Auctions

Yingkai Li

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Mechanism Design

A mechanism design instance is denoted as $\Gamma_M = \big(N, \Omega, (v_i)_{i\in N}, (\Theta_i)_{i\in N}, F\big)$ where

- N is the set of agents;
- Ω is the set of outcomes;
- Θ_i is the set of agent i's "types" where $\theta_i \in \Theta_i$ is private information of i;
- $v_i : \Omega \times \Theta_i \to \mathbb{R}$ is agent i's value function;
- $F = F_1 \times \cdots \times F_n$ prior distribution over types.

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Quasi-linear utility:

$$u_i(x, p_i, \theta_i) = v_i(x, \theta_i) - p_i.$$

Objectives

Welfare maximization:

$$\max \mathbf{E}\left[\sum_i v_i(x(\theta), \theta_i)\right].$$

Revenue maximization:

$$\max \mathbf{E}\left[\sum_i p_i(\theta)\right].$$

Revelation Mechanisms

A mechanism $M = (x, p)$ is a revelation mechanism if all agents are incentivized to report truthfully in mechanism M. I.e., $B_i = \Theta_i$ and

$$\mathbf{E}[v_i(x(\theta_i, \theta_{-i}), \theta_i) - p_i(\theta_i, \theta_{-i})] \geq \mathbf{E}[v_i(x(b_i, \theta_{-i}), \theta_i) - p_i(b_i, \theta_{-i})] \quad \forall i, \theta_i, b_i. \tag{IC}$$

$$\mathbf{E}[v_i(x(\theta_i, \theta_{-i}), \theta_i) - p_i(\theta_i, \theta_{-i})] \geq 0, \qquad \forall i, \theta_i. \tag{IR}$$

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Lemma (Revelation Principle [Myerson '81])

It is without loss to focus on revelation mechanisms.

Single-item Auctions

Auctions: a single item, n agents.

- each agent i has a private value $v_i \sim F_i \in \Delta(\mathbb{R}_+)$;
- each agent i has linear utility $u_i = v_i x_i - p_i$ where $x_i \in [0, 1], p_i \in \mathbb{R}$;
- feasibility: $\sum_i x_i \leq 1$.

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Welfare: $\mathbf{E}[\sum_i v_i x_i]$.

Revenue: $\mathbf{E}[\sum_i p_i]$.

Welfare Optimal Mechanisms

Second-Price Auction

Focus on single-item auctions.

Second-price auction:

- agent i^* with highest bid $b_{i^*} = \max_i b_i$ wins the item;
- agent i^* pays the second highest bid $p_{i^*} = \max_{i \neq i^*} b_i$;
- all other agents lose and pay zero.

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Second-price auction is incentive compatible and welfare optimal.

- it is a dominant strategy for all agents to bid $b_i(v_i) = v_i$.

VCG Mechanisms

Vickrey–Clarke–Groves (VCG) mechanism:
mechanism that implements efficient allocation in general environments.

- **allocation**: chooses outcome

$$\omega^* = \operatorname*{argmax}_{\omega \in \Omega} \sum_i v_i(\omega, \theta_i).$$

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$$p_i(\theta) = \max_{\omega \in \Omega} \sum_{j \neq i} v_j(\omega, \theta_j) - \sum_{j \neq i} v_j(\omega^*, \theta_j) \geq 0.$$

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VCG mechanism is incentive compatible, individually rational, and maximizes social welfare.

- specialize to second-price auction in single-item environment.

VCG Mechanisms

Agent i's utility in VCG mechanism:

$$v_i(\omega^*, \theta_i) - \left(\max_{\omega \in \Omega} \sum_{j \neq i} v_j(\omega, \theta_j) - \sum_{j \neq i} v_j(\omega^*, \theta_j) \right)$$
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Agent i's utility is maximized by truthfully reporting his type to choose the allocation ω^* that maximizes the welfare.

VCG Mechanisms

In the special case of single-item auction: item is allocated to the highest bidder

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VCG mechanism reduces to the second-price auction.

Revenue Optimal Mechanisms

Incentives

Given any $v > v'$:

$$v \cdot x(v) - p(v) \geq v \cdot x(v') - p(v')$$
$$v' \cdot x(v') - p(v') \geq v' \cdot x(v) - p(v)$$

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Combining inequalities:

$$v' \cdot (x(v) - x(v')) \leq p(v) - p(v') \leq v \cdot (x(v) - x(v')) \Rightarrow x(v) - x(v') \geq 0.$$

In any incentive compatible mechanism, allocation must be weakly increasing in values.

Revenue Curves: Single-agent Analysis

Quantile space: let $q(v) = \mathbf{Pr}_{z \sim F}[z \geq v]$ be the quantile for value v.

- $q \sim U[0,1]$ (assuming continuous type distribution)
- lower quantile $\Leftrightarrow$ higher willingness to pay.

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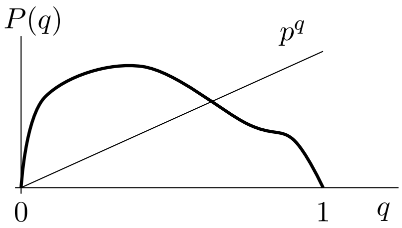
Price posting revenue curve $P(q)$: expected revenue from selling the item using market clearing price p^q.

- p^q: per-unit price that sells the item with total demand q, i.e., $q(p^q) = q$;
- $\bar{P}$: concave hull of P.



Pricing-based Mechanisms

Pricing-based mechanism in quantile space: thresholds $\{Q_i\}_{i\in[n]}$

- threshold $\hat{q}_i = Q_i(q_{-i})$ on quantiles for agent i;
- selling to quantiles lower than $\hat{q}_i \Leftrightarrow$ posting market clearing price $p^{\hat{q}_i}$ to agent i.

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Remarks:

- Incentive compatibility $\Leftrightarrow$ allocation is weakly increasing in value $\Leftrightarrow$ allocation is weakly decreasing in quantile;
- For each agent i, given Q_i, the distribution over thresholds $\hat{q}_i$ does not depend on the type distribution of other agents.

Marginal Revenue Maximization

Expected revenue from pricing-based mechanisms:

$$\sum_{i \in N} \mathbf{E}_{\forall j \neq i, q_j \sim U[0,1]} [P_i(Q_i(q_{-i}))] = \sum_{i \in N} \mathbf{E}_{\forall j, q_j \sim U[0,1]} \left[P_i'(q_i) x_i(q_i, q_{-i})\right]$$

$$= \mathbf{E}_{\forall j, q_j \sim U[0,1]} \left[\sum_{i \in N} P_i'(q_i) x_i(q_i, q_{-i})\right].$$

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- mechanism is incentive compatible if $P_i'(q_i)$ is weakly decreasing in q_i (regularity).

Marginal revenue maximization is optimal among all possible mechanisms.

Virtual Values

Define the virtual value for agent i as

$$\phi_i(v) = v - \frac{1 - F(v)}{f(v)}.$$

Note that $P_i'(q(v)) = \phi_i(v)$ for any v: virtual value is the marginal revenue for v.

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Theorem (Revenue Equivalence [Myerson '81])

The expected revenue of any mechanism M with allocation rule x is

$$\mathbf{E}_{v\sim F}\left[\sum_{i\in N} \phi_i(v_i) \cdot x_i(v)\right].$$

Equivalent formulation:

$$\mathrm{Rev}(M) = \mathbf{E}_{\forall j, q_j \sim U[0,1]}\left[\sum_{i\in N} P_i'(q_i) x_i(q_i, q_{-i})\right].$$

Envelope Theorem

For $t \in [a, b]$, any compact X, and any function $f(x, t)$ continuously differentiable in t for all $x \in X$, let

$$U(t) = \max_{x \in X} f(x, t), \quad x^*(t) = \operatorname*{argmax}_{x \in X} f(x, t).$$

Theorem (Milgrom & Segal '02)

$U(\cdot)$ is absolutely continuous and for all $a \leq t_1 < t_2 \leq b$,

$$U(t_2) - U(t_1) = \int_{t_1}^{t_2} \frac{\partial f}{\partial t}\big(x^*(t), t\big)\, dt.$$

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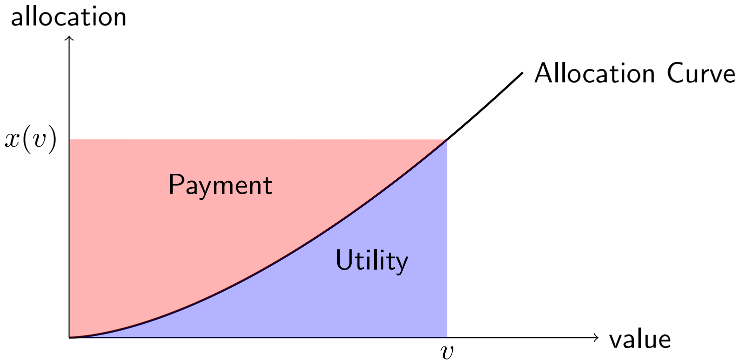
In single item auctions: $f(x, t) = x \cdot t \Rightarrow \frac{\partial f}{\partial t}\big(x^*(t), t\big) = x^*(t)$.

Alternative Geometric Proof / Envelope Theorem



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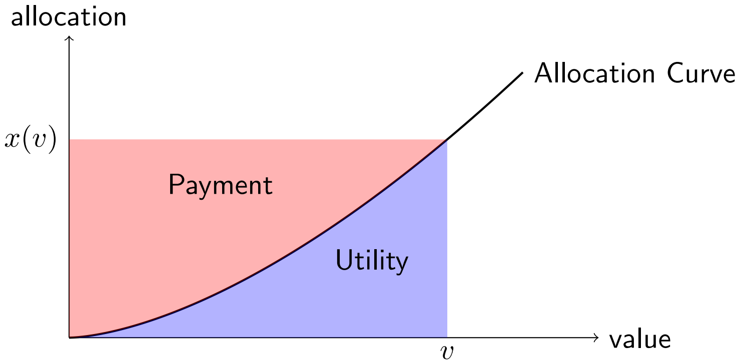

$$U(v) = \int_0^v x(z)\,\mathrm{d}z$$

$$\Rightarrow \mathbf{E}[p(v)] = \mathbf{E}[v \cdot x(v) - U(v)] = \mathbf{E}\left[v \cdot x(v) - \int_0^v x(z)\right]$$

$$= \mathbf{E}\left[\left(v - \frac{1-F(v)}{f(v)}\right) \cdot x(v)\right] = \mathbf{E}\left[P'(q) \cdot x(q)\right]$$

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If $P_i'(q_i)$ is not weakly decreasing

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Expected ironed marginal revenue is an upper bound for expected marginal revenue, and they have the same maximizer.

- ironed marginal revenue is always weakly decreasing.

Revenue Optimal Auctions

Focus on symmetric environments with regular distributions.

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- item is not sold if all agents have values below the reserve price;
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Remark: the optimal reserve price v^* does not depend on the number of agents.

- it is also the optimal price in the single agent problem.

Application of Revenue Equivalence: Equilibrium Analysis

First-price auction:

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Consider symmetric environments where values are identically and independently distributed.
Focus on symmetric equilibrium:

- interim allocation: $x_i(v_i) = F^{n-1}(v_i)$;
- revenue equivalence: $U_i(v_i) = \int_0^{v_i} x_i(z)\,\mathrm{d}z$;
- first-price auction: $b_i(v_i) \cdot x_i(v_i) = p_i(v_i)$.

$$b_i(v_i) = \frac{1}{x_i(v_i)} \cdot p_i(v_i) = \frac{1}{x_i(v_i)} \cdot \left(v_i \cdot x_i(v_i) - \int_0^{v_i} x_i(z)\,\mathrm{d}z \right) = v_i - \frac{1}{x_i(v_i)} \cdot \int_0^{v_i} F^{n-1}(z)\,\mathrm{d}z$$

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Example: two bidders, $v_i \sim [0,1]$ for $i \in \{1,2\}$.

- $b_i(v_i) = \frac{v_i}{2}$.