

Bayesian analysis for heavy quark studies

Yingru Xu

Duke University

yx59@phy.duke.edu

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In collaboration with :

Marlene Nahrgang

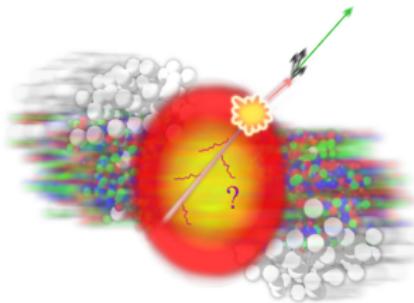
Jonah E. Bernhard

Shanshan Cao

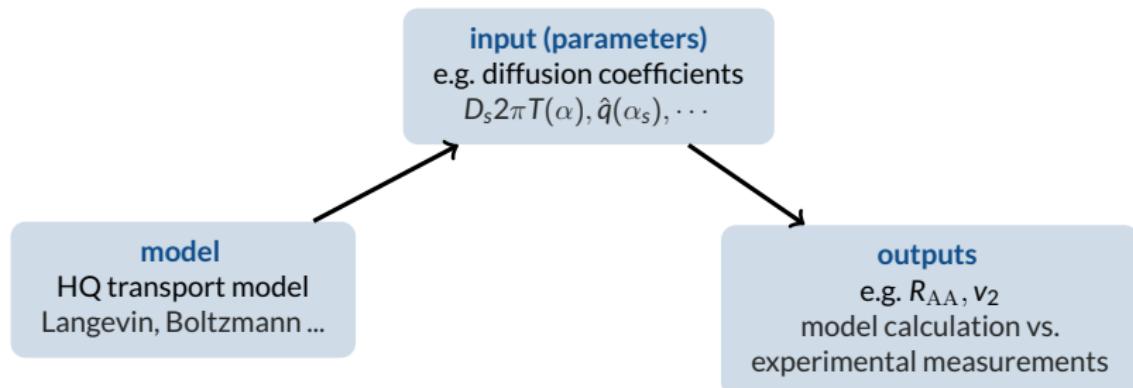
Steffen A. Bass

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[www2.lbl.gov/Science-Articles/
Archive/sabl/2008/Feb/jets.html](http://www2.lbl.gov/Science-Articles/Archive/sabl/2008/Feb/jets.html)

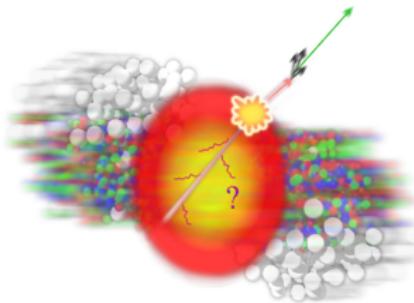


A modeling problem

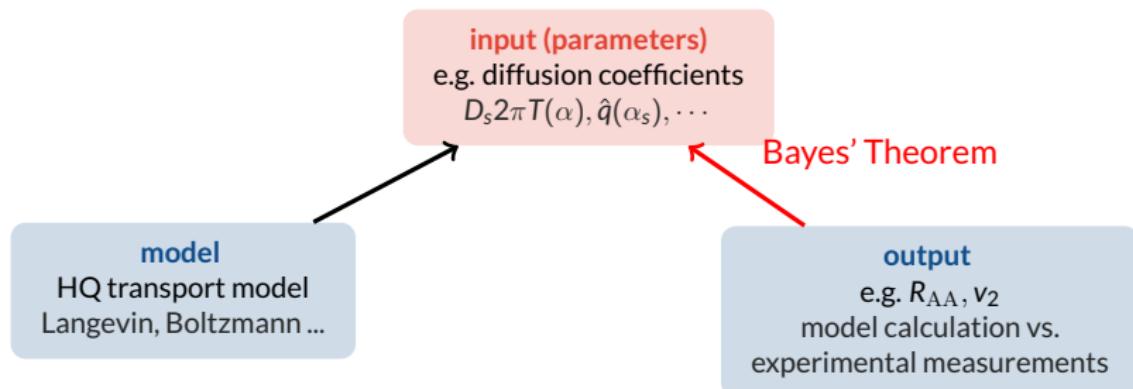


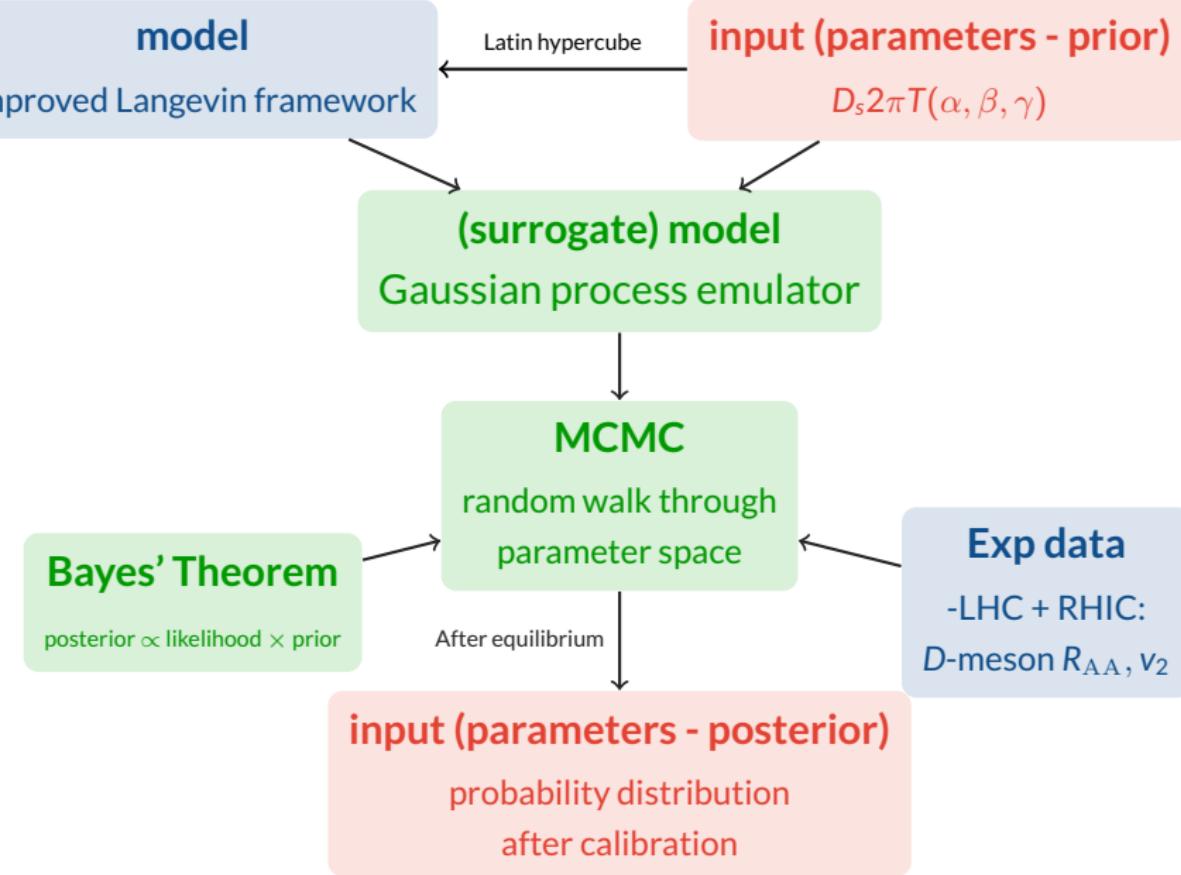
Heavy quark evolution in heavy-ion collisions

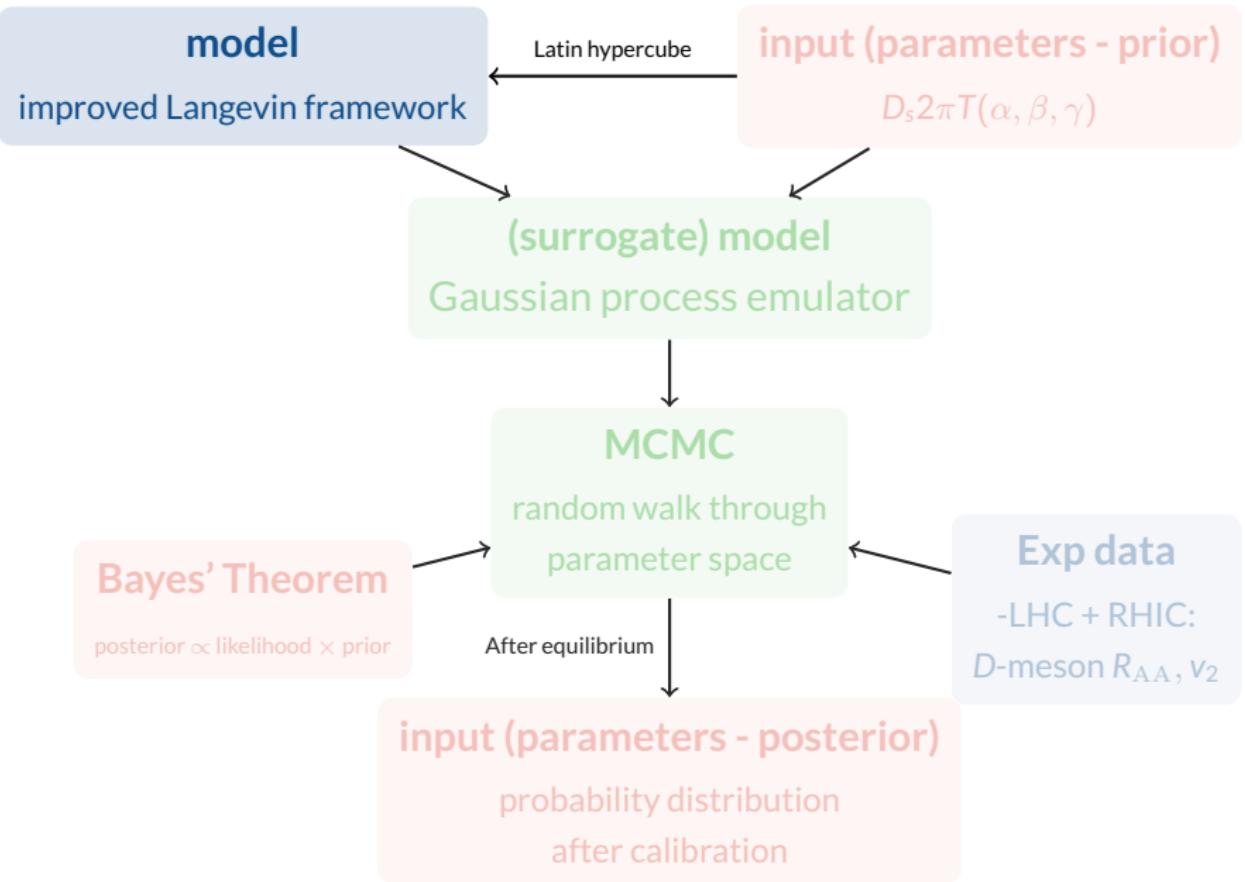
[www2.lbl.gov/Science-Articles/
Archive/sabl/2008/Feb/jets.html](http://www2.lbl.gov/Science-Articles/Archive/sabl/2008/Feb/jets.html)



A modeling problem (data-driven)

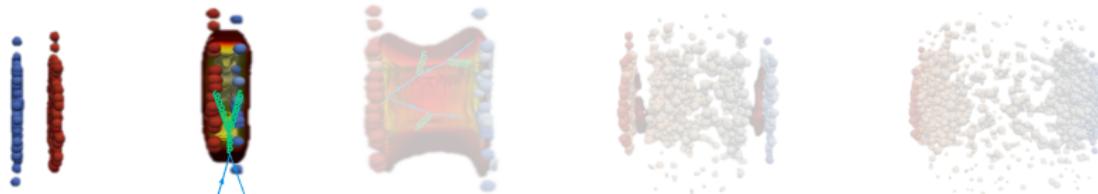






Model: initial condition

figure credit: Hannah Petersen

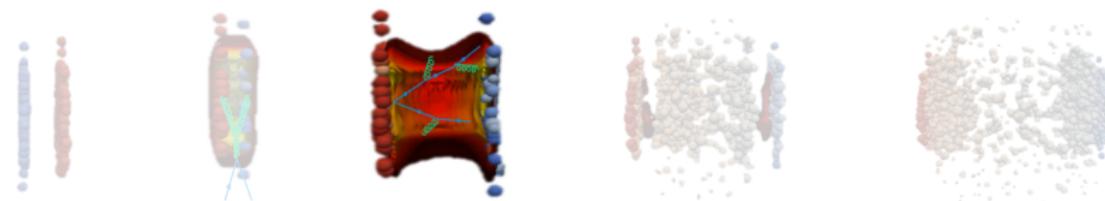


- **Heavy quark initial positions:**
 - scaled by the distribution of binary collisions
- **Heavy quark initial momentum:**
 - FONLL
 - nuclear shadowing effect: EPS09 NLO
- **Soft matter: T_RENTo**
 - parametric model of initial entropy deposition
 - consistent with a saturation-based picture: e.g. IP-Glasma

M.Cacciari, S.Frixione, and P.Nason,
[arxiv:hep-ph/0102134](https://arxiv.org/abs/hep-ph/0102134)

J.S.Moreland, J.Bernhard, and S.A.Bass,
Phys.Rev.C 92, 011901(2015)

Model: in-medium evolution



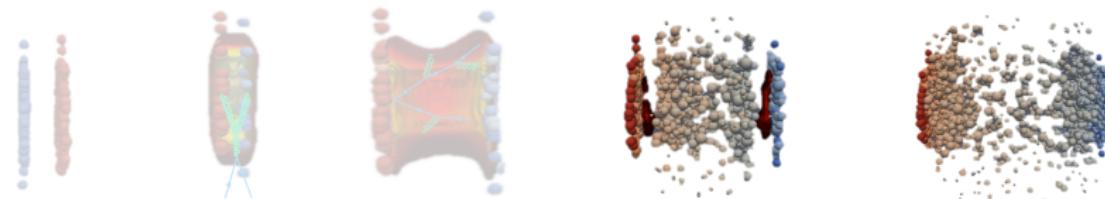
- Improved Langevin transport model:

S.Cao, G.Qin, and S.A.Bass,
Phys.Rev.C 92, 024907(2015)

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f}_g$$

- Drag force: $\eta_D(p) = \hat{q}/(4TE)$
- Thermal random force: $\langle \xi^i(t)\xi^j(t') \rangle = \frac{1}{2}\hat{q}\delta^{ij}\delta(t-t')$
- Recoil force from gluon radiation: $\vec{f}_g(\hat{q}) = -d\vec{p}_g/dt$
- Diffusion coefficient $D_s = 4T^2/\hat{q} \Rightarrow ?$

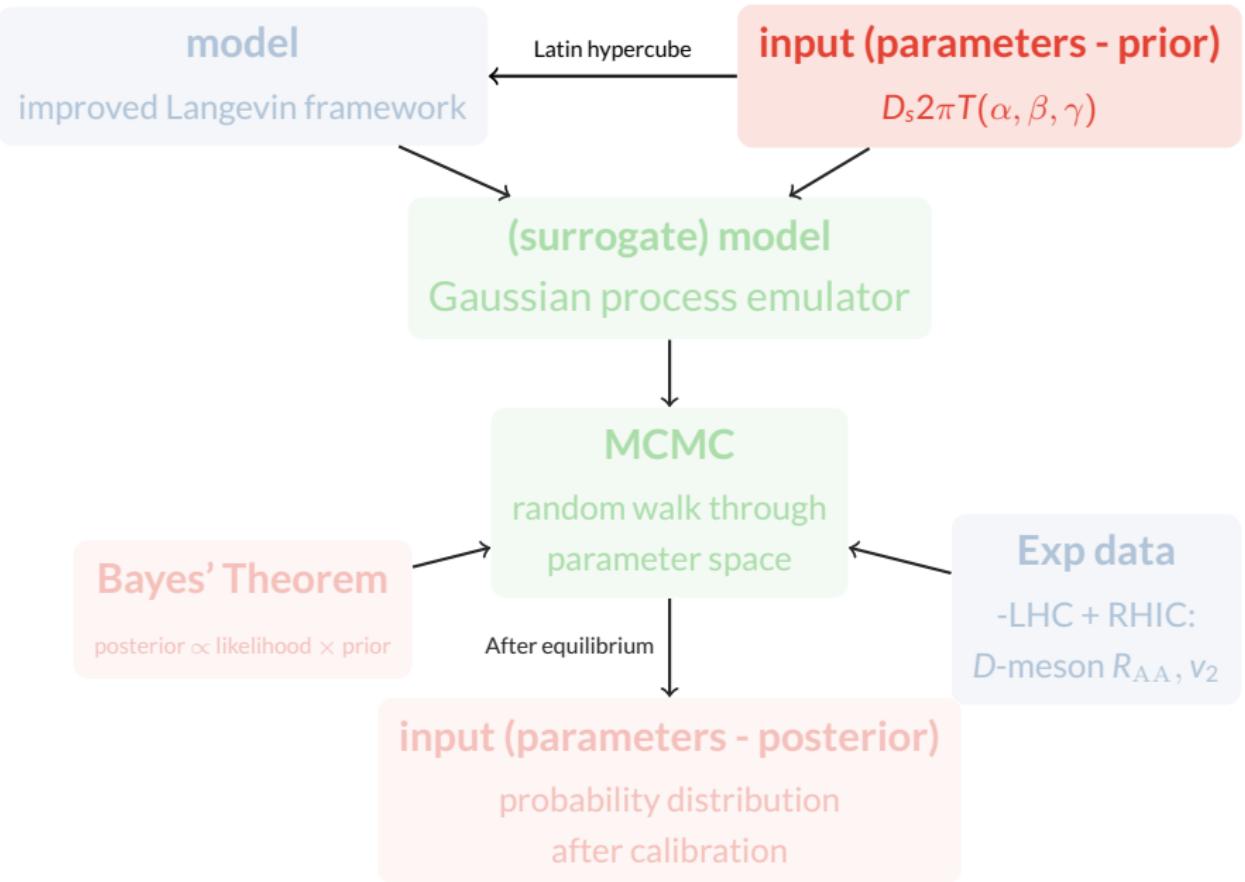
Model: hadronization + afterburner



- Hadronization:
Fragmentation (PYTHIA) + recombination
- Late stage hadronic interactions:
UrQMD
- QGP medium evolution:
 - (2+1)D viscous hydrodynamics model: iEoS-VISHNU
 - Shear and bulk viscosities: $\eta/s(T)$, $\zeta/s(T)$
 - All the soft medium related parameters (normalization, η/s , ζ/s etc.) are calibrated on soft hadronic observables

H.Song and U.W.Heinz,
Phys.Rev.C 77, 064901(2008)

J.Bernhard, J.S.Moreland, S.A.Bass, J.Liu, and U.Heinz
Phys.Rev.C 94, 024907(2015)



Input: HQ diffusion coefficient

Parametrization of $D_s 2\pi T(T, p; \alpha, \beta, \gamma)$:

- A combination of linear T -dependent and pQCD calculation

$$D_s 2\pi T(T, p) = \frac{1}{1 + (\gamma^2 p)^2} (D_s 2\pi T)^{\text{soft}} + \frac{(\gamma^2 p)^2}{1 + (\gamma^2 p)^2} (D_s 2\pi T)^{\text{pQCD}}$$

- $(D_s 2\pi T)^{\text{soft}}$: the linear- T component

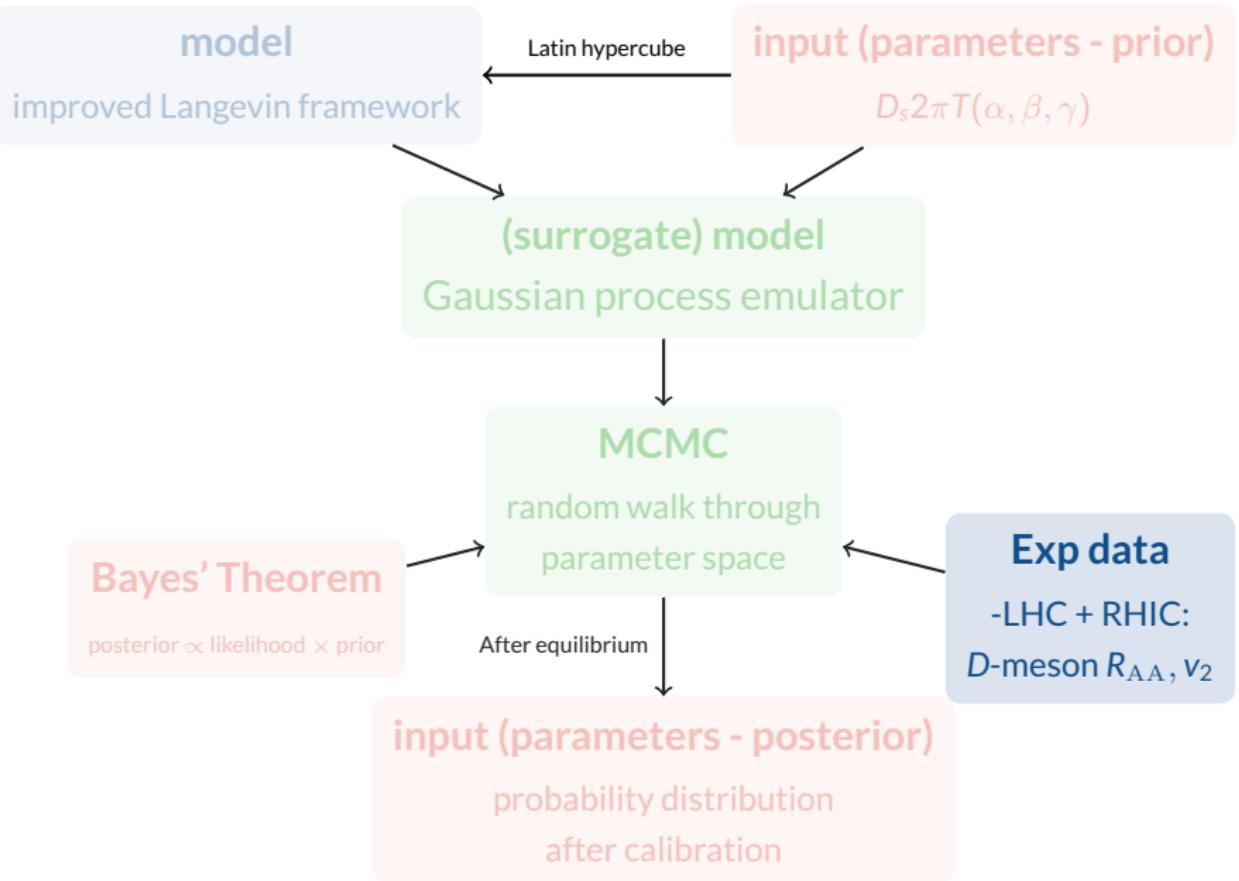
$$(D_s 2\pi T)^{\text{soft}} = \alpha \cdot \left[1 + \beta \cdot \left(\frac{T}{T_c} - 1 \right) \right]$$

- $(D_s 2\pi T)^{\text{pQCD}} = 8\pi T^3 / \hat{q}^{\text{pQCD}}$: the pQCD component

$$\hat{q}^{\text{pQCD}} = \left\langle (\vec{p}_{Q'})^2 - (\hat{p}_Q \cdot \vec{p}_{Q'})^2 \right\rangle \propto T^3 \ln\left(\frac{ET}{m_D^2}\right)$$

where $\langle X \rangle = \frac{\gamma q}{2E_Q} \int f_q(p_q) |\mathcal{M}|_{2 \rightarrow 2}^2 \cdot X$

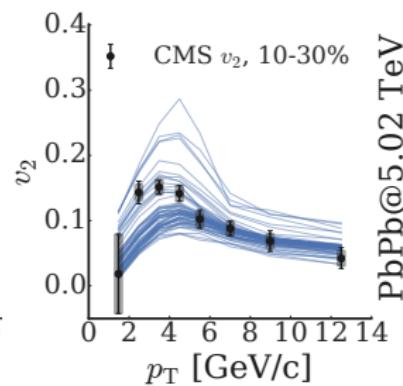
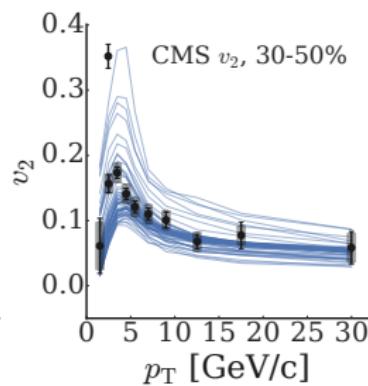
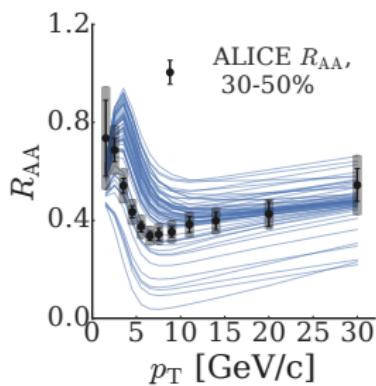
What is $D_s 2\pi T \Rightarrow$ what is (α, β, γ) ?



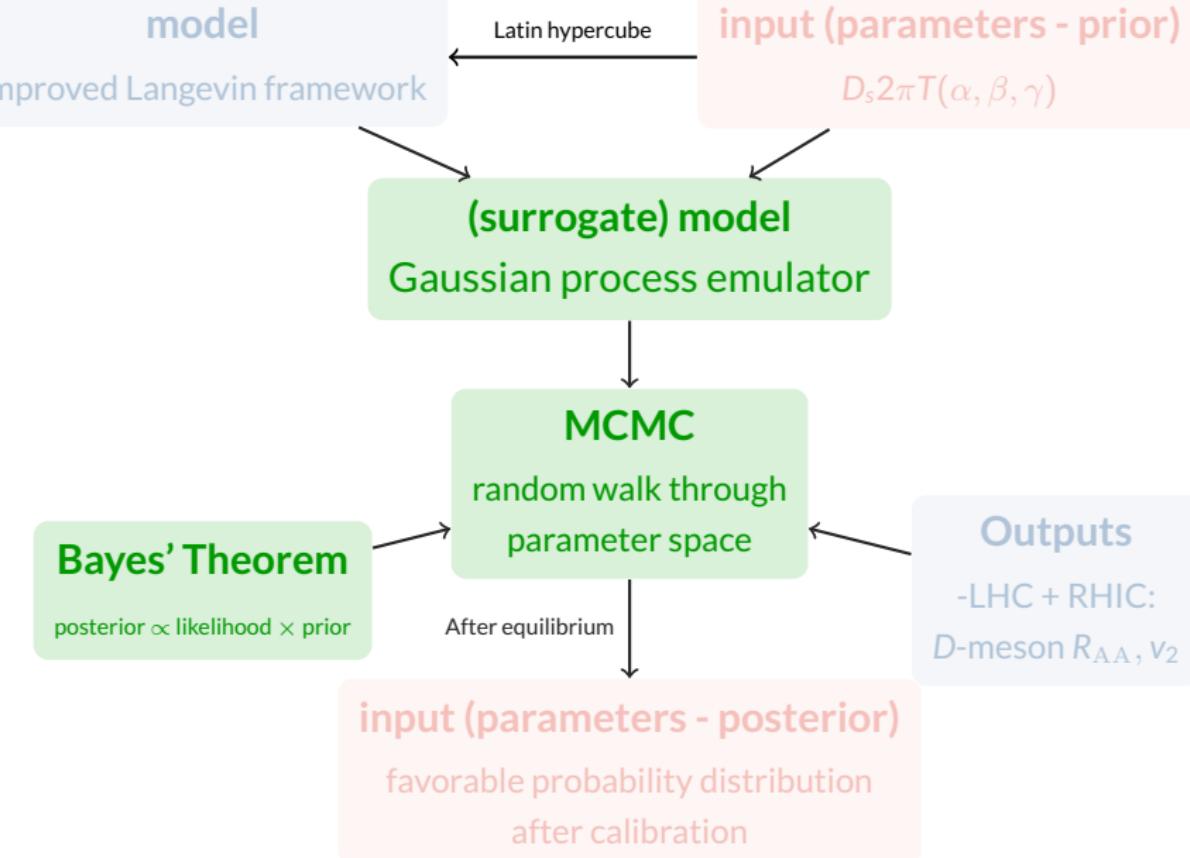
Output: experimental observables

D-meson R_{AA} , v_2 at RHIC and the LHC:

Experiment	variables	centrality/ p_T cut
PbPb@5.02 TeV	$R_{AA}(p_T)$	30-50%
	$v_2(p_T)$	10-30%, 30-50%
PbPb@2.76 TeV	$R_{AA}(n_{part})$	$p_T \sim 5-8, 8-16$ GeV/c
	$v_2(p_T)$	30-50%
AuAu@200 GeV	$R_{AA}(p_T)$	0-10%
	$v_2(p_T)$	0-80%, 10-40%



STAR: arxiv 1701.06060, 1601.00695
 ALICE: arxiv 1506.06604, PRC **90**, 034904 (2014)
 ALICE: prelim at QM'17
 CMS: prelim CMS-PAS-HIN-16-007



Bayes' Theorem

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

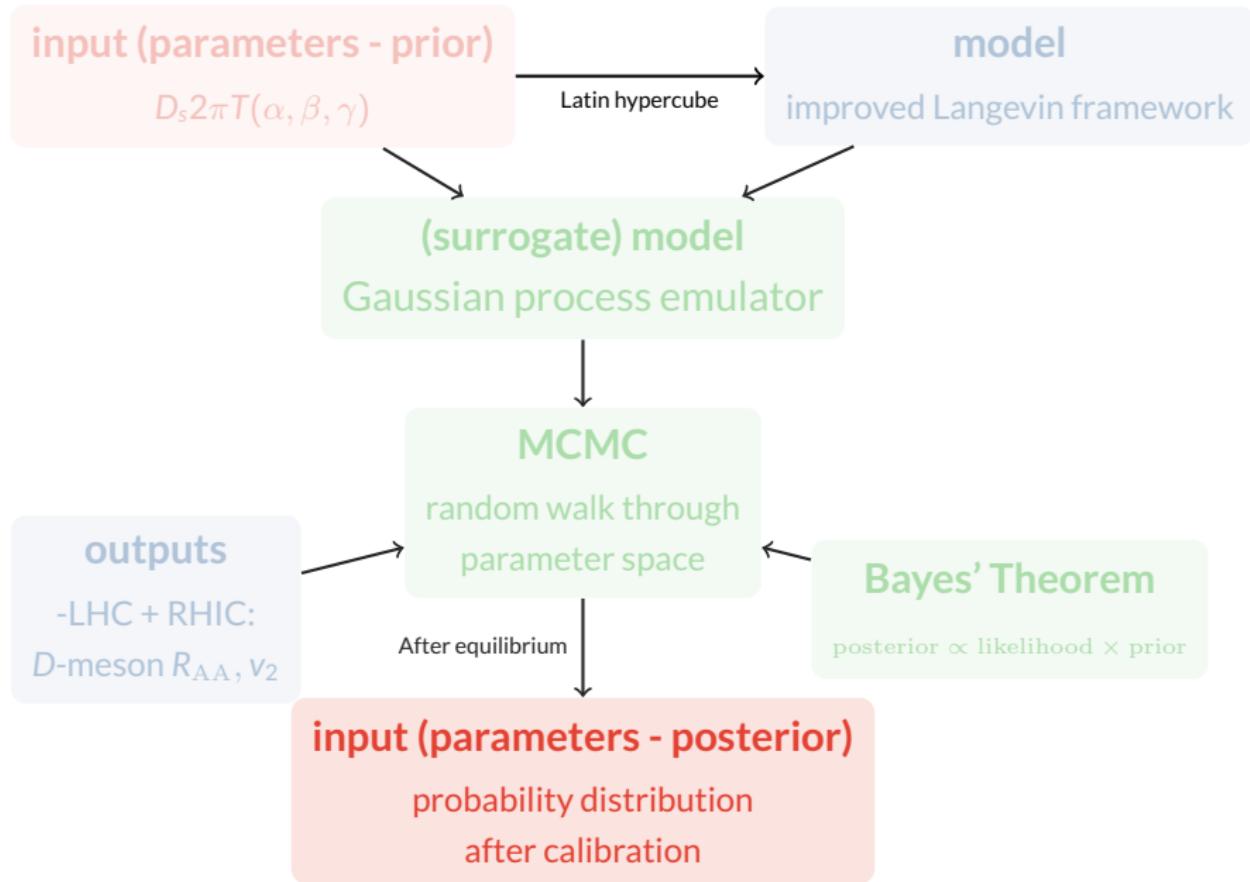
- **posterior**: distribution of input parameters favored by experimental observations
- prior: initial distribution of input parameters – uniform
- **likelihood**: $\propto \exp [(\mathbf{y} - \mathbf{y}_{\text{exp}})^T \Sigma^{-1} (\mathbf{y} - \mathbf{y}_{\text{exp}})]$
 - \mathbf{y} is the model's output at given input parameter $\mathbf{x} = (\alpha, \beta, \gamma)$
 - $\Sigma = \text{diag}(\sigma_{\text{stat}}^2) + \text{diag}(\sigma_{\text{sys}}^2) + \sigma_{\text{model}}^2$, covariance matrix

Gaussian process emulator

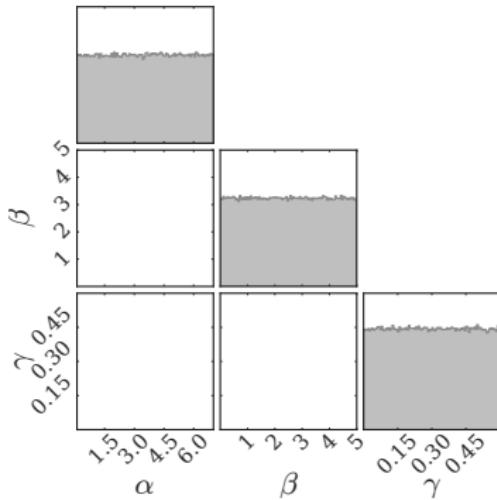
Markov chain Monte Carlo (MCMC)

- Random walk weighted by **likelihood**
- When reach equilibrium → **posterior distribution** of parameters

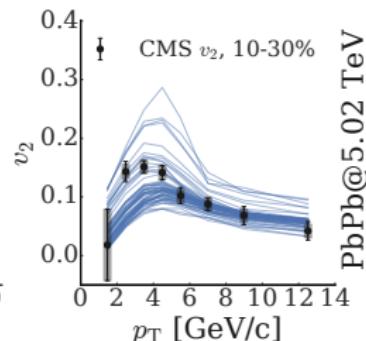
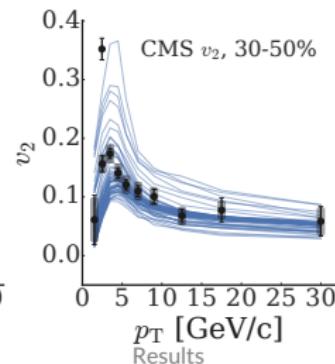
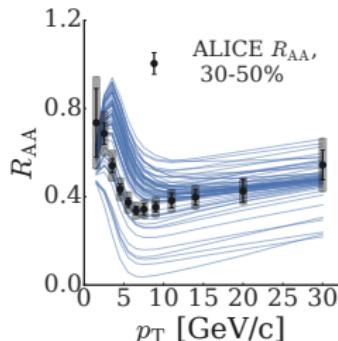
- Surrogate model to real Langevin model \Leftarrow necessary for computationally expensive model
- Returns not only prediction, also the uncertainty σ_{model}
- We make sure it works!



Results: before calibration (prior)

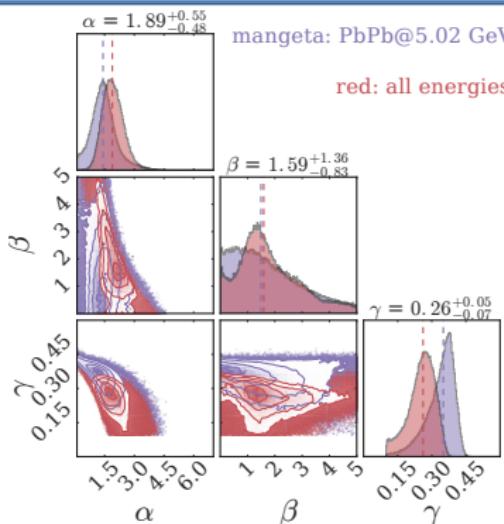


- $\alpha \in (0.1, 7.0)$, $\beta \in (0, 5.0)$,
 $\gamma \in (0, 0.6)$
- Uniform prior distributions
- Model outputs: widely spread

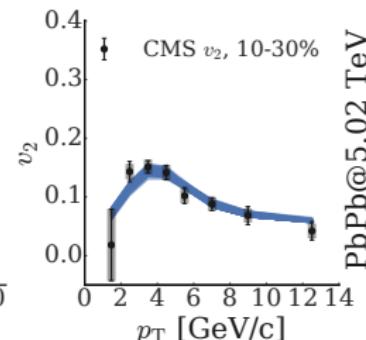
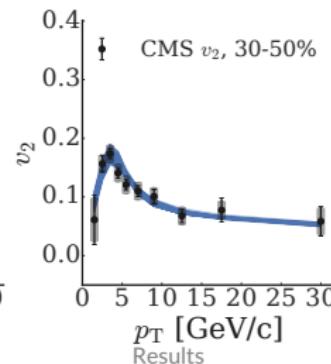
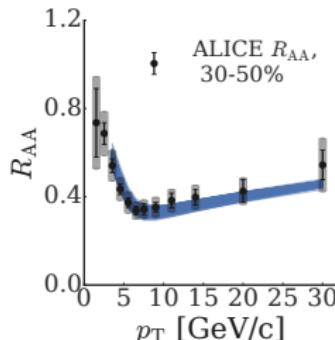


PbPb@5.02 TeV

Results: after calibration (posterior)

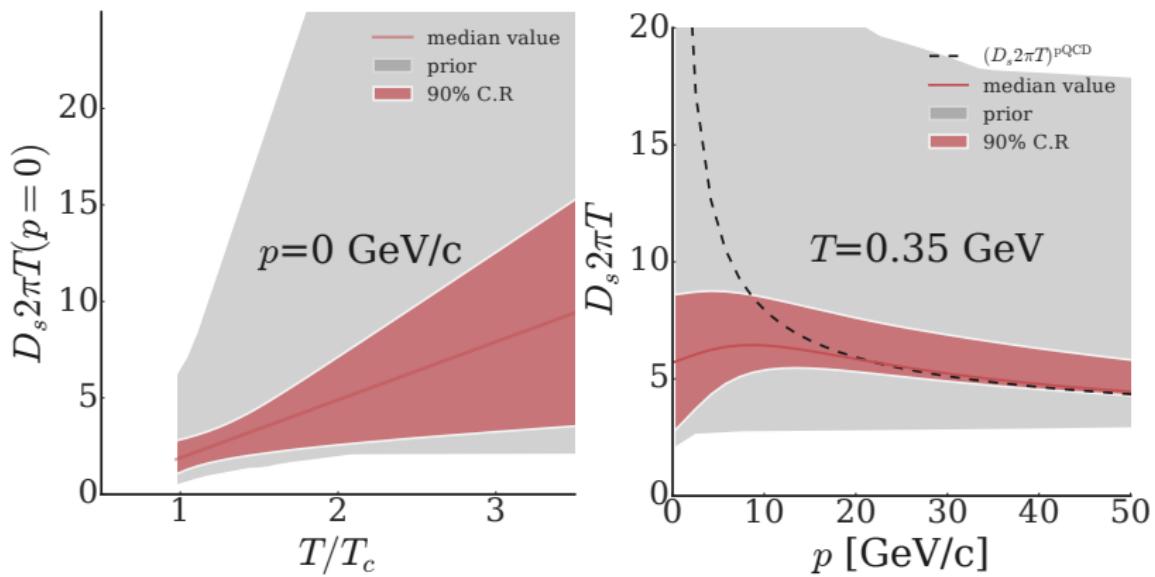


- (5-95)% percentile:
 $\alpha \in (1.14, 2.84), \beta \in (0.24, 4.04), \gamma \in (0.14, 0.34)$
- Distinguish peak for α, γ ; negative correlation between α, β
- Model output: calibrated on experimental data
- Distribution width \Leftarrow uncertainties:
how to improve it?



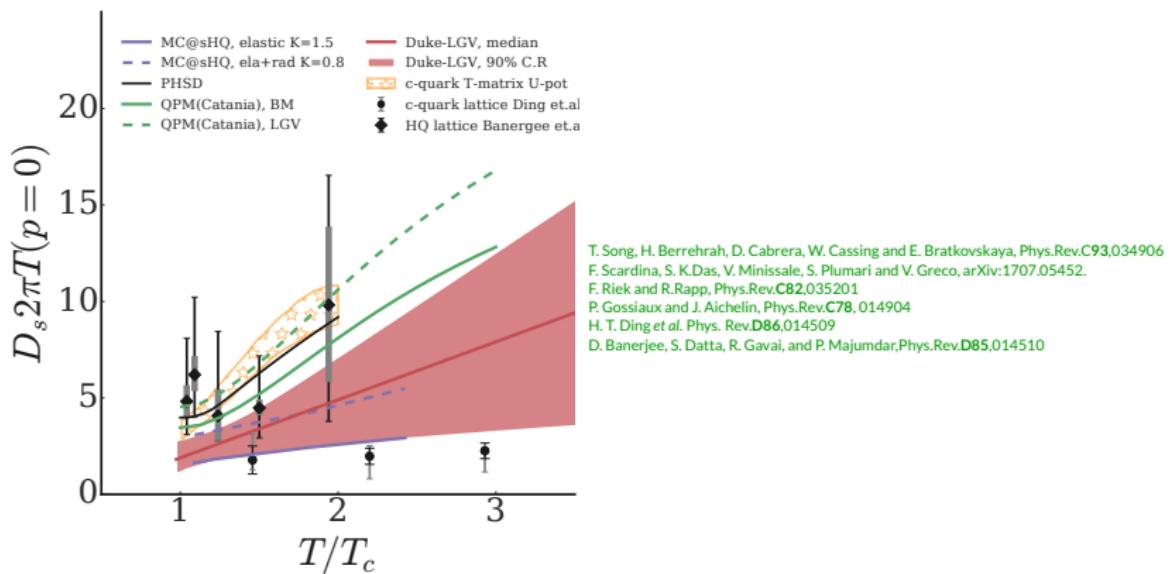
PbPb@5.02 TeV

Results: after calibration $D_s 2\pi T$



- Zero momentum $D_s 2\pi T$ has positive temperature dependence, mostly constrained between $1-2 T_c$
- Different behavior from $(D_s 2\pi T)^{\text{pQCD}}$ until mid- $p_T \Leftarrow$ significant non-perturbative effects

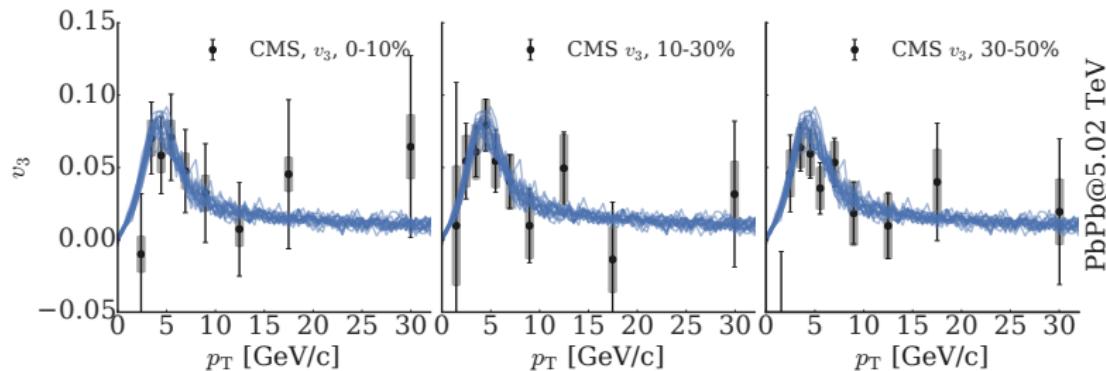
Results: this work vs. other models



- Comparing with other model's calculation, minimum of (1-3) around T_c
- Most models are converging with a minimum of (1-7) near T_c

What's next?

- **Observables beyond R_{AA} , v_2 :**
 D -meson v_3 in PbPb collisions at 5.02 TeV:



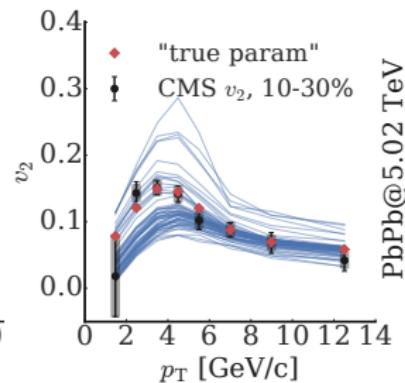
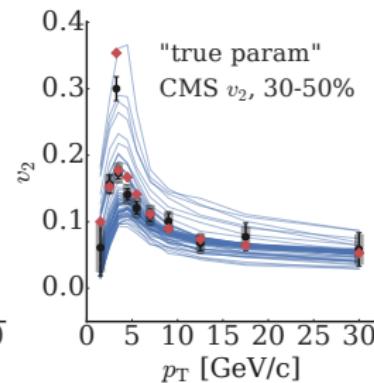
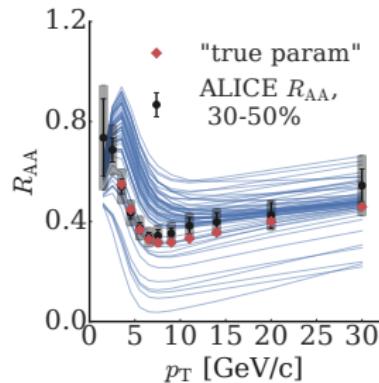
What's next?

- Observables beyond R_{AA}, v_2
- Improvement on constraints on transport coefficients:
 - Pursue higher precision on current observables
 - Explore other observables

Higher precision?

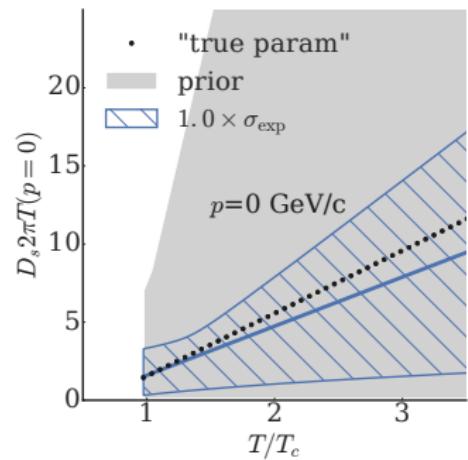
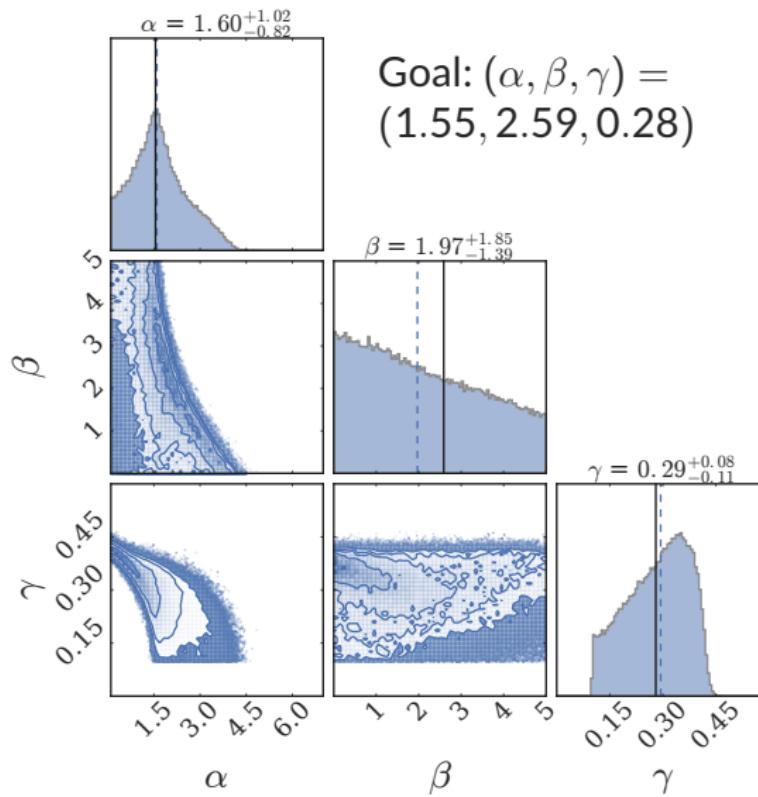
Question: To which degree do we mean high precision?

- **Test:** suppose our model describes perfectly the physics of heavy quarks; suppose a **true parameter set** exists, how accurate the data need to be to recover it from just R_{AA} and v_2 :
- A set of parameter: $(\alpha, \beta, \gamma) = (1.55, 2.59, 0.28)$

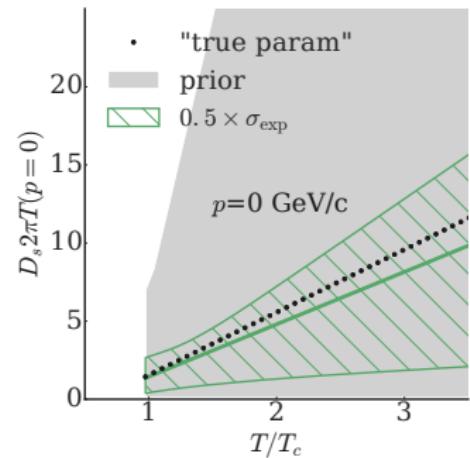
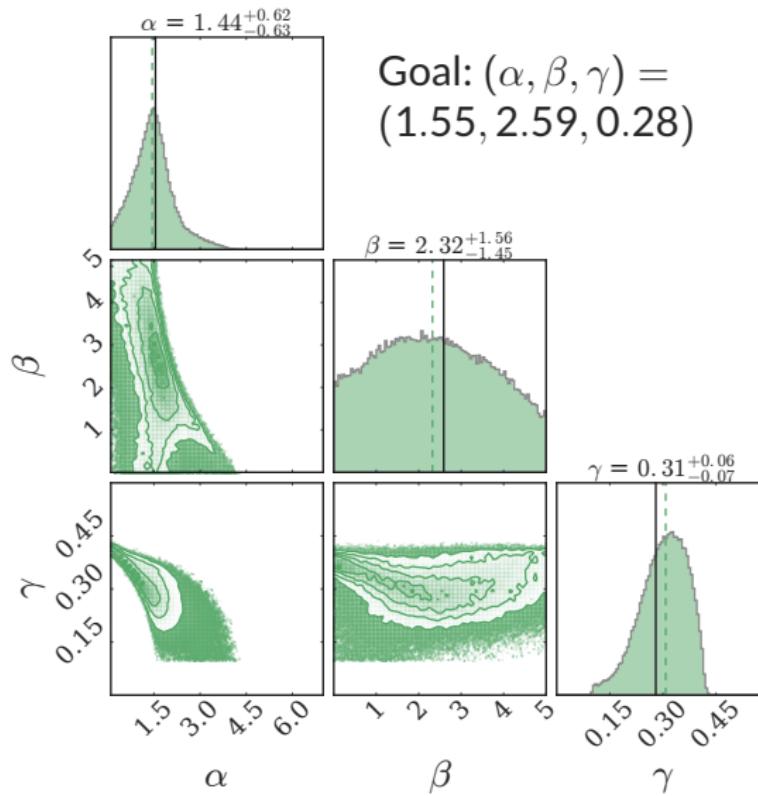


PbPb@5.02 TeV

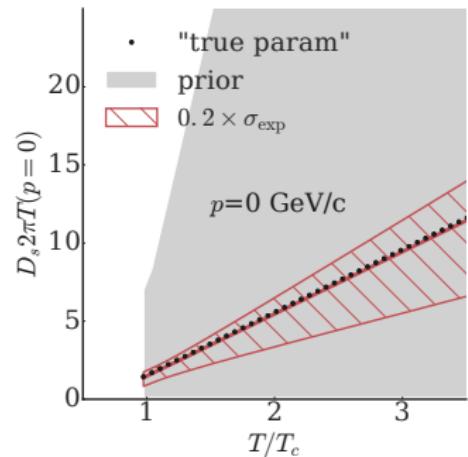
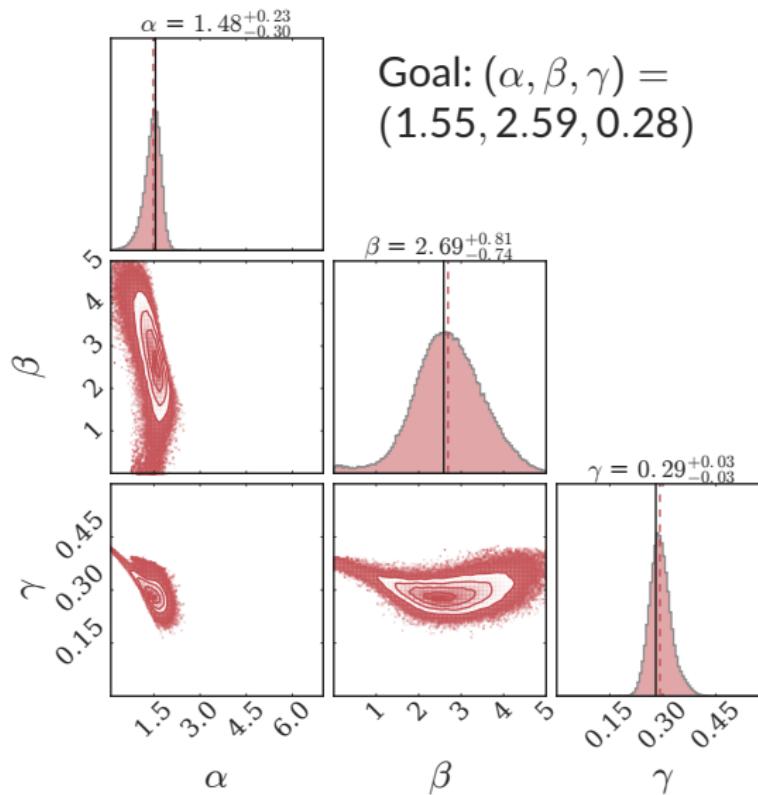
Higher precision: current σ_{exp}



Higher precision: $0.5\sigma_{\text{exp}}$



Higher precision: $0.2\sigma_{\text{exp}}$



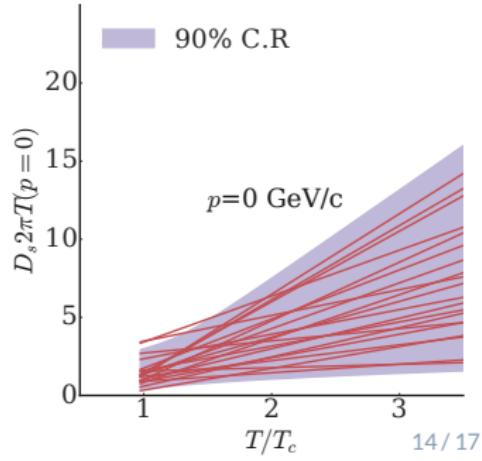
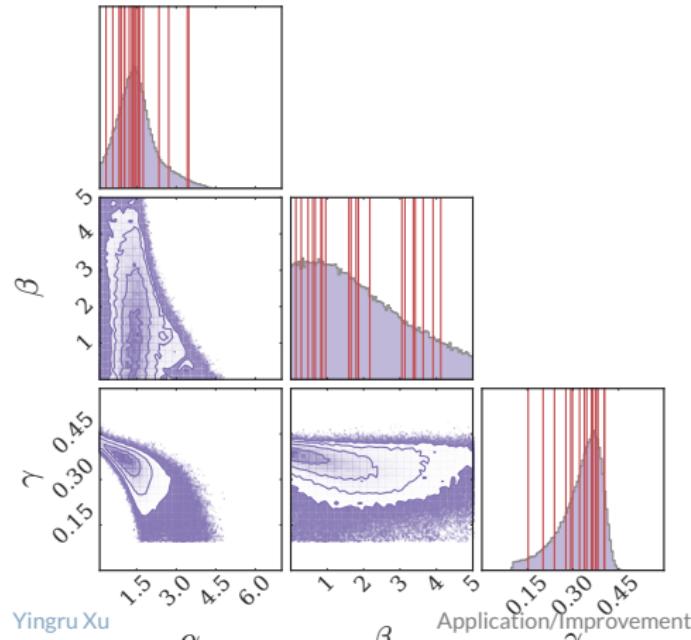
0.2 of current σ_{exp} !
 (considering PbPb @ 5.02 TeV only)

A different approach: more observables

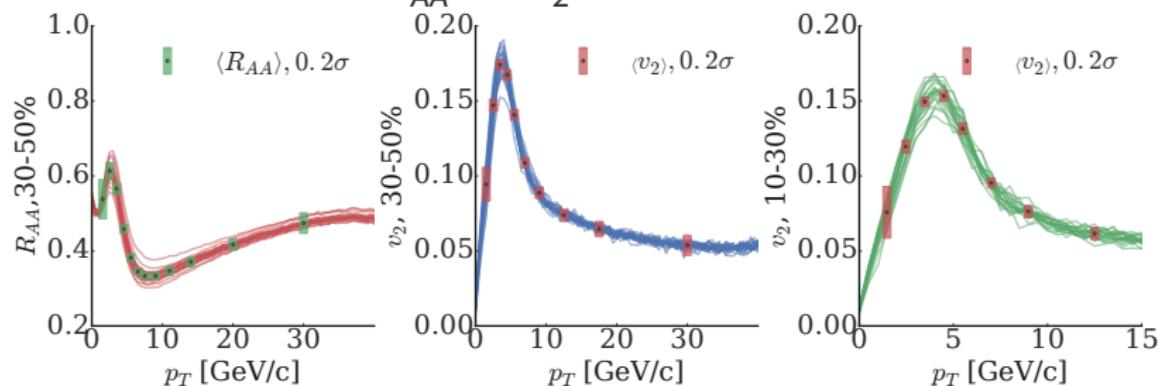
Question: what feature we expect from the new observables?

Wanted: observables that have larger variance/more sensitive
(than R_{AA} and v_2) → to obtain more information

- Take 20 sets of parameters from the posterior distribution of (α, β, γ)



- D-meson differential R_{AA} and v_2



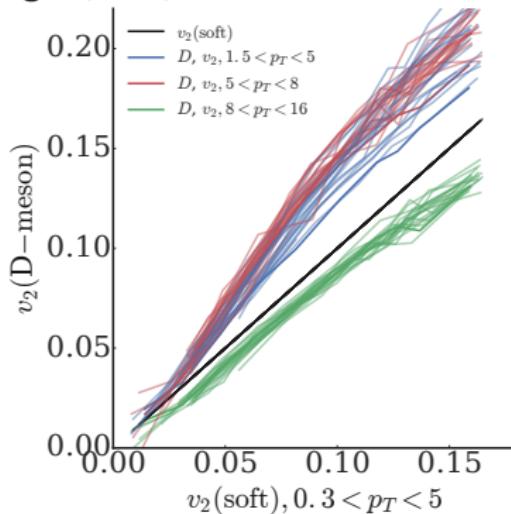
coefficient of variant (ratio of std and mean): $CV = \frac{\sigma}{\mu}$

Average over p_T bins:

- $- CV[R_{AA}(30 - 50\%)] = 0.030$
- $- CV[v_2(30 - 50\%)] = 0.051$
- $- CV[v_2(10 - 30\%)] = 0.0528$

soft-hard correlation

- light(soft) hadron and D -meson v_2



M.Nahrgang PhysRevC.91.014904
Prado et al. arXiv:1611.02965v1
P.B. Gossiaux, talk at QM2017

Average over centrality bins:

- $- CV[v_2(1.5 < p_T < 5)] = 0.2451$
- $- CV[v_2(5 < p_T < 8)] = 0.2434$
- $- CV[v_2(8 < p_T < 16)] = 0.2447$

- Minimum bias events, centrality bined by $q^2 = \frac{Q^2}{\sqrt{M}}$
- soft-hard correlation? **Need more investigation (on σ_{model})**. But **Bayesian analysis is a unique technique!**

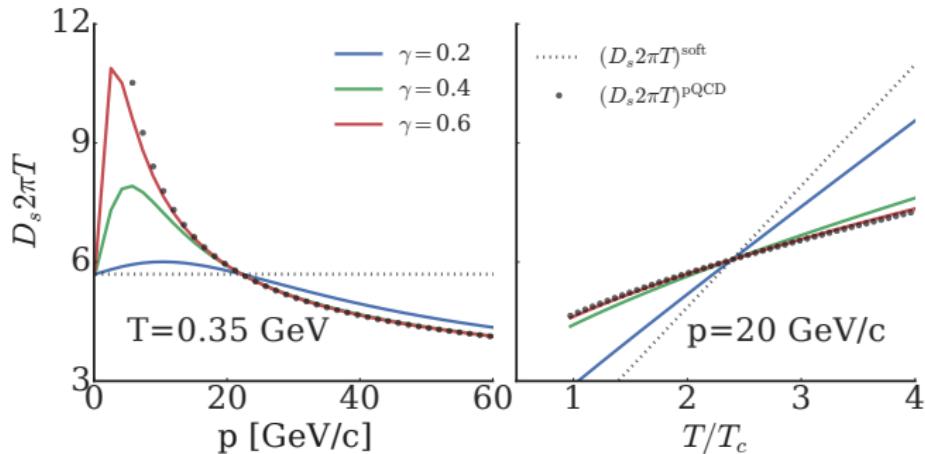
Summary

- Quantified charm quark diffusion coefficient $D_s 2\pi T$ using systematic Bayesian analysis
 - * $D_s 2\pi T(p = 0)$ minimum between (1-3) around T_c
 - * non-perturbative effects play a role for $p_T \sim (10-20)$ GeV/c
- Simultaneously described D -meson observables at RHIC and the LHC
- Improved experimental uncertainties is required
- Outlook:
 - * Explore other potential observables using Bayesian analysis
 - * Higher $p_T D$ -meson, B -meson calibration
 - * Applying on Boltzmann transport framework

backups

Input: prior parameter

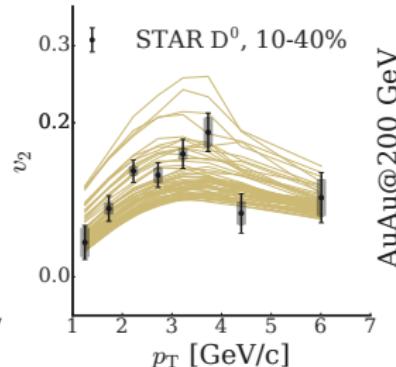
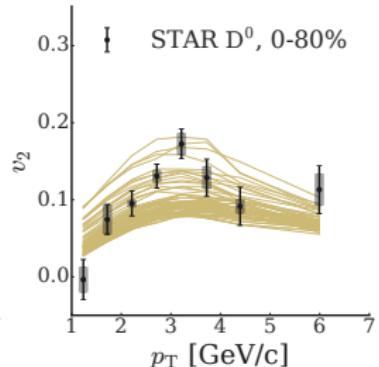
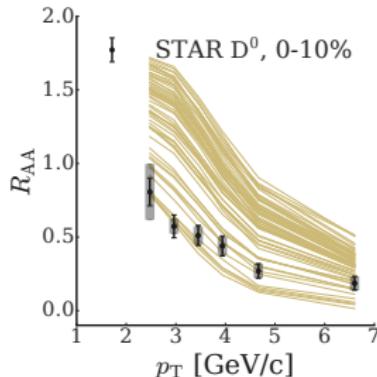
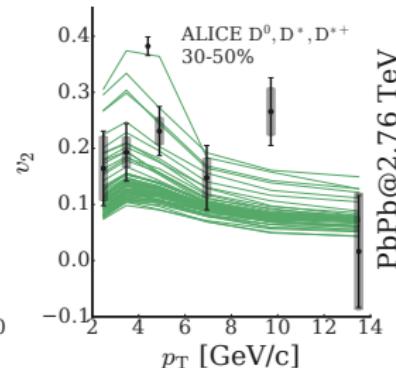
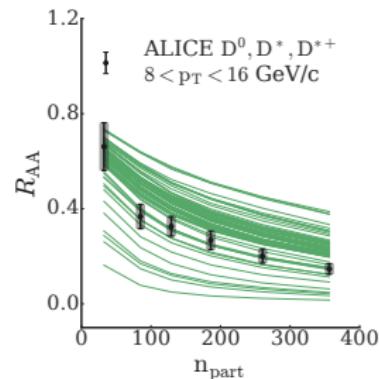
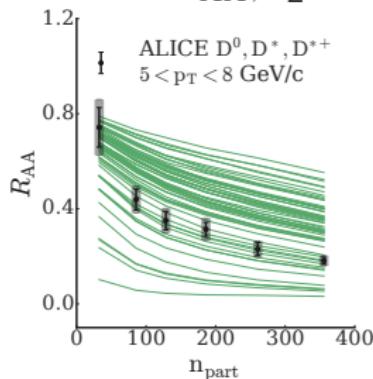
- An example of $D_s 2\pi T$ dependence on γ , at fixed $(\alpha, \beta) = (1.8, 1.7)$:
 - Larger γ , quicker conversion to $(D_s 2\pi T)^{\text{pQCD}}$
 - $p = 1/\gamma^2$, $(D_s 2\pi T)^{\text{soft}} = (D_s 2\pi T)^{\text{pQCD}}$



- Initially: no knowledge/constraint for the parameters → **uniform prior distribution**
- $\alpha \in (0, 7)$, $\beta \in (0, 5)$, $\gamma \in (0, 0.6)$

Output: experimental observables

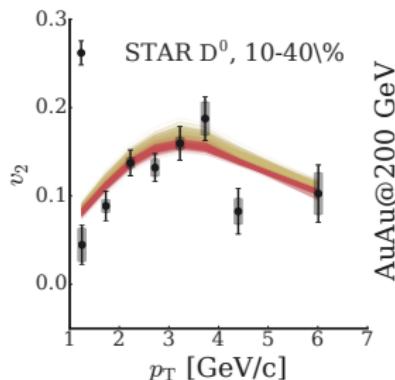
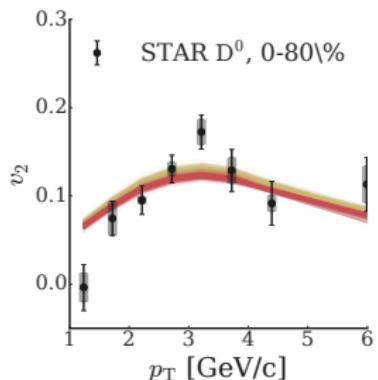
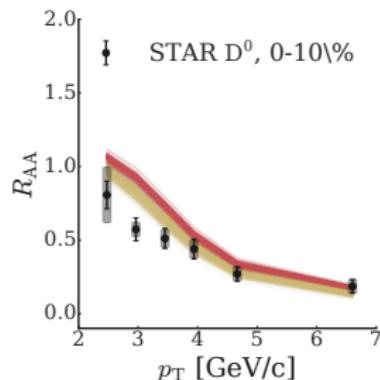
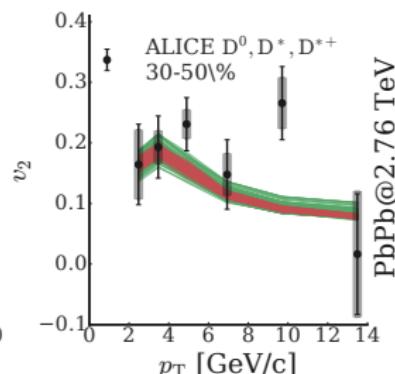
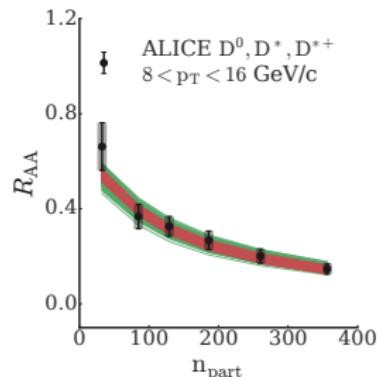
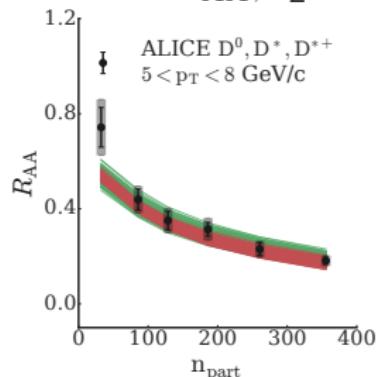
D-meson R_{AA} , v_2 at RHIC and the LHC:

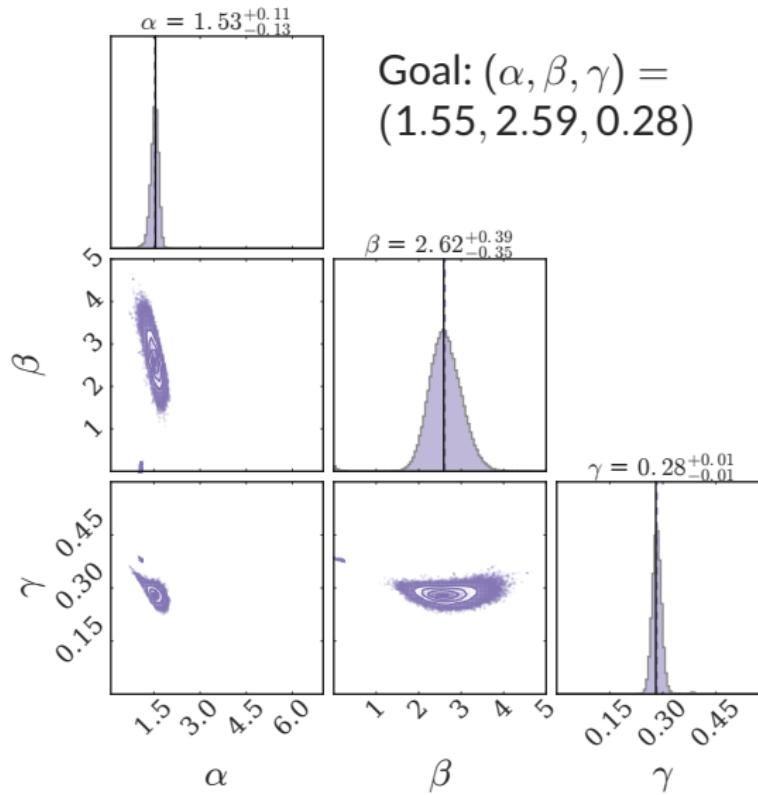


PbPb@2.76 TeV
 AuAu@200 GeV

Output: experimental observables

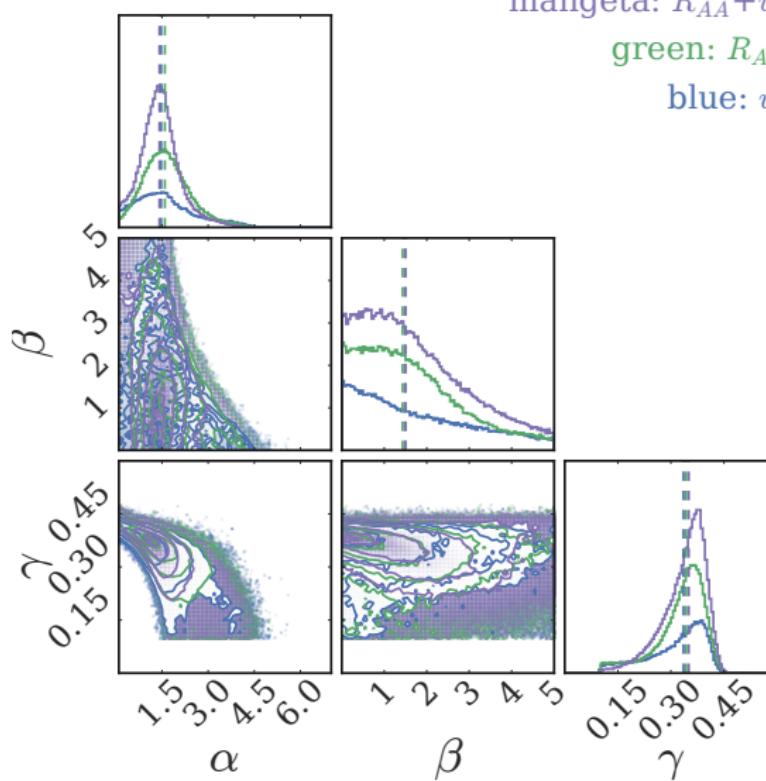
D-meson R_{AA} , v_2 at RHIC and the LHC:



Higher precision: $0.1\sigma_{\text{exp}}$ 

for higher precision,
even better
consistency between
desired value and
resulted value

R_{AA} vs. v_2



mangeta: $R_{AA} + v_2$

green: R_{AA}

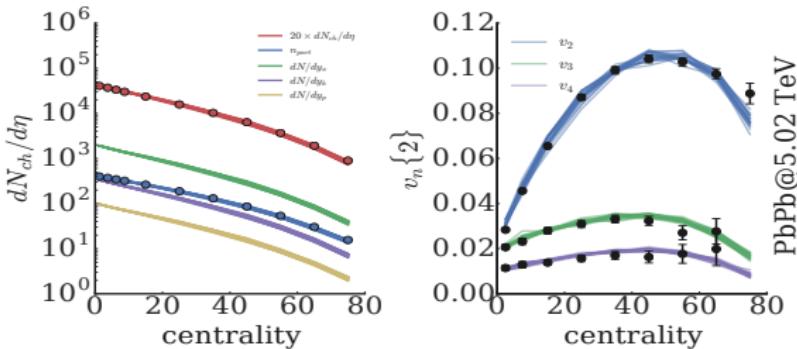
blue: v_2

The ability to constrain parameters:

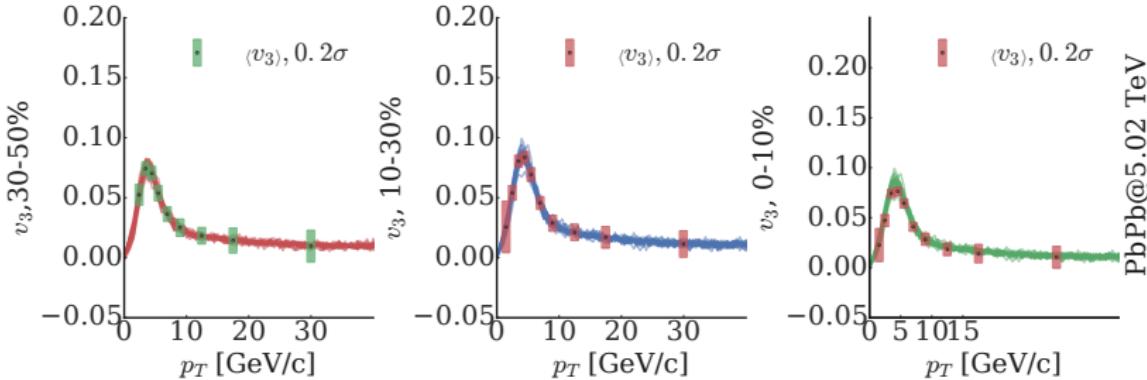
- its uncertainty
- the information it constrains

Maybe slightly better for R_{AA} in terms of (α, β, γ) posterior distribution, though not much of the difference from the posterior range of $D_s 2\pi T$; but **more is better!**

- Medium background

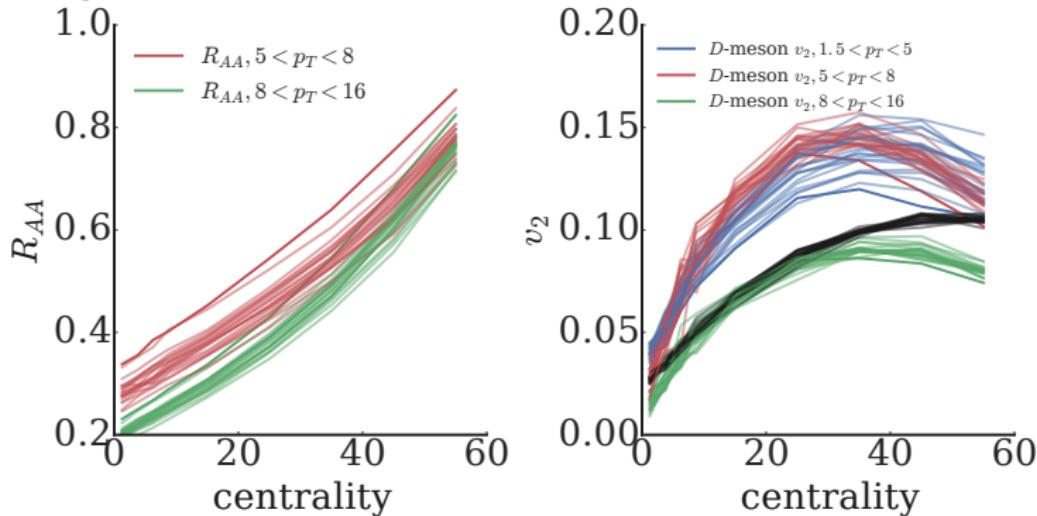


- D -meson v_3 :



Direction 2: integrated observables

- Integrated R_{AA} and v_2



- Integrated D -meson R_{AA}, v_2 tend to have larger variance than differential one. → models' uncertainty?
- Results from different centrality classes! → probing different average temperature