Note for Scattering Amplitude Computation

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1 4-point case

For the four point case $\mathcal{A}(V_2\Phi^\dagger V_1\Phi)$, we can construt the color-ordered amplitude from the residue. First, we cansider the $(+,-)$ helicity configuration. There are two feynman diagrams contributing to the color-ordered amplitude.

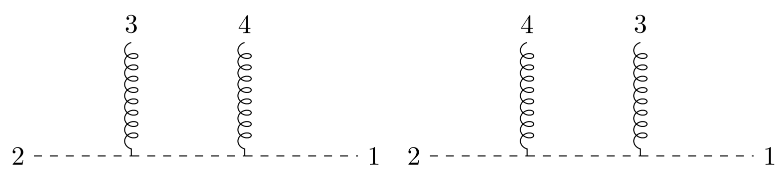

Figure 1: 4pt.

For the first diagram, the residue equals to

$$\mathcal{R}es|_{s_{12}=0} = \frac{[3I][23]}{[I2]} \times \frac{\langle I4\rangle \langle 41\rangle}{\langle 1I\rangle} = \frac{\langle 24\rangle [31] \langle 41\rangle [23]}{[42] \langle 24\rangle}$$

Similarly, the sencond one is

$$\mathcal{R}es|_{s_{13}=0} = \frac{\langle 4I\rangle \langle 24\rangle}{\langle I2\rangle} \times \frac{[31][I3]}{[1I]} = \frac{\langle 24\rangle [31] \langle 41\rangle [23]}{\langle 32\rangle [23]}$$

Then we can conclude that the four-point color-ordered amplitude $A[1,2,3^+,4^-]$ equals to

$$A[1,2,3^+,4^-] = \frac{\langle 24\rangle [31] \langle 41\rangle [23]}{\langle 32\rangle [23][42] \langle 24\rangle} = \frac{\langle 24\rangle \langle 14\rangle}{\langle 13\rangle \langle 23\rangle}$$

$\star Bonus$

It is still necessary to prove the color-ordered amplitude $A[1,2,3^+,4^+]$ equals to 0. Here we can use the color ordered Feynman rules to show the result.

$$A[1,2,3^+,4^+] \propto \frac{(\epsilon_3 \cdot p_2)(\epsilon_4 \cdot p_1)}{s_{23}} + \frac{(\epsilon_4 \cdot p_2)(\epsilon_3 \cdot p_1)}{s_{24}}$$

Here we can utilize the spinor-helicity virable to express polarization vector

$$\epsilon_2^{+\mu} = \frac{\langle r_1|\gamma^\mu|3]}{\sqrt{2}\,\langle r_1 3\rangle}, \qquad \epsilon_4^{+\mu} = \frac{\langle r_2|\gamma^\mu|4]}{\sqrt{2}\,\langle r_2 4\rangle}$$

here r_1 and r_2 represent the refrence spinor.
We can freely choose $r_1 = r_2 = 1$ or 2, then $\langle r_1 2\rangle$,$\langle r_2 1\rangle$,$\langle r_1 1\rangle$,$\langle r_2 2\rangle$, two of them equal to 0, so we can conclude that

$$A[1, 2, 3^+, 4^+] = 0$$

2 5-point case

For the 5-point case, we can utilize the BCFW recursion relation which can help us generate higher point amplitude from lower point on-shell subamplitudes. Here, we always consider the MHV (Maximal helicity violatio) amplitude.