A general elimination strategy for more related problems

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We show that our technique can be applied to more related problems and obtain the motion parameters directly, including minimal solution to the relative pose estimation with gravity direction [1], [2], [3], [4], [5], [6], homography-based egomotion with a common direction [7], [5], planar motion, etc.

A. Minimal solution to the relative pose estimation with gravity direction

This problem has been well studied in many papers [1], [2], [3], [4], [5], [6], and we will show that we only need several lines of Macaulay2 code to solve this problem. Given three point correspondences (the minimal case): $\mathbf{p}_1 \leftrightarrow \mathbf{p}_2, \mathbf{p}_3 \leftrightarrow \mathbf{p}_4, \mathbf{p}_5 \leftrightarrow \mathbf{p}_6$, where $\mathbf{p}_i = [u_i, v_i, 1]^\top$. The simplified essential matrix $\mathbf{E}_y$ with single-axis rotation is defined by

$$\mathbf{E}_y = [\mathbf{t}]_\times \mathbf{R}_y \triangleq \begin{bmatrix} -t_y s & -t_z & t_y c \\ t_z c + t_x s & 0 & t_z s - t_x c \\ -t_y c & t_x & -t_y s \end{bmatrix}. \tag{1}$$

We have three constraints

$$\mathbf{p}_2^\top \mathbf{E}_y \mathbf{p}_1 = 0,\ \mathbf{p}_4^\top \mathbf{E}_y \mathbf{p}_3 = 0,\ \mathbf{p}_6^\top \mathbf{E}_y \mathbf{p}_5 = 0. \tag{2}$$

The Macaulay2 code for directly obtaining the rotation is

```
R = QQ[c,s,tx,ty,tz,u1,v1,u2,v2,u3,v3,u4,v4,u5,v5,u6,v6];
eqs = {Eq.(2), tz=1, c^2+s^2=1};
J = eliminate({s,tx,ty,tz},ideal(eqs));
mingens J
```

In this case, we may obtain a univariate quartic equation in c, which can be solved in closed-form. Note that we have to add an addition constraint $t_z = 1$ since Eq.(2) is homogeneous.

B. Homography based egomotion estimation with a common direction

Minimal solutions to this problem was solved in [5] using the SVD-based method. We will show that it can be easily solved using our elimination technique. For points on the ground plane, we need at least two points and they should satisfy the ground plane induced homography constraint

$$[\mathbf{p}_2] \times \mathbf{H}_y \mathbf{p}_1 = 0,\ [\mathbf{p}_4] \times \mathbf{H}_y \mathbf{p}_3 = 0 \tag{3}$$

where

$$\mathbf{H}_y = \mathbf{R}_y - \mathbf{t}\mathbf{n}^\top,\ \mathbf{n} = [0,1,0]^\top. \tag{4}$$

The Macaulay2 code for directly obtaining the rotation is

```
R = QQ[c,s,tx,ty,tz,u1,v1,u2,v2,u3,v3,u4,v4];
eqs = {Eq.(3), c^2+s^2=1};
J = eliminate({s,tx,ty,tz},ideal(eqs));
mingens J
```

In this case, we may obtain a quadratic equation in c, and do not need to perform the SVD.

C. Planar motion

It is a special case of Sec.-A with $t_y = 0$. The essential matrix $\mathbf{E}_p$ under planar motion is defined by

$$\mathbf{E}_p = [\mathbf{t}]_\times \mathbf{R}_y \triangleq \begin{bmatrix} 0 & -t_z & 0 \\ t_z c + t_x s & 0 & t_z s - t_x c \\ 0 & t_x & 0 \end{bmatrix}. \tag{5}$$

We need two point correspondences to obtain the motion parameters.

$$\mathbf{p}_2^\top \mathbf{E}_y \mathbf{p}_1 = 0,\ \mathbf{p}_4^\top \mathbf{E}_y \mathbf{p}_3 = 0. \tag{6}$$

The Macaulay2 code is

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R = QQ[c,s,tx,tz,u1,v1,u2,v2,u3,v3,u4];
eqs = {Eq.(6), tz=1, c^2+s^2=1};
J = eliminate({s,tx,tz},ideal(eqs));
mingens J
```

In this case, we may also obtain a quadratic equation in c.

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