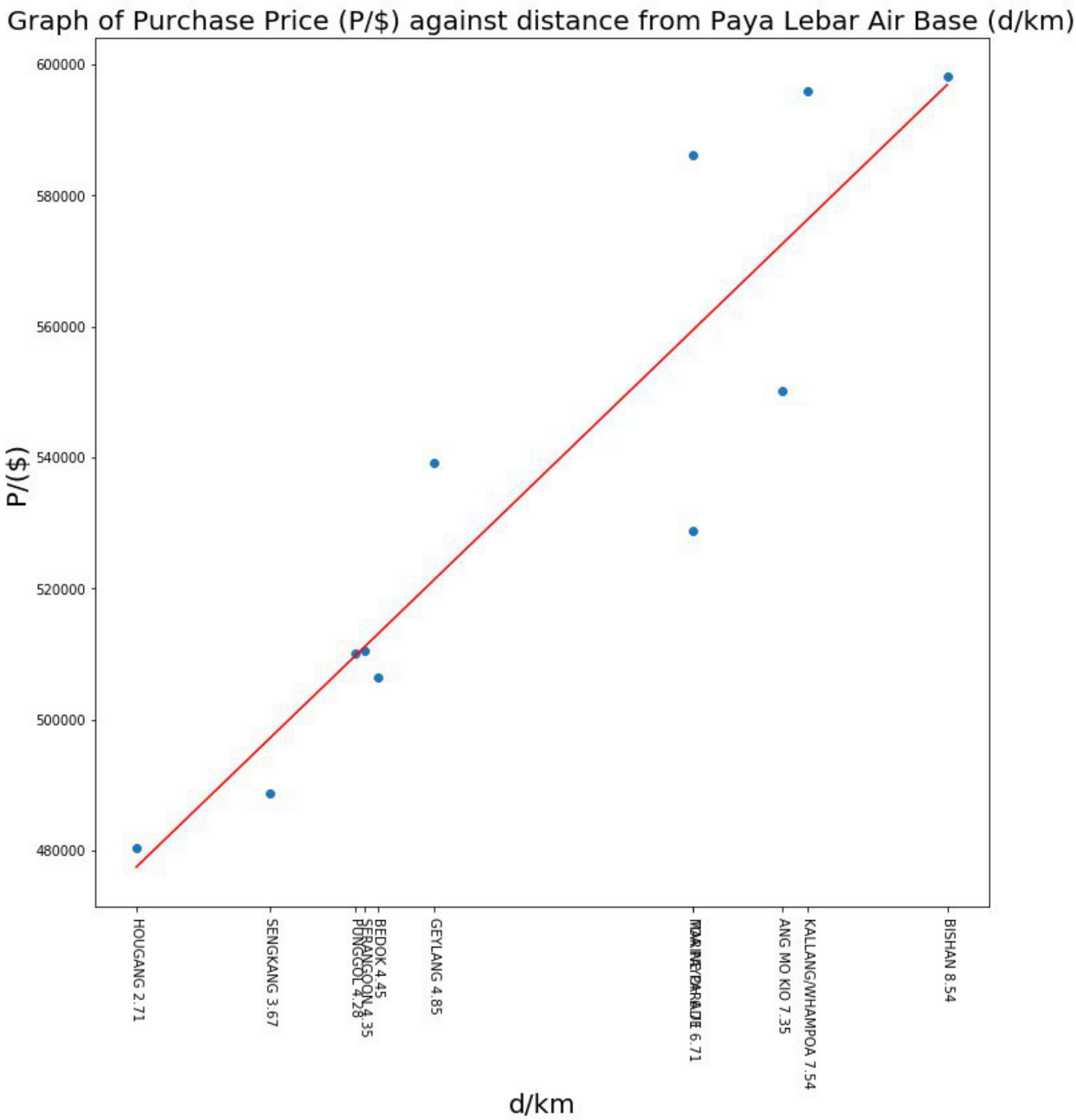


Figure 1: Graph of Purchase Price (P/$) against distance from Paya Lebar Air Base (d/km)

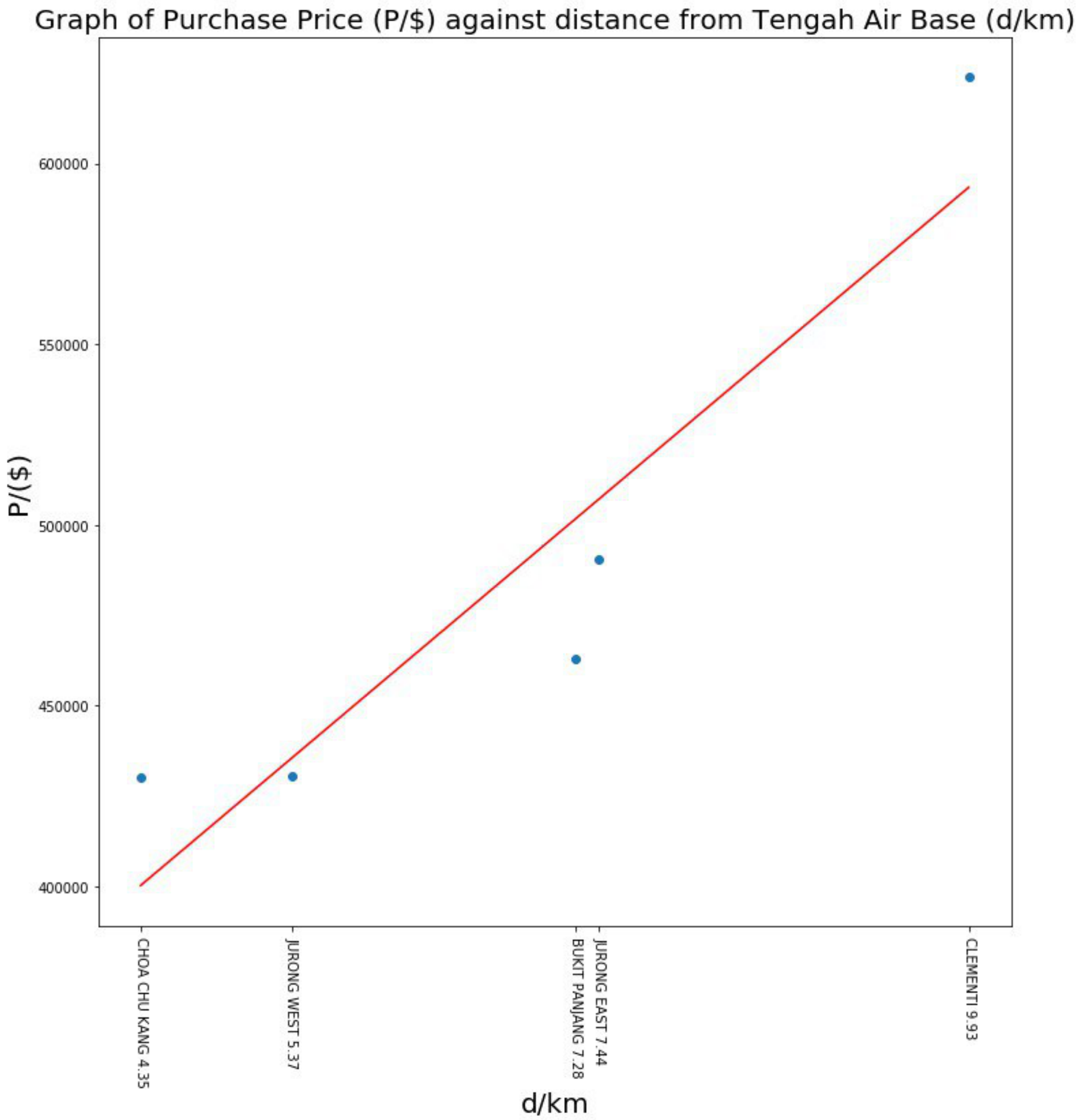


Figure 2: Graph of Purchase Price (P/$) against distance from Tengah Air Base (d/km)

Figure 1 and 2 shows a direct linear correlation between Purchase Price (P) and distance from air base (d). This agrees with equation (4) where our model states that P is directly proportional to d.

$$\text{Since } C \propto P \text{ and } C \propto d,$$
$$C \propto Pd - (5)$$

Hence, we will calculate C values for each housing district by multiplying P and d together. From the results above, we have concluded that as d increases, P increases. Therefore, C would also increase accordingly. This corresponds neatly with our initial hypothesis that as airplane noise levels (N) decreases with an increase in d, the more likely a housing area further away from the air base would be chosen.

Analysis and Assessment:

From the results above, we tabulated the respective C values (to 3s.f.) into a preference indicator as shown below, with red representing the least likely location that a Singaporean will choose for housing and green representing the most likely location to be chosen for housing.

Colour Legend:

Highest preference for housing location → Least preference for housing location

In increasing distance from Paya Lebar Air Base:

	Hougang	Sengkang	Punggol	Serangoon	Bedok	Geylang	Toa Payoh	Marine Parade	Ang Mo Kio	Kallang/ Whampoa	Bishan
C values/ 10^6	1.30	1.79	2.18	2.22	2.25	2.62	3.55	3.93	4.04	4.49	5.11

In increasing distance from Tengah Air Base:

	Choa Chu Kang	Jurong West	Bukit Panjang	Jurong East	Clementi
C values/ 10^6	1.87	2.31	3.37	3.65	6.20

As seen above, the C values show an increasing trend. This means that the closer the location is to the air base, the lower the preference for that particular location. With this preference indicator, we are able to effectively see the extent to which Singaporeans' choice of housing area is being affected by airplane noises. Hence, airplane noise does indeed affect Singaporean's choice of housing area to a large extent.

Strengths:

1) Our equation-based model depicts the relationship between airplane noise levels and choice of housing area as accurately as possible as it can be quantified through the relationship between the purchase price of housing and its distance away from an air base.
2) Our model is scalable and can be easily implemented to other air bases in the future.
3) We compared 2 areas for air bases which allow us to confirm our results, making it more reliable.
4) In our model, we calculated a weighted average price with different weightage of different types of houses to present a general, resprentable house price for each region.

Weaknesses:

1) We used the average distance from air base to the center of a town and average price for each town, which is not as representable as plotting the price of each resale house with respect to the distance from the resale house to the air base.
2) There are other factors influencing housing prices, not just airplane noise, hence there may be quite a few anomalies in the plotted graph.
3) Our model does not take into account the flight path of airplanes which may contribute to areas being more affected by airplane noise.

Appendix

(The code, data, a pdf version of this report and other supporting material are available at: https://github.com/YingjieQiao/10.004-Math-II-1D-Project)

10.004 Math II 1D Project

Dataset Processing and Data Visulization for Math Modeling

In [1]:
```
import csv
import pandas as pd
import numpy as np
```

In [2]:
```
df1 = pd.read_csv('resale-flat-prices-based-on-registration-date-from-

towns = [] #len = 26
for town in df1['town']:
    if town not in towns:
        towns.append(town)

ft = {}
'''
{'3 ROOM': 10020, '4 ROOM': 15189, '5 ROOM': 8734, '2 ROOM': 347, 'EXE
'''
for flat_type in df1['flat_type']:
    if flat_type not in ft.keys():
        ft[flat_type] = 0
    else:
        ft[flat_type] += 1

types = [key for key in ft.keys()]

print(towns)
print(types)
```

```
['ANG MO KIO', 'BEDOK', 'BISHAN', 'BUKIT BATOK', 'BUKIT MERAH', 'BUK
IT PANJANG', 'BUKIT TIMAH', 'CENTRAL AREA', 'CHOA CHU KANG', 'CLEMEN
TI', 'GEYLANG', 'HOUGANG', 'JURONG EAST', 'JURONG WEST', 'KALLANG/WH
AMPOA', 'MARINE PARADE', 'PASIR RIS', 'PUNGGOL', 'QUEENSTOWN', 'SEMB
AWANG', 'SENGKANG', 'SERANGOON', 'TAMPINES', 'TOA PAYOH', 'WOODLANDS
', 'YISHUN']
['2 ROOM', '3 ROOM', '4 ROOM', '5 ROOM', 'EXECUTIVE', '1 ROOM', 'MUL
TI-GENERATION']
```

In [3]:
```
#newdf = df1[(df1['town']=='ANG MO KIO') & (df1['flat_type']=='3 ROOM
#prices = [n for n in newdf['resale_price']]

price_distro = {}

def find_price(town_name):
    global df1,towns,typs,price_distro
```

```python
    dist = {}

    for ty in types:
        data = df1[(df1['town']==town_name) & (df1['flat_type']==ty)]
        prices = [n for n in data['resale_price']]
        if len(prices) != 0:
            avg = sum(prices)/len(prices)
            dist[ty] = round(avg, 2)
        else:
            dist[ty] = 0

        price_distro[town_name] = dist

    return None

for town in towns:
    find_price(town)

print(price_distro)
```


```
{'ANG MO KIO': {'2 ROOM': 219106.38, '3 ROOM': 299110.63, '4 ROOM':
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GENERATION': 759719.16}}

In [4]:
```
prices = {}

for town in price_distro.keys():
    s = 0
    for ty,pr in price_distro[town].items():
        if ty == '3 ROOM':
            s += pr/3
        elif ty == '4 ROOM':
            s += pr/4
        elif ty == '5 ROOM':
            s += pr/5
        elif ty == '2 ROOM':
            s += pr/2
        else:
            s += pr/10
    prices[town] = s

print(prices)
print(len(prices))
```

```
{'ANG MO KIO': 550181.7173333333, 'BEDOK': 506369.0695, 'BISHAN': 59
8076.1258333332, 'BUKIT BATOK': 350417.958, 'BUKIT MERAH': 587141.57
48333334, 'BUKIT PANJANG': 463101.60083333333, 'BUKIT TIMAH': 557957
.8893333334, 'CENTRAL AREA': 637365.2778333334, 'CHOA CHU KANG': 430
169.472, 'CLEMENTI': 623972.6781666668, 'GEYLANG': 539237.775, 'HOUG
ANG': 480396.57050000003, 'JURONG EAST': 490474.07450000005, 'JURONG
WEST': 430642.01283333334, 'KALLANG/WHAMPOA': 595854.4048333333, 'MA
RINE PARADE': 528722.7583333333, 'PASIR RIS': 358039.62700000004, 'P
UNGGOL': 510116.43816666666, 'QUEENSTOWN': 687146.0785, 'SEMBAWANG':
420567.44649999996, 'SENGKANG': 488778.0993333333, 'SERANGOON': 5104
18.9118333333, 'TAMPINES': 614978.2714999999, 'TOA PAYOH': 586108.13
66666667, 'WOODLANDS': 421938.10966666666, 'YISHUN': 519936.1855}
26
```

In [5]:
```
plab = {'ANG MO KIO': 7.35, 'BEDOK': 4.45, 'BISHAN': 8.54, 'GEYLANG':
tengah = {'BUKIT PANJANG': 7.28, 'CHOA CHU KANG': 4.35, 'CLEMENTI': 9.
```

In [6]:
```
import matplotlib.pyplot as plt

def plot(distance_dict,prices_dict):
    # To get the graph for changi airport first

    x = [town+' '+str(km) for town,km in distance_dict.items()] #conve
    x.sort(key = lambda x: float(x[-4:].strip()))               #sort
    print(x)
    x_ = [float(n[-4:].strip()) for n in x]
    print(x_)                                                   #x_ is
    y_towns = [town[:-4].strip() for town in x]
    y = [prices_dict[town] for town in y_towns]

    plt.figure(1, figsize=(12,12))

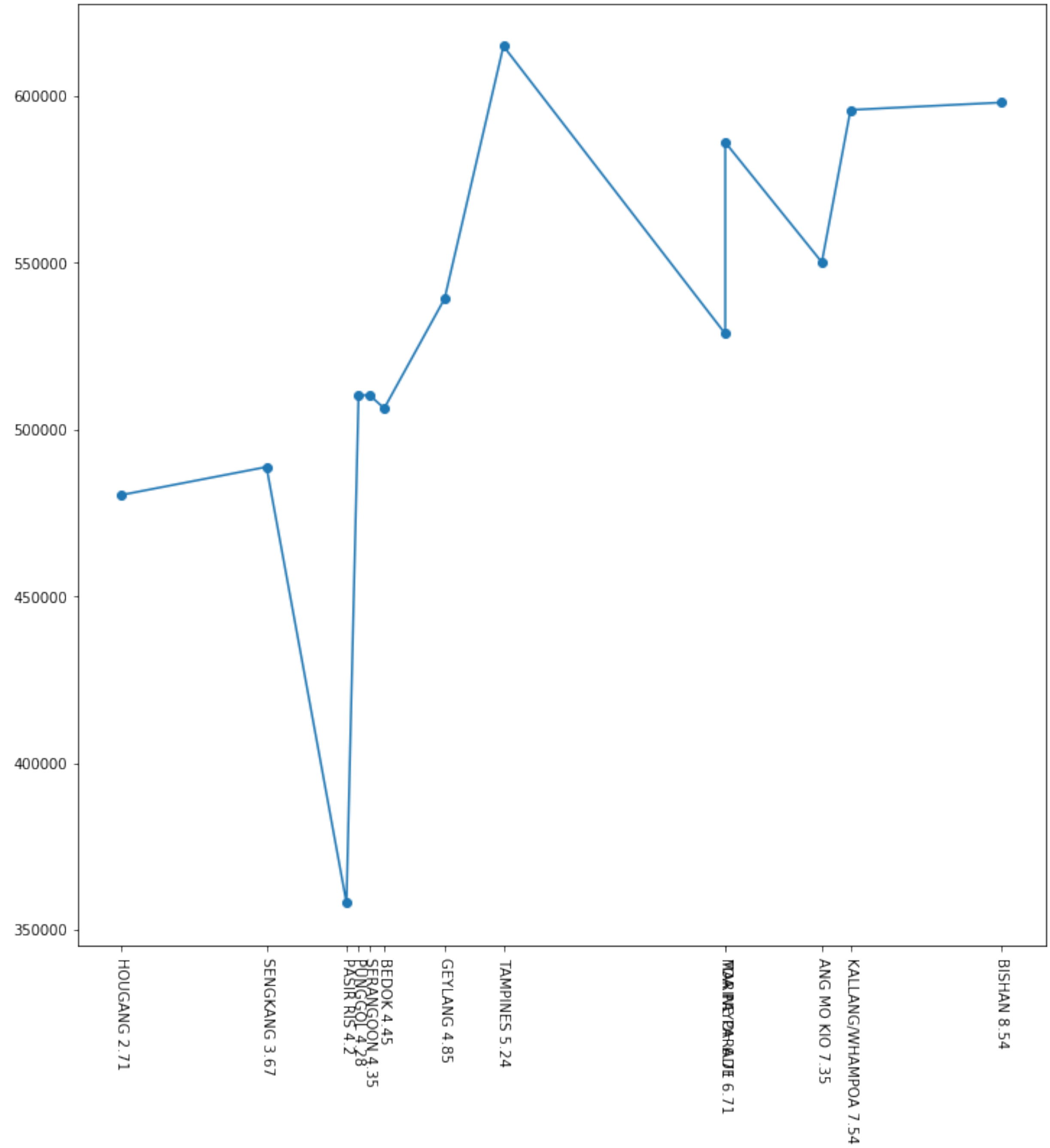
    graph = plt.plot(x_,y,'-o')
    plt.xticks(x_, x)

    plt.xticks(rotation=270)
    plt.show()

    return None
```

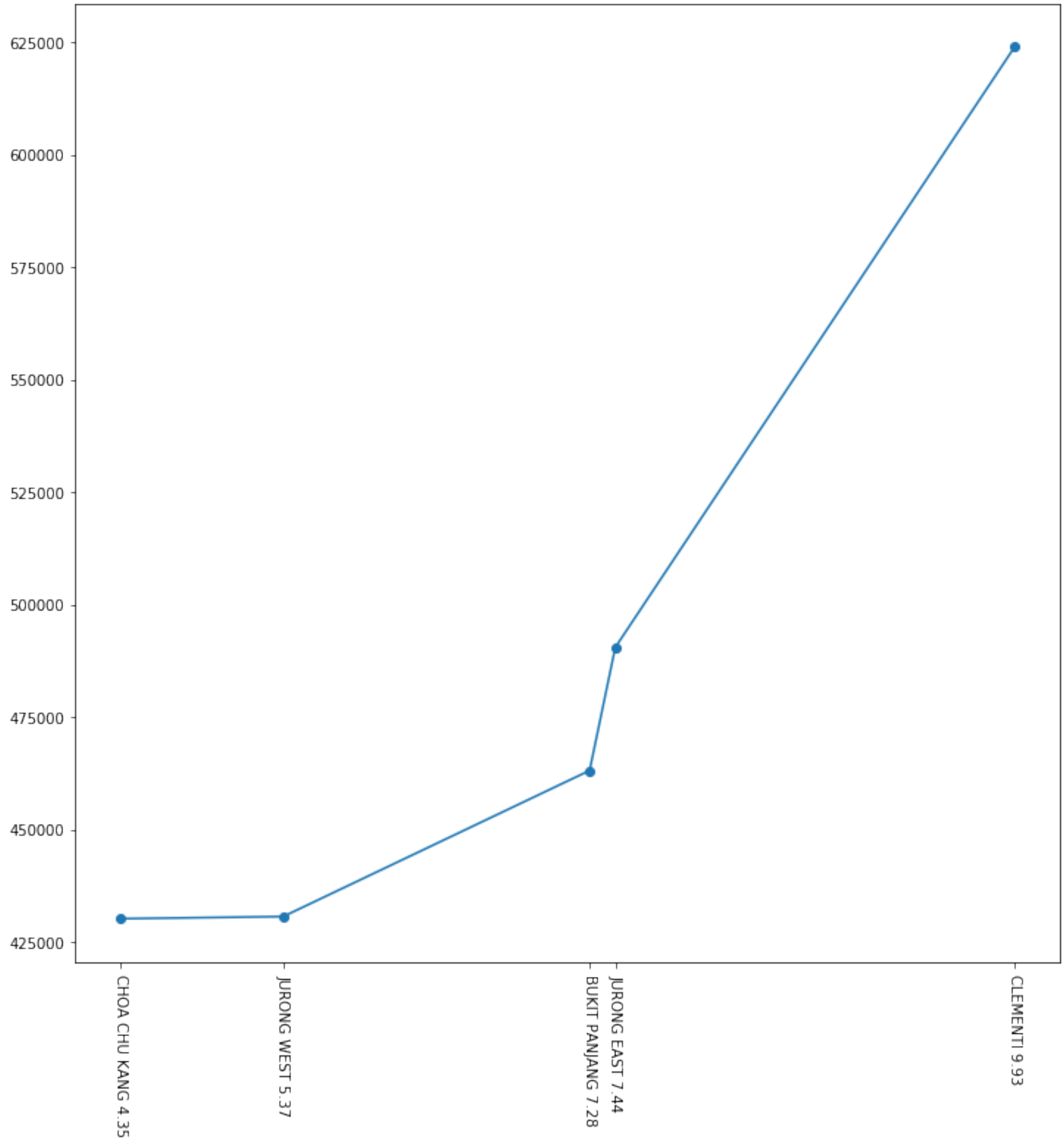
In [7]:
```
plot(plab,prices)
```

```
['HOUGANG 2.71', 'SENGKANG 3.67', 'PASIR RIS 4.2', 'PUNGGOL 4.28', '
SERANGOON 4.35', 'BEDOK 4.45', 'GEYLANG 4.85', 'TAMPINES 5.24', 'MAR
INE PARADE 6.71', 'TOA PAYOH 6.71', 'ANG MO KIO 7.35', 'KALLANG/WHAM
POA 7.54', 'BISHAN 8.54']
[2.71, 3.67, 4.2, 4.28, 4.35, 4.45, 4.85, 5.24, 6.71, 6.71, 7.35, 7.
54, 8.54]
```



In [8]:
```
plot(tengah,prices)
```

```
['CHOA CHU KANG 4.35', 'JURONG WEST 5.37', 'BUKIT PANJANG 7.28', 'JU
RONG EAST 7.44', 'CLEMENTI 9.93']
[4.35, 5.37, 7.28, 7.44, 9.93]
```



In [24]:
```
from sklearn.linear_model import LinearRegression
import numpy as np

linear_regressor = LinearRegression()  # create object for the class

def regress(X,Y):



    plt.figure(1, figsize=(12,12))



    X_ = [float(n[-4:].strip()) for n in X]
    plt.scatter(X_,Y)

    X_ = np.array(X_)
    Y = np.array(Y)

    X_ = X_.reshape(-1, 1)
    Y = Y.reshape(-1, 1)
    linear_regressor.fit(X_, Y)  # perform linear regression


    Y_pred = linear_regressor.predict(X_)  # make predictions
    figure = plt.plot(X_, Y_pred, color='red')
    plt.xticks(X_, X)

    plt.xticks(rotation=270)

    if len(X) == 5:
        plt.title('Graph of Purchase Price (P/$) against distance fron
    else:
        plt.title('Graph of Purchase Price (P/$) against distance fron

    plt.ylabel('P/($)',fontsize=20)
    plt.xlabel('d/km',fontsize=20)

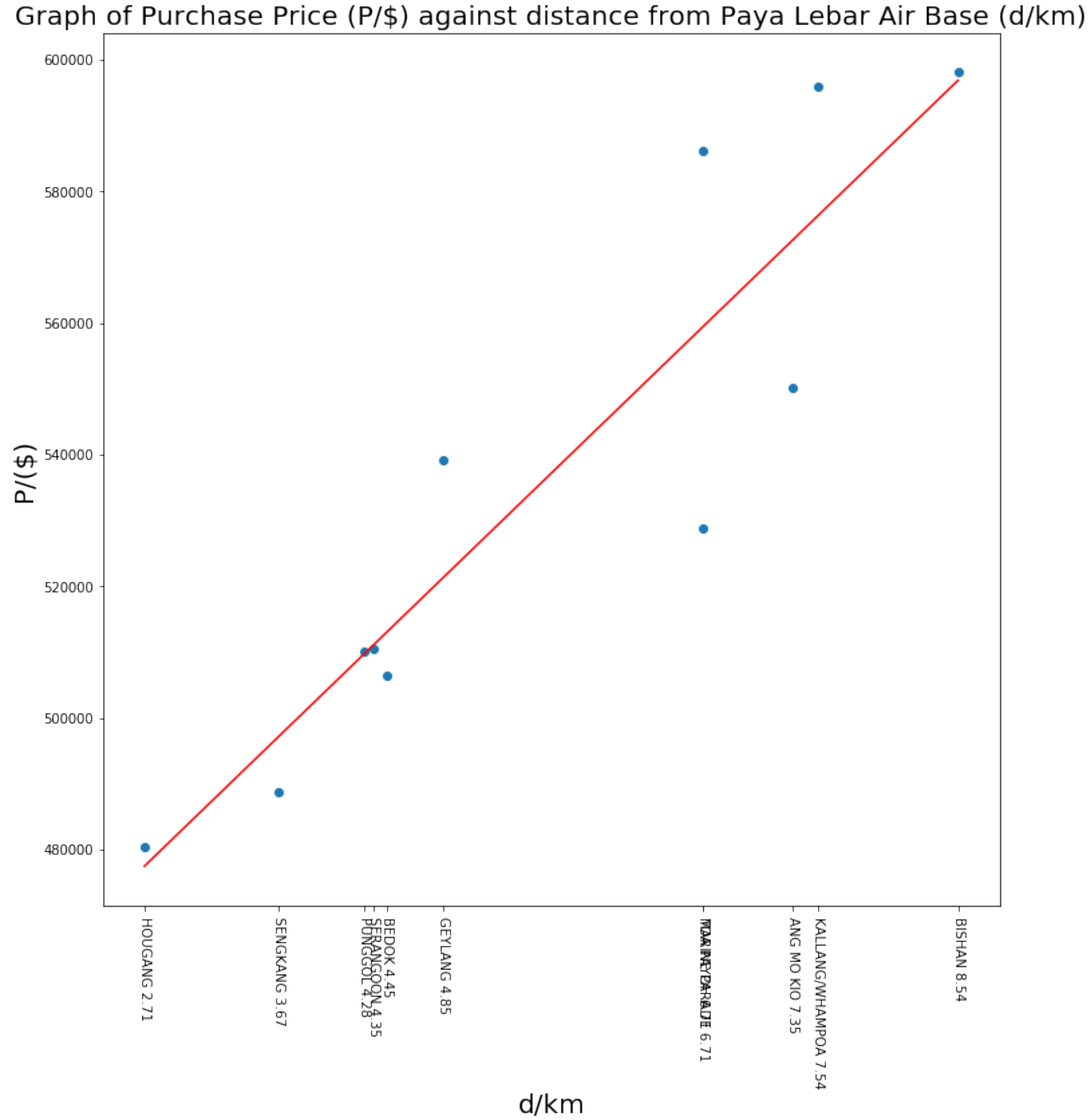
    plt.show()

    return None
```

In [25]:
```
p = ['HOUGANG 2.71', 'SENGKANG 3.67', 'PUNGGOL 4.28', 'SERANGOON 4.35'
y_towns2 = [town[:-4].strip() for town in p]
y2 = [prices[town] for town in y_towns2]

print(p,y2)
regress(p,y2)
```

```
['HOUGANG 2.71', 'SENGKANG 3.67', 'PUNGGOL 4.28', 'SERANGOON 4.35',
'BEDOK 4.45', 'GEYLANG 4.85', 'MARINE PARADE 6.71', 'TOA PAYOH 6.71'
, 'ANG MO KIO 7.35', 'KALLANG/WHAMPOA 7.54', 'BISHAN 8.54'] [480396.
57050000003, 488778.0993333333, 510116.43816666666, 510418.911833333
3, 506369.0695, 539237.775, 528722.7583333333, 586108.1366666667, 55
0181.7173333333, 595854.4048333333, 598076.1258333332]
```



In [26]:
```
t = ['CHOA CHU KANG 4.35', 'JURONG WEST 5.37', 'BUKIT PANJANG 7.28', '
y_towns3 = [town[:-4].strip() for town in t]
```

```
print(y_towns3)


y3 = [prices[town] for town in y_towns3]
print(y3)
print(t,y3)
regress(t,y3)
```

```
['CHOA CHU KANG', 'JURONG WEST', 'BUKIT PANJANG', 'JURONG EAST', 'CL
EMENTI']
[430169.472, 430642.01283333334, 463101.60083333333, 490474.07450000
005, 623972.6781666668]
['CHOA CHU KANG 4.35', 'JURONG WEST 5.37', 'BUKIT PANJANG 7.28', 'JU
RONG EAST 7.44', 'CLEMENTI 9.93'] [430169.472, 430642.01283333334, 4
63101.60083333333, 490474.07450000005, 623972.6781666668]
```