Heavy quark transport coefficient from a Bayesian analysis

Yingru Xu

Duke University

yx59@phy.duke.edu

July 13, 2017

In collaboration with :
Marlene Nahrgang
Jonah E. Bernhard
Shanshan Cao
Steffen A. Bass

This work has been supported by the U.S Department of Energy under grand DE-FG02-05ER41367. Computational resources were provided by the Open Science Grid (OSG).

HQ diffusion coefficients ($D_s, \hat{q}$) in heavy-ion collision

- ~~Not directly measurable~~ —— A modeling problem

model
eg. HQ transport
→
model parameters α, β, γ
eg. $D_s 2\pi T(\alpha, \beta, \gamma)$
↔
experimental obs
eg. R_{AA}, v_2

- Inputs: model parameters $\vec{x}$ – HQ diffusion coefficients
- Output: observables – $R_{\mathrm{AA}}, v_2, \cdots$
- Mapping: $\vec{x} \rightarrow \vec{y}^* = \mathrm{Model}(\vec{x}^*)$
- What if we have 10 parameters? Or 100 observables? How to increase precision?
- $\Rightarrow$ Bayesian inference

$$P(\vec{x}|\vec{y}) = \frac{P(\vec{y}|\vec{x}) \cdot P(\vec{x})}{\int P(\vec{y}|\vec{x}) \cdot P(\vec{x}) d\vec{x}} \propto P(\vec{y}|\vec{x}) \cdot P(\vec{x}) \tag{1}$$

Model parameters

$D_s 2\pi T(\alpha, \beta, \gamma)$

Latin hypercube

Physics model

HQ in-medium evolution

Gussian process emulator

Non-parametric interpolation

fast surrogate

STAR: arxiv 1701.06060, 1601.00695
ALICE: arxiv 1506.06604, PRC **90**, 034904 (2014)
ALICE: prelim at QM'17
CMS: prelim CMS-PAS-HIN-16-007

Experiemental data

-LHC + RHIC: R_{AA}, v_2

MCMC

random walk through

parameter space

Bayes' Theorem

posterior $\propto$ likelihood $\times$ prior

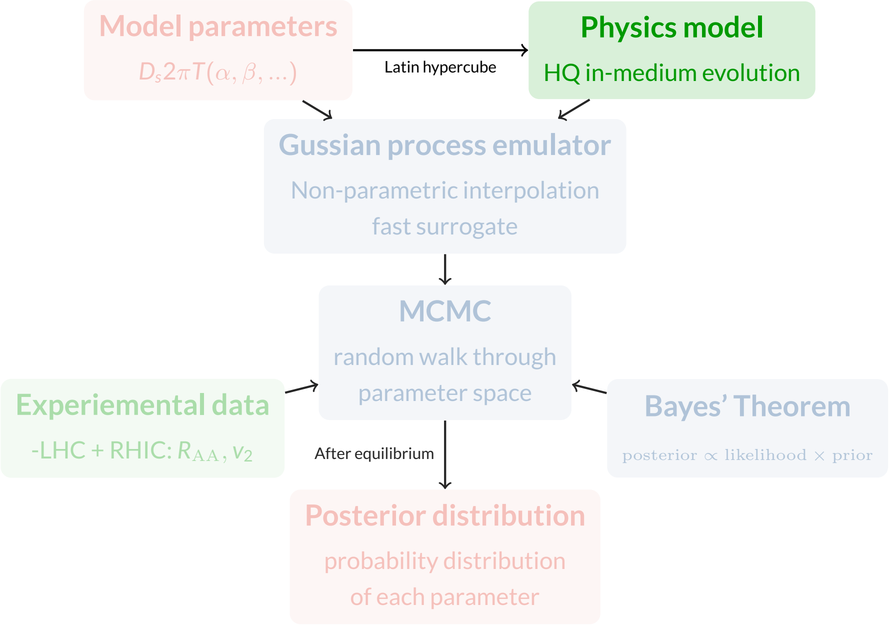
After equilibrium

Posterior distribution

probability distribution

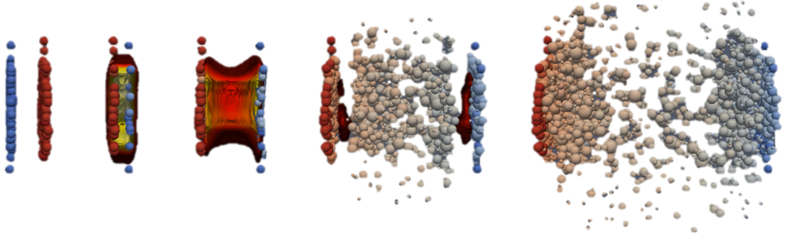
of each parameter

J.E.Bernhard, P.Marcy, C.Coleman-Smith, S.Huzurbazar, R.Wolpert and S.A.Bass, Phys.Rev.C 91, 054910(2015)


Model parameters
$D_s 2\pi T(\alpha, \beta, \ldots)$
Latin hypercube
Physics model
HQ in-medium evolution
Gussian process emulator
Non-parametric interpolation
fast surrogate
MCMC
random walk through
parameter space
Experiemental data
-LHC + RHIC: R_{AA}, v_2
Bayes' Theorem
posterior $\propto$ likelihood $\times$ prior
After equilibrium
Posterior distribution
probability distribution
of each parameter

HQ in heavy-ion collisions

figure credit: Hannah Petersen(Au-Au collisions)



Initial condition:
Spatial IC:
– TRENTo
Momentum IC:
– FONLL

In-medium evol:
HQ transport:
Langevin (col + rad)
Medium:
hydrodynamic

Hadronization:
fragmentation +
recombination

after-burner:
UrQMD

Initial condition

Position space: T$_{\mathbf{R}}$ENTo (A parametric IC model)

- Entropy deposition proportional to eikonal parameterization

$$\left.\frac{ds}{dy}\right|_{\tau=\tau_0} \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

J.S.Moreland,J.Bernhard, and S.A.Bass, Phys.Rev.C 92, 011901(2015)

- $p = 0 \Rightarrow ds/dy \propto \sqrt{T_A T_B}$ (mimic the behavior of IP-Glasma)
- Heavy quark initial production probability: $\left.\frac{dN}{dy}\right|_{\tau=\tau_0} \propto T_{\mathrm{AA}}$

Momentum space: FONLL

- Parton distribution function: CTEQ6
- Nuclear shadowing effect: EPS09 NLO

M.Cacciari,S.Frixione, and P.Nason, arxiv:hep-ph/0102134

Calibration of the medium

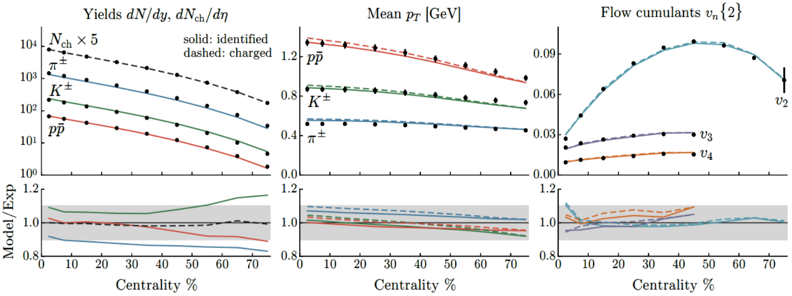
(2+1)D viscous hydro: iEbE-VISHNU

H.Song and U.W.Heinz, Phys.Rev.C 77, 064901(2008)

- Equation of state from lattice QCD (HotQCD collaboration)
- Temperature-dependent shear + bulk vis correction

$$(\eta/s)(T) = (\eta/s)_{\min} + (\eta/s)_{\rm slope}(T - T_c),\, T_c = 154\mathrm{MeV}$$
$$(\zeta/s)(T) = (\zeta/s)_{\rm norm} \times f(T)$$

- All the initial/medium related parameters $(\mathrm{norm}, p, \eta/s$ etc.) are calibrated by Bayesian analysis with experimental data



J.Bernhard,J.S.Moreland,S.A.Bass,J.Liu, and U.Heinz Phys.Rev.C 94, 024907(2015)

Improved Langevin transport model

S.Cao, G.Qin, and S.A.Bass, Phys.Rev.C 92, 024907(2015)

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f_g} \tag{2}$$

- Drag force: $\eta_D(p) = \kappa/(2TE)$
- Thermal random force: $\langle \xi^i(t)\xi^j(t') = \kappa\delta^{ij}\delta(t-t') \rangle$
- Recoil force from gluon radiation: $\vec{f_g} = -d\vec{p_g}/dt$
- Gluon emission probability:

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s P(x)\hat{q}_g}{\pi k_\perp^4} \sin^2\left(\frac{t-t_i}{2\tau_f}\right)\left(\frac{k_\perp^2}{k_\perp^2 + x^2M^2}\right)^4 \tag{3}$$

- $\hat{q}_g = \hat{q}C_A/C_F = 2\kappa C_A/C_F, D_s = 2T^2/\kappa$
- **Diffusion coefficient D_s=?**

A combination of linear temperature dependence and pQCD calculation:

$$D_s 2\pi T(T,p) = \frac{1}{1+(\gamma^2 p)^2} D2\pi T^{\mathrm{linear}} + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2} D2\pi T^{\mathrm{pQCD}} \tag{4}$$

- $D_s 2\pi T^{\mathrm{linear}}$: the linear component, defined as:

$$D2\pi T = \alpha \cdot (1 + \beta \cdot (\frac{T}{T_c} - 1)) \tag{5}$$

- $D_s 2\pi T^{\mathrm{pQCD}} = 4\pi T^3/\kappa$: the pQCD component

$$\kappa_{//} = \kappa_{\perp} = \left\langle p_{\perp}^2 \right\rangle - \langle p_{\perp} \rangle^2 \tag{6}$$

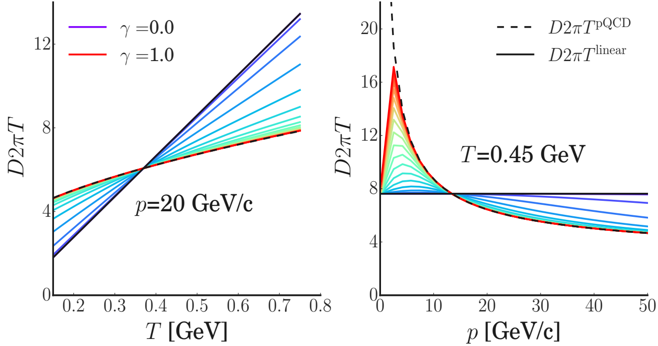
where

$$\langle X \rangle = \frac{\gamma_q}{2E_Q} \int \frac{d^3 p_q}{(2\pi)^3 2E_q} \frac{d^3 p_{q'}}{(2\pi)^3 2E_{q'}} \frac{d^3 p'_Q}{(2\pi)^3 2E'_Q} f_q(p_q)(2\pi)^4 \delta(p_Q + p_q - p'_Q - p'_q) |\mathcal{M}|^2_{Q+q\to Q'+q'}$$

a parametrization of $D_s 2\pi T$

$$D_s 2\pi T(T,p) = \frac{1}{1+(\gamma^2 p)^2} D2\pi T^{\rm linear} + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2} D2\pi T^{\rm pQCD}$$

- $D_s 2\pi T(p=0) = D_s 2\pi T^{\rm linear}$, $D_s 2\pi T(p>>0) = D_s 2\pi T^{\rm pQCD}$
- $p < 1/\gamma^2$: linear component contributes more
  $p > 1/\gamma^2$: pQCD component contributes more
  $p = 1/\gamma^2$: contributes equally
- An example of varying parameter γ: (with α, β fixed)



Prior calculation

HQ spatial diffusion coefficients:

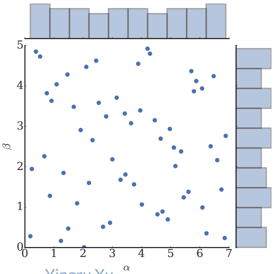
$$D_s 2\pi T(T,p;\alpha,\beta,\gamma) = \frac{1}{1+(\gamma^2 p)^2} D2\pi T^{\mathrm{linear}}(\alpha,\beta) + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2} D2\pi T^{\mathrm{pQCD}}$$

Inputs: $X_{60\times 3}$

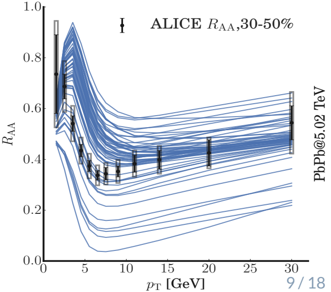
Parameters	Range
α	0.1-7.0
β	0-5.0
γ	0.0-0.6

Outputs: $Y_{60\times 69}$

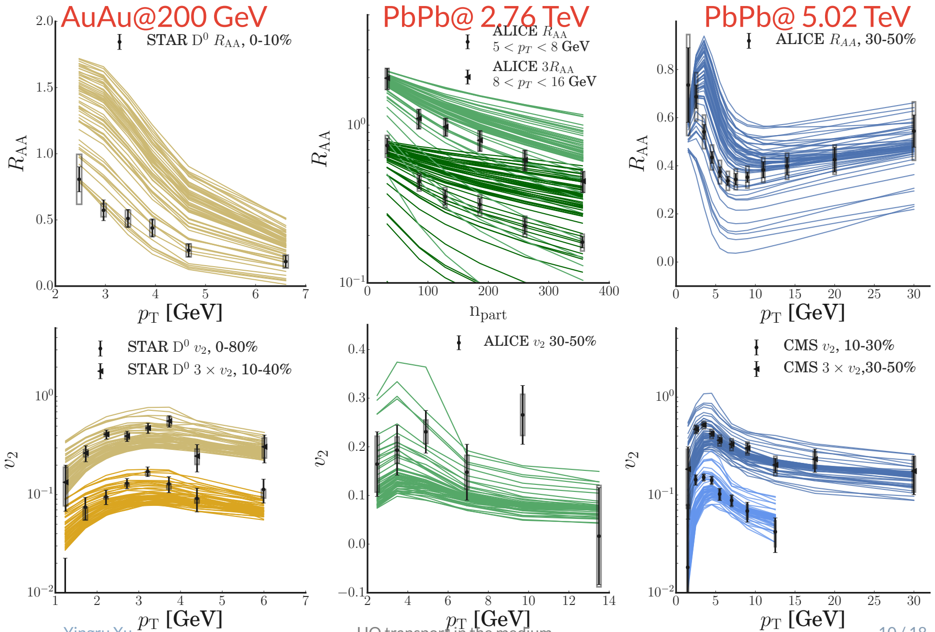
Experiment	variables	cut
AuAu@200 GeV	$R_{AA}(p_T)$	0-10%
	$v_2(\mathrm{EP})(p_T)$	0-80%,10-40%
PbPb@2.76 TeV	$R_{AA}(n_{part})$	$p_T \sim$ 5-8, 8-16 GeV/c
	$v_2(EP)(p_T)$	30-50%
PbPb@5.02 TeV	$R_{AA}(p_T)$	30-50%
	$v_2\{2\}(p_T)$	10-30%, 30-50%



⇒ 60 samples $Y = Model(X)$ ⇒



Prior & training data



Model parameters

$D_s 2\pi T(\alpha, \beta, \ldots)$

Latin hypercube

Physics model

HQ in-medium evolution

Gussian process emulator

Non-parametric interpolation

fast surrogate

MCMC

random walk through

parameter space

Experiemental data

-LHC + RHIC: R_{AA}, v_2

Bayes' Theorem

posterior $\propto$ likelihood $\times$ prior

After equilibrium

Posterior distribution

probability distribution

of each parameter

GP emulator

Difficulties

Full Langevin framework run $\propto$ 8hrs for 100 events produced

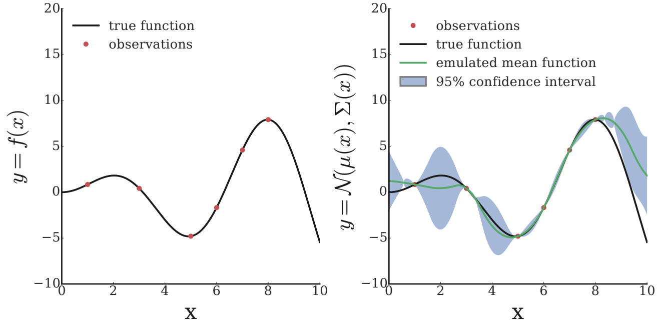
$\Downarrow$

5000 events needed for an event-by-event study

$\Downarrow$

$O(10^3)$ CPU hours to evaluate one parameter point $\vec{x}$

GP emulator: a mapping from $\vec{x} \rightarrow y = \mathcal{N}(\mu(\vec{z}), \Sigma(\vec{x}))$



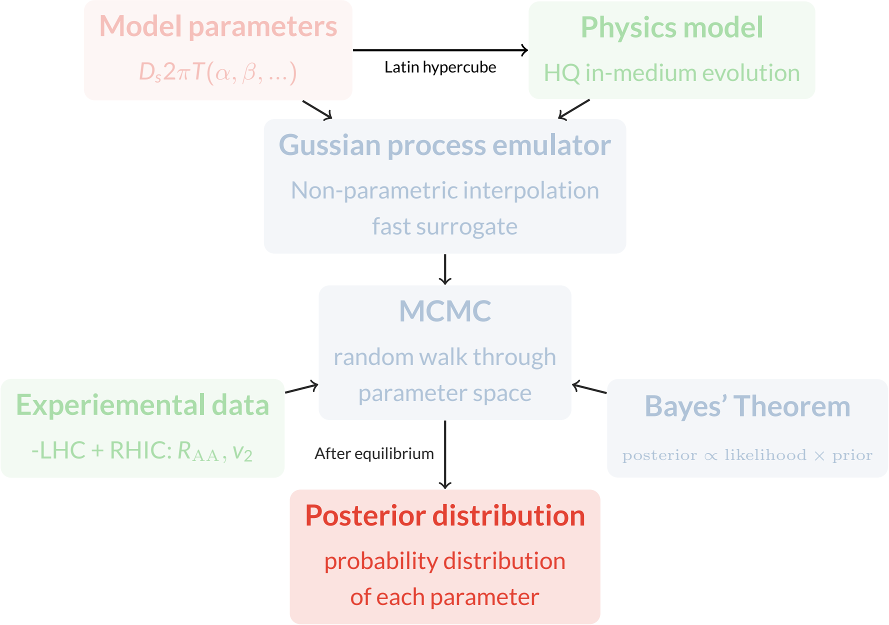
Bayes' Theorem:

$$\text{posterior} \propto \text{likelihood} \times \text{prior} \qquad (7)$$

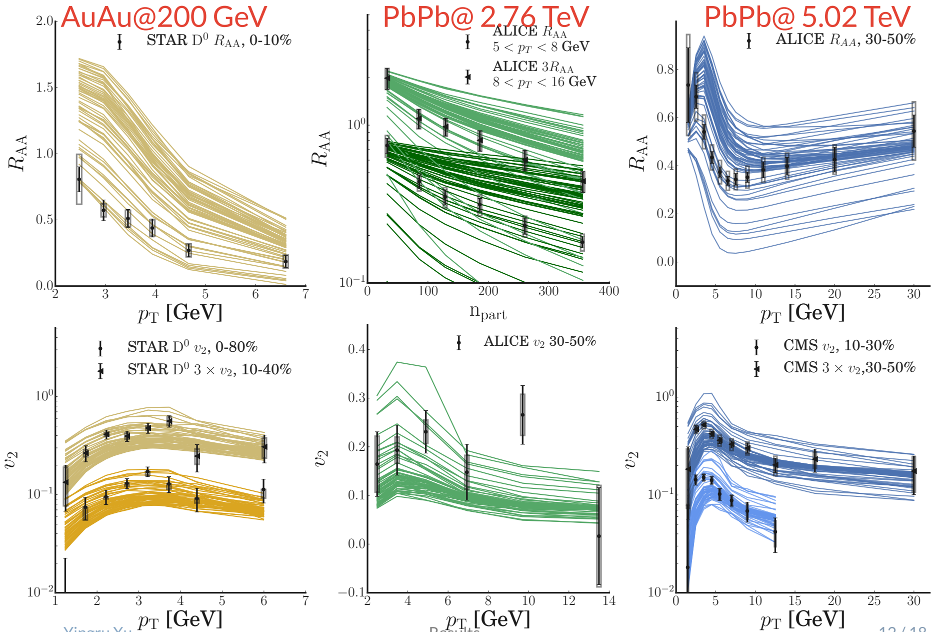
- Posterior: $P(\vec{x}|\vec{y}_{\rm exp})$ – the distribution for parameter $\vec{x}$ given the observation of $\vec{y}_{\rm exp}$
- Prior: $P(\vec{x})$ – flat (uniform) distribution
- Likelihood: $P(\vec{y}_{\rm exp}|\vec{x}) \propto \exp[(\vec{y} - \vec{y}_{exp})\Sigma^{-1}(\vec{y} - \vec{y}_{exp})^T]$
- Covariance matrix: $\Sigma = \mathrm{diag}(\sigma^2_{stat}) + \mathrm{diag}(\sigma^2_{sys})$

MCMC

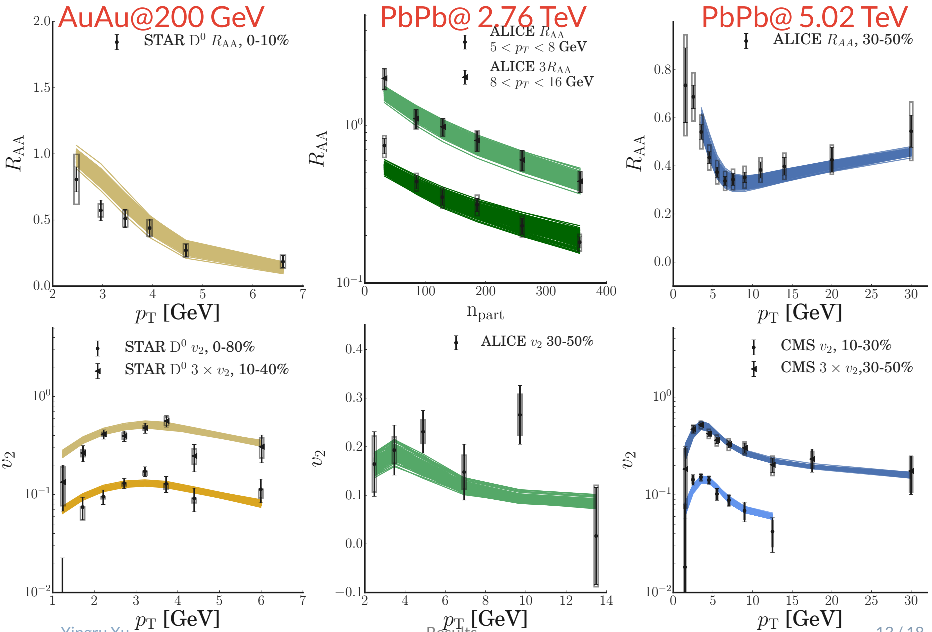
- Random walk weighted by likelihood $P(\vec{y}_{\rm exp}|\vec{x})$
- Each step is accepted or rejected according to the relative likelihood
- Acceptance rate for this study: 40-50%
- When reach equilibrium $\rightarrow$ **posterior distribution**


Model parameters
$D_s 2\pi T(\alpha, \beta, \ldots)$
Latin hypercube
Physics model
HQ in-medium evolution
Gussian process emulator
Non-parametric interpolation
fast surrogate
MCMC
random walk through
parameter space
Experiemental data
-LHC + RHIC: R_{AA}, v_2
Bayes' Theorem
posterior $\propto$ likelihood $\times$ prior
After equilibrium
Posterior distribution
probability distribution
of each parameter

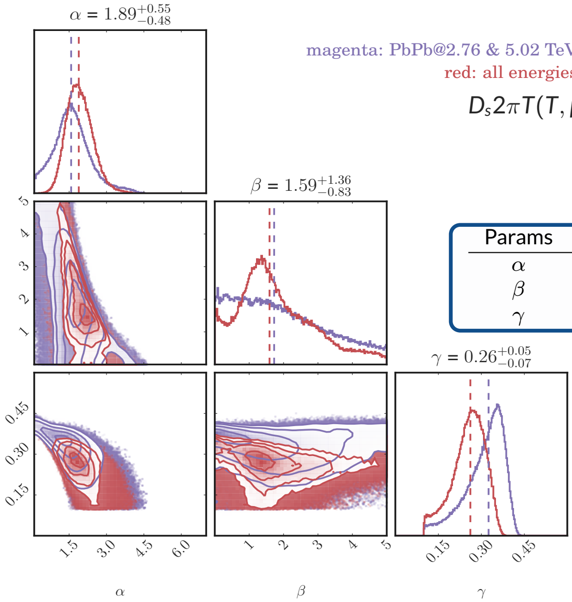
Prior & training data



Posterior results: calibated observables



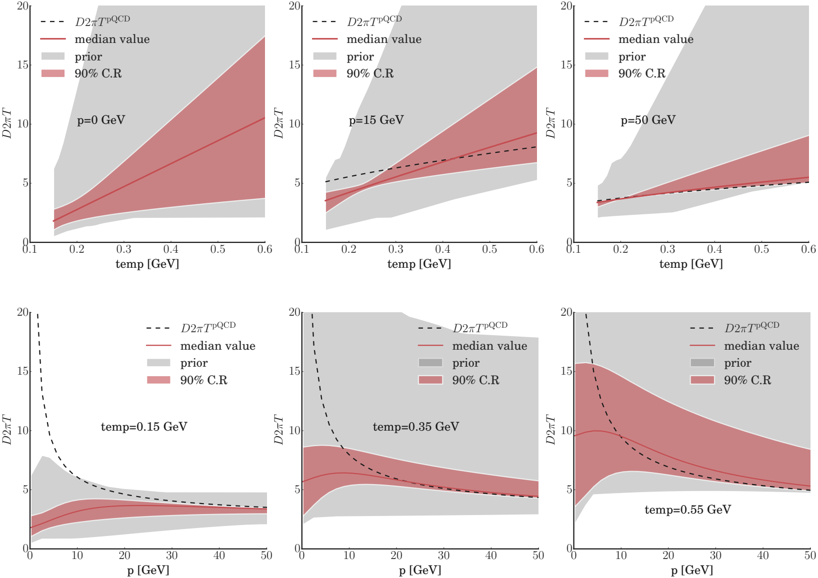
Posterior results: parameter distributions



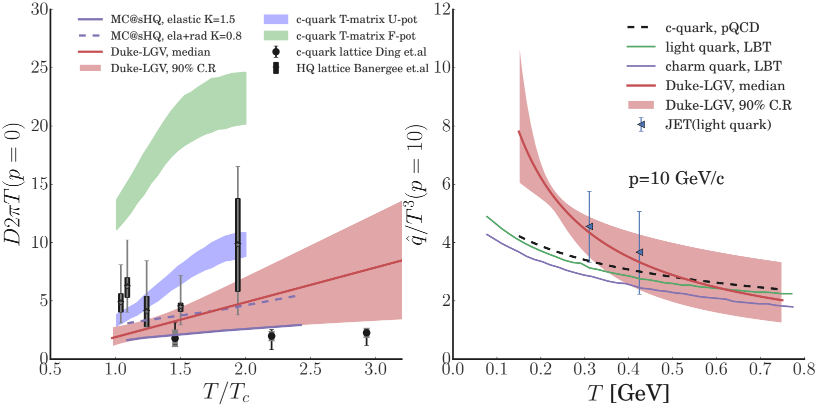
$$D_s 2\pi T(T,p) = \frac{1}{1+(\gamma^2 p)^2}\left(\alpha \cdot \left(1+\beta\frac{T}{T_c}-1\right)\right) + \frac{\gamma^2 p)^2}{1+(\gamma^2 p)^2} D2\pi T^{\mathrm{pQCD}}$$

Params	Prior	LHC	All
α	0.1-7.0	0.45-3.00	1.14 - 2.84
β	0-5.0	0.17-4.39	0.24 - 4.04
γ	0.0-0.6	0.17-0.39	0.14-0.34

Posterior results: $D_s 2\pi T$



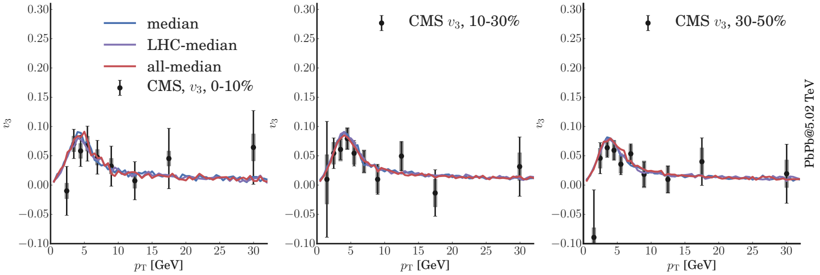
Posterior results: $D_s 2\pi T(p=0)$ & $\hat{q}$



- Compatible with lattice QCD calculation
- $D_s 2\pi T(p=0)$ best constrained around $T \sim 200-250$ MeV, with the value (1-3) near T_c, and positive slope for temperature dependence above T_c
- Charm quark $\hat{q}/T^3$ obvious negative slope for temperature dependence

Prediction beyond the calibration

D-meson v_3 in PbPb collisions at 5.02 TeV:



Summary

- Bayesian analysis can be utilized to systematically and quantitatively estimate the heavy flavor diffusion coefficients; the improved Langevin model is able to simultaneously describe D-meson $R_{\mathrm{AA}}, v_2, v_3$
- $D_s 2\pi T(p=0)$ compatible with lQCD calculation, with the value in the range of (1-3) near T_c, and a positive slope for temperature dependence above T_c
- Non-perturbative contribution ($D_2 \pi T^{\mathrm{linear}}$) still plays a role at $p \sim 10-20$ GeV. Higher momentum region compatible with pQCD calculation.
- Future work:
  - ⋆ application of Bayesian analysis to comparative study of Boltzmann vs. Langevin transport
  - ⋆ application to other rare probes and observables
  - ⋆ simultaneously calibration on soft and hard sector