Mechanism Design with Endogenous Principal Learning

Daniel Clark (UCLA)

Yingkai Li (NUS)

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Information Sharing in Supply Chains

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Optimal mechanism for the retailer:

- joint design of the information structures (e.g., for demand forecasts) and contracts;
- incentive compatible for both the retailer and the manufacturer;
- maximize ex ante payoff of the retailer.

Strategic Ignorance

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Full learning can still be strictly suboptimal for the retailer even if it is payoff irrelevant for the manufacturer.

- e.g., retailer learns the inventory costs, or the demand function of consumers that purchase from the retailer.

Illustrative Example

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Example relies on two factors: (1) richness of the unknown feature space; (2) no transfers.

Key Findings

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- Class of environments in which there are just 2 possible features.
- Class of environments that nests familiar class of environments with "independent private values with transfers."

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In many environments outside of these classes, no optimal mechanism features full learning by principal.

In some environments, every optimal mechanism involves fairly sophisticated information design with relatively large amount of "on-path" beliefs.

- Contrasts with various findings from standard information design [Kamenica and Gentzkow '11].

Related Literature

Information sharing in supply chains.
Lee and Whang [2000]; Fiala [2005]; Ebrahim-Khanjari, Hopp, Iravani [2012]; Cui, Allon, Bassamboo and Van Mieghem [2015].

Joint mechanism-information design problems.

- Information design concerning just information received by agent:
  Bergemann and Pesendorfer [2007], Eső and Szentes [2007], Li and Shi [2017]; Bergemann, Bonatti and Smolin [2018]; Bergemann, Heumann, Morris, Sorokin and Winter [2022]; Li [2022]; Wei and Green [2022]; and Yang [2022].
- Information design concerning information received by principal but ignoring issues of principal incentive compatibility:
  Bergemann, Brooks and Morris [2015]; Haghpanah and Siegel [2023]; and Karik and Zhong [2023].

Related Literature (cont.)

Information acquisition by a sender in sender-receiver games without mechanism design.

- Cheap-talk: Ivanov [2010], Kreutzkamp [2022], Lou [2022], and Lyu and Suen [2022].
- Other games: Pavan and Tirole [2023] and Li and Xu [2024].

Informed principal problems.

Myerson [1981], Maskin and Tirole [1990,1992], Mylovanov and Tröger [2012,2014], Koessler and Skreta [2016,2023], Clark [2023], Clark [2024], and Clark and Yang [2024].

- In our framework, principal can become privately informed only after they have committed to and implemented a mechanism.

Model

Single principal and single agent.

Non-empty and finite set of possible features Ω and agent types Θ, where $(\omega, \theta) \in \Omega \times \Theta$ is distributed ex-ante according to common prior $F \in \Delta(\Omega \times \Theta)$.

Compact allocation space X.

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Both parties have an outside option.

- Each party's outside option gives them a payoff of 0 regardless of $(\omega, \theta) \in \Omega \times \Theta$.
- Use $o \notin X$ to represent outside options being realized and abuse notation by having $U(\omega, \theta, o) = V(\omega, \theta, o) = 0$ for all $(\omega, \theta) \in \Omega \times \Theta$.

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An *information structure* is a tuple (S, σ) consisting of a signal space S and a mapping $\sigma : \Omega \to \Delta(S)$.

- Signal space S must be a non-empty and compact metric space.
- Ultimate signal realization $s \in S$ can be viewed as the principal's endogenously acquired "type."

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Principal controls information structure through their choice of mechanism.

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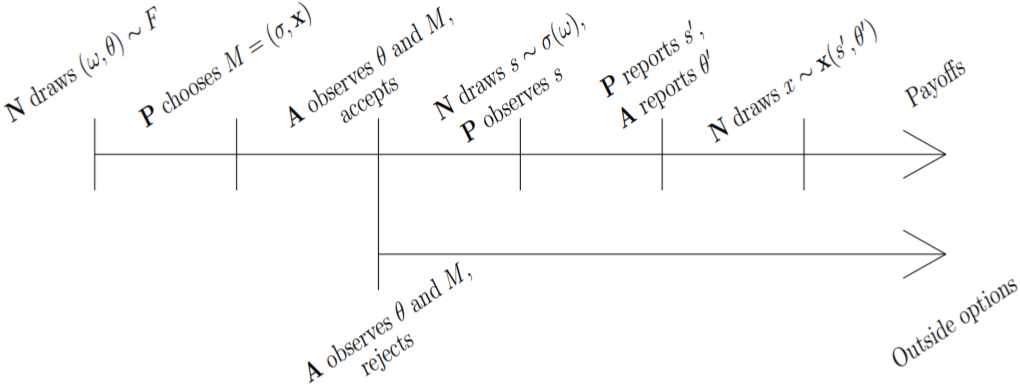
Mechanism offered by principal needs to be

- Incentive compatible for both principal and agent.
- Individually rational for agent.

Timing


N draws $(\omega,\theta) \sim F$
P chooses $M = (\sigma, \mathbf{x})$
A observes θ and M,
accepts
N draws $s \sim \sigma(\omega)$,
P observes s
P reports s',
A reports θ'
N draws $x \sim \mathbf{x}(s', \theta')$
Payoffs
A observes θ and M,
rejects
Outside options

Principal Incentive Compatibility

Principal incentive compatibility constraint can be expressed as

$$\text{PIC:} \quad s \in \arg\max_{s' \in \Delta(\Omega)} \mathbb{E}_{(\omega,\theta)\sim G(s)}[\mathbb{E}_{x\sim \mathbf{x}(s',\theta)}[U(\omega,\theta,x)]] \quad \forall s \in \Delta(\Omega).$$

where $G(s) \in \Delta(\Omega \times \Theta)$ is belief that principal would hold over $\Omega \times \Theta$ after observing signal s.

Agent Incentive Compatibility and Individual Rationality

Agent incentive compatibility constraint can be expressed as

$$\text{AIC:} \quad \theta \in \arg\max_{\theta' \in \Theta} \mathbb{E}_{(\omega,s)\sim H(\theta,\sigma)}[\mathbb{E}_{x\sim \mathbf{x}(s,\theta')}[V(\omega,\theta,x)]] \quad \forall \theta \in \Theta.$$

where $H(\theta,\sigma) \in \Delta(\Omega \times \Delta(\Omega))$ is belief that type θ agent would hold over ultimate $(\omega, s) \in \Omega \times \Delta(\Omega)$ after observing principal pick information structure σ.

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Agent individual rationality constraint can be expressed as

$$\text{AIR:} \quad \mathbb{E}_{(\omega,s)\sim H(\theta,\sigma)}[\mathbb{E}_{x\sim \mathbf{x}(s,\theta)}[V(\omega,\theta,x)]] \geq 0 \quad \forall \theta \in \Theta.$$

Outside Option Consistency

Mechanism offered by principal needs to be such that, for every $\theta \in \Theta$ and all $s, s' \in \Delta(\Omega)$, the probability of o occurring given principal type s and agent type θ equals the probability of o occurring given principal type s' and agent type θ.

Outside option consistency constraint given by

$$\text{OOC:} \quad \mathbf{x}(s,\theta)[o] = \mathbf{x}(s',\theta)[o] \quad \forall s, s' \in \Delta(\Omega), \theta \in \Theta.$$

Principal's Problem

Principal's problem is to maximize their ex-ante expected payoff across all direct mechanisms that satisfy incentive compatibility, individual rationality, and outside option consistency:

$$\max_{(\sigma,\mathbf{x})\in\mathcal{M}} \mathbb{E}_{(\omega,\theta)\sim F}[\mathbb{E}_{s\sim\sigma(\omega)}[\mathbb{E}_{x\sim\mathbf{x}(s,\theta)}[U(\omega,\theta,x)]]]$$

s.t. PIC, AIC, AIR, OOC.

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Question: ~~characterization of the optimal mechanism~~

- when is it optimal to learn nothing / fully learn?

Fully Uninformative Information Structure

The *fully uninformative* information structure is the information structure $\sigma^{FU} \in \mathcal{I}$ given by $S = \{\perp\}, \sigma^{FU}(\omega) = \delta_{\perp}$ for all $\omega \in \Omega$.

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Proposition

Given any environment, if there is an optimal mechanism with fully uninformative information structure σ^{FU}, for any information structure σ, there is an optimal mechanism with information structure σ.

Consider mechanisms that ignore the report from the principal.

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Consider mechanisms that ignore the report from the principal.

It is never strictly optimal for the principal to learn nothing.

Fully Revealing Information Structure

The *fully revealing* information structure is the information structure $\sigma^{FR} \in \mathcal{I}$ given by $\sigma^{FR}(\omega) = \delta_\omega$ for all $\omega \in \Omega$.

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- less information alleviates the requirements of incentive compatibility for the principal.

Study classes of environments in which there are optimal mechanisms with fully revealing information structures.

- Class of environments in which set of possible features is binary.
- Class of "IAPV" environments which nests familiar class of environments with "independent private values with transfers."

Binary Features

Suppose $\Omega = \{0, 1\}$.

Theorem

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- allow interdependent values;
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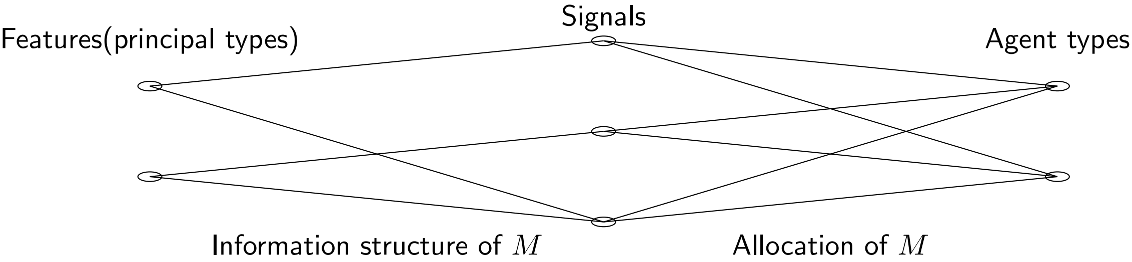
In supply-chain applications, if the retailer can only forecaster whether the demand is high or low, there is an optimal mechanism with the fully revealing information structure.

Binary Features

Intuition: simulate arbitrary mechanism M using mechanisms $\widehat{M}$ with fully revealing information structure.



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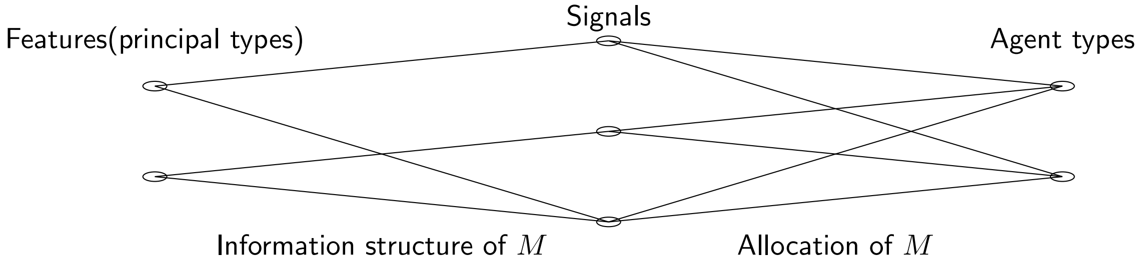

By construction, mechanism $\widehat{M}$ must satisfy AIC, AIR, OOC.

- verify PIC.

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To verify PIC: binary feature structure implies that

- for any signal $s \geq s'$, PIC of M
  $\Rightarrow$ principal with feature 1 weakly prefer outcome of s
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PIC for mechanisms with fully revealing information structures that directly simulate other mechanisms highly relies on the binary features assumption.

Independent Agent Private Values

An environment has quasilinear transfers iff

1. $X = Y \times \mathbb{R}$ for some non-empty compact metric space Y and
2. There are continuous functions $u : \Omega \times \Theta \times Y \to \mathbb{R}$ and $v : \Omega \times \Theta \times Y \to \mathbb{R}$ such that, for each $(\omega, \theta, y, t) \in \Omega \times \Theta \times Y \times \mathbb{R}$,

$$U(\omega, \theta, (y, t)) = u(\omega, \theta, y) + t,$$
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$$\begin{aligned} U(\omega, \theta, (y, t)) &= u(\omega, \theta, y) + t, \\ V(\omega, \theta, (y, t)) &= v(\omega, \theta, y) - t. \end{aligned}$$

An environment with quasilinear transfers is an independent agent private values (IAPV) environment iff

1. The feature $\omega \in \Omega$ and agent type $\theta \in \Theta$ are statistically independent under F and
2. The associated agent value function $v : \Omega \times \Theta \times Y \to \mathbb{R}$ satisfies

$$v(\omega, \theta, y) = v(\omega', \theta, y) \quad \forall \omega, \omega' \in \Omega, \theta \in \Theta, y \in Y.$$

Optimality of Full Revelation in IAPV Environments

Theorem

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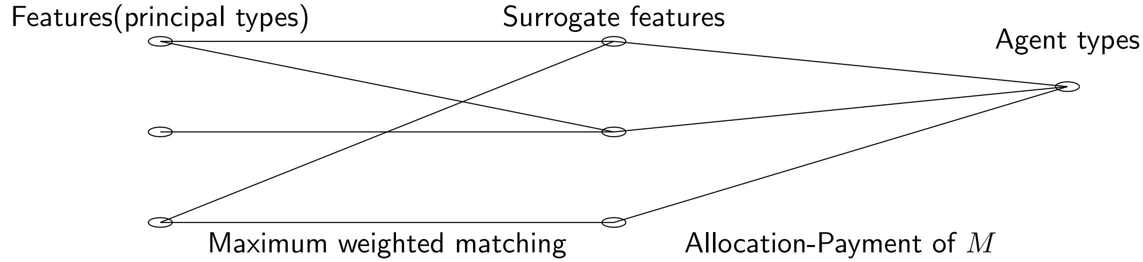
Intuition: Given arbitrary mechanism M satisfying PIC, AIC, AIR constraints (ignoring OOC which is easy to handle), construct $\widehat{M}$ as follows [Hartline, Kleinberg and Malekian '15]:



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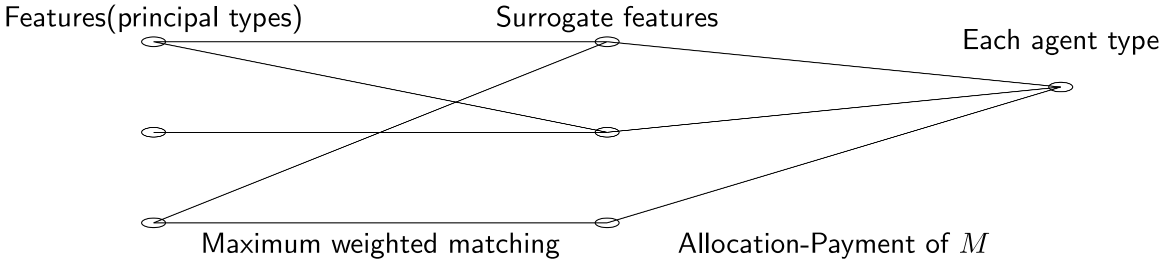

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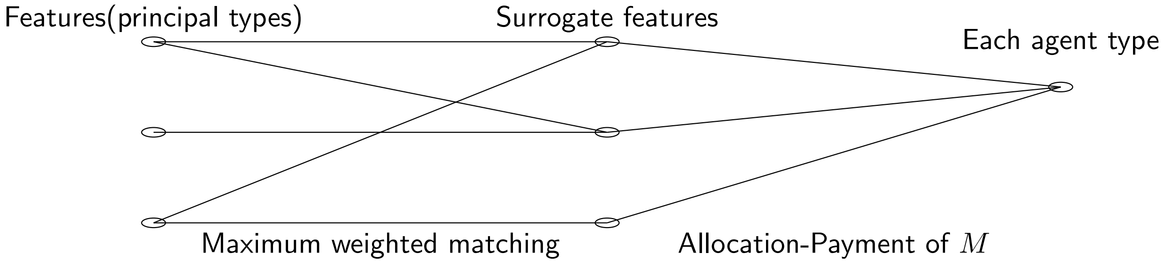

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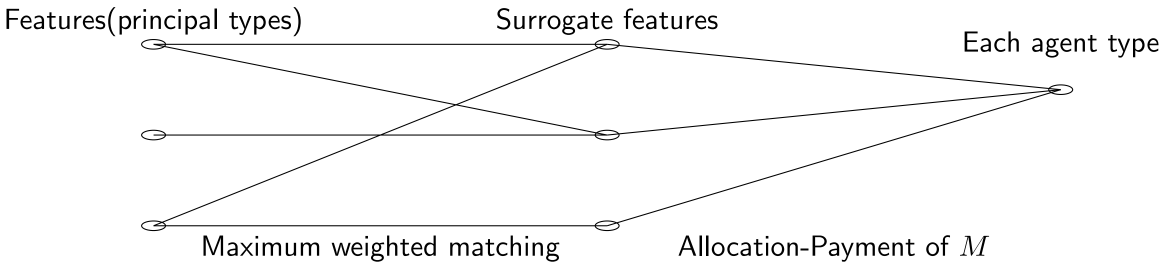

- view the matching as a general eqm model: exist prices on virtual features that supports maximum matching (via duality arguments), which guarantees PIC;
- shifting payments simultaneously for all features given each agent type to ensure expected utility of each agent type is the same in M and $\widehat{M}$, thus guarantees AIC and AIR;

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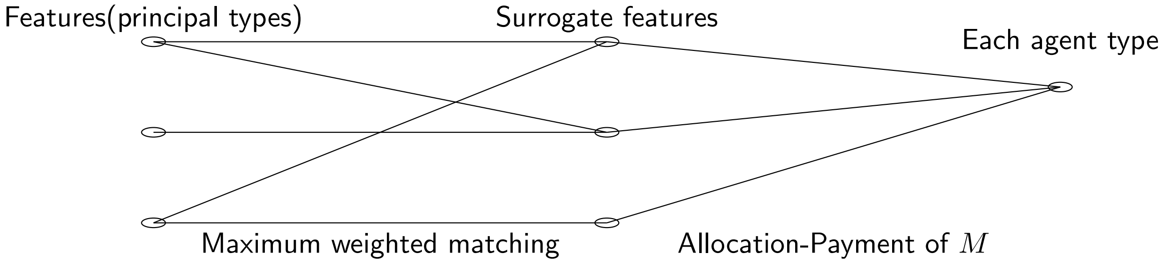

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- shifting payments simultaneously for all features given each agent type to ensure expected utility of each agent type is the same in M and $\widehat{M}$, thus guarantees AIC and AIR;
- higher efficiency, same agent utility $\Rightarrow$ higher revenue.

Optimal Mechanisms

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Theorem

There exists an example in which, for all optimal mechanisms, at least $|\Omega| + 1$ *distinct signals must be induced with strictly positive probability.*

Contrasts with Bayesian persuasion models [Kamenica and Gentzkow '11] where the cardinality of signals in the optimal information structure is at most $|\Omega|$ via Carathéodory's theorem.

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Interpretation: optimal mechanisms require complex information structures to mitigate the principal's incentives.

Lemon Markets

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- 1: selling the item; 0: no sale.

We assume linear utilities for tractability:

- $\Omega, \Theta \subset [0, 1]$;
- $U(\omega, \theta, y, t) = \omega(1 - y) + t$;
- $V(\omega, \theta, y, t) = (c(\omega) + \theta)y - t$.

Lemon Markets

Proposition

In the lemon's problem, fixing the utility function of both the principal and the agent and the distribution over unknown features,

1. $c(\omega) - \omega$ *is non-increasing in* ω *for all* ω *in the support of* F_Ω*; or*
2. $c(\omega) - \omega$ *is linearly increasing in* ω *for all* ω *in the support of* F_Ω*.*

if and only if, for every agent type set and corresponding type distribution, there exists a mechanism with fully revealing information structure.

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if and only if, for every agent type set and corresponding type distribution, there exists a mechanism with fully revealing information structure.

- Case 1: interests are aligned (low principal's value $\Rightarrow$ higher surplus from trade);
  $\Rightarrow$ ex ante optimal mechanism ignoring PIC automatically satisfies PIC;

Lemon Markets

Proposition

In the lemon's problem, fixing the utility function of both the principal and the agent and the distribution over unknown features,

1. $c(\omega) - \omega$ *is non-increasing in* ω *for all* ω *in the support of* F_Ω*; or*
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if and only if, for every agent type set and corresponding type distribution, there exists a mechanism with fully revealing information structure.

- Case 1: interests are aligned (low principal's value $\Rightarrow$ higher surplus from trade);
  $\Rightarrow$ ex ante optimal mechanism ignoring PIC automatically satisfies PIC;
- Case 2: interests are misaligned in a linear relationship;
  $\Rightarrow$ optimal mechanism requires full pooling given any information structure;
  $\Rightarrow$ any information structure, including fully revealing information structure, is optimal.

Correlation

We consider an environments where the unknown features are signals that are informative about the agent's private type.

- $u(\omega, \theta, y) = u(\omega', \theta', y)$ for all $\omega, \omega' \in \Omega, \theta, \theta' \in \Theta, y \in Y$.

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There exists environments with non-binary agent type set such that mechanisms with a fully revealing information structure are strictly suboptimal.

Correlation

Corollary

If the agent type set is binary and the unknown features are payoff irrelevant for the principal and the agent, and if the agent's type distribution is binary and non-degenerate, the principal cannot extract full surplus from the agent.

Contrast to [Cremer and Mclean '1988] where the principal can extract full surplus under correlation.

Summary

Formulated problem of mechanism design with endogenous principal learning.

In IAPV environments or environments with binary feature sets, there exist optimal mechanisms with fully revealing information structures.

In many environments outside of these classes, no optimal mechanism features full learning by principal.

In some environments, every optimal mechanism uses an information structure with sophisticated garbling and relatively large “support.”

Other Applications

- Auto insurance companies can monitor real-time driving behavior of policyholders and provide personalized insurance rates.
- Online platforms algorithmically forecast clicks and provide these forecasts to bidders participating in relevant auctions.
- Manufacturers use automated vision systems for detecting defects in product lines.

Future Directions

Further study properties of optimal mechanisms, particularly characteristics of information structures they use in environments which do not have optimal mechanisms that are fully revealing.

Many issues related to learning that could be considered.

- Learning by ex-ante informed principal.
- Costly information acquisition.
- Learning interaction between principal and agent.
- Richer information design considerations with multiple parties.

Allow for moral hazard, particularly on part of principal.