Project 2 (iii) Effective control of room heating

1. Project overview

This project explores the mathematical modelling of an automated heating system. You are asked to consider a room comprising a heating device, which is triggered by a thermostat, also located in the room. The overall aim will be to produce a plausible model, and to use it to determine effective control strategies for the thermostat, as well as effective placements of the thermostat within the room.

Some mathematical concepts you may find useful include:

- *Newton's Law of Cooling*, which models the temperature $T(t)$ of a room as time-dependent only:
$$\frac{dT}{dt} = -k(T(t) - T_a) + u(t). \tag{1}$$
where T_a is the ambient outside temperature and $u(t)$ is the heat source. See also [1].

- *The heat equation*, where $T = T(x, y, t)$ also varies spatially
$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + u(x, y, t). \tag{2}$$
α is a diffusivity constant, and u is a heat source. See also [2].

- *Control theory:* The term $u(t)$ above is controlled by the thermostat and you may choose strategies where it is simply constant, or varies as a function of the temperature perceived by the thermostat. The latter relies on *feedback*, an important concept in control theory [3]. Real-world thermostats often operate as simple on/off switches. This is sometimes known as *Bang-Bang Control*:
$$u(T) = \begin{cases} U_{max} & \text{if } T < T_{set} \\ 0 & \text{if } T \geq T_{set}. \end{cases} \tag{3}$$
This approach includes discrete and continuous terms in the same model, which is often termed *hybrid system* [4]. This may lead to unique phenomena which can be explained mathematically, for instance the "Zeno effect", i.e. the occurrence of an infinite number of switches over a finite time interval (see [4]).

Examples of tasks you may choose to undertake include (but are not limited to):

- Consider a one-dimensional model as proof of concept, based for instance on symmetry arguments on the geometry of the room.
- Investigate different shapes of two-dimensional rooms, with different locations for the heat source and the thermostat.
- Does the Zeno effect occur for some formulations of the model?
- Consider more complex situations, with multiple rooms in a building, with multiple heat sources and thermostats.

3. Deliverables

You are required to provide an oral report supported by a Powerpoint presentation of your project outcomes, to be submitted via moodle by 1pm on Monday 9 March 2026, giving the oral presentation during the Workshop on Monday 16 March, at which you should be prepared to answer questions. It is expected that the presentation will be a max of 10 mins, with up to 10 minutes for questions. The written report should have a maximum length of approximately 25 pages including executive summary, table of contents, figures, tables, calculations, references, appendices (but not including title page or code listings).

References

[1] Wikipedia (accessed 18/12/2025): `https://en.wikipedia.org/wiki/Newton%27s_law_of_cooling`

[2] Wikipedia (accessed 18/12/2025): `https://en.wikipedia.org/wiki/Heat_equation`

[3] Wikipedia (accessed 18/12/2025): `https://en.wikipedia.org/wiki/Feedback`

[4] Goebel R, Sanfelice RG, Teel AR. Hybrid dynamical systems. IEEE control systems magazine. 2009 Mar 27;29(2):28-93..