# Basics

## Estimation of the direction of arrival with a Radar array

Using techniques like FMCW, it is possible to estimate range and velocity of objects using Radar sensors. For complete knowledge oft he environment of the Radar sensor, it is however necessary to estimate the angle of the objects along with the range.

Angle estimation, also called the estimation of the direction of arrival (DOA) of the reflected Radar signal, requires the use of multiple antennas in an array, as the angle estimation is based on the differences in the arrival time of the reflected wave front at the individual antennas.

[Fig.1.1 Show incoming wave front with multiple antennas]

As you can see in fig. 1.1, a wave front that is not parallel to the array will result in different travel times of the signal to the individual antennas. This leads to phase differences of the received signals at the individual antennas, when evaluated at the same time.

The signals at the individual antennas are then formulated as

[Eq. 1.1 Single antenna signal with delay depending on scalar product of position vector and angle vector]

In general, the

## 1.2 Expansion to MIMO antennas

Multiple-Input-Multiple-Output (MIMO) antennas are a method to design arrays with a higher diversity as well as a higher virtual aperture.

The virtual aperture is the value [Z/lambda], which is the physical aperture of the array normalized over the wavelength. Using multiple transmitting antennas will increase the virtual array aperture of the antenna array by the virtual array of the transmitting antennas.

Fig. 1.2 shows a visualization of a simple MIMO array with a visualization of the expanded aperture.

[Fig. 1.2 Show a transmitting array and a receiving array. Then show the resulting virtual array (e.g. replicate receiving array at each transmitting antenna)]

## 1.3 Multi carrier signal model

## 1.4 Maximum likelihood estimation

## 1.5 Beam pattern

## 1.6 Performance metrics

[CRB and SLL, similar to Lange]

## 1.6 Combined CRB of multi carrier arrays for range and angle

In section 1.4 it was shown that the estimation of range and angle can not be separated. However, [Michael-CRB] demonstrates that the CRB of range and angle estimation can be separated if the origin of the coordinate system is set to the location of the array centroid.

In [Michael-CRB] the CRB has the form

[show matrix form from paper, but with the E matrix replaced]

The origin of the coordinate system can be set to the location of the array centroid by performing the operation

[P\_decoupled = P – p\_centroid]

or for MIMO arrays

[P\_decoupled = P – 2\*1\_vector \* p\_centroid]

This can be easily done, as it only affects the coordinate system used in the estimation and does not impose any constraints or limits on the rest of the system.

When this condition is fulfilled, the CRB of angle and range is decoupled. This means that the CRB matrix becomes block diagonal as in eq. […].

[Equation of CRB with p\_centroid = 0]

The top left block in eq. […] is the CRB for the angles (two-dimensional, as general angle estimation has two angles), while the bottom right block (1x1) is the CRB for the range estimation. Now, that the CRB of range and angle estimation are decoupled, they can be studied individually.

## 1.7 Individual CRB of multi carrier arrays for angle

Using the results of [Michael-CRB], the CRB for angle estimation can be derived for the decoupled case.

[Eq. … Important part, since this adds my own results]

# On Grating Lobes

## Grating lobes

Grating lobes are side lobes of the same size as the main lobe. They are characterized by a visible periodicity in the beam pattern as it can be seen in fig. […]. Naturally, grating lobes within the ROI are very undesirable, as they lead to ambiguous estimation results, since it is impossible to discern between the main lobe and the grating lobe.

[Fig. Beam pattern mit grating lobes]

A standard ULA with a spacing of d=lambda/2 does not show grating lobes within the region of u=-1,1. The edge cases u=-1 and u=1, are the first occurrence of grating lobes for such arrays.

[Fig. beam pattern for ULA]

Widening the spacing d of an ULA, will introduce grating lobes within the region u=-1,1. For example a spacing d=lambda, which is double the value of a standard ULA, will result in a beam pattern as in fig. […]. It can be seen, that the unambiguous ROI of an ULA with a doubled spacing is half the size of the ROI of the standard ROI.

[Fig. beam pattern for ULA with d=2 \* d\_standard = lambda]

For an easier understanding of grating lobes, it is useful to observe the parallels between angle estimation and Fourier transformations. In most cases, Fourier transformations are used to generate frequency spectrums from signal values sampled in time. This frequency spectrum can then be used to estimate the dominating frequencies.

In the case of angle estimation, the antenna positions are equivalent to the time at which samples of the signal are taken, while the beam pattern is equivalent to the frequency spectrum. Instead of estimating the dominating frequencies, it is the angles that are estimated.

This analogy between angle estimation and Fourier transformation becomes clearer by looking at the signal model of angle estimation (in eq. …) and the transformation matrix of discrete Fourier transformation (in eq. …).

[Eq. derivation of angle estimation signal model to a form that correlates to transformation matrix]

[Eq. of transformation matrix in DFT]

## Grating lobes in multicarrier systems

# Overview of models for the optimization of antenna array for angle estimation

[give a short overview of the different models]

# Discrete models

[note how discrete models work using the origin ULA etc. Fill UL slots with antennas. Randomly (CS) or deterministically (Sparse Sensing).]

## Compressive Sensing

* + 1. Basics of Compressive Sensing
    2. The CS model for multi carrier DOA estimation
       1. The sparse parameter space
       2. Linearizing the signal space and the sparse parameter space
       3. The sensing matrix A
    3. Coherence and side lobe level
    4. The Q-Matrix

## Sparse Sensing

* + 1. Sensor selection
    2. Discrete optimization

# Continuous models

* 1. Continuous Optimization Model for multi carrier arrays
  2. Genetic algorithm
     1. Basic algorithm

[Explain the basic genetic algorithm.]

* + 1. Application to multi carrier array optimization
       1. Applied constraints
       2. Calculation of SLL