Problem Set 7

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1 Background

I am modeling a firm's stocking decision. In operations management, firms have to make decisions in terms of how much inventory to order for each period t to an infinite horizon. To simply the problem, I assume that there is no lead time. Other parameters needed are as follows:

x_t: the state variable, initial inventory level at each time period

y_t: the control variable, inventory level at each time period after ordering

h: holding cost rate. Cost incurred when a firm holds onto the inventory.

b: back order cost rate. Cost incurred when a firm fails to meet the current demand.

c: unit cost for each piece of inventory ordered

D_t: random demand at each time period (assume i.i.d.)

β: discount rate

2 Equations

Transition from t to t+1:

$$x_{t+1} = y_t - D_t \tag{1}$$

For each time period, the cost on existing inventory is:

$$E[h(y_t - D_t)^+ + b(y_t - D_t)^-] \tag{2}$$

The Bellman equation is:

$$V_t(x) = min_{x \leq y} c(y-x) + E[h(y_t - D_t)^+ + b(y_t - D_t)^-] + \alpha * E[V_{t+1}(y-D)] \tag{3}$$

Solve backwards:

$$V_{T-1}(x) = min_{x \leq y} c(y-x) + E[h(y-D)^+ + b(y-D)^-] + 0 \tag{4}$$

The FOC w.r.t. y is:

$$cy = hE[d/dy(y)^+] + bE[d/dy(y)^-] \tag{5}$$

$$cy = hE[y^+] + bE[y^-] \tag{6}$$