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Algorithms Lab

Exercise – *San Francisco*

After traveling half the globe for more than sixty days, Phileas arrives in San Francisco together with Passepartout and Fix. Their plan is to use the *Pacific Railroad* for their journey to New York. From ocean to ocean—as the Americans say; these four words being a synonym for the 'great trunk line' which crosses the entire width of the United States.

However, unforeseen events start to unroll. The train is ambushed by a band of Sioux warriors and Passepartout is captured. Phileas, a true gentleman, cannot leave a man behind. He pursues the band across the vast white plains and finally manages to catch up with them.

After a lengthy brawl, the chief of the tribe, being a reasonable man, agrees with Phileas upon the following. The two will play an ancient Sioux board game: if Phileas manages to score at least as many points as the chief, he may take Passepartout and they are free to leave; otherwise, they both become slaves for an unspecified amount of time—at the current stage of their journey clearly a suboptimal event.

The game is a single player game, played with a single marble on a wooden board with n carved holes and m carved canals between these holes. An arrow is engraved in each canal to indicate the direction in which the marble may be moved through the canal. Furthermore, each canal carries a nonnegative number of *points*, which the player *scores* whenever rolling the marble through the canal (every canal can be used and scored multiple times throughout a game). Given only a limited number of moves, the goal of the game is to maximise the score, of course.

The chief explains the rules of the game to Phileas. There is a unique starting hole, called *Angvariationu-toke* (a Sioux word for 'another day'). The marble, called *Canowicakte* (a Sioux word for 'forest hunter'), starts at Angvariationu-toke. In each move, the player rolls the marble from the current hole to a neighboring hole through one of the incident canals, while respecting the direction of the engraved arrow. Doing so, (s)he scores as many points as the canal carries. A hole with no outgoing canal is called *Weayaya* (a Sioux word for 'setting sun') and from such a hole the player may take the marble back to Angvariationu-toke as a *free action*. Such a free action does not count as a move and it yields no score.

The chief makes the bold claim that he can achieve a score of x in k moves. Phileas' goal is to beat the chief dramatically: either find the minimum number of moves in order to score at least as much as the chief, or prove that it is impossible to achieve the score of x in k moves. It may be noted that the same canal can be scored more than once.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- The first line contains four integers `n m x k`, separated by a space. They denote
  - n, the number of holes in the game board ($2 \leq n \leq 10^3$);
  - m, the number of canals between the holes ($1 \leq m \leq 4 \cdot 10^3$);
  - x, the claimed score of the chief ($1 \leq x \leq 10^{14}$);
  - k, the maximum number of moves allowed ($1 \leq k \leq 4 \cdot 10^3$).

Hole 0 always corresponds to Angvariationu-toke.

- The following m lines define the canals. Each line consists of three integers `u v p`, separated by a space, and such that $0 \leq u, v \leq n-1$ and $0 \leq p < 2^{31}$. This means that the arrow engraved in the canal points from u to v. The player can roll the marble from hole u to hole v, thereby scoring p points. Note that (1) there can be more than one canal from hole u to hole v and (2) possibly $u = v$.

Output For each test case output one line containing a single integer that denotes the minimum number of moves to get at least x points. If it is not possible to score at least x points in k moves, output '`Impossible`'.

Points There are three groups of public test sets, worth 80 points in total. For each of the first two group of test sets there is also a corresponding hidden test set that is worth 5 points. For the final group of test sets there is a corresponding hidden test set that is worth 10 points. So, there are $80 + 5 + 5 + 10 = 100$ points in total.

1. For the first group of test sets, worth 20 points, you may assume $n \leq 40$ and $k \leq 20$. Furthermore, you may assume that all routes from Angvariationu-toke to a Weayaya hole use exactly k canals.
2. For the second group of test sets, worth 30 points, you may assume that all routes from Angvariationu-toke to a Weayaya hole use exactly k canals.
3. For the third group of test sets, worth 30 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`i`.in/out`, for $i \in \{1, 2, 3\}$.

Sample Input

```
3
6 6 7 3
0 1 1
0 2 1
1 4 2
2 3 1
3 5 5
4 5 2
6 8 7 5
0 1 0
0 2 2
0 2 1
0 5 1
1 3 0
2 4 0
3 5 4
4 5 0
4 4 1 100
0 1 0
1 2 0
2 3 0
3 1 0
```

Sample Output

```
3
5
Impossible
```