Problem Set 7

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1 Background

I am modeling a firm's stocking decision. In operations management, firms have to make a decision in terms of how much inventory to order for each period t (t = 1, 2, ..., T). To simply the problem, I assume that there is no lead time, this is a finite horizon inventory problem and that at time T (i.e. the firm is out of business), there is no salvage value at all for any leftover inventory, so the terminal condition is that:

$$V_T(x) = 0 \tag{1}$$

Other parameters needed are as follows:
x_t: the state variable, initial inventory level at each time period
y_t: the control variable, inventory level at each time period after ordering
h: holding cost rate. Cost incurred when a firm holds onto the invenotry.
b: back order cost rate. Cost incurred when a firm fails to meet the current demand.
c: unit cost for each piece of inventory ordered
D_t: random demand at each time period (assume i.i.d.)
α: discount rate

2 Equations

Transition from t to t+1:

$$x_t + 1 = y_t - D_t \tag{2}$$

For each time period, the cost on existing inventory is:

$$E[h(y_t - D_t)^+ + b(y_t - D_t)^-] \tag{3}$$

The Bellman equation is:

$$V_t(x) = min_{x \leq y} c(y - x) + E[h(y_t - D_t)^+ + b(y_t - D_t)^-] + \alpha * E[V_{t+1}(y - D)] \tag{4}$$

Solve backwards:

$$V_{T-1}(x) = min_{x \leq y} c(y-x) + E[h(y-D)^+ + b(y-D)^-] + 0 \tag{5}$$

The FOC w.r.t. y is:

$$cy = hE[d/dy(y)^+] + bE[d/dy(y)^-] \tag{6}$$

$$cy = hE[y^+] + bE[y^-] \tag{7}$$