# Review

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# Exchange Economy

Exchange economy: a finite set $A$ of agents, $\ell$ commodities.

For each agent $a \in A$:

- utility function $U^a : \mathbb{R}^\ell_+ \to \mathbb{R}$
- endowment $\omega^a = (\omega^a_1, \omega^a_2, ..., \omega^a_l)$ in $\mathbb{R}^\ell_+$.

Assume $U^a$ satisfies: (P1) continuous, (P2) strongly monotone, and (P3) strictly quasiconcave

Aggregate endowment:

$$\bar{\omega} = \sum_{a \in A} \omega^a \gg 0.$$

# Excess Demand

Aggregate (or market) demand at price $p$ is

$$X(p) = \sum_{a \in A} \hat{x}^a(p).$$

The aggregate excess demand function $Z : \mathbb{R}^\ell_{++} \to \mathbb{R}^\ell$ is given by

$$Z(p) = X(p) - \bar{\omega}.$$

# Exchange Economy: Excess Demand

**Theorem**

*The excess demand function* $Z : \mathbb{R}^\ell_{++} \to \mathbb{R}^\ell$ *of the economy* $\mathcal{E}$ *(under assumption (P1), (P2), (P3)) has the following properties:*

(1) *it is zero-homogenous,*

(2) *it obeys Walras' Law,*

(3) *it is continuous,*

(4) *it satisfies the boundary condition,*

(5) *it is bounded below.*

**Theorem (Arrow and Debreu '54; McKenzie '59)**

*Suppose* $Z$ *satisfies properties (1) to (5). Then there is* $p^* \gg 0$ *such that* $Z(p^*) = 0$.

# Pareto Optimality

**Feasible allocations:** In an economy with $\ell$ commodities with total endowment $\bar{\omega}$, an allocation $\{y^a\}_{a \in A}$ with $y^a \in \mathbb{R}^\ell_+$ is *feasible* if $\sum_{a \in A} y^a = \bar{\omega}$.

**Definition**

An allocation $\{z^a\}_{a \in A}$ is a Pareto improvement of another allocation $\{y^a\}_{a \in A}$ if $U^a(z^a) \geq U^a(y^a)$ for all $a \in A$ and the inequality is strict for at least one agent. Moreover, an allocation $\{y^a\}_{a \in A}$ is Pareto optimal if it cannot be Pareto-improved by another feasible allocation.

# The First Welfare Theorem

**Definition**

An allocation $\{x^a\}_{a \in A}$ is a Walrasian allocation if there exists $p \in \mathbb{R}_+ +^{\ell}$ such that $Z(p) = 0$ and $x^a = \hat{x}(p)$.

**Theorem**

*Suppose $U^a$ is monotone for all agent $a \in A$. Then every Walrasian allocation is Pareto optimal.*