Test Design without Commitment

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Job Market Hiring Process

In job markets, universities schedule interviews with candidates in order to learn their qualities for the subsequent decision of making job offers.

There is a **conflict of interests** between the (mid-ranged) universities and the candidates:

- universities wish to hire high qualities candidates;
- candidates would reject the current offer in hope of being hired by better universities if they learn that they are high-quality candidates.

No commitment: cannot commit to hiring decisions based on observed information.

Job Market Hiring Process

Question: optimal design of the "interviews" to alleviate the conflict of interests issue when lack of commitment

- hire high quality candidates in equilibrium.

Preview of Results:

- public test is optimal, i.e., no value of screening the agent;
- optimal test reveals the lemons.

Model

Binary states $\Theta = \{l, h\}$; prior $d \in (0, 1)$: prob of state h;

Binary action space $A = \{0, 1\}$.

Utilities (Conflict of Interests):

- normalize utility for action 0 in all states to 0 for both principal and agent;
- for action 1:
  - agent: utility $u_l > 0 > u_h$;
  - principal: utility $v_l < 0 < v_h$.

Information:

- agent has subjective belief p of state being h:
  - $p \sim F$ and $\mathbf{E}_{p \sim F}[p] = d$.
- principal holds no additional information beyond the prior.

Test Design without Commitment

Principal can schedule tests for learning the true state.

- principal cannot credibly disclose received information to the agent.

Timeline of the model:

1. Principal commits to a menu of information structures $\Sigma = \{\sigma_i\}_{i=1}^k$.
2. State θ and belief p are realized, and agent chooses $\sigma \in \Sigma$ based on belief p.
3. Principal receives signal $s \sim \sigma(\theta)$, and decides whether to take action 0.
   - optional: principal sends cheap-talk message;
4. If the principal takes action 0, the game ends. Otherwise, agent chooses whether to accept action 1.

Related Work

Optimal Information in Cheap Talk Games:
[Ivanov '10; Lou '22; Kreutzkamp '23; Lyu and Suen '23]

Optimal Principal Information: [Clark and Li '23]

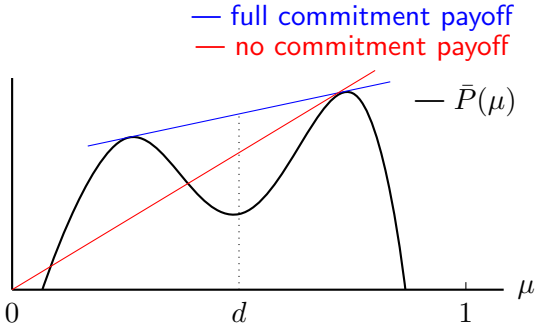
General Information Design:

- Bayesian Persuasion: [Rayo and Segal '10; Kamenica and Gentzkow '11; Kolotilin, Mylovanov, Zapechelnyuk and Li '17; Guo and Shmaya '19; Candogan and Strack '21; Lipnowski, Ravid and Shishkin '22; Lin and Liu '22; . . .]
- Cheap Talk: [Crawford and Sobel '82; Green and Stokey '07; Aumann and Hart '03; Chakraborty and Harbaugh '10; Lipnowski and Ravid '20; . . .]
- Information Design in Games: [Bergemann, Brooks and Morris '15; Roesler and Szentes '17; Deb and Roesler '21; Bobkova '21; . . .]

Intuition: Public Test

Principal commits to a single test σ for all types of the agent.

- unlike a menu of tests, cannot further infer the agent's type or the state based on the selection of the test.



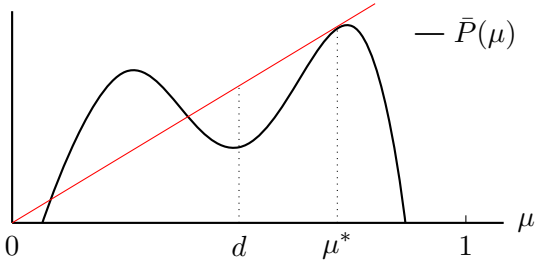
To ensure principal IC without commitment, realization of the low signal is sufficiently low such that the principal do not make an offer given low realization.

- detect and reject lemons; stars may not be recognized.

No Value of Screening

Theorem

When agents have private types, public test with binary signal is optimal. Moreover, the public test induces binary posteriors $\{0, \mu^*\}$ *with* $\mu^* = \operatorname{argmax}_{\mu \geq d} \frac{d}{\mu} \cdot \bar{P}(\mu)$.



Even through the principal can implement broader set of outcomes through screening, it is never beneficial to do so.

Overqualification

Let $p^* = \frac{u_l}{u_l - u_h}$ be the belief such that the agent is indifferent between action 0 and 1.

An agent with belief p is overqualified if $p > p^*$.

Lemma

Given any menu of test, an overqualified agent never accepts action 1 in equilibrium.

Principal IC ensures that:

- in each test of the menu, only binary signal is generated;
- principal only chooses action 1 for the high signal.

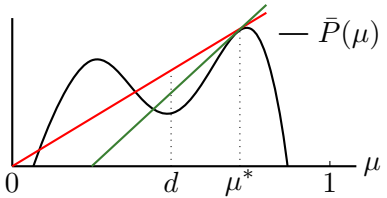
Agent with initial belief $p > p^*$, upon receiving action 1, will update to posterior $\mu \geq p > p^*$, and reject it.

No Value of Screening: Proof Idea

Optimality of public test:

- Principal IC: Overqualified agents never accept action 1 in any equilibrium;
- Agent IC: For underqualified agents, the extreme points of the implementable outcomes can be attained through public test with binary signals;
- Principal's payoff is linear in the outcomes.

Detecting lemons:



Lowering the posterior belief for low signal, increases the probability of the high signal, which increases the expected payoff.

Fraud Proof

In job markets, candidates can deliberately ruin an interview by simply not performing well.

- agent can manipulate the test outcome by stochastically changing the input to the test from high type h to low type l.

Proposition

When the principal does not have commitment power, the optimal test is fraud proof, i.e., agent never has incentive to manipulate the test.

Intuition: in equilibrium, low type leads to low signal realization with higher probability, and no offer is made in equilibrium for low signals.

Extensions

Non-binary states with conflict of interests:

- public test is optimal if only partitional signal structures are allowed.

Full commitment

- public test with binary signals is optimal;
- both signals can be partially revealing.

Persuasion, i.e., principal can only send cheap-talk message, agent fully controls the action,

- for any menu of test: no information is communicated in equilibrium, and agent chooses action solely based on his prior belief.

Summary

Characterization of the optimal test when the principal does not have commitment power

- public test is optimal;
- detecting lemons;
- fraud proof.

Applications: games with conflict of interests

- job market;
- coordinations and cooperations.

Thanks!